Growth and climate change: Threshold and multiple equilibria

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Abstract

In this paper we analyze a basic growth model where we allow for global warming. As concerns global warming we assume that the climate system is characterized by feedback effects such that the ability of the earth to emit radiation to space is reduced as the global surface temperature rises. We first study the model assuming that abatement spending is fixed exogenously and demonstrate with the use of numerical examples that the augmented model may give rise to multiple equilibria and thresholds. Then, we analyze the social optimum where both consumption and abatement are set optimally and show that the long-run equilibrium is unique in this case. In the context of our model with multiple equilibria initial conditions are more important for policy actions than discount rates.

Keywords: Basic growth model, global warming, multiple equilibria, thresholds
JEL: C61, O13, Q30

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1 Introduction

Meanwhile, it is widely accepted that the emission of greenhouse gases (GHGs), such as carbon dioxide (CO_2) or methane (CH_4) just to mention two, considerably affects the atmosphere of the earth and, thus, climate on earth. One consequence of a higher concentration of GHGs in the atmosphere is an increase in the global average surface temperature on the earth. According to the Intergovernmental Panel on Climate Change (IPCC) it is certain that the global average surface temperature of the earth has increased since 1861. Over the 20th century the temperature has risen by about 0.6 degree Celsius, and it is very likely that the 1990s was the warmest decade since 1861 (IPCC, 2001, 26), where very likely means that the level of confidence is between $90 - 99$ percent. Eleven of the twelve years from 1995-2006 rank among the 12 warmest years since 1850 and for the next one hundred years the IPCC expects that the mean temperature will rise by about 3 degrees Celsius (IPCC, 2007). Besides the increase of the average global surface temperature, heavy and extreme weather events, primarily in the Northern Hemisphere have occurred more frequently. It is true that changes in the climate may occur as a result of both internal variability within the climate system and as a result of external factors where the latter can be natural or anthropogenic. But, there is strong evidence that most of the climate change observed over the last 50 years is the result of human activities such as the emission of greenhouse gases.

The rise in the average global surface temperature will not only have immediate effects for the natural environment but it will also affect economies. This holds because, on the one hand, agricultural production has been adapted to the current climatic situation and deviations from it will be associated with costs. On the other hand, more extreme weather events will cause immediate damages that imply costs and may reduce GDP. Therefore, economists have constructed models that incorporate climatic interrelations with the economic subsystem. Examples for this type of models are CETA (see Peck, 1992), FUND (see Tol, 1999), RICE and DICE (see Nordhaus and Boyer, 2000 and Nordhaus 2008), WIAGEM (see Kemfert, 2001) or DART (see Deke et al., 2001). The aim of these studies is to evaluate different abatement scenarios as to economic welfare and as to their effects on GHG emissions. However, it must be pointed out that the results partly are very sensitive with respect to the assumptions made. Popp (2003), for example, demonstrates that the outcome in Nordhaus and Boyer (2000) changes considerably when technical change is taken into account.

A recent study that has received great attention in the economics literature as well as in press media is the report by Stern (2008). Stern strongly argues that decisive actions should be undertaken now that aim at reducing GHG emissions in order to avoid catastrophic possibilities that could along with major economic costs. Otherwise, future generations will suffer from extremely high costs that are much larger than costs of avoiding GHG emissions today in present values. But the Stern review has also bee in part heavily criticized. Weitzman (2007) argues that the outcome obtained by the Stern review heavily depends on the low discount rate that is resorted to in the latter report. In addition, there is large uncertainty about structural parameters such that makes the predictions of the Stern review rather uncertain.

Another important research direction, undertaken by scientists, studies the impact of greenhouse gas emissions on climate change through the change of ocean circulations. The papers by Deutsch et al. (2002) and Keller et al. (2000), for example, describe how the gulf stream and the North Atlantic current, part of the North Atlantic thermohaline circulation (THC), transport a large amount of heat from warm regions to Europe. As those papers show, due to the heating up of surface water, the currents could suddenly change and trigger a change in temperature. The THC collapse and the sudden cooling of regions would most likely have a strong economic impact on Europe and Africa. An event like this would have an impact on the climate in these regions and would also likely affect economic growth. Further results on THC mechanisms are given in Broecker (1997). Although a breakdown of the gulf stream is to be considered as rather unlikely meanwhile, this does not hold for the existence of feedback effects of a change in the global climate that affect the ability of earth to emit radiation to space.

The goal of our contribution is different from the above economic studies and we do not intend to evaluate abatement policies as to their welfare effects. We want to study, in the context of a basic growth model, the long-run effects of the interaction of global warming and economics and, in particular, the transitions dynamics that might occur with global warming. More specifically, we want to study the question of whether there possibly exist multiple equilibria and thresholds that separate basins of attraction for optimal paths to some long-run steady state. In order to study such a problem, we take a basic growth model and integrate a simple climate model. Our approach is related to the one presented in Greiner and Semmler (2005) where an endogenous growth model is studied. However, in contrast to the latter we analyze an exogenous growth model and we rigourously prove that initial conditions can be decisive as concerns the question of to which equilibrium the economy converges in the long-run. In our context the urgency of actions is given less by a low discount rate but rather by initial conditions.

The remainder of the paper is organized as follows. In the next section we present the model with non-optimal abatement spending and analyze its dynamics. In section 3, we study the social optimum where both consumption and abatement are chosen optimally and section 4, finally, concludes.

2 The neoclassical growth model with non-optimal abatement spending

In this section we present the neoclassical growth model where we integrate a climate system of the earth and where abatement is not chosen optimally. First, we present the structure of the model and, then, we analyze its dynamics.

2.1 The structure of the model

Our economy is represented by one household with household production that chooses consumption in order to maximize a discounted stream of utility over an infinite time horizon subject to its budget constraint.

Economic activities of the household generate emissions of GHGs. As regards emissions of GHGs we assume that these are a by-product of capital used in production and expressed in CO_2 equivalents. Hence, emissions are a function of per-capita capital, K, relative to per-capita abatement activities, A. This implies that a higher capital stock goes along with higher emissions for a given level of abatement spending. This assumption is frequently encountered in environmental economics (see e.g. Smulders, 1995, or Hettich, 2000). We should also like to point out that the emission of GHGs does not affect utility and production directly but only indirectly by affecting the climate of the earth which leads to a higher surface temperature and to more extreme weather situations. Formally, emissions are described by

$$E = \left(a \frac{LK}{LA} \right)^{\gamma}, \tag{1}$$

with L the amount of labour, $\gamma > 0$ and $a > 0$ are constants. The parameter a can be interpreted as a technology index describing how polluting a given technology is. For large values of a a given stock of capital (and abatement) goes along with high emissions implying a relatively polluting technology and vice versa.

The effect of emissions is to raise the GHG concentration, M, in the atmosphere. The concentration of GHGs evolves according to the following differential equation

$$\dot{M} = \beta_1 E - \mu M, M(0) = M_0, \tag{2}$$

where μ is the inverse of the atmospheric lifetime of CO_2. As to the parameter μ we assume a value of $\mu = 0.1$.[1] β_1 captures the fact that a certain part of GHG emissions are taken up by oceans and do not enter the atmosphere. According to IPCC $\beta_1 = 0.49$ for the time period 1990 to 1999 for CO_2 emissions (IPCC, 2001, 39).

The evolution of per-capita capital is described by the following differential equation that gives the budget constraint of the household,

$$\dot{K} = Y - C - A - (\delta + n)K, \ K(0) = K_0, \tag{3}$$

with Y per-capita production, K per-capita capital, A per-capita abatement activities and δ is the depreciation rate of capital. L is labour, which grows at rate n.

As concerns abatement activities we assume that these are determined exogenously. One can assume that the government levies a non-distortionary tax, like a lump-sum tax or a tax on consumption in our model, and uses its revenue to finance abatement spending.[2]

The production function giving per-capita output is given by

$$Y = B\, K^{\alpha} D(T - T_0), \tag{4}$$

[1]The range of μ given by IPCC is $\mu \in (0.005, 0.2)$, see IPCC (2001, 38).

[2]We are not interested in distortions arising from taxation but in the dynamics of the model. Therefore, we can limit our considerations to the income effect of taxation.

with $\alpha \in (0,1)$ the capital share and B is a positive constant. $D(T - T_0)$ is the damage due to deviations from the normal pre-industrial temperature T_o. As concerns the damage function $D(\cdot)$ we assume the function

$$D(\cdot) = \left(a_1 \, (T - T_o)^2 + 1\right)^{-\psi}, \tag{5}$$

with $a_1 > 0$, $\psi > 0$. This function shows that the damage is the higher the higher the deviation of the actual temperature, T, from the pre-industrial temperature T_o.

To model the climate system of the earth we use the simplest way and resort to a so-called energy balance models (EBM). According to an EBM the change in the average surface temperature on earth is described by[3]

$$\frac{dT(t)}{dt} \, c_h \equiv \dot{T}(t) \, c_h = S_E - H(t) - F_N(t), \; T(0) = T_0, \tag{6}$$

with $T(t)$ the average global surface temperature measured in Kelvin[4] (K), c_h the heat capacity[5] of the earth with dimension $J \, m^{-2} \, K^{-1}$ (Joule per square meter per Kelvin)[6] which is considered a constant parameter. Since most of the earth's surface is covered by seawater, c_h is largely determined by the oceans. Therefore, the heat capacity of the oceans is used as a proxy for that of the earth. The numerical value of this parameter[7] is $c_h = 0.1497 \, J \, m^{-2} \, K^{-1}$. S_E is the solar input, $H(t)$ is the non-radiative energy flow, and $F_N(t) = F \uparrow (t) - F \downarrow (t)$ is the difference between the outgoing radiative flux and the incoming radiative flux. S_E, $H(t)$ and $F_N(t)$ have the dimension Watt per square meter ($W m^{-2}$). t is the time argument which will be omitted in the following as long as no ambiguity can arise. $F \uparrow$ follows the Stefan-Boltzmann-Gesetz, which is

$$F \uparrow = \epsilon \, \sigma_T \, T^4, \tag{7}$$

with ϵ the emissivity that gives the ratio of actual emission to blackbody emission. Blackbodies are objects that emit the maximum amount of radiation and that have $\epsilon = 1$. For

[3] This subsection follows Roedel (2001) chap. 10.2.1 and chap. 1 and Henderson and McGuffie (1987), chap. 1.4 and chap. 2.4. See also Gassmann (1992) and Harvey (2000).

[4] 273 Kelvin are 0 degree Celsius.

[5] The heat capacity is the amount of heat that needs to be added per square meter of horizontal area to raise the surface temperature of the reservoir by 1K.

[6] 1 Watt is 1 Joule per second.

[7] For more details concerning the calculation of this parameter see Harvey (2000).

the earth ϵ can be set to $\epsilon = 0.95$. σ_T is the Stefan-Boltzmann constant that is given by $\sigma_T = 5.67\,10^{-8}\,Wm^{-2}K^{-4}$. Further, the flux ratio $F\uparrow/F\downarrow$ is given by $F\uparrow/F\downarrow = 109/88$. The difference $S_E - H$ can be written as $S_E - H = Q(1 - \alpha_1(T))/4$, with $Q = 1367.5 Wm^{-2}$ the solar constant, $\alpha_1(T)$ the planetary albedo, determining how much of the incoming energy is reflected to space.

According to Henderson and McGuffie (1987) and Schmitz (1991) the albedo $\alpha_1(T)$ is a function that negatively depends on the temperature on earth. This holds because deviations from the equilibrium average surface temperature have feedback effects that affect the reflection of incoming energy. Examples of such feedback effects are the ice-albedo feedback mechanism and the water vapour 'greenhouse' effect (see Henderson and McGuffie, 1987, chap. 1.4). With higher temperatures a feedback mechanism occurs, with the areas covered by snow and ice likely to be reduced.[8] This implies that a smaller amount of solar radiation is reflected when the temperature rises tending to increase the temperature on earth further. Therefore, Henderson and McGuffie (1987, chap. 2.4) and Schmitz (1991, 194) propose a function as shown in figure 1.

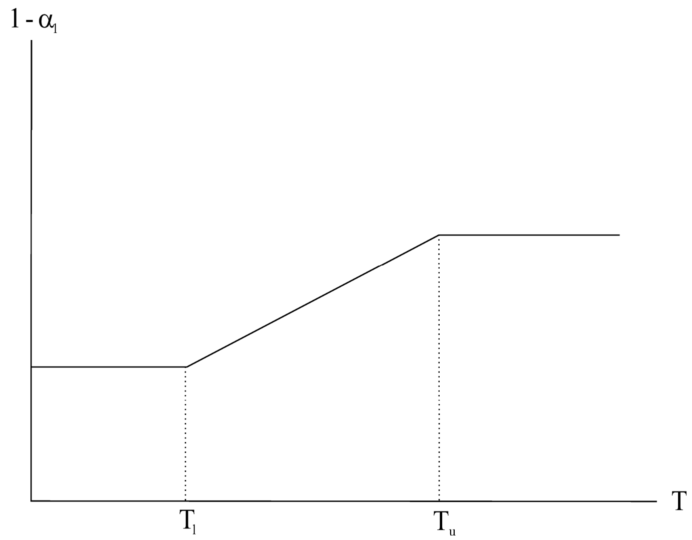

Figure 1: Albedo as a function of the temperature.

Figure 1 shows $1 - \alpha_1(T)$, that part of energy that is not reflected by earth. For the

[8]For a further detailed discussion of positive feedback effects from temperature to higher temperature, see Lovelock (2006).

average temperature smaller than T_l the albedo is a constant, then the albedo declines linearly, so that $1 - \alpha_1(T)$ rises until the temperature reaches T_u from which point on, the albedo is constant again. Here, we should like to point out that other feedback effects may occur, such as a change in the flux ratio of outgoing to incoming radiative flux for example. However, we do not take into account these effects since the qualitative result would remain the same.

The effect of emitting GHGs is to raise the concentration of GHGs in the atmosphere according to equation (2). The effect of a higher concentration of GHGs on the temperature is obtained by calculating the so-called radiative forcing, which is a measure of the influence a GHG, such as CO_2 or CH_4, has on changing the balance of incoming and outgoing energy in the earth-atmosphere system. The dimension of the radiative forcing is Wm^{-2}. For example, for CO_2 the radiative forcing, which we denote by F, is approximately given by

$$F = 6.3 \ln \frac{M}{M_o} , \tag{8}$$

with M the actual CO_2 concentration, M_o the pre-industrial CO_2 concentration and ln the natural logarithm (see IPCC, 1996, 52-53).[9] For other GHGs other formulas can be given describing their respective radiative forcing and these values can be converted in CO_2 equivalents.

Incorporating (8) in (6) gives

$$\dot{T}(t)\, c_h = \frac{1367.5}{4} (1 - \alpha_1(T)) - 0.95 \left(5.67\, 10^{-8}\right) (21/109) T^4 + (1 - \xi)\, 6.3 \ln \frac{M}{M_o},\ T(0) = T_0. \tag{9}$$

The parameter ξ captures the fact that a certain part of the warmth generated by the greenhouse effect is absorbed by the oceans which transport the heat from upper layers to the deep sea. We set $\xi = 0.23$.

According to Roedel (2001), $(1 - \alpha_1(T)) = 0.21$ holds in equilibrium, for $\dot{T} = 0$ with $M = M_o$, giving a surface temperature of about 288 Kelvin which is about 15 degree Celsius.

[9] The CO_2 concentration is given in parts per million (ppm).

2.2 The dynamics of the model

In order to analyze the dynamics of our model, we first have to solve the optimization problem of the household. The household maximizes a discounted stream of utility arising from per-capita consumption, C, times the number of household members subject to the budget constraint and taking into account that emissions affect the climate. As to the utility function we assume a logarithmic function $U(C) = \ln C$.

Thus, the agent's optimization problem can be written as

$$\max_{C} \int_{0}^{\infty} e^{-\rho t} L_0 e^{nt} \, \ln C \, dt, \tag{10}$$

subject to (2), (3) and (9). ρ in (10) is the subjective discount rate, and L_0 is labour supply at time $t = 0$.

To find the optimal solution we form the current-value Hamiltonian[10] which is

$$\begin{aligned} H(\cdot) \;=\; & \ln C + \lambda_1 (B\, K^{\alpha} D(T - T_o) - C - A - (\delta + n)K) + \lambda_2 \left(\beta_1\, a^{\gamma} K^{\gamma} A^{-\gamma} - \mu M\right) \\ & + \lambda_3 \,(c_h)^{-1} \;\cdot \\ & \left(\frac{1367.5}{4}(1 - \alpha_1(T)) - \left(5.6710^{-8}\right)(19.95/109) T^4 + (1 - \xi) 6.3 \ln \frac{M}{M_o}\right), \end{aligned} \tag{11}$$

where λ_i, $i = 1, 2, 3$, are the shadow prices of K, M and T, respectively, and $E = a^{\gamma} K^{\gamma} A^{-\gamma}$. Note that λ_1 is positive while λ_2 and λ_3 are negative.

As to the albedo, $\alpha_1(T)$, we use a function as shown in figure 1. We approximate the function shown in figure 1 by a differentiable function. More concretely, we use the function

$$1 - \alpha_1(T) = k_1 \left(\frac{2}{\Pi}\right) \mathrm{ArcTan}\left(\frac{\Pi\,(\mathrm{T} - 293)}{2}\right) + \mathrm{k}_2. \tag{12}$$

k_1 and k_2 are parameters that are set to $k_1 = 5.6 \cdot 10^{-3}$ and $k_2 = 0.2135$.

[10] For an introduction to the optimality conditions of Pontryagin's maximum principle, see Feichtinger and Hartl (1986) or Seierstad and Sydsaeter (1987).

The necessary optimality conditions, then, are obtained as

$$\begin{aligned}
\frac{\partial H(\cdot)}{\partial C} &= C^{-1} - \lambda_1 = 0, && (13)\\
\dot{\lambda}_1 &= (\rho+\delta)\,\lambda_1 - \lambda_1\,\alpha K^{\alpha-1} B\,D(\cdot) - \lambda_2\,\beta_1\,\gamma\,a^{\gamma}\,K^{\gamma-1}\,A^{-\gamma}, && (14)\\
\dot{\lambda}_2 &= (\rho-n)\,\lambda_2 + \lambda_2\,\mu - \lambda_3\,(1-\xi)\,6.3\,c_h^{-1}\,M^{-1}, && (15)\\
\dot{\lambda}_3 &= (\rho-n)\,\lambda_3 - \lambda_1\,B\,K^{\alpha}\,D'(\cdot) + \lambda_3\,(c_h)^{-1}\,341.875\,\alpha_1'(\cdot) + \\
&\quad \lambda_3\left(5.67\,10^{-8}(19.95/109)\,4\,T^3\right)(c_h)^{-1}, && (16)
\end{aligned}$$

with $\alpha_1' = -k_1(1+0.25\Pi^2(T-293)^2)^{-1}$. Further, the limiting transversality condition $\lim_{t\to\infty} e^{-(\rho+n)t}(\lambda_1 K + \lambda_2 T + \lambda_3 M) = 0$ must hold.

Combining (13) and (14) the economy is completely described by the following differential equations:

$$\begin{aligned}
\dot{C} &= C\left(B\,\alpha K^{\alpha-1} D(\cdot) + \lambda_2\,\beta_1\,\gamma\,a^{\gamma}\,K^{\gamma-1}\,A^{-\gamma} - (\rho+\delta)\right), && (17)\\
\dot{K} &= B\,K^{\alpha} D(\cdot) - C - A - (\delta+n)K,\ K(0) = K_0, && (18)\\
\dot{M} &= \beta_1\,a^{\gamma} K^{\gamma} A^{-\gamma} - \mu M\,M, M(0) = M_0, && (19)\\
\dot{T} &= c_h^{-1}\left(341.875(1-\alpha_1(T)) - 5.67\,10^{-8}(19.95/109)T^4 + 6.3\,(1-\xi)\ln\frac{M}{M_o}\right), \\
&\quad T(0) = T_0, && (20)\\
\dot{\lambda}_2 &= (\rho-n)\,\lambda_2 + \lambda_2\,\mu - \lambda_3\,(1-\xi)\,6.3\,c_h^{-1}\,M^{-1}, && (21)\\
\dot{\lambda}_3 &= (\rho-n)\,\lambda_3 - \lambda_1\,B\,K^{\alpha}\,D'(\cdot) + \lambda_3\,(c_h)^{-1}\,341.875\,\alpha_1'(\cdot) + \\
&\quad \lambda_3\left(5.67\,10^{-8}(19.95/109)\,4\,T^3\right)(c_h)^{-1}. && (22)
\end{aligned}$$

where $C(0)$, $\lambda_2(0)$ and $\lambda_3(0)$ can be chosen by society. A rest point of the dynamic system (17)-(22) gives a steady state for our economy, where we are only interested in solutions with $M^\star \geq M_o$.[11] In order to get additional insight we resort to a numerical analysis where we use the following parameter values.

We consider one time period to comprise one year. The discount rate is set to $\rho = 0.035$, the population growth rate is assumed to be $n = 0.03$, and the depreciation rate of capital is $\delta = 0.075$. The pre-industrial level of GHGs is normalized to one (i.e. $M_o = 1$) and we set $\gamma = 1$. ξ is set to $\xi = 0.23$ (see the previous subsection) and the capital share

[11] The $\star$ denotes steady state values.

is set to $\alpha = 0.18$. The parameter a is set to $a = 3.5 \cdot 10^{-4}$, abatement is $A = 0.0012$ and B is normalized to one, i.e. $B = 1$. As concerns the parameters in the damage function $D(\cdot)$, specified in (5), we assume $a_1 = 0.025$ and $\psi = 0.025$.

In order to find rest points of the system (17)-(22) we first solve $\dot{\lambda}_2 = 0$ with respect to M giving $M = M(\lambda_2, \lambda_3, \cdot)$ and $\dot{M} = 0$ with respect to λ_3 that yields $\lambda_3 = \lambda_3(K, \lambda_2, \cdot)$. Next, we solve $\dot{C}/C = 0$ with respect C leading to $C = C(K, T, \lambda_2, \cdot)$ and setting $\dot{K} = 0$ gives $\lambda_2 = \lambda_2(K, T, \cdot)$. Thus, we end up with the two differential equations $\dot{T}$ and $\dot{\lambda}_3$ that only depend on the two variables K and T and a solution $\dot{T} = \dot{\lambda}_3 = 0$ with respect to K and T gives a steady state for our economy. In order to find possible steady states we plot the $\dot{T} = 0$ isocline, denoted by $Q1$, and the $\dot{\lambda}_3 = 0$ isocline, denoted by $Q2$, in the $(T - K)$ plane. A point where the isoclines intersect gives a rest point for our dynamical system (17)-(22) and, thus, a steady state for our economy.

Figure 2 shows the $Q1$ and the $Q2$ isoclines in the $(T - K)$ plane. One realizes that there are 3 solutions for $Q1 = Q2$.

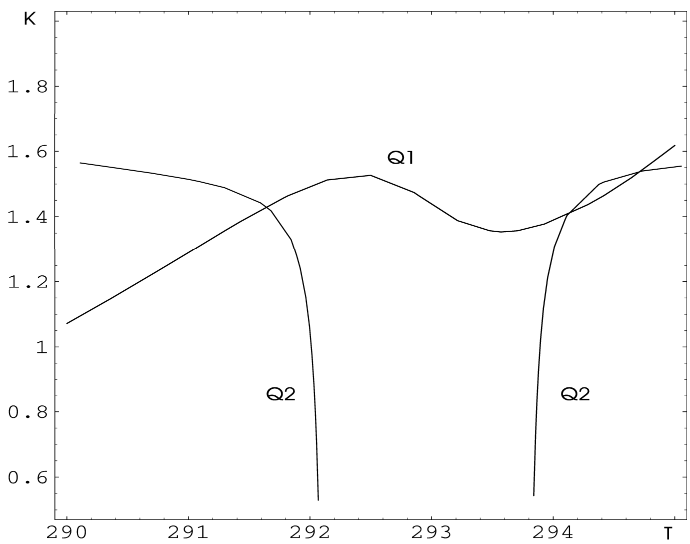

Figure 2: $\dot{T} = 0$ isocline ($Q1$) and $\dot{\lambda}_3 = 0$ isocline ($Q2$) in the $(T - K)$ plane.

Table 1 gives the steady state values for $T^\star$, $K^\star$ and $C^\star/Y^\star$ as well as the eigenvalues of the Jacobian matrix evaluated at the corresponding rest point of (17)-(22).

Table 1. Steady state values and eigenvalues of the Jacobian matrix.

Steady state	$T^\star$	$K^\star$	$C^\star/Y^\star$	eigenvalues
I	291.9	1.47	85.4%	$3.88, -3.88, 0.202 \pm 0.059\,i, -0.197 \pm 0.059\,i$
II	294.1	1.4	85.8%	$3.71, -3.70, 0.3, -0.3, 0.003 \pm 0.115\,i$
III	294.6	1.5	84.9%	$5.39, -5.39, 0.25, -0.25, 0.079, -0.074$

This table shows that the first and third long-run steady states (I and III) are saddle point stable, while the second is unstable, with the exception of a two-dimensional stable manifold. Thus, there are two possible long-run steady states to which the economy can converge where the initial values of consumption, $C(0)$, of the shadow price of GHGs, $\lambda_2(0)$, and of the shadow price of the temperature, $\lambda_3(0)$, must be chosen such that these values lie on the stable manifold leading either to the first or to the third steady state. The first steady state implies a temperature increase of about 3.9 degrees and a steady state consumption share of 85.4 percent; the third steady state corresponds to a temperature increase of about 6.6 degrees and a steady state consumption share of 84.9 percent. The GHG concentration associated with the first steady state is 2.1 and that associated with the third steady state is 2.16.

Before we calculate the value function (10) in order to see which of the two saddle point stable steady states is optimal, we want to study how variations in the abatement spending A affects the outcome. When we reduce abatement spending the qualitative picture as shown in figure 2 does not change. That means there still exist three steady states, where we let looked at the range $A \in [7 \cdot 10^{-4}, 1.21 \cdot 10^{-3}]$. But the steady state value of the temperature becomes larger both for the first and for the third steady state. For example, with $A = 7 \cdot 10^{-4}$ the temperature increase at the first steady state is 4.3 degree Celsius and it is 8.5 degree Celsius at the third steady state. Both steady states are again saddle point stable. When we increase abatement spending the left branch of the $Q2$ isocline in figure 2 moves to the left and the right branch of the $Q2$ isocline moves to the right and, once abatement spending exceeds a certain threshold, only the left intersection point of the $Q2$ isocline with the $Q1$ isocline remains. For $A \in (1.21 \cdot 10^{-3}, 3 \cdot 10^{-3}]$ the steady state is unique and saddle point stable, where $A = 3 \cdot 10^{-3}$ was the largest value we looked at. For example, setting $A = 3 \cdot 10^{-3}$ gives a temperature increase of 0.3 degree Celsius in steady state.

It should also be noted $1 - \alpha_1(\cdot)$ takes the value 0.2098 for $T^\star = 291.9$ and 0.2178 for

$T^{\star} = 294.6$ demonstrating that the quantitative decrease in the albedo does not have to be large for the occurrence of multiple equilibria.

Our result suggests that there exists a threshold such that the initial conditions determine whether it is optimal to converge to steady state I or III. In order to answer the questions of for which initial values of the capital stock, of the GHG concentration and of the temperature it is optimal to converge to the first or to third steady state, respectively, we numerically compute the value function (10).

Doing so allows to calculate the so-called Skiba plane that separates the domains of attraction of the two steady states. The trajectories were computed using a dynamic programming algorithm with adaptive grid as described in Grüne (1997) and Grüne and Semmler (2004). Note that the adaptive gridding technique is particularly suited to compute the domains of attractions of multiple optimal equilibria, see also the example in Grüne and Semmler (2004), section 5.2. The boundaries of the domains of attraction have been computed from the numerically simulated optimal trajectories using bisection for 50 K-values in the 2d example and for 1024 (K,T)-values in the 3d example. Figure 3 shows the Skiba plane in the $(T - K - M)$ space.

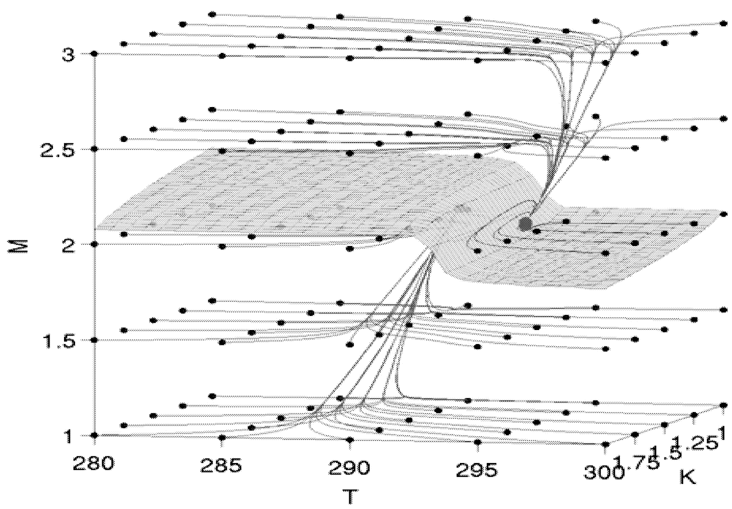

Figure 3: Skiba plane in the $(T - K - M)$ space.

According to IPCC estimations, most projections predict that the GHG concentration in the atmosphere will stabilize at values between 450 ppm and 750 ppm (see e.g. Metz et al., 2007, p. 12). Normalizing the value of pre-industrial GHGs to one, i.e. $M_o = 1$ as in our model, this implies that GHGs stabilize at values of M between 1.6 and 2.7. A GHG concentration of 1.6 implies a temperature increase of about 1.1 to 2.9 degree Celsius and a concentration of 2.7 goes along with a rise in the average global surface temperature of 2.4 to 6.4 degree Celsius.

Figure 3 shows that for initial values of GHGs smaller than about 1.7, convergence to steady state I, with the relatively low temperature increase and the relatively high capital stock, will be the long-run outcome, independent of which temperature increases is associated with these levels of GHGs and independent of the initial physical capital stock. On the other hand, figure 3 also demonstrates that for initial values of GHGs larger than about 2.4, convergence to steady state III, with the relatively high temperature increase and the relatively low capital stock, will be the long-run outcome, independent of which temperature increases is associated with these levels of GHGs and independent of the initial physical capital stock.

It should also be noted that our model has important policy implications. If the government waits too long with actions against GHG emissions, the GHG concentration may rise above the threshold so that the initial condition $M(0)$ is above the Skiba plane in figure 3. If $M(0)$ is above the threshold, private agents will find it optimal to consume, save and invest in a way such that the economy converges to steady state III, when the government starts to take actions against GHG emissions. However, when the government now takes measures against GHG emissions, as long as the level of GHGs is below the threshold, so that the economy will stabilize at a GHG level below the threshold, the economy will converge to steady state I where the long-run temperature is smaller and production is higher, leading to higher welfare. Hence, governments should not wait too long with taking actions against global warming. Thus, the urgency of policy actions is defined more by initial conditions than by a low discount rate.

Only if stabilization of GHGs occurs between about 1.7 and 2.4, the temperature associated with a certain GHG concentration and, possibly, the initial condition with respect to physical capital may be crucial as concerns the question of to which steady state the economy finally converges. Thus, for a certain range of GHGs, it will be the climate sensitivity[12] that is decisive as to whether the economy converges to steady state I or to steady state III. In order to see this, we assume a doubling of GHGs and set $M(0) = 2$ which is in between the boundaries of the IPCC estimates. For that value, figure 4 shows the Skiba curve, drawn as the solid black line, that separates the domains of attraction of the two steady states in the $(T - K)$ plane.

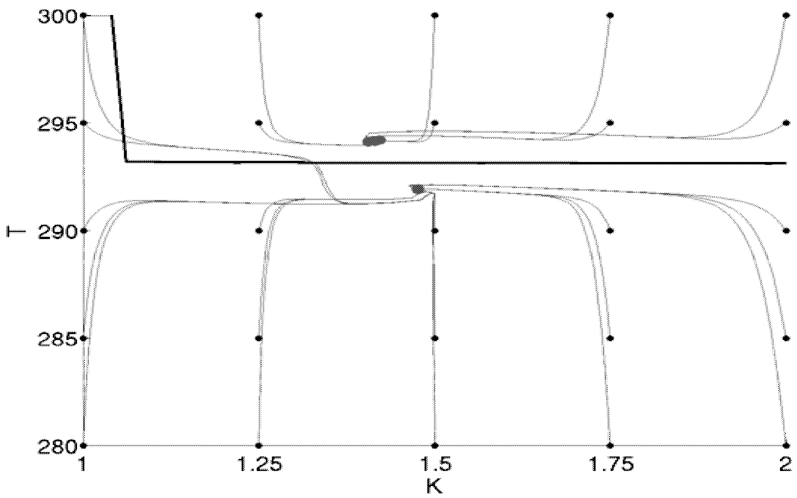

Figure 4: Skiba curve in the $(T - K)$ plane with $M(0) = 2$.

From figure 4 it can be realized that for values of physical capital, K, smaller than about 1.05 convergence to steady state I is always optimal because the Skiba curve becomes almost vertical at $K = 1.05$. This implies that for relatively small initial capital stocks the economy will always converge to the steady state with the relatively small temperature increase and the relatively high capital stock. If the capital stock is larger than about 1.05 it is the temperature increase going along with a doubling of GHGs that determines whether the economy will converge to steady state I or to steady state III. Hence, if a doubling of GHGs implies a temperature larger than 293 Kelvin the economy converges to steady state III with a relatively small capital stock and a relatively large temperature increase. If the temperature is smaller than about 293 Kelvin the economy converges to steady state I with the relatively large capital stock and the relatively small

[12]The climate sensitivity determines by how much the average surface temperature rises as a result of a higher GHG concentration in the atmosphere.

temperature increase.

3 The social optimum

In formulating the optimization problem for the social optimum, a social planner needs to take account that both consumption and abatement have to be set optimally. Consequently, the optimization problem is

$$\max_{C,A} \int_0^{\infty} e^{-\rho t} L_0 e^{nt} \, \ln C \, dt, \tag{23}$$

subject to (2), (3) and (9).

To find necessary optimality conditions we formulate the current-value Hamiltonian which is

$$\begin{aligned} H(\cdot) \;=\; & \ln C + \lambda_4 (B\, K^{\alpha} D(T-T_o) - C - A - (\delta+n)K) + \lambda_5 \left(\beta_1\, a^{\gamma} K^{\gamma} A^{-\gamma} - \mu M\right) \\ & + \lambda_6\, (c_h)^{-1} \, \cdot \\ & \left(\frac{1367.5}{4}(1-\alpha_1(T)) - \left(5.6710^{-8}\right)(19.95/109)T^4 + (1-\xi) 6.3 \ln \frac{M}{M_o}\right), \end{aligned} \tag{24}$$

where λ_i, $i = 4, 5, 6$, are the shadow prices of K, M and T with $\alpha_1(T)$ given by (12) and where $D(\cdot)$ is again given by (5). Again λ_4 is positive while λ_5 and λ_6 are negative.

The necessary optimality conditions are obtained as

$$\frac{\partial H(\cdot)}{\partial C} = C^{-1} - \lambda_4 = 0, \tag{25}$$

$$\frac{\partial H(\cdot)}{\partial A} = -\lambda_5\, \beta_1\, a^{\gamma}\, K^{\gamma}\, \gamma A^{-\gamma-1} - \lambda_4 = 0, \tag{26}$$

$$\dot{\lambda}_4 = (\rho+\delta)\, \lambda_4 - \lambda_4\, \alpha K^{\alpha-1} B\, D(\cdot) - \lambda_5\, \beta_1\, \gamma\, a^{\gamma}\, K^{\gamma-1}\, A^{-\gamma}, \tag{27}$$

$$\dot{\lambda}_5 = (\rho-n)\, \lambda_5 + \lambda_5\, \mu - \lambda_6\, (1-\xi)\, 6.3\, c_h^{-1}\, M^{-1}, \tag{28}$$

$$\begin{aligned} \dot{\lambda}_6 = \; & (\rho-n)\, \lambda_6 - \lambda_5\, B\, K^{\alpha}\, D'(\cdot) + \lambda_6\, (c_h)^{-1}\, 341.875\, \alpha_1'(\cdot) + \\ & \lambda_6 \left(5.67\, 10^{-8} (19.95/109)\, 4\, T^3\right) (c_h)^{-1}, \end{aligned} \tag{29}$$

with $\alpha_1' = -k_1(1 + 0.25\Pi^2(T-293)^2)^{-1}$. Further, the limiting transversality condition $\lim_{t\to\infty} e^{-(\rho+n)t}(\lambda_4 K + \lambda_5 T + \lambda_6 M) = 0$ must hold.

From (25) and (26) we get the optimal abatement spending as,

$$A = \left(a^{\gamma}\, \beta_1\, C\, \gamma\, K^{\gamma}\, (-\lambda_5)\right)^{1/(1+\gamma)} . \tag{30}$$

The dynamics of the social optimum is described by equations (17)-(22) where abatement spending is replaced by its optimal value given in (30). As for the non-optimal economy a steady state is given for variables $C^\star$, $K^\star$, $T^\star$, $M^\star$, $\lambda_5^\star$ and $\lambda_6^\star$ such that $\dot{C} = \dot{K} = \dot{T} = \dot{M} = \dot{\lambda}_5 = \dot{\lambda}_6 = 0$ holds.

To find steady states for the social optimum we recursively solve system (17)-(22), with A given by (30), and end up with the three differential equations $\dot{C}$, $\dot{K}$ and $\dot{\lambda}_6$ that are nonlinear functions of the variables C, K and λ_5. A rest point of these equations then yields a steady state for the social optimum. Analyzing that system demonstrates that there exists a unique steady state with the values given in table 2.

Table 2. Steady state values and eigenvalues of the Jacobian matrix.

$T^\star$	$K^\star$	$A^\star$	$C^\star/Y^\star$	eigenvalues
288.4	1.79	0.00274	82.8%	$6.411, -6.406, 0.263, -0.258, 0.221, -0.216$

Table 2 demonstrates that the value of optimal abatement spending is $A = 2.74 \cdot 10^{-3}$. If abatement spending is less than that value, as it was the case in the last section, the rise in the average temperature is larger than the socially optimal increase which is about 0.4 degree Celsius and there may be more than one steady state as also demonstrated above. If abatement spending is larger than the socially optimal value the steady state is unique and the increase in the temperature is smaller than in the social optimum. It should also be pointed out that the consumption share in the social optimum is smaller than in the non-optimal economy where abatement spending is below its optimum, implying that a higher share of GDP is invested in the social optimum.

4 Conclusion

In this paper we have analyzed a basic growth model with global warming. In modelling the climate change we allowed for feedback effects going along with a higher average global surface temperature, implying that the ability of the earth to emit radiation to space decreases as the average surface temperature on earth rises.

Assuming that greenhouse gases stabilize at values between 450 ppm and 750 ppm, which is plausible according to the IPCC, we could show that the initial condition with respect to the GHG concentration can be crucial as regards the questions of to which steady state the economy converges in the long-run. This outcome can be observed if abatement spending is set to a value smaller than the socially optimal value. In this case, multiple equilibria can emerge and there may exist a threshold determining whether the economy converges to the steady state with a relatively low increase in the average global surface temperature or whether it converges to the steady state with a large rise in the temperature. If GHGs stabilize within a certain corridor the climate sensitivity will be decisive to which steady state the economy converges in the long-run, independent of government policy.

Our model has also important policy implications. When governments wait too long with taking actions against GHG emissions, the GHG concentration may reach a level so that the economy always converges to the steady state with the higher temperature and with a small capital stock and low production. On the contrary, when governments act soon and achieve a stabilization of GHGs below the critical value, the economy will converge to the steady state with a more moderate temperature increase and with a higher capital stock and higher production. The latter scenario also yields higher welfare because this outcome is closer to that of the social optimum. Hence, our analysis, even without reference to a low discount rate, gives support to the policy recommendation reached by the Stern report (2006, 2007) that measures against global warming should be taken soon.

Further, we could also demonstrate that multiple equilibria and thresholds cannot be observed in the social optimum. In this case, the steady state is unique and saddle point stable. In addition, the steady state temperature is smaller and the capital stock is larger compared to the economy with lower abatement spending.

Comparing our results with those obtained for an endogenous growth model, as studied in Greiner and Semmler (2005), one realizes that the outcomes are the same from a qualitative point of view. In the latter model, the social optimum is also characterized by a unique steady state,[13] but of course with ongoing growth, whereas the market economy with non-optimal abatement spending may give rise to multiple equilibria. Hence, independent of whether the long-run growth rate is an exogenous or an endogenous variable,

[13]Unless damages of the temperature increase are extremely small such that damages are virtually non-existent; a case that was not analyzed here.

multiple equilibria and thresholds may emerge when abatement spending is set lower than its socially optimal value.

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