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Watt's better? Pay-as-bid vs. Uniform pricing in electricity market[☆]

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ABSTRACT

Rising electricity prices of recent years have reopened the debate on replacing uniform with pay-as-bid pricing in electricity markets. This paper contributes to this ongoing political debate by comparing peak prices, consumer surplus, and welfare under pay-as-bid and uniform pricing in a game-theoretic model of suppliers' strategic bidding behaviour. For this comparison, it derives supply function equilibria for both pricing rules in a model that matches four stylised facts of electricity markets: Suppliers have (1) oligopolistic market power, (2) increasing marginal costs, (3) face a downward-sloping demand, and (4) have uncertainty over time-varying demand but common knowledge of production costs. In the model, peak prices are lower under pay-as-bid pricing. Both pay-as-bid and uniform pricing maximise welfare with zero profits for producers when marginal costs are flat, or there is an infinite number of producers. Restricting attention to the case where marginal costs and demand are linear with a uniformly distributed intercept of demand, pay-as-bid pricing results in a higher expected consumer surplus. The welfare comparison is ambiguous even in this linear model. Pay-as-bid pricing results in higher expected welfare if and only if demand variation is sufficiently low. The findings of this paper suggest that regulators should seriously consider pay-as-bid pricing to raise consumer surplus and curb price peaks.

1. Introduction

On January 8th, 2025, the price of a MWh of electricity on the intraday market for Great Britain peaked at more than £1000 (Ambrose, 2025) – roughly ten times the going rate for the whole month (EPEX, 2025), and several times the estimated production costs for even the most expensive technologies (International Energy Agency, 2020; Department for Energy Security & Net Zero, 2023a). Such price peaks raise questions about the adequacy of the current market institutions, particularly the way prices are set in electricity markets.

Currently, under the uniform-price rule, every unit trades at the same price, which is determined by the price of the most expensive unit. Intuitively, this approach places the burden of high-price situations disproportionately on consumers, who pay high prices even for units that are inexpensive to produce. Consequently, this price rule has come under fire amid the rising energy prices of recent years. For example, in her State of the Union address in 2022, Ursula von der Leyen, president of the European Commission, argued that "[t]he current electricity market design – based on merit order – is not doing justice to consumers anymore. They should reap the benefits of low-cost renewables. So, we have to decouple the dominant influence of gas [usually the most expensive technology] on the price of electricity" (European Commission, 2022). An obvious way to do so would be to change the pricing rule to what is called pay-as-bid, or discriminatory pricing, where every unit trades at "its own" price, which corresponds to the ask-price of the producer. Thus, regulators have recently started debating whether pay-as-bid pricing should replace uniform pricing.[1]

This paper contributes to this ongoing debate by comparing peak prices, consumer surplus and welfare under pay-as-bid and uniform pricing in a game-theoretic model of strategic bidding behaviour.

The key challenge in the formal analysis of firms' equilibrium behaviour in electricity markets is that they submit bids as combinations of quantities and corresponding prices, rather than choosing a single price or quantity. Thus, a proper analysis of electricity markets requires finding so-called "supply function equilibria". Doing so is particularly complex for pay-as-bid pricing, where a firm's revenue depends on its entire supply function, as each unit is remunerated at the bid price for that unit. So far, the literature has not established equilibrium behaviour for pay-as-bid pricing under assumptions that plausibly reflect

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[1] For example, the UK government's Review of Electricity Market Arrangements considered and rejected such a switch because of "the risk of tactical bidding" (Department for Energy Security & Net Zero, 2023b). This paper compares price rules accounting for this strategic behaviour.

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electricity markets.

This paper is the first to derive equilibrium bids under pay-as-bid pricing in a model that incorporates four important stylised facts of electricity markets in a single model. First, there are typically only a few suppliers. Thus, electricity suppliers strategically exert market power (e.g., Krzywnicka and Barner, 2025; Bushnell et al., 2008; Borenstein et al., 2002, among others), as opposed to the models in Song et al. (2025), Willems and Yueting (2023) and Zhao et al. (2023), or Federico and Rahman (2003). Second, even though demand is rather inelastic, it does react to prices even in the very short term (Hirth et al., 2024), which, for example, the models in Pycia and Woodward (2026), Zhao et al. (2023), Holmberg (2009), Genc (2008), Holmberg (2008), and Hästö and Holmberg (2006) do not reflect. Third, suppliers are well-informed about their competitors' costs/bidding behaviour (Doraszelski et al., 2018) but face uncertainty over the exact level of demand (see Laitsos et al., 2024, for a review of the literature on forecasting demand in electricity markets). Some literature analyses minimal information environments regarding competitors' cost (e.g., Galgana and Golrezaei, 2025; Kasberger and Woodward, 2025) or abstracts from uncertainty over demand (e.g., Caragiannis et al., 2025; Galgana and Golrezaei, 2025; Kasberger and Woodward, 2025; Vanelli et al., 2025; Aussel et al., 2017a,b; Son et al., 2004).[2] Fourth, some models (e.g., Caragiannis et al., 2025; Fabra and Llobet, 2023; Holmberg and Wolak, 2018; Genc, 2008; Fabra et al., 2006) do not account for the fact that – apart from purely renewable electricity suppliers – suppliers usually have increasing marginal costs (their so-called "merit order") as they own a portfolio of power plants with differing production technologies (see, e.g., Hortaçsu and Puller, 2008; Borenstein and Bushnell, 1999, for corresponding empirical estimates).

Accommodating these stylised facts in a single model, this paper derives the supply function equilibrium under pay-as-bid pricing when suppliers with commonly known, (weakly) increasing marginal costs compete for a time-varying downward-sloping demand in an oligopolistic market. For uniform pricing, the equilibrium is known from Klemperer and Meyer (1989). Comparing pay-as-bid and uniform pricing in this general setup, this paper finds that peak prices are lower under pay-as-bid pricing. Moreover, both pricing maximise welfare with zero profits under perfect competition, i.e., if marginal costs are flat or there is an infinite number of competitors. To derive insights into consumer surplus and welfare outside these extremely competitive cases, the paper also considers a linear version of the model, where marginal costs and demand are linear with a uniformly distributed intercept of demand. In this linear model, the expected consumer surplus is higher under pay-as-bid pricing. Expected welfare increases through pay-as-bid pricing if and only if the uncertainty over demand is sufficiently low. A naive back-of-the-envelope calibration of the linear model to German electricity market data suggests that pay-as-bid pricing would result in 13% lower peak prices and a 1.2% increase in consumer surplus at the cost of a (negligible) 0.007% decrease in expected welfare. While this example shows that applying the model in this paper is feasible, it also highlights the model's limitations.

This paper proceeds as follows. Section 2 discusses related literature. Section 3 presents the general model and derives equilibrium bids for arbitrary demand and cost functions. Based on this equilibrium characterisation, it shows that peak prices are lower under pay-as-bid pricing. Both price rules implement the first-best under perfect competition. Section 4 compares consumer surplus and welfare in the linear model. Section 5 discusses the implications and limitations of the model, before Section 6 concludes.

[2] Kasberger and Woodward (2025) and Vanelli et al. (2025) do so because they do not consider electricity markets.

2. Related literature

Finding optimal bidding strategies in electricity markets is mathematically complex. Many studies rely on simulation models where computer agents learn through repeated interactions (e.g., Viehmann et al., 2021; Sugianto and Liao, 2014; Liu et al., 2012; Guerci et al., 2007; Hailu and Thoyer, 2007; Bakirtzis and Tellidou, 2006; Xiong et al., 2004; Bower and Bunn, 2001). A strength of this approach is that it can incorporate market features such as grid constraints (Guerci and Rastegar, 2012) or dynamic technological learning (Anatolitis and Welisch, 2017), which are difficult to capture in analytic models. On the downside, such studies only identify equilibrium behaviour in a specific setting with numeric values for relevant parameters such as costs and demand. Thus, it remains unclear under which conditions their findings generalise.

Analytical models partly address this concern by deriving equilibrium behaviour mathematically, making the role of specific assumptions more transparent. In the model of this paper, *oligopolistic* electricity suppliers with *increasing* marginal costs compete for a *time-varying/uncertain*, *downward-sloping* demand. Hence, the model expands on previous literature that has not considered these aspects in a single model.[3] Each of these assumptions is relevant for electricity markets and has important implications for equilibrium behaviour.

A minority of the related literature compares uniform and pay-as-bid pricing under the assumption that electricity suppliers are price-takers and do not strategically influence prices (e.g., Song et al., 2025; Willems and Yueting, 2023; Zhao et al., 2023; Federico and Rahman, 2003). Although this is certainly true for a subset of suppliers, the empirical evidence shows that models of imperfect competition better describe behaviour in electricity markets (e.g., Krzywnicka and Barner, 2025; Bushnell et al., 2008; Borenstein et al., 2002, among others). As is intuitively plausible, both pricing rules perform identically and maximise welfare under perfect competition in the model of this paper (Corollary 2).

Many other models assume that demand is completely inelastic (e.g., Pycia and Woodward, 2026; Zhao et al., 2023; Holmberg, 2009; Genc, 2008; Holmberg, 2008; Swider and Weber, 2007; Hästö and Holmberg, 2006). Even though electricity demand is rather inelastic, it does react to price increases even in the (very) short term (see Hirth et al., 2024, for a recent analysis, or Labandeira et al., 2017, for a meta-analysis). Accounting for this demand-side reaction is important as it limits the exercise of market power (Cramton, 2004). For example, Heim and Götz (2021) empirically document very high prices under pay-as-bid pricing in the German market for reserve power, where regulation implies that demand is completely inelastic. Similarly, models of uniform-price auctions with inelastic demand regularly feature *binding* price caps (e.g., Zhao et al., 2023; Holmberg, 2009; Genc, 2008; Holmberg, 2008).[4] The model of this paper nests the case of inelastic demand as a limiting case.

Some models assume that marginal production costs are constant (e.g., Caragiannis et al., 2025; Fabra and Llobet, 2023; Holmberg and Wolak, 2018; Genc, 2008; Fabra et al., 2006). Generally, this assumption does not accurately reflect that suppliers usually own a fleet of power plants with differing production technologies and, therefore, have increasing marginal costs (see, e.g., Hortaçsu and Puller, 2008; Borenstein and Bushnell, 1999, for corresponding empirical estimates). When comparing consumer surplus under pay-as-bid and uniform pricing, accounting for increasing marginal costs is crucial, as the main disadvantage of uniform pricing for consumers is that they pay high

[3] Outside the context of electricity markets, some authors also restrict attention to single-unit demand/supply of bidders (e.g., Bougt et al., 2025; Anderson and Holmberg, 2023; Krishna, 2010).

[4] In the model of this paper, inelastic could lead to infinite prices under uniform pricing but not pay-as-bid pricing.

Table 1
Overview of assumptions in the related literature.

	Oligopoly	Elastic demand	Increasing marginal costs	Stochastic uncertainty
Aussel et al. (2017a,b)	✓		✓	
Caragiannis et al. (2025)	✓			
Fabra et al. (2006)	✓	(✓) (extension)		(✓) Over demand (extension)
Federico and Rahman (2003)	(✓) Only monopoly	✓	(✓) Only across producers	✓ Over demand
Galgana and Golrezaei (2025)	✓		✓	Deep uncertainty on comp.bids
Genc (2008)	✓			✓ over demand
Hästö and Holmberg (2006) & Holmberg (2009)	✓		✓	✓ Over demand
Holmberg and Wolak (2018)	✓			✓ Over own and competitors' cost and demand
Kasberger and Woodward (2025)	✓		✓	Deep uncertainty on comp. bids
Pycia and Woodward (2026)	✓		✓	✓ Over demand
Son et al. (2004)	✓		✓	
Song et al. (2025)		Random prices		✓ Over equilibrium prices
Vanelli et al. (2025)	✓	✓	✓	
Willems and Yueting (2023)		✓	(✓) Only across producers	✓ Over demand
Zhao et al. (2023)				✓ Over own production capacity
This paper	✓	✓	✓	✓ Over demand

prices for low-cost units when a high-cost technology sets the price. By contrast, the assumption of constant (zero) marginal costs becomes plausible for markets approaching 100% renewable electricity generation. Such purely renewable markets introduce a new complexity because suppliers have uncertainty over their own production capacity, which, for example, depends on meteorological conditions (see Zhao et al., 2023, for such a model). If renewable electricity suppliers can eliminate uncertainty over their own production capacity through aggregation of production facilities (e.g., Song et al., 2025), such 100% renewable markets correspond to the limiting case of this model with constant marginal costs. However, if this uncertainty persists, such markets deserve an entirely separate treatment, as uncertainty over their own production introduces fundamentally new considerations into bidding behaviour (Fabra and Llobet, 2023).

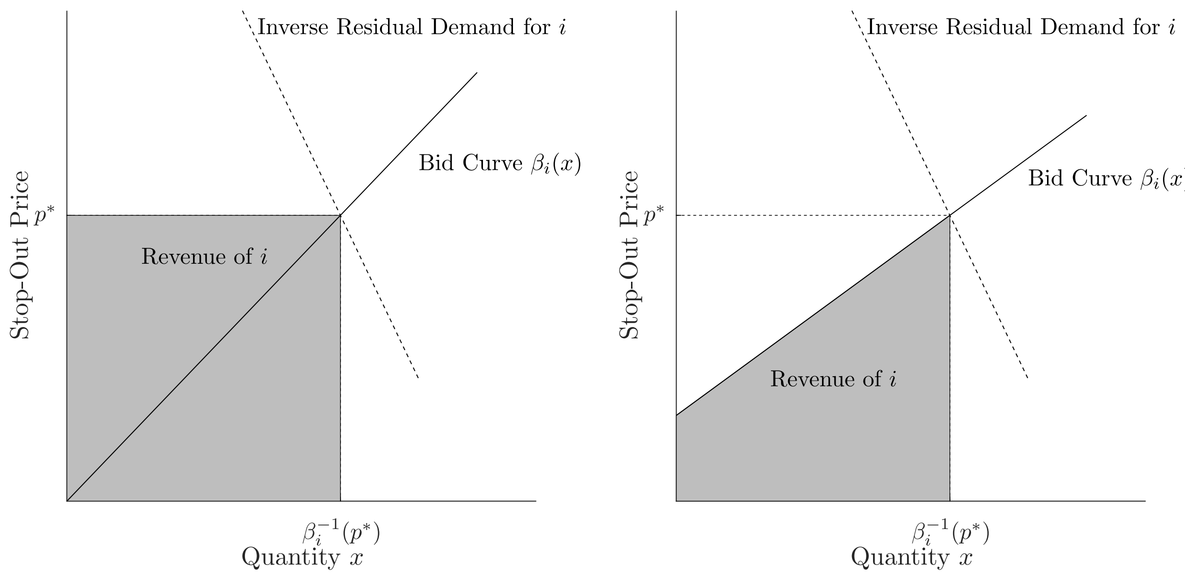
The last important component of electricity market models is the information structure. Some recent literature (e.g., Galgana and Golrezaei, 2025; Kasberger and Woodward, 2025) considers minimal information environments. Bidders operate under "deep uncertainty" over competitors' costs and cannot even attach probabilities to different possibilities. Thus, they do not maximise expected profits. Instead, they minimise their loss in profits compared to a situation where they know their competitors' costs (regret). However, such models are mostly adequate for very rare interactions (Kasberger and Woodward, 2025). As suppliers interact repeatedly in electricity markets, they are well-informed about their competitors' costs (Hortaçsu and Puller, 2008), especially as all producers employ similar production technologies. Thus, empirically, suppliers in electricity seem to bid according to a Nash equilibrium, where they know their competitors' costs (Doraszelski et al., 2018).[5] In line with the substantial literature on demand forecasting and its error (see Laitsos et al., 2024, for a review) and much of the theoretical literature (e.g., Willems and Yueting, 2023; Holmberg and Wolak, 2018; Holmberg, 2009; Genc, 2008; Hästö and Holmberg, 2006; Federico and Rahman, 2003), this paper assumes that suppliers are left with uncertainty over the time-varying demand.[6] Some models (e.g., Caragiannis et al., 2025; Vanelli et al., 2025; Aussel et al., 2017a,b; Son et al., 2004) also eliminate this source of uncertainty. Incorporating demand uncertainty not only seems empirically more plausible but also leaves the no-uncertainty case as the limiting case where the distribution of demand approaches a point mass.

Table 1 presents the assumptions made in previous literature. As discussed, the novelty of this paper lies in its integration of four plausible electricity market assumptions: supply-side market power, increasing marginal costs, and time-varying, downward-sloping demand. According to Table 1, this paper is most closely related to the work of Federico and Rahman (2003), who compare pay-as-bid and uniform pricing with linear marginal costs and random, linear demand in a monopoly. This paper extends their work in two directions: First, Section 3 derives equilibrium behaviour in oligopoly without any of these functional form restrictions. Second, Section 4 extends their comparison with linear functional forms to the setting of oligopolistic market power.

For expositional clarity, this paper discusses the results of previous literature in the following sections. This approach makes it easier

[5] The close alignment of cost curves estimated from bidding data with engineering estimates in Wolak (2003) also indirectly supports this view, suggesting that externally accessible engineering estimates of costs can reliably predict bids.

[6] From a pure optimisation perspective, Swider and Weber (2007) optimise the bids of a bidder who knows the inelastic demand but considers market prices a random variable with their distribution given by historic data. Game-theoretically, this approach is inconsistent with a Nash equilibrium under known costs of competitors. Either their optimisation results in a best response in a game, where competitors' cost have random time-specific shocks, or, if competitors' costs are not random, their strategy does not describe a Nash equilibrium because the focal bidder does not correctly anticipate competitors' equilibrium strategy.



(a) Uniform pricing. (b) Pay-as-bid pricing.

Fig. 1. Illustration of uniform and pay-as-bid pricing.

to see how the paper generalises earlier findings and what explains differences.

3. General model

3.1. Model

Demand.— The demanded quantity in the entire market at price p is given by $D(p,\varepsilon)$, where ε is a demand-shifter. Without loss of generality, assume that $\frac{\partial D(p,\varepsilon)}{\partial \varepsilon} > 0$. The demand-shifter ε is a random variable that follows the cumulative distribution $F(\varepsilon)$ on $[\underline{\varepsilon}, \bar{\varepsilon}]$ with $F'(\varepsilon) = f(\varepsilon) > 0$. This paper assumes that demand is downward-sloping, i.e., that $\frac{\partial D(p,\varepsilon)}{\partial p} < 0$. However, the case of inelastic demand is nested in the derivation of equilibrium behaviour. Throughout the paper, assume that $D(p,\varepsilon)$ reflects the demand side's (referred to as consumers) true willingness to pay.

Supply.— There are $n \geq 2$ symmetric producers that compete in the market. Each producer has the same, commonly known marginal cost curve $c(x)$ with $c'(x) \geq 0$.[7] Every bidder i submits a weakly increasing bidding function $\beta_i(x)$ that reflects, how much money they demand when supplying x units.

Market Clearing.— The market clears at a stop-out price p^* that equalises demand and aggregate supply, i.e.,

$$D(p^*,\varepsilon) = \sum_i \beta_i^{-1}(p^*), \tag{1}$$

where $\beta_i^{-1}(p)$ is the inverse of $\beta_i(x)$. Thus, as long as bidding functions are strictly increasing, a producer supplies $\beta_i^{-1}(p^*)$ units, i.e., all units for which their bidding function is weakly below p^*. Throughout the paper, assume that the minimal demand level is high enough for trade to occur for every demand realisation under both pricing rules.

Pricing Rules.— Under the uniform-price rule, a buyer is paid the stop-out price p^* for all units they supply. When they bid $\beta_i(x)$ and the stop-out price is p^*, they make profits of

$$\pi_{UP} = p^* \; \beta_i^{-1}(p^*) - C\left(\beta_i^{-1}(p^*)\right),$$

where $C(x) = \int_0^x c(y)\,dy$. Under the pay-as-bid rule, they are paid the price specified by their bidding function $\beta_i(x)$ for each supplied unit; their profits with stop-out price p^* are

$$\pi_{PAB} = \int_0^{\beta_i^{-1}(p^*)} \beta_i(x)\,dx - C\left(\beta_i^{-1}(p^*)\right).$$

Fig. 1 illustrates a producer's revenue under the two pricing rules.

Discussion of Model Assumptions.— A few elements of this model setup, though shared with next to all of the literature discussed in Section 2, are worth stressing.

First, the producers know each others' production costs. This assumption seems plausible for electricity markets given that a producer's power plants are generally observable to its competitors, who use the same production technologies. As certainty over the costs of competitors is implausible in many other settings, some recent contributions in auction theory (Linnenbrink, 2025 for the pay-as-bid and Burkett and Woodward, 2020 for the uniform-price auction) develop models with private costs.[8] For electricity markets, Holmberg and Wolak (2018) consider private costs but restrict bid functions to be flat.

Second, demand is exogenous and not strategically determined by the auctioneer. As regulators are not active in the market, they cannot change demand, which is, therefore, exogenous to their decision (unlike in the comparison of Pycia and Woodward, 2026). Even though the demand side does not determine the pricing rule, it presumably reacts strategically to it (see Kamat and Oren, 2002, for such a model); this paper neglects strategic demand.

Third, producers in the model do not condition their bidding strategies on past behaviour of their competitors; they behave as if they only interacted once. This is a critical assumption. So-called Folk Theorems state that, by conditioning behaviour on past outcomes, less

[7] Equilibrium bidding strategies do not change when including fixed costs.

[8] At first sight, many simulation studies seem to assume private information on costs, with bidders not explicitly using their competitors' costs as inputs to their bidding strategy (e.g., Viehmann et al., 2021; Sugianto and Liao, 2014; Liu et al., 2012; Guerci et al., 2007; Hailu and Thoyer, 2007; Bakirtzis and Tellidou, 2006; Xiong et al., 2004; Bower and Bunn, 2001). However, as they *fix* producers' costs, agents learn a best response to a competitor with that particular cost type. Game-theoretically, this is equivalent to finding a Nash equilibrium with known costs (Bakirtzis and Tellidou, 2006).

competitive behaviour than in the "one-shot" game can become an equilibrium (Mas-Collel et al., 1995).

3.2. Equilibrium under pay-as-bid pricing

Under pay-as-bid pricing, the bid on a unit only influences the remuneration for that unit. Thus, increasing a bid is particularly profitable if the bidder is very likely to supply that unit. Intuitively, this implies that mark-ups are highest for low quantities that bidders are (almost) guaranteed to win.

Bidder's Problem.— Under pay-as-bid pricing, a producer i earns their margin $\beta_i(x) - c(x)$ on some unit x whenever they supply more than x units. Assume all other producers $j \neq i$ follow the strategy $\beta_j(x)$. In that case, producer i supplies more than x units, if demand at price $\beta_i(x)$ is large enough to cover the x units of producer i and the $(n-1)\,\beta_j^{-1}(\beta_i(x))$ units the other producers supply at this price. Thus, producer i earns the margin $\beta_i(x) - c(x)$ if and only if $D(\beta_i(x), \varepsilon) \geq x + (n-1)\beta_j^{-1}(\beta_i(x))$. This is equivalent to the demand-shifter ε being weakly larger than the value for which demand at price $\beta_i(x)$ would exactly equal aggregate supply at that price. Formally, it must be the case that

$$\varepsilon \geq e\Big(\beta_i(x), x + (n-1)\beta_j^{-1}(\beta_i(x))\Big),$$

where $e(p, Q)$ it the unique value of the demand-shifter ε for which demand at price p equals Q, i.e., $D\big(p, e(p, Q)\big) = Q$. Thus, bidder i's equilibrium bidding strategy maximises

$$\int_{\underline{x}_i}^{\bar{x}_i} [\beta_i(x) - c(x)] \times \Big[1 - F\Big(e\big(\beta_i(x), x + (n-1)\beta_j^{-1}(\beta_i(x))\big)\Big)\Big] dx, \tag{2}$$

with respect to $\beta_i(x)$, where $\underline{x}_i$ and $\bar{x}_i$ denote the quantities bidder i gets to supply with their strategy when demand is at its minimal level $\underline{\varepsilon}$ and maximum level $\bar{\varepsilon}$, respectively. The objective function in (2) corresponds to the margin on unit x, weighted by the probability that the demand-shifter ε is large enough for the producer to supply more than x units.

The maximisation problem in (2) can be solved by point-wise maximisation of the integrand; the first-order condition is

$$1 - F(\cdot) = [\beta_i(x) - c(x)]\, f(\cdot) \times \Big(e_p(\cdot) + e_Q(\cdot)(n-1)\beta_j^{-1\prime}(\beta_i(x))\Big), \tag{3}$$

where $e_p(\cdot)$ and $e_Q(\cdot)$ denote the partial derivatives of $e(\cdot)$ with respect to p and Q, respectively. The producer benefits from increasing their bid on some unit x by making additional profits whenever unit x is inframarginal, i.e., whenever they win more than x units (left-hand side); on the downside, they lose the margin on the unit in the case where they do not get to supply the unit because of the higher bid, i.e., when the unit is marginal (right-hand side). As the unit $\bar{x}_i$, which the producer only supplies when $\varepsilon = \bar{\varepsilon}$, is never inframarginal, there is no incentive to bid above marginal costs on that unit. Consequently, producer i's optimal bidding function satisfies

$$\beta_i(\bar{x}_i) = c(\bar{x}_i). \tag{4}$$

The trade-off in the first-order condition from (3) only captures the producer's problem correctly if they are at risk of losing the unit in question: In that case, the producer trades off a positive probability of realising the margin on that unit (positive left-hand side) with the positive probability of losing that margin if demand is sufficiently low (positive right-hand side). However, for units $x < \underline{x}_i$, there is no risk of losing them, as the producer supplies $\underline{x}_i$ units even if demand is at its minimal level. Evidently, the supplier should increase the bid on these units until they become marginal for the minimal demand level. Thus, the bid on all units $x \leq \underline{x}_i$ is identical and the optimal bidding schedule of producer i is

$$\beta_i(x) = \begin{cases} b_i(\underline{x}_i) & \text{for } x < \underline{x}_i \\ b_i(x) & \text{for } x \geq \underline{x}_i, \end{cases} \tag{5}$$

where $b_i(x)$ denotes the solution to (3) and $\underline{x}_i$ is the quantity they supply at the minimal demand level under this solution, i.e., $\underline{x}_i = D\big(b_i(\underline{x}_i), \underline{\varepsilon}\big) - (n-1)\beta_j^{-1}\big(b_i(\underline{x}_i)\big)$.

Bidding Behaviour in the Symmetric Equilibrium.— Eqs. (3)–(5) describe the optimal bid function of an individual bidder i, whose competitors all follow the same bidding strategy $\beta_j(x)$. Based on this description of optimal bids, Proposition 1 characterises the symmetric equilibrium, i.e., the situation in which bidder i's best response it to follow the strategy $\beta_j(x)$ as well.

Proposition 1 (*Equilibrium Under Pay-as-Bid Pricing*). *Consider the bidding behaviour of $n \geq 2$ symmetric bidders with commonly known, weakly increasing marginal costs $c(x)$ under pay-as-bid pricing. Assume that market demand is given by some, weakly downward-sloping demand function $D(p, \varepsilon)$ such that $D_\varepsilon(\cdot) > 0$ where ε follows the cumulative distribution $F(\cdot)$ on $[\underline{\varepsilon}, \bar{\varepsilon}]$. The symmetric equilibrium in pure strategies is given by*

$$\beta^*_{PAB}(x) = \begin{cases} b^*_{PAB}(\underline{x}) & \text{for } x < \underline{x} \\ b^*_{PAB}(x) & \text{for } x \geq \underline{x}, \end{cases} \tag{6a}$$

with

$$b^{*\prime}_{PAB}(x) = \frac{(n-1)\, e_Q\big(b^*_{PAB}(x), nx\big)\, [b^*_{PAB}(x) - c(x)]\, f(\cdot)}{1 - F(\cdot) - e_p(\cdot)\, [b^*_{PAB}(x) - c(x)]\, f(\cdot)}, \tag{6b}$$

$$b^*_{PAB}(\bar{x}) = c(\bar{x}), \tag{6c}$$

$$\bar{x} = \frac{D(c(\bar{x}), \bar{\varepsilon})}{n}, \tag{6d}$$

$$\underline{x} = \frac{D(b^*_{PAB}(\underline{x}), \underline{\varepsilon})}{n}, \tag{6e}$$

where $e(p, Q)$ is such that $D\big(p, e(p, Q)\big) = Q$.

Proof. Eqs. (6a)–(6c) follow from using the definition of a symmetric equilibrium – i.e., $x_i = x_j = x$, $p = \beta(x) = \beta_i(x) = \beta_j(x)$ and, consequently, $\beta^{-1\prime}(p) = 1/\beta'(x)$ – in Eqs. (3)–(5). Eqs. (6d) and (6e) formalise the definition of the minimal and maximum quantity a bidder receives in the symmetric equilibrium.

If demand is completely inelastic, we have that $e(p, Q) = Q$, in which case (6b) corresponds to the solution in Pycia and Woodward (2026). The derivation of Proposition 1 also explains why Vanelli et al. (2025), in their setting without demand uncertainty, find that pay-as-bid pricing implements perfect competition when demand and marginal costs are linear. In a best response, a bidder bids truthfully on the highest quantity they ever receive. Hence, without demand uncertainty, bids are truthful on the marginal unit even beyond linear functional forms.

3.3. Equilibrium under uniform pricing

This section briefly revisits equilibrium behaviour under uniform pricing, which has been established by Klemperer and Meyer (1989).

Under uniform pricing, the bid on a unit is only relevant if it sets the market price, i.e, it is marginal. In this case, a higher bid not only raises the revenue for that specific unit (as under pay-as-bid pricing) but also that for all other (inframarginal) units the bidder supplies. Thus, increasing a bid is particularly profitable and margins are particularly high if the bidder expects to supply a large number of units, the opposite pattern from that for pay-as-bid pricing.

Bidder's Problem.— Klemperer and Meyer (1989) show that – for given bids of the other suppliers – a bidder's equilibrium bids maximise the bidder's profits for every realisation of the demand-shifter ε, i.e., they are an ex-post equilibrium. Therefore, we can model an individual producer i as maximising their profits for a given realisation of ε. If the other producers $j \neq i$ bid according to $\beta_j(p)$, producer i will supply $x_i = D(p, \varepsilon) - (n-1)\beta_j^{-1}(p)$ units at price p. Thus, they solve

$$\max_p \quad \pi_{UP} = \big(D(p, \varepsilon) - (n-1)\beta_j^{-1}(p)\big)\, p - C\big(D(p, \varepsilon) - (n-1)\beta_j^{-1}(p)\big). \tag{7}$$

The first-order-condition of this problem is

$$x_i = \left(p - c(x_i)\right)\left((n-1)\beta_j^{-1'}(p) - D_p(p,\varepsilon)\right), \tag{8}$$

where $D_p(\cdot)$ is the derivative of $D(\cdot)$ with respect to p. The left-hand side of (8) reflects the upside of marginally raising the price, or equivalently, raising the bid on the x_ith unit; the producer obtains additional marginal earnings on the x_i inframarginal units. However, they also lose marginal units and the corresponding margins, which the right-hand side of (8) reflects. Hence, producers bid truthfully on the first unit, as they do not benefit from increased margins on any inframarginal units. Proposition 2 describes bids in a symmetric equilibrium under uniform pricing based on the first-order condition in (8).

Proposition 2 (*Klemperer and Meyer, 1989: Equilibrium Under Uniform Pricing*)**.** *The bidding strategies in a symmetric equilibrium under uniform pricing with $n \geq 2$ symmetric bidders with commonly known, weakly increasing marginal costs $c(x)$, when market demand is given by some demand function $D(p,\varepsilon)$ satisfy the initial value problem*

$$\beta_{UP}^{*'}(x) = \frac{(n-1)[\beta_{UP}^*(x) - c(x)]}{x + D_p\left(\beta_{UP}^*(x),\varepsilon\right)[\beta_{UP}^*(x) - c(x)]} \tag{9a}$$

$$\beta_{UP}^*(0) = c(0). \tag{9b}$$

Proof. Use the definition of a symmetric equilibrium, i.e., $x_i = x_j = x$, $p = \beta(x) = \beta_i(x) = \beta_j(x)$, and $\beta^{-1'}(p) = 1/\beta'(x)$ in the first-order condition from (8).

Multiplicity of Equilibria and Equilibrium Selection.— Klemperer and Meyer (1989) show that there are infinitely many solutions to the initial value problem in (9), i.e., there are infinitely many symmetric equilibria under uniform pricing.[9] Intuitively, producers' pay-offs only depend on a single point on their bid function for a given demand realisation; the rest of the bidding function stabilises the outcome (see Back and Zender, 2001).

When comparing price rules, this leaves a problem of equilibrium selection. Klemperer and Meyer (1989) show that there is (potentially) only one equilibrium, in which producers – for sufficiently high demand – (i) offer their entire, infinite production capacity and (ii) do so at weakly positive margins. The rest of this paper focuses on such equilibria. All other equilibria seem – at least to some extent – implausible in electricity markets. If (ii) is violated, margins become negative at some point, resulting in negative profits for high demand levels.[10] If an equilibrium violates (i), there is a strict upper bound to the number of units the capacity-unconstrained producers supply; for high enough demand levels, producers would even curb the quantity they supply as demand increases.[11] Competition authorities most likely would prevent such behaviour. Pycia and Woodward (2026) interpret the remaining equlibria as uncertainty-robust equilibria; no matter, how much demand potentially increases through the demand-shifter ε, i.e., independently of the distribution $F(\varepsilon)$, these equilibria remain economically meaningful. This robustness makes these equilibria focal for the comparison to pay-as-bid pricing.

[9] Formally, (9a) does not satisfy a Lipschitz condition in the environment of $x = 0$.

[10] Of course, real-world producers do not have infinite production capacity. Thus, some of the equilibria that violate (ii) might be plausible in the real world.

[11] Formally, equilibria violating (i) and (ii) correspond to the denominator and the numerator in (9a) going to zero such that the slope of bids goes to zero or infinity, respectively.

3.4. Peak prices, consumer surplus, and welfare

Peak Prices.— Price peaks, such as the one in January 2025 (see Section 1), are focal in the debate on switching to pay-as-bid pricing. The equilibrium behaviour under pay-as-bid and uniform pricing established in Propositions 1 and 2 directly implies that peak prices are lower under pay-as-bid pricing.

Intuitively, peak prices, i.e., prices when demand is at its maximum, depend on the bidders' margins for large quantities. Under pay-as-bid pricing, bidders do not make a positive margin on the price-setting unit when demand is at its maximum. They only benefit from a higher margin on that unit when demand is at its maximum, which – with a continuous distribution of the demand shock – has a probability weight of zero. By contrast, uniform pricing results in particularly high margins in that situation: Raising bids raises the margin on the large number of inframarginal units.

Corollary 1 formalises this intuition.

Corollary 1 (*Lower Peak Prices Under Pay-as-Bid Pricing*)**.** *Consider an electricity market with $n \geq 2$ symmetric bidders with commonly known, weakly increasing marginal costs $c(x)$. Assume that market demand is given by some, weakly downward-sloping demand function $D(p,\varepsilon)$ such that $D_\varepsilon(\cdot) > 0$ where ε follows the cumulative distribution $F(\cdot)$ on $[\underline{\varepsilon},\bar{\varepsilon}]$. If profitable trade is possible, peak prices, i.e., prices when demand is $D(p,\bar{\varepsilon})$, are strictly lower under pay-as-bid pricing than in the uncertainty-robust equilibrium under uniform pricing.*

Proof. Equilibrium bidding functions are weakly increasing (for uniform pricing: in an uncertainty-robust equilibrium). Thus, the highest prices result for $\varepsilon = \bar{\varepsilon}$. The quantity an individual bidder supplies in that case solves $D\left(\beta_{PAB/UP}^*(\bar{x}_{PAB/UP}),\bar{\varepsilon}\right) = n\,\bar{x}_{PAB/UP}$. By (6c), $\beta_{PAB}^*(\bar{x}_{PAB}) = c(\bar{x}_{PAB})$. If profitable trade is possible, $\bar{x}_{PAB} > 0$. Corollary 1 claims that $\beta_{UP}^*(\bar{x}_{UP}) > \beta_{PAB}^*(\bar{x}_{PAB})$. Prove this inequality by contradiction. Suppose that $\beta_{UP}^*(\bar{x}_{UP}) \leq \beta_{PAB}^*(\bar{x}_{PAB})$. If peak prices were weakly lower under uniform pricing, it would need to be the case that $\bar{x}_{UP} \geq \bar{x}_{PAB}$ because demand is (weakly) downward-sloping. As costs are weakly increasing, it follows that $c(\bar{x}_{UP}) \geq c(\bar{x}_{PAB}) = \beta_{PAB}^*(\bar{x}_{PAB})$. Thus, peak prices are only lower under uniform pricing if $\beta_{UP}^*(\bar{x}_{UP}) \leq c(\bar{x}_{UP})$, which – with increasing bids – is inconsistent with optimal bidding behaviour under uniform pricing for $\bar{x}_{UP} > 0$ according to the first-order condition in (8).

While peak prices play a focal role in the public debate, they are not an accurate measure of the performance of a price rule. They only focus on an extreme demand situation and, under pay-as-bid pricing, do not even measure actual market prices for this demand situation, as inframarginal units trade at a lower price. To address these limitations, the following considers consumer surplus and welfare as comprehensive measures of the performance of price rules.

Consumer Surplus and Welfare under Perfect Competition.— The minimal functional form assumptions of the model impose significant constraints on ranking the two pricing rules regarding consumer surplus and welfare. In fact, Section 4 shows that the welfare ranking is ambiguous even under the substantially more restrictive functional form assumptions of the linear model. However, Corollary 2 shows that both pay-as-bid and uniform pricing result in first-best welfare and zero profits under perfect competition, i.e., if there is an infinite number of suppliers or marginal costs are constant.

With perfect competition, the threat of displacement eliminates the incentive to bid above marginal cost under both pricing rules. If marginal costs are constant, equally efficient competitors undercut any bidder attempting to secure a positive margin. With infinitely many suppliers, no price above the (low) costs of the first marginal unit can be sustained, as there is always an additional competitor available to enter the market and provide the unit at cost.

Corollary 2 (*Welfare & Consumer Surplus Under Perfect Competition*). *Consider $n \geq 2$ symmetric producers with the commonly known marginal cost curve $c(x)$, where $c(x) \geq 0$ and $c'(x) \geq 0$. The producers compete under uniform or pay-as-bid pricing with market demand at price p given by the downward-sloping demand function $D(p, \varepsilon)$, where ε is a random variable. For $n \to \infty$, or constant marginal costs, both price rules implement the first-best welfare. Producers do not make profits and consumer surplus corresponds to first-best welfare.*

Proof. Consider the case of infinitely many suppliers. Under both pricing rules, the quantity supplied by an individual supplier, x_i, converges to zero as $n \to \infty$ for every demand realisation. It follows from Proposition 2 that the entire traded quantity is provided to consumers at price $\beta^*_{UP}(0) = c(0)$ under uniform pricing. Similarly, it follows from Proposition 1 that the entire traded quantity is provided at costs $\beta^*_{PAB}(\bar{x}_{PAB}) = c(\bar{x}_{PAB}) = c(0)$ to consumers under pay-as-bid pricing. The results on welfare and consumer surplus follow immediately.

Next, consider the case of constant unit costs. In that case, producers bid their true costs under both pricing rules. Thus, both pricing rules trade the first-best quantity and, given that marginal costs are constant, suppliers make zero profits. For pay-as-bid pricing, the solution to (6b) with constant marginal costs is $b^*_{PAB}(x) = c(x)$. Producers can only make positive margins to the degree that their competitors cannot undercut this price because they have higher costs when they produce more units. With constant marginal costs undercutting is always profitable and pay-as-bid pricing collapses into Bertrand competition. Similarly, it is easy to verify that truthful bidding is also an equilibrium under uniform pricing; it solves the initial value problem in (9). More precisely, truthful bidding is the uncertainty-robust equilibrium.

The result of Corollary 2 for constant marginal cost is the elastic-demand equivalent of the result in Genc (2008). By contrast, the result that pay-as-bid pricing achieves first-best welfare when there is an infinite number of suppliers contrasts with the findings of Federico and Rahman (2003), who conclude that pay-as-bid pricing does not implement first-best welfare under perfect competition. As noted by Willems and Yueting (2023), even though Federico and Rahman (2003) assume that suppliers are price-takers, their perfect competition benchmark reflects monopolistic competition rather than traditional perfect competition. In their model, every producer corresponds to a point on the increasing industry cost curve. Consequently, producers compete with competitors who have higher costs. Thus, less efficient competitors cannot undercut the price offered by a more efficient supplier even if that supplier has a positive margin. By contrast, in the model of this paper, there is an infinite number of competitors, who can supply an additional unit at a price of $c(0)$. Hence, it is impossible to charge a higher price.

4. Linear model

To derive more insights regarding consumer surplus and welfare outside the extremely competitive cases considered in Corollary 2, this section considers a linear model, where

$$D(p, \varepsilon) = \varepsilon - mp, \tag{10a}$$

$$\varepsilon \sim U[\underline{\varepsilon}, \bar{\varepsilon}], \tag{10b}$$

and

$$c(x) = \alpha x \tag{10c}$$

with $m, \alpha, \underline{\varepsilon} > 0$ and $\bar{\varepsilon} > \underline{\varepsilon} > 0$.

4.1. Equilibrium bids and market outcomes

Equilibrium Bids in the Linear Model.— Corollary 3 characterises the symmetric equilibria under pay-as-bid and uniform pricing in the linear model from (10).

Corollary 3 (*Equilibrium Bids in the Linear Model*). *Consider the case where demand is $D(p, \varepsilon) = \varepsilon - mp$ with $\varepsilon \sim U[\underline{\varepsilon}, \bar{\varepsilon}]$ and there are $n \geq 2$ symmetric producers, each with the commonly known marginal cost curve $c(x) = \alpha x$ with $\alpha, m > 0$. The symmetric equilibrium in pure strategies under pay-as-bid pricing is given by the bidding function*

$$\beta^*_{PAB}(x) = \begin{cases} \lambda + \delta_{PAB}\,\underline{x}_{PAB} & \text{for } x < \underline{x}_{PAB} \\ \lambda + \delta_{PAB}\,x & \text{for } x \geq \underline{x}_{PAB} \end{cases} \tag{11a}$$

with

$$\lambda = \frac{(\alpha - \delta_{PAB})\bar{\varepsilon}}{(n + m\alpha)}, \tag{11b}$$

$$\delta_{PAB} = \frac{\sqrt{2\alpha m\,(2n-3) + (1-2n)^2 + \alpha^2 m^2}}{4m} + \frac{\alpha m + 1 - n}{4m}, \tag{11c}$$

and

$$\underline{x}_{PAB} = \frac{\underline{\varepsilon} - \lambda}{n + m\delta_{PAB}} \tag{11d}$$

assuming that the traded quantity is positive even with minimal demand, i.e., $\underline{\varepsilon} > \lambda$. Bids in the only symmetric equilibrium in pure strategies under uniform pricing, in which bids are increasing for all $x > 0$, i.e., in the uncertainty-robust equilibrium, satisfy

$$\begin{aligned}\beta^*_{UP}(x) &= \delta_{UP}\,x \\ &= \frac{2 + \alpha\,m - n + \sqrt{4 - 4n + (\alpha m + n)^2}}{2m}\,x.\end{aligned} \tag{12}$$

Proof. Noting that $e(p, Q) = Q + m\,p$ for the linear model from (10), it is straightforward to verify that the bid function in (11) satisfies the equilibrium conditions from Proposition 1. Similarly, it is straightforward to verify that (12) satisfies the initial value problem from Proposition 2 and is a uncertainty-robust equilibrium. The proof of its uniqueness is analogous to that for the linear example with two suppliers in Klemperer and Meyer (1989).

Fig. 2 illustrates the equilibrium bids from Corollary 3. Bids in the linear model are linear. Under uniform pricing, bids start at marginal costs but then continue steeper than marginal costs.[12] For higher quantities, raising the price becomes increasingly attractive, as it increases the price on many inframarginal units. By contrast, under pay-as-bid pricing, suppliers have large margins on low quantities, with bids being less steep than costs. For the last unit, which the producer only supplies in the highest-demand situation, bids converge to marginal costs.[13]

Market Outcomes.— With equilibrium bids in place, we can characterise market outcomes. In the symmetric equilibrium, an individual producer gets to supply $D(\beta(x), \varepsilon) - (n-1)x$ units when the demand level is ε. Thus, they supply

$$x^*_{UP}(\varepsilon) = \frac{\varepsilon}{n + m\delta_{UP}}$$

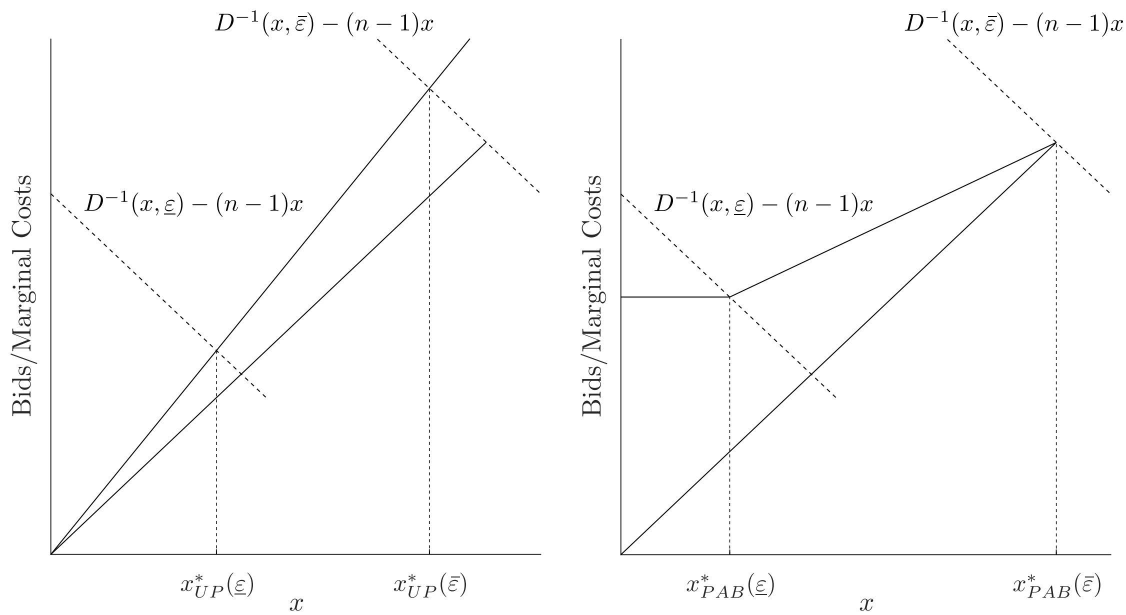
and

$$x^*_{PAB}(\varepsilon) = \frac{\varepsilon - m\lambda}{n + m\delta_{PAB}}$$

units under uniform and pay-as-bid pricing, respectively. Consequently, the market trades $n\,x^*_{UP}(\varepsilon)$ and $n\,x^*_{PAB}(\varepsilon)$ at a stop-out price of β^*_{UP}

[12] Formally, bound the term in the square root in (12) by $(\alpha m + n - 2)^2$ and $(\alpha m + n)^2$, respectively, to get that $\alpha < \delta_{UP} < \alpha + 1/m$.

[13] Formally, replace the term in the square root in (11c) with $(\alpha m + 2n - 3)^2$ and $(\alpha m + 2n - 1)^2$, respectively, to verify that $(\alpha m - 1)/2m < \delta_{PAB} < \alpha/2$.



(a) Uniform pricing. (b) Pay-as-bid pricing.

Fig. 2. Bids (solid line) and costs (dotted line) in the linear model under uniform and pay-as-bid pricing.

$(x^*_{UP}(\varepsilon))$ and $\beta^*_{PAB}(x^*_{PAB}(\varepsilon))$, respectively. By contrast, under the first-best, when the traded quantity and stop-out price are determined by marginal costs, an individual producer would supply

$$x^*_{FB}(\varepsilon) = \frac{\varepsilon}{n + m\alpha}$$

units, and the market would trade $n\,x^*_{FB}(\varepsilon)$ units at a stop-out price of $\alpha\,x^*_{FB}(\varepsilon)$.

4.2. Consumer surplus & welfare

This characterisation of market outcomes makes it possible to compare consumer surplus and welfare in the linear model. Without loss of generality, the comparisons assume that $m = 1$.

Consumer Surplus.— Fig. 3 illustrates consumer surplus for different demand levels under both pricing rules. It shows that the comparison of consumer surplus becomes more favourable to pay-as-bid pricing as demand increases: First, bids are steeper under uniform pricing, such that the stop-out price under uniform pricing increases faster in demand. Second, the rising stop-out price is more detrimental to consumer surplus under uniform pricing. In contrast to pay-as-bid pricing, consumers pay the increasing stop-out price on all units. Consequently, consumer surplus is higher under pay-as-bid pricing for the highest demand realisation, as illustrated by the lightly shaded areas in Fig. 3. The stop-out price is lower under pay-as-bid pricing (see Corollary 1) and, additionally, consumers pay less than the stop-out price on inframarginal units. By contrast, they pay the higher stop-out price on all units under uniform pricing. When demand is low, the comparison of consumer surplus is ambiguous. In Fig. 3, uniform pricing realises a higher consumer surplus in the minimal demand situation (darkly shaded areas) but this is not necessarily the case.

A meaningful comparison of pricing rules must account for all demand situations, which requires comparing expected consumer surplus over all possible demand realisations. Proposition 3 demonstrates that, in the expectation of all demand realisations, consumer surplus is higher under pay-as-bid than under uniform pricing. As is intuitive from Fig. 3, the reason is not that expected stop-out prices are lower under pay-as-bid pricing but that consumers pay less than the stop-out price on inframarginal units.

Formally, consumer surplus under uniform pricing for a given demand realistion is $CS_{UP}(\varepsilon) = \int_0^{nx^*_{UP}} D^{-1}(x,\varepsilon)\,dx - nx^*_{UP}\,\beta^*_{UP}(x^*_{UP})$, where $D^{-1}(x,\varepsilon)$ is the inverse demand function for a given demand level ε. Building the expectation over all demand realisations, the expected consumer surplus under uniform pricing is

$$\mathbb{E}[CS_{UP}] = \frac{n^2}{2(n+\delta_{UP})^2}\,\mathbb{E}[\varepsilon^2].$$

Under pay-as-bid pricing, consumer surplus for a given demand level is $CS_{PAB}(\varepsilon) = \int_0^{nx^*_{PAB}} D^{-1}(x,\varepsilon)\,dx - n\int_0^{x^*_{PAB}} \beta^*_{PAB}(x)\,dx$. In expectation, this gives a consumer surplus of

$$\mathbb{E}[CS_{PAB}] = \frac{n}{2(n+\delta_{PAB})}\,\mathbb{E}[\varepsilon^2] - \frac{n\lambda}{(n+\delta_{PAB})}\,\mathbb{E}[\varepsilon] + \frac{n\left(n\lambda^2 - \underline{\varepsilon}(\underline{\varepsilon} - 2\lambda)\delta_{PAB}\right)}{2(n+\delta_{PAB})^2}.$$

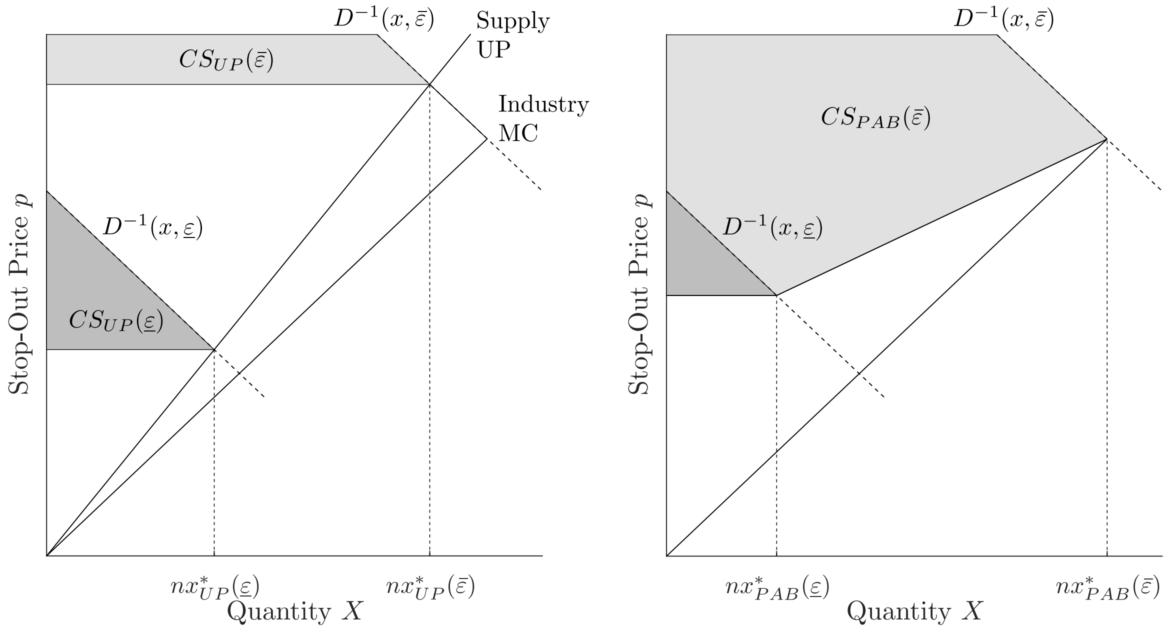
Proposition 3 shows that $\mathbb{E}[CS_{PAB}] > \mathbb{E}[CS_{UP}]$.

Proposition 3 (*Linear Model: Higher Expected Consumer Surplus Under Pay-as-Bid-Pricing*)**.** *Consider the case where demand is $D(p,\varepsilon) = \varepsilon - mp$ with $\varepsilon \sim U[\underline{\varepsilon},\bar{\varepsilon}]$ and there are $n \geq 2$ symmetric producers, each with the commonly known marginal cost curve $c(x) = \alpha x$ with $\alpha, m > 0$. Without loss of generality, assume that $m = 1$. Expected consumer surplus is strictly higher under pay-as-bid pricing than in the uncertainty-robust equilibrium under the uniform pricing assuming that the traded quantity undeer pay-as-bid pricing is positive, i.e., that $\underline{\varepsilon} \geq \lambda$.*

Proof. See Appendix A.

The result of Proposition 3 is unsurprising in the context of theoretical literature. For example, Pycia and Woodward (2026) show that for inelastic demand the *optimally designed* pay-as-bid auction results in a higher expected consumer surplus than the *optimally designed* uniform price auction without making any substantial functional form assumptions.[14] The big difference to the result in Proposition 3 is that they have the auctioneer choose the distribution of demand (and stochastic reserve prices) to maximise the expected consumer surplus

[14] This is, translating their results to the reverse auction.



(a) Uniform pricing. (b) Pay-as-bid pricing.

Fig. 3. Consumer surplus under uniform and pay-as-bid pricing. The dotted and solid lines represent industry-wide marginal costs and supply, respectively; the darkly and lightly shaded areas correspond to the consumer surplus for the lowest and highest demand realisation, respectively.

under either pricing rule. Thus, their result does not say much about the policy choice between pricing rules in electricity markets: In the context of electricity markets, regulators have to take demand and its variation as given. Proposition 3 speaks to that situation.

Welfare.— The ranking of the two pricing rules regarding expected welfare is generally ambiguous. Comparing welfare between the two pricing rules is equivalent to comparing the deadweight losses, i.e., the welfare loss compared to the first-best. Fig. 4 represents the deadweight loss for varying demand levels under both pricing rules as shaded triangles. Formally, Fig. 4 makes clear that the deadweight loss under pay-as-bid and uniform pricing is

$$DWL_{PAB/UP} = 0.5 \cdot (n\, x^*_{PAB/UP} - n\, x_{FB}) \cdot (\beta^*_{PAB/UP}(x^*_{PAB/UP}) - \alpha\, x^*_{PAB/UP}). \tag{13}$$

Eq. (13) shows that welfare only depends on how close the traded quantity is to its first-best level (second factor).[15] The actual payment for electricity is only a welfare-neutral transfer from consumers to producers.

Under either pricing rule, the traded quantity converges to its first-best when the stop-out price is close to marginal costs. Thus, as illustrated in Fig. 4, pay-as-bid pricing eliminates the deadweight loss when demand is at its maximum, as the bid on the "last" unit is truthful. More generally, the deadweight loss decreases with an increase in demand under pay-as-bid pricing as bids approach marginal costs. The opposite is true for uniform pricing. Margins and, consequently, the deadweight loss, increase with demand. Thus, pay-as-bid pricing results in higher welfare for high-demand situations, whereas, potentially, uniform pricing realises higher welfare for low demand levels (in Fig. 4, it does).

Intuitively, the ranking of the two pricing rules regarding expected welfare depends on demand variability: Pay-as-bid pricing delivers higher expected welfare when demand variability is sufficiently low, i.e., when all realisations lie in a small neighbourhood of the maximum-demand case. If demand variability is high, there are sufficiently many low-demand situations for uniform pricing to result in higher expected welfare.

Formally, both price rules result in the same welfare when they trade the same quantity, i.e., $x^*_{PAB} = x^*_{UP}$, which is the case for the demand level

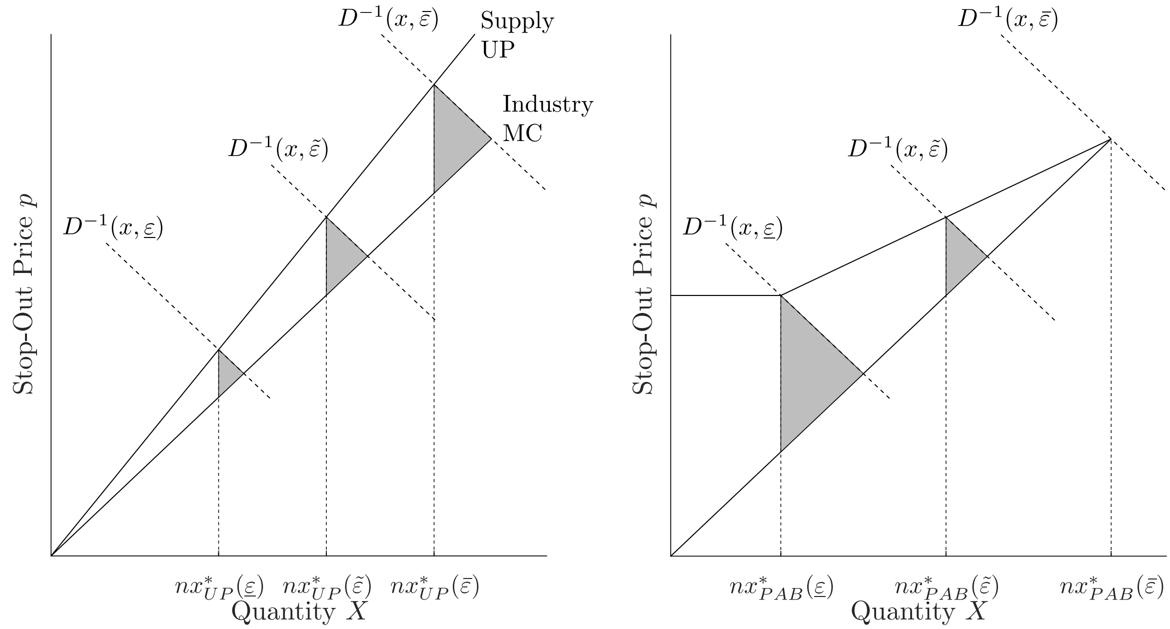
$$\tilde{\varepsilon} = \underbrace{\frac{(\alpha - \delta_{PAB})(n + \delta_{UP})}{(\delta_{UP} - \delta_{PAB})(n + \alpha)}}_{\equiv \tilde{\mu} < 1} \bar{\varepsilon}. \tag{14}$$

Pay-as-bid pricing results in higher welfare if and only if $\varepsilon > \tilde{\varepsilon}$ and uniform pricing outperforms when $\varepsilon < \tilde{\varepsilon}$. Eq. (14) shows that $\tilde{\varepsilon}$ is always strictly lower than $\bar{\varepsilon}$, which confirms that there are always some (high-demand) situations in which pay-as-bid pricing does better than uniform pricing regarding welfare. Interestingly, none of the terms on the right-hand side of Eq. (14) depend on $\underline{\varepsilon}$. Thus, we can have $\underline{\varepsilon} > \tilde{\varepsilon}$, in which case pay-as-bid pricing results in higher welfare for every single demand realisation. This is the case for any demand distribution for which the ratio of the minimum and maximum demand levels, $\mu = \underline{\varepsilon}/\bar{\varepsilon}$, is sufficiently large; i.e., that has $\mu > \tilde{\mu}$, where – according to Eq. (14) – $\tilde{\mu}$ does not depend on the demand distribution.

Expected welfare can be higher under pay-as-bid pricing even when there are some low-demand situations where uniform pricing results in higher welfare. Proposition 4 shows that there is a cut-off value for μ such that pay-as-bid pricing results in higher expected welfare than uniform pricing if and only if $\mu > \tilde{\mu}^E$, where $\tilde{\mu}^E > \tilde{\mu}$. An increase in μ is an increase in the lowest demand level $\underline{\varepsilon}$ for a given maximum demand level $\bar{\varepsilon}$; thus, it relatively raises the expected welfare under pay-as-bid pricing by shifting probability mass towards those cases where pay-as-bid pricing does better.

The upshot is that changes in the demand distribution affect the welfare comparison if and only if they change μ. Intuitively, μ captures demand uncertainty by measuring the distance between the minimum and maximum demand levels multiplicatively. Formally, μ measures

[15] As bids are linear, the third factor increases linearly in the difference of the traded quantity to the first best quantity.



(a) Uniform pricing. (b) Pay-as-bid pricing.

Fig. 4. Welfare under uniform and pay-as-bid pricing in the linear model. The dotted and solid lines represent industry-wide marginal costs and supply, respectively; the shaded triangles correspond to the deadweight loss for different demand levels.

the geometric variance of the demand level ε, or, equivalently, the variance of $\log(\varepsilon)$.[16]

Proposition 4 formalises these results on welfare.

Proposition 4 (*Linear Model: Expected Welfare*)**.** *Consider the case where demand is $D(p,\varepsilon)=\varepsilon-mp$ with $\varepsilon \sim U[\underline{\varepsilon},\bar{\varepsilon}]$, where $\underline{\varepsilon}=\mu\bar{\varepsilon}$ with $0<\mu<1$. There are $n \geq 2$ symmetric producers, each with the commonly known marginal cost curve $c(x)=\alpha x$ with $\alpha, m>0$, that compete either under pay-as-bid or uniform pricing. Without loss of generality, assume that $m=1$. Furthermore, assume that the traded quantity under pay-as-bid pricing is positive, i.e., that $\mu > \underline{\mu} = \frac{(\alpha-\delta_{PAB})}{(\alpha+n)}$. If $\mu > \tilde{\mu}$, welfare is higher under pay-as-bid pricing for every single demand realisation, i.e., $W^*_{PAB}(\varepsilon) > W^*_{UP}(\varepsilon)\ \forall \varepsilon \in [\underline{\varepsilon},\bar{\varepsilon}]$ if*

$$\mu > \tilde{\mu} = \frac{(\alpha-\delta_{PAB})(n+\delta_{UP})}{(\delta_{UP}-\delta_{PAB})(n+\alpha)}.$$

By contrast, a complete welfare-dominance of uniform pricing is impossible. The critical demand level for the dominance of pay-as-bid pricing, $\tilde{\mu}$, is increasing in α. For a given maximum level of demand $\bar{\varepsilon}$, the expected welfare under pay-as-bid pricing rises relative to uniform pricing when the minimum demand level increases, i.e., $\frac{\partial \mathbb{E}[\Delta W]}{\partial \mu} > 0$, where $\mathbb{E}[\Delta W] = \mathbb{E}[W^*_{PAB}] - \mathbb{E}[W^*_{UP}]$. Thus, if the minimal demand level is sufficiently high, pay-as-bid pricing results in higher expected welfare, i.e.,

$$\mu > \max\left\{\underline{\mu};\tilde{\mu}^E\right\} \qquad \Rightarrow \qquad \mathbb{E}\left[W^*_{PAB}\right] > \mathbb{E}\left[W^*_{UP}\right],$$

where $\mu > \underline{\mu}$ corresponds to the assumption of positive trade under pay-as-bid pricing and

$$\tilde{\mu}^E = \frac{\frac{(\delta_{UP}-\alpha)}{(n+\delta_{UP})}\sqrt{\frac{12(\alpha-\delta_{PAB})^2}{(n+\delta_{PAB})^2}-\frac{3(\delta_{UP}-\alpha)^2}{(n+\delta_{UP})^2}}}{2\left(\frac{(\delta_{UP}-\alpha)^2}{(n+\delta_{UP})^2}-\frac{(\alpha-\delta_{PAB})^2}{(n+\delta_{PAB})^2}\right)} - \frac{\frac{(\delta_{UP}-\alpha)^2}{(n+\delta_{UP})^2}+\frac{2(\alpha-\delta_{PAB})^2}{(n+\delta_{PAB})^2}}{2\left(\frac{(\delta_{UP}-\alpha)^2}{(n+\delta_{UP})^2}-\frac{(\alpha-\delta_{PAB})^2}{(n+\delta_{PAB})^2}\right)}, \quad (15)$$

with $\mu^E < \tilde{\mu}$.

Proof. See Appendix B.

In addition to the previously discussed, Proposition 4 claims that a complete welfare-dominance of pay-as-bid pricing is "more likely", i.e., can be realised for more parametrisations of the demand distribution if marginal costs are relatively flat. Federico and Rahman (2003) obtain a similar result in a monopoly.

4.3. A stylised example based on the german electricity market

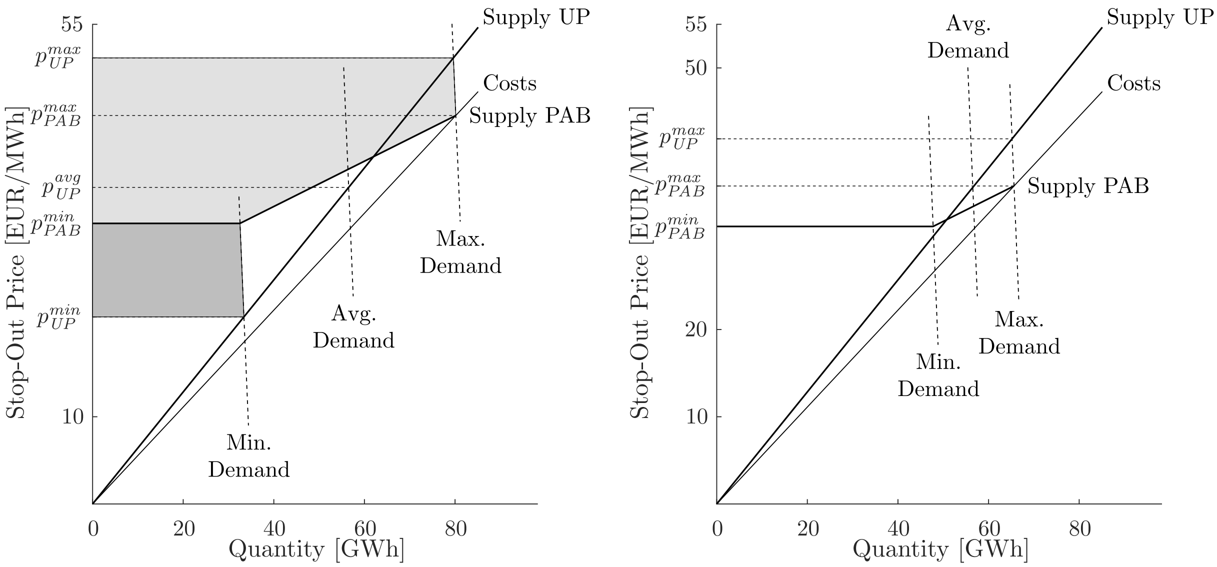
To get a feeling for the orders of magnitudes of the effects of changing the pricing rule, consider a naive calibration of the model that reproduces some core facts of the German electricity market, which uses uniform pricing. This calibration exercise also highlights the challenges in directly translating the model to the real world.

For their hourly data for the German electricity market from 2015 to 2019, Hirth et al. (2024) report that the German electricity market on average traded 56.52 GWh per hour at an average price of 36.3 €/MWh. Moreover, they estimate a linear short-term reduction of demand of roughly 80 MWh for a one €/MWh price increase on the spot market. Measuring the traded quantity in GWh and the market price in €/MWh, this estimate implies that $m = 0.08$ in the model. Intuitively, with this estimate of the slope of the demand function, we can calibrate the average level of demand and the slope of marginal costs to reproduce these averages under uniform pricing. Furthermore, we can use the difference between the maximum and minimum demand level, $\bar{\varepsilon}-\underline{\varepsilon}$, to reproduce the fact that the traded quantity varies between 31.31 and 77.55 GWh. The model cannot exactly match these figures,

[16] Thus, μ is a very specific measure of demand uncertainty; it measures the extent to which ε multiplicatively deviates from its mean. By contrast, the arithmetic variance of ε – which measures the additive deviation of ε from its mean – depends on μ *and* $\bar{\varepsilon}$. Hence, the two pricing rules can perform differently regarding (expected) welfare, because they differ in μ, but still have the same arithmetic variance (if $\bar{\varepsilon}$ varies accordingly).

Table 2
Core descriptives of hourly data from the German electricity market between 2015 and 2019 (see Hirth et al., 2024, Table 2) and results in the linear model under uniform and pay-as-bid pricing. The model is calibrated to reproduce these core descriptives under uniform pricing, whereas the results for pay-as-bid pricing are a counterfactual.

	German market (2015–2019)	Model (Uniform pricing)	Model (Pay-as-bid pricing)
Average (Stop-out) price [€/MWh]	36.3	36.3	38.36
Average traded quantity [GWh]	56.52	56.52	56.36
Minimum traded quantity [GWh]	31.31	33.4	32.54
Maximum traded quantity [GWh]	77.55	79.64	80.17
Minimum (Stop-out) price [€/MWh]	−130.39	21.45	32.17
Maximum (Stop-out) price [€/MWh]	163.52	51.15	44.54



(a) Linear model of the German market.

(b) Linear model of the German market with reduced demand variability.

Fig. 5. Market equilibrium under uniform and pay-as-bid pricing in the linear model calibrated to the German electricity market.
Note: Numeric values for the stop-out prices and the corresponding quantities for the model of the German market in the left panel are provided in Table 2. The shaded areas illustrate the difference in consumer surplus between the two pricing rules. The dark area shows the additional surplus under uniform pricing at minimal demand, while the light area shows the additional surplus under pay-as-bid pricing at maximal demand.

as the traded quantity in the real world is not symmetric around its mean. To complete the calibration of the model, we need to know the number of suppliers. As the model, unrealistically, assumes that suppliers are symmetric, it is not clear what the right number of suppliers is. For simplicity, assume that $n = 8$, which extrapolates the roughly 60%-market share of the five biggest supplier (see, Bundesnetzagentur, 2025) to the entire market. In this case, under uniform pricing, the linear model with $m = 0.08$, $\underline{\varepsilon} \approx 35.12$, $\bar{\varepsilon} \approx 80.17$, $\alpha \approx 4.445$, and $n = 8$ reproduces the average price and traded quantity of the German market and closely mirrors the range of traded quantities. Table 2 reports the core empirical descriptives of the German market in the sample of Hirth et al. (2024) and the corresponding variables in the calibrated model under uniform pricing (calibrated to mirror the actual data) and pay-as-bid pricing (counterfactual). The left panel of Fig. 5 graphically depicts the calibrated model.

Table 2 shows the limits of the restrictive functional form assumptions in the linear model. It substantially underestimates how much prices vary between low and high demand situations under uniform pricing. Particularly, the model does not, and cannot, capture hours with negative prices.

Turning to the comparison of uniform and pay-as-bid pricing, Table 2 shows that in the model, pay-as-bid pricing substantially lowers the variation in stop-out prices. The range of realised stop-out prices decreases by 58%. The reason for this reduced variation of the stop-out price is that the aggregated supply curve is substantially flatter under pay-as-bid pricing, as illustrated in the left panel of Fig. 5.

Peak prices fall 13% under pay-as-bid pricing. As all but the marginal unit trade at an even lower price under pay-as-bid pricing, this figure underestimates how much consumers benefit from pay-as-bid pricing in high demand situations. The entire gain in consumer surplus through pay-as-bid pricing for the maximum demand level corresponds to the light grey area in the left panel of Fig. 5. However, in low demand situations consumers suffer from pay-as-bid pricing. For example, the dark grey area in the left panel of Fig. 5 measures the decrease in consumer surplus through pay-as-bid pricing when demand is at is lowest level. In expectation over all demand realisations, pay-as-bid pricing raises consumer surplus by 1.2% in this example. As one might guess from the slight increase in the average stop-out price/decrease in the average traded quantity reported in Table 2, in the example, this gain in consumer surplus comes at the cost of a (negligible) decrease in average welfare of 0.007%.

This calibration vastly overestimates the uncertainty over demand suppliers face when bidding: It assumes that they cannot narrow down possible demand realisations based on seasonal, daily and hourly patterns before bidding.[17] Instead, any scenario that has realised over a four year period seems possible to them.

A key prediction of the (linear) model is that switching to pay-as-bid pricing would raise average welfare if demand variability was lower. The right panel of Fig. 5 illustrates this change by recalibrating the model without demand below and above the lower and upper quartile. Under uniform pricing, supply remains unchanged. By contrast, under pay-as-bid pricing, the upward-sloping part of the supply curve shifts downward to meet the cost curve at a reduced maximum quantity. As a result, the average stop-out price under pay-as-bid pricing decreases by roughly 11% compared to the model without reduced demand uncertainty, whereas the average price under uniform pricing remains unchanged. With reduced demand variability, even the maximum stop-out price under pay-as-bid pricing is almost identical to the unchanged average price under uniform pricing. Consequently, when shifting to pay-as-bid pricing, the model predicts that the increase in average consumer surplus (now 1.3%) is accompanied by an increase in average welfare (0.003%) when demand variability is reduced.

5. Implications & discussion

Some Notes on External Validity.— When considering the policy implications of this paper's model, it is important to acknowledge its limitations. The calibration of the model in Section 4.3 points to two groups of limitations. First, the model had to extend the market power of the five biggest suppliers to the other suppliers because it assumes symmetry of suppliers. Second, the model cannot explain negative prices because it is static.

Electricity suppliers are not symmetric. Thus, there are not eight electricity suppliers in Germany, but five large ones producing around 60% of the electricity (Bundesnetzagentur, 2025), and more than 4000 other agents that act as electricity suppliers, direct marketers and wholesale electricity traders (Bundesnetzagentur, 2026). Even though there is substantial market power, not all suppliers have market power. Moreover, suppliers do not only differ in size but also in production technologies, i.e., costs. In principle, the methodology used in deriving first-order conditions for optimal bidding behaviour in this paper readily extends to asymmetric suppliers. However, deriving meaningful insights from such an extension requires extensive market data and, presumably, numeric approximation of the equilibrium bidding behaviour implied by the first-order conditions. Thus, the analytic approach of this paper can serve as an input for future simulation work using realistic market data.

Furthermore, the model is static. Producers behave as if they were bidding on the market only once. Consequently, the model cannot make sense of the negative electricity prices that we sometimes observe in the real world (if they are not due to subsidies): Part of the explanation for negative electricity prices is that there is a cost to shutting down production capacity that generates profits in the near future. In the model, suppliers do not consider the future and, therefore, would never generate power that fetches a negative price. Suppliers in the model also do not react to past market outcomes, which creates two further limitations. First, producers' profits, which, in the model of this paper, essentially create market inefficiency, have an important societal value in directing long-term investments (Cramton and Stoft, 2007). Willems and Yueting (2023) and Fabra et al. (2011) compare pricing rules regarding the resulting investment incentives with differing results. Second, as discussed in Section 3.1, unlike in the model, producers in the real world can condition their bidding strategies on past behaviour of their competitors, which allows more collusive behaviour to become an equilibrium. Dechenaux and Kovenock (2007) and Fabra (2003) suggest that pay-as-bid pricing is less prone to such outcomes.

With these caveats in mind, the model has several important findings.

Strategic Tractability.— First, Proposition 1 shows that optimal behaviour under pay-as-bid pricing is tractable under fairly general conditions. Hence, fears of "tactical bidding", which were one of the reasons why the UK government's Review of Electricity Market Arrangements rejected a switch to pay-as-bid pricing (Department for Energy Security & Net Zero, 2023b), should not keep regulators from considering pay-as-bid pricing. Bidding under pay-as-bid pricing is strategically tractable and, therefore, predictable.

Price Peaks and Consumer Surplus.— Second, uniform pricing results in higher peak prices in the model. As this holds for the general model with few functional form assumptions, it seems very likely that uniform pricing exacerbates price peaks, such as the one in January of 2025, discussed in Section 1. Furthermore, this paper suggests that consumers are better off under pay-as-bid pricing.

Welfare.— However, this paper also confirms that this increase in consumer surplus does not necessarily go along with an increase in welfare. Hence, a switch to pay-as-bid pricing is not an obvious decision. In the model, lower demand uncertainty favours pay-as-bid pricing in terms of welfare. Consequently, experiments with pay-as-bid pricing are most promising in markets with little uncertainty.

Areas for Future Research.— While the model in this paper contributes to our understanding of strategic bidding behaviour in electricity markets, further research is necessary evaluate the effects of a switch to pay-as-bid pricing reliably. First, empirical research could bring data to the model, which could enable realistic welfare predictions under either pricing rule that are firmly grounded in economic theory. This way, future research could substantially lower policymakers' uncertainty regarding the effects of price rules. Second, extensive theoretical and simulation work is necessary to overcome the limitations of the model, which range from only considering symmetric producers, neglecting long-term investment and bidding incentives, abstracting from the intermittent character of increasingly important renewable electricity sources, capacity and grid constraints, to ignoring the interaction between different electricity markets and strategic demand behaviour. While some of these limitations, such as not considering asymmetric bidders, are easily captured in analytic work, others, such as grid constraints and optimising bidding behaviour across several interacting auction markets presumably can only be addressed in simulations.

6. Conclusion

This paper has derived the equilibrium strategies under pay-as-bid pricing for an oligopoly of symmetric suppliers with an arbitrary, increasing marginal cost function and an arbitrary, random, downward-sloping demand. This general model showed that peak prices are lower under pay-as-bid pricing and that both pricing rules implement the first-best welfare with zero profits for suppliers when competition is intense, because marginal costs are constant or there are infinitely many suppliers.

To derive more detailed results, this paper has also considered a linear model, where marginal costs and demand are linear, and the intercept of the demand curve follows a uniform distribution. In the linear model, pay-as-bid pricing results in a higher expected consumer surplus because it lowers prices on inframarginal units. By contrast, the comparison of expected welfare remains ambiguous. Pay-as-bid pricing outperforms uniform pricing for at least some demand realisations. It results in higher expected welfare if demand variability is sufficiently low.

This paper has shown that optimal bidding is tractable – and, thus, in principle, predictable – under pay-as-bid pricing under general conditions; the risk of strategic bidding should not keep regulators from considering it. A switch to pay-as-bid pricing could lower peak prices and benefit consumers, whereas its effects on welfare are unclear. Positive welfare effects are most likely in markets with low demand uncertainty.

[17] By contrast, the month, day of week and hour explain 83% of the variation in realised demand in the data set of Hirth et al. (2024).

CRediT authorship contribution statement

Claudio Rottner: Writing – review & editing, Writing – original draft, Visualization, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Proof of Proposition 3

The equilibrium under uniform pricing maximises consumer surplus conditional on the pricing rule and bids: A unit is traded if and only if the value it creates for consumers is larger than the associated price. In contrast to uniform pricing, trading a unit does not impact the price of other units such that the equilibrium quantity maximises consumer surplus. Therefore, to prove Proposition 3, it is sufficient to show that $CS_{PAB}(nx^*_{UP}) > CS^*_{UP}$ given that $CS^*_{PAB} > CS_{PAB}(nx^*_{UP})$. As both pricing rules would generate the same gross value to consumers when trading nx^*_{UP}, we can prove Proposition 3 by showing that consumers' total payments would be lower under pay-as-bid pricing when trading nx^*_{UP}, i.e., that

$$\Delta P = \underbrace{\mathbb{E}\Big[nx^*_{UP}\ \beta^*_{UP}\big(x^*_{UP}(\varepsilon)\big)\Big]}_{\text{Expected Payment in UP}} - \underbrace{\mathbb{E}\Big[n\int_0^{x^*_{UP}(\varepsilon)}\beta^*_{PAB}(x)\,dx\Big]}_{\text{Exp. Pay. in PAB when trading } nx^*_{UP}} > 0. \quad \text{(A.1)}$$

Because we do not know, for what demand levels $x^*_{UP}(\varepsilon) > x^*_{PAB}(\underline{\varepsilon})$ such that the increasing part of the bidding function $\beta^*_{PAB}(\cdot)$ becomes relevant. We can overestimate payments under pay-as-bid pricing by assuming that $x^*_{UP}(\underline{\varepsilon}) > x^*_{PAB}(\underline{\varepsilon})$, i.e.,

$$\mathbb{E}\Big[n\int_0^{x^*_{UP}(\varepsilon)}\beta^*_{PAB}(x)\,dx\Big] < \mathbb{E}\left[n\left(\int_0^{x^*_{UP}(\varepsilon)}\Big(\lambda+\delta_{PAB}\,x\Big)dx + 0.5\,\delta_{PAB}\,\big(x^*_{PAB}(\underline{\varepsilon})\big)^2\right)\right],$$

where $\lambda = b^*_{PAB}(0) = (\alpha-\delta_{PAB})x^*_{PAB}(\bar{\varepsilon}) = \frac{(\alpha-\delta_{PAB0})}{(n+m\alpha)}\,\bar{\varepsilon}$. Using this bound in (A.1) and dividing by n, we get that for $\Delta P > 0$ it is sufficient that

$$\frac{(2\delta_{UP}-\delta_{PAB})}{2(n+\delta_{UP})^2}\,\mathbb{E}[\varepsilon^2] - \frac{2\lambda}{2(n+\delta_{UP})}\,\mathbb{E}[\varepsilon] - \frac{\delta_{PAB}\,\big(\underline{\varepsilon}-\lambda\big)^2}{2(n+\delta_{PAB})^2} > 0, \quad \text{(A.2)}$$

where we used $x^*_{UP}(\varepsilon) = \frac{\varepsilon}{n+m\delta_{UP}}$. Rewriting the expected values by setting $\underline{\varepsilon} = \mu\bar{\varepsilon}$, where $\mu \in [\underline{\mu}, 1)$ with $\underline{\mu} = \lambda/\bar{\varepsilon} = \frac{(\alpha-\delta_{PAB})}{(\alpha+n)}$ by the assumption of positive trade, i.e., $\underline{\varepsilon} > \lambda$ and dividing by $(\bar{\varepsilon})^2$, we get

$$\begin{aligned}\Delta\tilde{P} = &\frac{(2\delta_{UP}-\delta_{PAB})(1+\mu+\mu^2)}{6(n+\delta_{UP})^2} - \frac{(\alpha-\delta_{PAB})(1+\mu)}{2(n+\delta_{UP})\,(n+\alpha)}\\ &- \frac{\delta_{PAB}}{2(n+\delta_{PAB})^2}\left(\mu - \frac{(\alpha-\delta_{PAB})}{(\alpha+n)}\right)^2 > 0,\end{aligned} \quad \text{(A.3)}$$

which is equivalent to (A.2) under the assumptions of Proposition 3. Given that (A.3) is a polynomial of degree two in μ, (A.3) holds if (a) $\Delta\tilde{P}(\underline{\mu}) > 0$, (b) $\Delta\tilde{P}'(\underline{\mu}) > 0$ and (c) $\Delta\tilde{P}(1) > 0$.

Proof of (a). For $\Delta\tilde{P}(\underline{\mu})$, we get

$$\frac{(2\delta_{UP}-\delta_{PAB})(1+\underline{\mu}+\underline{\mu}^2)}{6(n+\delta_{UP})^2} - \frac{(\alpha-\delta_{PAB})(1+\underline{\mu})}{2(n+\delta_{UP})\,(n+\alpha)} > 0$$

$$\overset{\substack{\delta_{PAB}<\alpha/2\\ \delta_{PAB}>(\alpha-1)/2}}{\Leftarrow} \frac{(2\delta_{UP}-\delta_{PAB})}{6(n+\delta_{UP})^2} - \frac{(\alpha-\delta_{PAB})}{2(n+\delta_{UP})\,(\alpha+n)} + \frac{(2\delta_{UP}-\delta_{PAB})}{6(n+\delta_{UP})^2}\,\frac{\alpha^2}{2(\alpha+n)}\,\frac{1}{3\alpha+2n+1} > 0$$

$$\overset{\substack{n>1\\ \delta_{PAB}<\alpha/2}}{\Leftarrow} (2\delta_{UP}-\alpha/2)\,\frac{6(\alpha+n)^2+\alpha^2}{6(\alpha+n)} - 3(n+\delta_{UP})(\alpha-\delta_{PAB}) > 0$$

$$\overset{\substack{\delta_{UP}>\alpha\\ \delta_{PAB}\delta_{UP}\geq\alpha^2/2}}{\Leftarrow} (2n-\alpha)\delta_{UP} + 3n\delta_{PAB} + a^2 - \frac{7n}{2}\alpha + \frac{\alpha^3}{4(\alpha+n)} > 0 \quad \text{(A.4)}$$

To prove that (A.4) holds, we transform it into easy-to verify polynomials: For $n = 2$, (A.4) resolves to

$$\alpha^2 - 7\alpha + \frac{a^3}{2(a+2)} + (4-\alpha)\sqrt{\alpha(4+\alpha)} + 3\sqrt{9+\alpha(2+\alpha)} - 9 > 0.$$

For $n \geq 3$, use $\delta_{PAB} > \frac{(n-1)}{(2n-1)}\alpha$, to obtain the sufficient condition

$$(2n-\alpha)\delta_{UP} + \alpha^2 + \frac{\alpha^3}{4(a+n)} - \frac{(8n-1)n\alpha}{2(2n-1)} > 0. \quad \text{(A.5)}$$

To reduce this problem to a simple polynomial, we can bound δ_{UP} distinguishing $2n \lesseqgtr \alpha$. *If* $2n < \alpha$, δ_{UP} enters negatively, and we get a sufficient condition by using $\delta_{UP} < \alpha + 1$. Hence, a sufficient condition for (A.5) in this case, is

$$\alpha^3 + \frac{(4-14n)}{(2n-1)}\,\alpha^2 + \frac{2n(n-2)}{(2n-1)}\,\alpha + 8n^2 > 0.$$

If $2n \geq \alpha$, we can use the additional restriction on α to give a particularly tight lower bound on δ_{UP}, i.e., $\delta_{UP} > \alpha + \frac{277(n+2)}{400n(n+\alpha)}\alpha$ to rewrite the sufficient condition from (A.5) as

$$100n(2n-1)\alpha^2 + (-1154n^2 - 831n + 554)\alpha + 508n^3 + 1662n^2 - 1108n.$$

Proof of (b). $\Delta\tilde{P}'(\underline{\mu}) > 0$ follows directly from (a). We have,

$$\Delta\tilde{P}'(\underline{\mu}) = \frac{(2\delta_{UPA}-\delta_{PABA})(1+2\underline{\mu})}{6(n+\delta_{UPA})^2} - \frac{(\alpha-\delta_{PABA})}{2(n+\delta_{UPA})\,(n+\alpha)},$$

which is larger than $\Delta\tilde{P}(\underline{\mu})$ given that $\underline{\mu} > 0$ and $\underline{\mu}^2 < \underline{\mu}$ as $\underline{\mu} < 1$.

Proof of (c). We need to show that

$$\Delta\tilde{P}(1) = \frac{(2\delta_{UP}-\delta_{PAB})}{2(n+\delta_{UP})^2} - \frac{(\alpha-\delta_{PAB})}{(n+\delta_{UP})\,(n+\alpha)} - \frac{\delta_{PAB}}{2(n+a)^2} > 0.$$

$\Delta\tilde{P}(1)$ is decreasing in δ_{PAB}. Thus, $\Delta\tilde{P}(1)$ is strictly positive as long as $\delta_{PAB} < \frac{2n(\alpha+n)}{(\delta_{UP}-\alpha)}$, which is satisfied as $(\delta_{UP}-\alpha)\,\delta_{PAB} < (\alpha+1-\alpha)\,\delta_{PAB} = \delta_{PAB} < \alpha < 2n(\alpha+n)$.

Appendix B. Proof of Proposition 4

Critical Value for Dominance of Pay-as-Bid Pricing $\tilde{\mu}$ decreases in α.— Proposition 4 claims that $\frac{\partial\tilde{\mu}}{\partial\alpha} > 0$, or equivalently,

$$\begin{aligned}&-(\delta_{UP}-\delta_{PAB}) + \frac{(n+\alpha)}{(n+\delta_{UP})}\,(\alpha-\delta_{PAB})\frac{\partial\delta_{UP}}{\partial\alpha}\\ &\quad + \frac{(n+\alpha)}{(n+\delta_{PAB})}\,(\delta_{UP}-\alpha)\frac{\partial\delta_{PAB}}{\partial\alpha} < 0.\end{aligned} \quad \text{(B.1)}$$

A sufficient condition for (B.1) to hold follows from replacing the first factors in the second and third summand by one, which, after rewriting the derivatives, gives

$$\begin{aligned}&-(\delta_{UP}-\delta_{PAB}) + (\alpha-\delta_{PAB})\,\frac{(\delta_{UP}+n-1)}{(2\delta_{UP}-a+n-2)}\\ &\quad + (\delta_{UP}-\alpha)\frac{(\delta_{PAB}+n-1)}{(4\delta_{PAB}-a+2n-1)} < 0.\end{aligned} \quad \text{(B.2)}$$

For $\alpha < 1/2$, use that $\frac{(n-1)}{(2n-1)}\alpha < \delta_{PAB} < \frac{\alpha}{2} - \frac{\alpha}{2(2n-1+\alpha)}$ in the first two summands, and $a/4 < \delta_{PAB} < a/2$ in the third summand. After multiplying by $2\sqrt{\alpha^2+2\alpha n+n^2-4n+4}$, adding $(1/4-a/2)(a^2/(2n-1))>0$, and multiplying by $2(2n-1)(2n-1+\alpha)$, we get that a sufficient condition for (B.2) is

$$\begin{aligned}&\underbrace{\left(2n+\frac{1}{2}\right)}_{>0}\alpha^3+\underbrace{\left(3n^2-5n+\frac{7}{2}\right)}_{>0}\alpha^2\\&\quad+\underbrace{\left(-4n^3+n^2+4n-4\right)}_{<0}\alpha-2n(n-2)^2(2n-1)\\&<\left(a^3+(3n-5)\alpha^2-(n-2)\alpha-n(4n^2-10n+4)\right)\\&\quad\times\underbrace{\sqrt{\alpha^2+2\alpha n+n^2-4n+4}}_{>0}.\end{aligned}\tag{B.3}$$

The left-hand side is negative; the first two summands are dominated by the third one given that $\alpha < 1/2$. The right-hand side is negative if and only if $n \geq 3$. Thus, (B.3) holds for $n = 2$. For $n \geq 3$, squaring both sides and dividing by $\alpha^2 > 0$, (B.3) is equivalent to

$$\begin{aligned}0>\;&\alpha^6+(8n-10)\alpha^5+\underbrace{\frac{(72n^2-232n+131)}{4}}_{>0}\alpha^4\\&+\frac{(8n^3-134n^2+254n-127)}{2}\alpha^3\\&+\frac{(-96n^4+360n^3+140n^2-804n+447)}{4}\alpha^2\\&+(-16n^5+140n^4-264n^3+149n^2+52n-52)\alpha\\&-(8n^5-176n^4+490n^3-408n^2+104n),\end{aligned}$$

where we can obtain an upper bound for the right-hand side by using the fact $\alpha < 1$, i.e., it is sufficient that

$$\begin{aligned}0>\;&\alpha^2+(8n-10)\alpha^2+\frac{(72n^2-232n+131)}{4}\alpha^2\\&+\frac{(8n^3-134n^2+254n-127)}{2}\alpha^3\\&+\frac{(-96n^4+360n^3+140n^2-804n+447)}{4}\alpha^2\\&+(-16n^5+140n^4-264n^3+149n^2+52n-52)\alpha\\&-(8n^5-176n^4+490n^3-408n^2+104n)\alpha.\end{aligned}$$

Replacing α by $2\alpha^2$ in the last two summands, after multiplying by two and dividing by α^2, it is sufficient to show that, for $\alpha < 1/2$ and $n \geq 3$,

$$\begin{aligned}0>&(8n^3-134n^2+254n-127)\,\alpha\\&+(-96n^5-192n^4+1084n^3-930n^2+122n+63).\end{aligned}$$

For $\alpha \geq 1/2$, a sufficient condition for (B.2) is $-(\delta_{UP}-\delta_{PAB})+(\alpha-\delta_{PAB})\frac{\partial\delta_{UP}}{\partial\alpha}+\frac{(\delta_{UP}-\alpha)}{2}<0$, given that $\delta_{PAB} > (a-1)/2$, or equivalently,

$$(\alpha-\delta_{PAB})-\left(\frac{(n-2)+2(\delta_{UP}-\delta_{PAB})+\alpha}{2}\right)(\delta_{UP}-\alpha)<0.$$

Using the bounds $\frac{(n-1)}{(2n-1)}\alpha < \delta_{PAB} < \alpha/2$ for the first and second δ_{PAB}, respectively, we get the alternative sufficient condition

$$(n-2)^2+\left(\frac{2n^2-n-2}{(2n-1)}\right)\alpha>(n-2)\sqrt{\alpha^2+2\alpha n+n^2-4n+4}.\tag{B.4}$$

For $n = 2$, (B.4) evidently holds, as only the positive second summand on the left-hand side is non-zero. For $n \geq 3$ both the left- and right-hand side of (B.4) are positive. Thus, after squaring both sides, (B.4) is equivalent to

$$\left(\left(\frac{2n^2-n-2}{(2n-1)(n-2)}\right)^2-1\right)\alpha-\frac{4}{(2n-1)}>0.$$

Relative Expected Welfare under Pay-as-Bid Pricing Increases with Higher Minimal Demand.— The difference in welfare between the pricing rules for a given demand realisation $\varepsilon = z\bar{\varepsilon}$ with $z \in [\underline{\mu}, 1)$, where from the assumption of positive trade $\underline{\mu} = \frac{(\alpha-\delta_{PAB})}{(\alpha+n)}$, is the difference in the respective deadweight loss, i.e.,

$$\begin{aligned}\Delta W=\;&(z\bar{\varepsilon})^2\frac{n}{2}\underbrace{\left(\frac{1}{(n+m\alpha)}-\frac{1}{(n+\delta_{UP})}\right)}_{>0}\times\underbrace{\left(\frac{(\delta_{UP}-\alpha)}{(n+\delta_{UP})}\right)}_{>0}\\&-(\bar{\varepsilon}^2)\frac{n}{2}\underbrace{\left(\frac{z}{(n+m\alpha)}-\frac{(z-\tilde{\lambda})}{(n+\delta_{PAB})}\right)}_{>0}\\&\times\underbrace{\left(\frac{(\delta_{PAB}-\alpha)(z-\tilde{\lambda})}{(n+\delta_{PAB})}+\tilde{\lambda}\right)}_{>0},\end{aligned}$$

where $\tilde{\lambda} = \lambda/\bar{\varepsilon} = \frac{(\alpha-\delta_{PAB})}{(\alpha+n)}$. The derivative of ΔW with respect to μ is

$$\begin{aligned}\frac{\partial\Delta W}{\partial z}=\;&z(\bar{\varepsilon})^2 n\underbrace{\left(\frac{1}{(n+m\alpha)}-\frac{1}{(n+\delta_{UP})}\right)}_{>0}\underbrace{\left(\frac{(\delta_{UP}-\alpha)}{(n+\delta_{UP})}\right)}_{>0}\\&-(\bar{\varepsilon}^2)\frac{n}{2}\underbrace{\left(\frac{z}{(n+m\alpha)}-\frac{(z-\tilde{\lambda})}{(n+\delta_{PAB})}\right)}_{>0}\underbrace{\left(\frac{(\delta_{PAB}-\alpha)}{(n+\delta_{PAB})}\right)}_{<0}\\&-(\bar{\varepsilon}^2)\frac{n}{2}\underbrace{\left(\frac{1}{(n+m\alpha)}-\frac{1}{(n+\delta_{PAB})}\right)}_{<0}\\&\times\underbrace{\left(\frac{(\delta_{PAB}-\alpha)(z-\tilde{\lambda})}{(n+\delta_{PAB})}+\tilde{\lambda}\right)}_{>0}>0.\end{aligned}$$

From $\frac{\partial\Delta W}{\partial z} > 0$, it follows that $\frac{\partial\mathbb{E}[\Delta W]}{\partial\mu} > 0$: Expected welfare can be written as

$$\mathbb{E}[\Delta W]=\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\Delta W(\varepsilon;\bar{\varepsilon})\,\frac{1}{\bar{\varepsilon}-\underline{\varepsilon}}\,d\varepsilon=\bar{\varepsilon}^2\int_{\mu}^{1}\Delta\tilde{W}(z)\,\frac{1}{(1-\mu)}\,dz,\tag{B.5}$$

where $\Delta\tilde{W}(z) = \Delta W/(\bar{\varepsilon})^2$. Hence, $\frac{\mathbb{E}[\Delta W]}{\partial\mu} = \frac{\bar{\varepsilon}^2}{(1-\mu)}\left(\frac{1}{(1-\mu)}\int_{\mu}^{1}\Delta\tilde{W}(z)\,dz - \Delta\tilde{W}(\mu)\right) > 0$.

Conditions for Higher Expected Welfare under Pay-as-Bid Pricing.— Based on (B.5), pay-as-bid pricing results in higher expected welfare if and only if $\int_{\mu}^{1}\Delta\tilde{W}(z)\,dz > 0$. Given that $\frac{\partial\mathbb{E}[\Delta W]}{\partial\mu} > 0$ the critical value of μ for (B.6) to hold is $\tilde{\mu}^E < \tilde{\mu}$. Formally $\tilde{\mu}^E$ solves

$$\int_{\tilde{\mu}^E}^{1}\Delta\tilde{W}(z)\,dz=0,\tag{B.6}$$

or equivalently, $\frac{(\delta_{UP}-\alpha)^2}{(n+\delta_{UP})^2}(1-(\tilde{\mu}^E)^3)-\frac{(\alpha-\delta_{PAB})^2}{(n+\delta_{PAB})^2}(1-\tilde{\mu}^E)^3=0$. Given that $\tilde{\mu}^E = 1$ is an obvious solution to (B.6) but not the desired solution with $\tilde{\mu}^E < \tilde{\mu}$, after polynomial long division with $(1-\tilde{\mu}^E)$, we have that

$$\begin{aligned}&\left(\frac{(\delta_{UP}-\alpha)^2}{(n+\delta_{UP})^2}-\frac{(\alpha-\delta_{PAB})^2}{(n+\delta_{PAB})^2}\right)\left((\tilde{\mu}^E)^2+1\right)\\&\quad+\left(\frac{(\delta_{UP}-\alpha)^2}{(n+\delta_{UP})^2}+\frac{2(\alpha-\delta_{PAB})^2}{(n+\delta_{PAB})^2}\right)(\tilde{\mu}^E)=0.\end{aligned}\tag{B.7}$$

Distinguish two cases: If $\frac{(\delta_{UP}-\alpha)^2}{(n+\delta_{UP})^2} \geq \frac{(\alpha-\delta_{PAB})^2}{(n+\delta_{PAB})^2}$ the left-hand side of (B.7) is positive for all $\tilde{\mu}^E > 0$, i.e., expected welfare under pay-as-bid pricing exceeds expected welfare under uniform pricing for any μ.[18] If $\frac{(\delta_{UP}-\alpha)^2}{(n+\delta_{UP})^2} < \frac{(\alpha-\delta_{PAB})^2}{(n+\delta_{PAB})^2}$ (B.7) has one solution for which $\tilde{\mu}^E < 1$, which is given in (15).

[18] Expected welfare is higher if under pay-as-bid pricing if and only if $(1-\tilde{\mu}^E)$ times the left-hand side is of (B.7) is positive with $\mu = \tilde{\mu}^E$.

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