Redesign techniques for nonlinear sampled-data systems

Entwurfstechniken für nichtlineare Abtastsysteme

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Im Emulationsansatz zum Entwurf einer Regelung über ein Kommunikationsnetz wird der Regler zunächst in kontinuierlicher Zeit ohne Berücksichtigung des Netzwerks entworfen und dann als Abtastregler implementiert. Dieser Ansatz ist attraktiv wegen seiner Einfachheit, benötigt aber i.A. hinreichend kleine Abtastraten für ein gutes Regelverhalten des resultierenden Abtastsystems. Unter Bandbreitenbeschränkungen kann daher die Regelgüte bis hin zur Instabilität beeinträchtigt werden. In dieser Arbeit stellen wir verschiedene analytische und numerische Entwurfstechniken zur Anpassung des Emulationsreglers an die Abtastsituation vor, mit denen die Regelgüte des Emulationsreglers verbessert wird um so die benötigte Kommunikationsbandbreite zu verringern.

In the emulation approach to controller design for networked control systems the controller is first designed in continuous time ignoring the network and then implemented as a sampled-data controller. While very attractive for its simplicity, typically sufficiently small sampling periods are needed in order to ensure satisfactory performance of the resulting sampled-data closed loop. Thus, in the presence of network bandwidth constraints performance loss up to instability may occur. In this paper we present a variety of analytical and numerical techniques for the redesign of sampled-data controllers which improve the sampled-data performance of the non-redesigned controller and aim at reducing the necessary communication bandwidth.

Schlagwörter: Reglerentwurf, Abtastsystem, Bandbreitenbeschränkung, nichtlinear, Konvergenzrate, Taylor Entwicklung, Stabilität, modellprädiktive Regelung, Referenzlösung, Lyapunov Funktion

Keywords: controller design, sampled-data system, bandwidth constraint, nonlinear, convergence rate, Taylor expansion, stabilisation, model predictive control, reference solution, Lyapunov function

1 Introduction

Emerging technologies, such as drive-by-wire cars and fly-by-wire aircraft, lead to novel control system architectures in which numerous control loops are closed over a single network that may also be used to transmit information from unrelated users. Motivation for this set-up comes from reduced cost, volume and weight of the system, as well as easier installation and maintenance. This motivates research into the class of Networked Control Systems (NCS) in which multiple actuators and sensors are connected to the network (modelled as a serial communication channel) and only a subset of these can access the network to transmit their values at any transmission instant.

Design of NCS is currently attracting a lot of attention in the literature. For instance, an emulation-like approach to design controllers for NCS was considered in [26, 27], i.e., the controller is designed ignoring the network and then implemented over the network with a sufficiently small sampling period. While very attractive for its simplicity, a common problem in emulation is that the sampling period needed to guarantee stability of the closed-loop may be too small to be implementable on the actual network. In other words, the required network bandwidth[1] may have to be higher than what is available on the real network and, hence, the emulation approach can not be used in such cases and more sophisticated techniques are needed.

In this paper we consider nonlinear sampled-data systems as the simplest example of nonlinear NCS in which all sensors and actuators get access to the network at each transmission instant. This allows us to concentrate our study on the effect of bandwidth constraints, however, our results can be modified in order to cover more general NCS which is subject of our current research. For linear sampled-data systems, a large range of advanced controller discretisation and redesign techniques have been developed, see, e.g., [1, 3, 16]. In the nonlinear case, results are more scarce and often restricted to ad hoc solutions. Notable exceptions are methods addressing specific control tasks like feedback linearisation, see, e.g., [2] and the references therein, or using structural assumptions on the input-output behaviour, for instance on the relative degree, see [21] and the references therein.

For general classes of nonlinear systems, the authors have recently started developing a framework for controller redesign to be used within the emulation framework of sampled-data nonlinear systems [10, 11, 12, 23, 24, 33]. This approach aims at redesigning the emulated controller so that stability of the closed-loop can be preserved with larger sampling periods (i.e. smaller bandwidth) than with the non-redesigned emulated controllers.

In this paper we discuss several analytical and numerical techniques derived within this framework. The analytical approaches [12, 23, 33] are based on Fliess and Taylor expansions and are attractive because they yield closed analytic formulas for the resulting sampled-data feedback laws and theoretical insight into the possibilities and limitations of sampled-data controls, in particular from a geometrical point of view. The numerical approaches [10, 11, 24, 33] are based on online optimisation techniques. Compared to the analytical techniques they are in general more flexible and often show better performance, however, they require fast and efficient online optimisation algorithms in order to be applicable in practice.

The paper is organised as follows: In Section 2 we define the setup and give a more detailed overview of the methods. In Section 3 we present the analytical techniques either based on considerations of solution trajectories only or utilising suitable continuous-time Lyapunov functions. The numerical techniques are discussed in Section 4 starting with the sketch of a simple least squares approach followed by a more detailed treatment of a general model predictive control scheme. In Section 5 we illustrate the techniques by numerical examples and Section 6 concludes the paper.

[1] Sampling period is inversely proportional to bandwidth.

2 Preliminaries and Overview

We consider a nonlinear plant model

$$\dot{x}(t) = f(x(t), u(t)) \tag{1}$$

with vector field $f : \mathbb{R}^n \times \mathbb{U} \to \mathbb{R}^n$ which is continuous and locally Lipschitz in x, state $x(t) \in \mathbb{R}^n$ and control $u(t) \in \mathbb{U} \subset \mathbb{R}^m$. Throughout the paper we will use the following standing assumption:

> We assume that a locally Lipschitz continuous static state feedback $u_0 : \mathbb{R}^n \to \mathbb{R}^m$ has been designed which solves some given control problem for the continuous-time closed-loop system
>
> $$\dot{x}(t) = f(x(t), u_0(x(t))) \qquad x(0) = x_0 \tag{2}$$
>
> whose solution we denote by $\phi(\cdot, x_0)$.

Our goal is now to design $u_T(x)$ such that the corresponding sampled-data solution $\phi_T(\cdot, x_0, u_T)$ of the closed-loop system using a sampler and zero order hold

$$\dot{x}(t) = f(x(t), u_T(x(kT))), \quad t \in [kT, (k+1)T) \tag{3}$$

$k = 0, 1, \ldots$, reproduces the behaviour of the continuous-time system — either in terms of the system trajectories or in terms of a Lyapunov function — and thus improves the performance of the sampled-data closed loop system. Note that our solution concept for (3) coincides with the S-solution (sampled solution) proposed in [4] and that we omit the explicit listing of the control u_0 in $\phi(\cdot, x_0)$ to distinguish it from the emulated implementation used for (3).

2.1 Techniques based on an asymptotic series expansions

This paper gives a survey of several techniques for the redesign of u_T. Our first approach, developed in [12, 23, 33], uses an asymptotic analysis in order to study the difference between the continuous-time model (2) and the sampled-data model (3), i.e., we study the sampled-data system's behaviour for $T \to 0$. To this end, for a function $a : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ we write $a(T, x) = O(T^q)$, if for any compact set $K \subset \mathbb{R}^n$ there exists a constant $C > 0$ (which may depend on K) such that the inequality $a(T, x) \le CT^q$ holds for all elements $x \in K$. If we consider a specific set K we explicitly write $a(T, x) = O(T^q)$ on K. For our analysis we consider an "output" function $h : \mathbb{R}^n \to \mathbb{R}$ and derive series expansions for the difference

$$\Delta h(T, x_0, u_T) := |h(\phi(T, x_0)) - h(\phi_T(T, x_0, u_T))|. \tag{4}$$

Note that h here is not a physical output of the system but rather a scalar auxiliary function which can be chosen in different ways. For instance, establishing $\Delta h_i(T, x_0, u_T) = O(T^q)$ for $h_i(x) = x_i$ and $i = 1, \ldots, n$ one obtains $\Delta\phi(T, x_0, u_T) = O(T^q)$ for the difference

$$\Delta\phi(T, x_0, u_T) := \|\phi(T, x_0) - \phi_T(T, x_0, u_T)\|_\infty \quad (5)$$

measured in the maximum norm $\|\cdot\|_\infty$. From this estimate it follows by a standard induction argument that on each interval $[0, t^*]$ we obtain $\Delta\phi(t, x_0, u_T) \leq O(T^{q-1})$ for all times $t = kT$, $k \in \mathbb{N}$ with $t \in [0, t^*]$ which in particular allows to carry over stability properties from ϕ to ϕ_T, see [29, 30].

If u_0 is a stabilising feedback law with a corresponding Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}$, then another possible choice of the auxiliary function is $h = V$. In this case we study (4) for $h = V$ in order to estimate

$$\Delta V(T, x_0, u_T) := |V(\phi_T(T, x_0, u_T)) - V(\phi(T, x_0))|. \quad (6)$$

Here one may again choose u_T in order to establish $\Delta V(T, x_0, u_T) = O(T^q)$ but the fact that V is a Lyapunov function leads to alternative choices. In particular, a reasonable design objective for u_T would be to make the Lyapunov difference $V(\phi_T(T, x_0, u_T)) - V(x)$ as negative as possible in order to achieve a faster decrease of the Lyapunov function V along the sampled-data trajectory. This approach leads to sampled-data feedback laws with correction terms of the well known $-L_g V$ structure.

In order to facilitate the analytical computations for the representation of (4) we restrict ourselves to control affine systems where f in (1)–(3) takes the form

$$f(x, u) = g_0(x) + g_1(x)u \quad (7)$$

with vector fields $g_0, g_1 : \mathbb{R}^n \to \mathbb{R}^n$. For simplicity of exposition, in the analytical approach we consider single input systems, i.e., $u \in \mathbb{U} \subseteq \mathbb{R}$. Corresponding results for the multi input case can be found in [14].

2.2 Methods using an optimisation approach

The second class of methods we discuss in this paper relies on numerical optimisation techniques and was investigated in [10, 11, 24, 33]. These techniques need fewer structural assumptions, typically work better for larger sampling times and allow to explicitly include the inter sampling behaviour in the redesign — at the price of replacing the analytical formulas for u_T by a numerical (online) optimisation. The idea of the methods is to use the continuous-time trajectory $\phi(t, x_0)$ as a reference and to minimise the deviation of $\phi_T(t, x_0, u_T)$ from this reference. The simplest approach uses a least squares optimisation which iteratively minimises the difference on each sampling interval. A more powerful — but also computationally more demanding — optimisation criterion is the minimisation of an infinite horizon functional given by

$$\sum_{j=0}^{\infty} \int_{jT}^{(j+1)T} l(\phi_T(t) - \phi(t, x_0), u_T(\phi_T(jT)))dt \quad (8)$$

using the abbreviation $\phi_T(t) = \phi_T(t, x_0, u_T)$. This criterion explicitely takes into account the whole future of the systems' behaviour and thus minimises the averaged error over $[0, \infty)$. In order to solve this optimisation problem and to compute the feedback u_T online we employ a model predictive control (MPC) scheme using a receding horizon technique.

2.3 Notations and basic definitions

Concerning the feedback u_T, we consider the following general class of functions.

Definition 2.1 *An* admissible sampled-data feedback law *u_T is a family of maps $u_T : \mathbb{R}^n \to \mathbb{R}^m$, parameterised by the sampling period $T \in (0, T^*]$ for some maximal sampling period T^*, such that for each compact set $K \subset \mathbb{R}^n$ the inequality*

$$\sup_{x \in K,\, T \in (0, T^*]} |u_T(x)| < \infty$$

holds.

Note that for existence and uniqueness of the solutions to (3), we do not need any continuity assumptions on u_T. Boundedness is, however, imposed, because unbounded feedback laws are physically impossible to implement and often lead to closed-loop systems which are very sensitive to modelling or approximation errors, cf., e.g., the examples in [9, 25, 29].

Throughout the paper we use the following notation: a function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{K}$ if it is continuous, zero at zero and strictly increasing. It is of class $\mathcal{K}_\infty$ if it is also unbounded. It is of class $\mathcal{L}$ if it is strictly positive and it is decreasing to zero as its argument tends to infinity. A continuous function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{KL}$ if for every fixed $t \geq 0$ the function $\beta(\cdot, t)$ is of class $\mathcal{K}$ and for each fixed $s > 0$ the function $\beta(s, \cdot)$ is of class $\mathcal{L}$. For subsets $D \subset \mathbb{R}^n$ we use the notation $cl\, D$, $int\, D$ for the closure and the interior of D. The notation $|\cdot|$ stands for the Euclidean norm while $\|x\|_\infty = \max_{i=1,\ldots,n} |x_i|$ denoted the maximum norm in $\mathbb{R}^n$. Furthermore (cf. [15]) we denote the directional derivative of a function $h : \mathbb{R}^n \to \mathbb{R}$ in the direction of $g : \mathbb{R}^n \to \mathbb{R}^n$ by

$$L_g h(x) := \frac{d}{dx} h(x) \cdot g(x)$$

and the Lie bracket of vector fields g_0, $g_1 : \mathbb{R}^n \to \mathbb{R}^n$ by

$$[g_0, g_1] = \frac{d}{dx} g_1 \cdot g_0 - \frac{d}{dx} g_0 \cdot g_1.$$

3 Analytical redesign techniques

In this section we describe the analytical redesign techniques from [12, 23, 33]. We consider nonlinear control affine systems (7) and assume that all functions involved are smooth with sufficiently high degree of smoothness such that the derivatives taken in what follows are well defined and continuous. While the admissible sampled-data feedback u_T we are looking for may in principle be completely unrelated to u_0, in the sequel it will turn out that a certain relation between u_0 and u_T must hold. More precisely, we will see that the resulting sampled-data feedback (if existing) will be of the form

$$u_T(x) = \sum_{i=0}^{M} T^i u_i(x) \tag{9}$$

with u_0 from (2) and $u_1, \ldots, u_M : \mathbb{R}^n \to \mathbb{R}$ being locally bounded functions. This structure appears to be rather natural and was also obtained as the outcome of the design procedure in several other papers, cf. [2, 20, 28].

All our analytical approaches are based on the following theorem ([23, Theorem 3.1]) for a closed-loop control affine system (2) of the form (7) with controller (9).

Theorem 3.1 *Consider the vector field* (7)*, a smooth function* $h : \mathbb{R}^n \to \mathbb{R}$*, the continuous-time closed-loop system* (2) *and the sampled-data closed-loop system* (3) *with controller* u_T *given by* (9)*. Then, for sufficiently small* T*, we can write:*

$$\begin{aligned} h(\phi_T(T, x, u_T)) = h(x) + \sum_{s=0}^{M} T^{s+1}[L_{g_1} h \cdot u_s \\ + p_s(x, u_0, \ldots, u_{s-1})] + O(T^{M+2}). \end{aligned} \tag{10}$$

The proof of this theorem relies on a comparison of the Taylor series expansion of the continuous-time closed-loop system (2)

$$h(\Phi(T, x)) = h(x) + \sum_{i=1}^{\infty} \frac{T^i}{i!} \left. \frac{d^i h(\Phi(T, x))}{dt^i} \right|_{t=0} \tag{11}$$

with the Fliess series expansion of the sampled-data system (3) at the sampling time T. Explicit formulas for the p_i-terms can be found in [23] and [33, Sections 3.1 and 3.2], here we only restate $p_0(x) = L_{g_0} h(x)$ and

$$\begin{aligned} p_1(x, u_0) = \frac{1}{2} \Big[L_{g_0} L_{g_0} h(x) + (L_{g_0} L_{g_1} + L_{g_1} L_{g_0}) \\ h(x) u_0(x) + L_{g_1} L_{g_1} h(x) u_0(x)^2 \Big]. \end{aligned}$$

The next theorem is a rather straightforward consequence from Theorem 3.1 performing a careful evaluation of the p_i-terms. For its proof see [23, Theorem 4.11] for the cases (i) and (ii) and [12, Theorem 3.1] for case (iii). For its formulation we use the notation

$$u_i(x) = \frac{1}{(i+1)!} \left. \frac{d^i u_0(\phi(t, x))}{dt^i} \right|_{t=0}. \tag{12}$$

Theorem 3.2 *Consider the vector field* (7)*, the continuous-time closed-loop system* (2)*, the sampled-data closed-loop system* (3)*, a smooth function* $h : \mathbb{R}^n \to \mathbb{R}$ *and a compact set* $K \subset \mathbb{R}^n$*. Then the following assertions hold for* u_i *from* (12)*.*

(i) $\Delta h(T, x_0, u_T) = O(T^2)$ *holds on* K *for*

$$u_T(x) = u_0(x).$$

(ii) $\Delta h(T, x_0, u_T) = O(T^3)$ *holds on* K *for*

$$u_T(x) = u_0(x) + T u_1(x).$$

(iii) If the condition

$$\left| L_{[g_0, g_1]} h(x) \cdot L_{g_0 + g_1 u_0} u_0(x) \right| \leq c |L_{g_1} h(x)| \tag{13}$$

holds for some constant $c \geq 0$ *and all* $x \in K$*, then*

$$\Delta h(T, x_0, u_T) = O(T^4)$$

holds on K *for* $u_T(x) =$

$$\begin{cases} u_0(x) + T u_1(x) + T^2 u_2(x) + \dfrac{T^2}{12} \alpha_h(x), & x \in \operatorname{cl} \widetilde{K} \\ u_0(x) + T u_1(x), & x \notin \operatorname{cl} \widetilde{K} \end{cases}$$

with $\alpha_h(x) := \dfrac{L_{[g_0, g_1]} h(x) \cdot L_{g_0 + g_1 u_0} u_0(x)}{L_{g_1} h(x)}$ *and* $\widetilde{K} := \{x \in K \mid L_{g_1} h(x) \neq 0\}$.

Here the feedback laws u_T *in (i) and (ii) are unique up to higher order terms in* T *on* K *and the feedback law in (iii) is unique up to higher order terms on* $\widetilde{K}$*. Furthermore, on* $\operatorname{cl} \widetilde{K}$ *the sufficient condition* (13) *is also necessary for the existence of* u_T *in (iii).*

Remark 3.3 *On* $\widetilde{K}$*, the necessary and sufficient condition* (13) *can be interpreted as follows: For* $x \in \widetilde{K}$ *one can always induce any third order correction. However, if* $L_{g_1} h(x_n) \to 0$ *for some sequence* $x_n \in \widetilde{K}$*, then the control effort needed for this purpose may be unbounded which causes the resulting feedback to be not admissible in the sense of Definition 2.1. Condition* (13) *guards against this situation.*

3.1 High order approximations of trajectories

In this section, following [12], we use Theorem 3.2 for finding a feedback $u_T(x)$ such that the trajectory of the sampled-data system is close to the one of the continuous-time closed-loop system (2). More precisely, we want to find u_T such that the difference

$\Delta\phi(T, x_0, u_T)$ after one sampling period becomes as small as possible, i.e., of order $O(T^p)$ for p as large as possible.

The corresponding theorem [12, Theorem 3.6] for $\Delta\phi$ is obtained by applying Theorem 3.2 to $h(x) = x_i$, $i = 1, \dots, n$:

Theorem 3.4 *Consider the vector field* (7)*, the continuous-time closed-loop system* (2)*, the sampled-data closed-loop system* (3) *and a compact set* $K \subset \mathbb{R}^n$ *satisfying* $K = \operatorname{cl}\operatorname{int} K$*. Then the following assertions hold for* u_i *from* (12)*.*

(i) $\Delta\phi(T, x_0, u_T) = O(T^2)$ *holds on* K *for*

$$u_T(x) = u_0(x).$$

(ii) $\Delta\phi(T, x_0, u_T) = O(T^3)$ *holds on* K *for*

$$u_T(x) = u_0(x) + T u_1(x).$$

(iii) If there exists a bounded function $\alpha : K \to \mathbb{R}$ *satisfying*

$$[g_0, g_1](x) L_{g_0 + g_1 u_0} u_0(x) = \alpha(x) g_1(x) \tag{14}$$

then

$$\Delta\phi(T, x_0, u_T) = O(T^4)$$

holds on K *for* $u_T(x) =$

$$\begin{cases} u_0(x) + T u_1(x) + T^2 u_2(x) + \dfrac{T^2}{12}\alpha(x), & x \in \operatorname{cl}\widetilde{K} \\ \textit{arbitrary}, & x \notin \operatorname{cl}\widetilde{K} \end{cases}$$

with $\widetilde{K} := \{x \in K \mid g_1(x) \neq 0\}$*.*

Here the feedback laws u_T *in (i) and (ii) are unique up to higher order terms in* T *on* K *and the feedback law in (iii) is unique up to higher order terms on* $\widetilde{K}$*. Furthermore, on* $\operatorname{cl}\widetilde{K}$ *the sufficient condition* (14) *is also necessary for the existence of* u_T *in (iii).*

Remark 3.5 *Necessary and sufficient conditions for* $O(T^p)$*,* $p \geq 5$*, can be obtained in a similar way but become more and more involved, because the number of higher order Lie brackets to be considered grows exponentially, cf. [33, Chapter 5]. However, despite this growing complexity, for a given continuous-time closed-loop system it is possible to give a rather simple recursive* MAPLE *procedure which checks the conditions for arbitrary order and calculates the corresponding sampled-data feedback. This* MAPLE *code is available on* `www.math.uni-bayreuth.de/~lgruene/publ/redesign_tra.html`*.*

Remark 3.6 *If we disregard the necessity and focus on merely sufficient conditions for* $O(T^p)$*,* $p \geq 5$*, the derivation is considerably easier. For instance, in [33] (see also [12]) it was shown that the feedback law*

$$u_T(x) = \sum_{i=0}^{p-2} T^i u_i(x)$$

with u_i *from* (12) *realises* $\Delta\phi = O(T^p)$ *for arbitrary* $p \geq 2$ *if the vector fields* g_0 *and* g_1 *commute, i.e.,* $[g_0, g_1] = 0$*. It should be noted that this condition is also well known in the numerical approximation theory of control systems, cf., e.g., [8, 32].*

This commutativity condition could be considerably weakened in [22], where it was shown that if the vector fields $[g_0, g_1]$ *and* g_1 *are parallel, then sampled-data feedback laws* u_T *realising* $\Delta\phi = O(T^p)$ *exist for all* $p \geq 2$*. Here, however, the resulting formulas for* u_T *again become very complicated.*

Observe that while (13) ensuring $\Delta h = O(T^4)$ is still relatively easy to satisfy at least in parts of the state space, cf. [12, Remark 3.5], assumption (14) is rather strong. It demands that the direction generated by the Lie bracket $[g_0, g_1]$ must be contained in the span of g_1 whenever $L_{g_0 + g_1 u_0} u_0(x) \neq 0$.

On the other hand, while the estimate $\Delta\phi(T, x_0, u_T) \leq O(T^p)$ allows an immediate inductive[2] extension to arbitrary compact time intervals yielding $\Delta\phi(t, x_0, u_T) \leq O(T^{p-1})$ (due to the fact that an estimate for the whole state is available), this is in general not possible for the estimate $\Delta h(T, x_0, u_T) \leq O(T^p)$. A notable exception is the case when $h = V$ is a Lyapunov function for the closed-loop system, a situation studied in [23] which we will present next.

3.2 Redesign using Lyapunov functions

For several classes of systems (7) nowadays there exist systematic methods to design a stabilising continuous-time control law of the form $u = u_0(x)$ and an associated Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$, i.e., there exist $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$ such that

$$\alpha_1(|x|) \;\leq\; V(x) \leq \alpha_2(|x|) \tag{15}$$

$$\frac{\partial V}{\partial x}[g_0(x) + g_1(x) u_0(x)] \leq -\alpha_3(|x|) \quad \forall\, x \in \mathbb{R}^n. \tag{16}$$

Examples of such methods are backstepping [6, 18] and forwarding [31] or methods based on control Lyapunov functions, such as Sontag's formula [15]. We now illustrate how the knowledge of V can be used in the redesign procedure, an approach which can be extended to more general dissipation inequalities [19].

Integrating (16) one obtains an estimate for the Lyapunov difference

$$V(\phi(t, x_0)) - V(x) \leq \underbrace{-\int_0^t \alpha_3(|\phi(\tau, x_0)|) d\tau}_{=:\alpha(t,x)}. \tag{17}$$

[2] cf. [29, Section 2.2.2]. The procedure is similar to the convergence proof of numerical one-step approximations. The necessary numerical stability condition follows from our Lipschitz assumptions on f and u_0.

Using (17) and (15), there exists a function $\beta \in \mathcal{KL}$ such that solutions of the closed-loop system (2) satisfy:

$$|\phi(t, x_0)| \leq \beta(|x_0|, t) \qquad \forall\, x_0 \in \mathbb{R}^n,\ t \geq 0\ . \tag{18}$$

Note that β can be computed from α_1, α_2 and (17), cf. [17, Section 4.4]. In what follows we assume that the estimate on $|\phi(t, x_0)|$ from (18) satisfies all desired performance specifications in terms of overshoot and convergence speed. This motivates our redesign objective, which is to ensure that $\phi_T(T, x_0, u_T)$ satisfies (17) approximately for $t = T$ with as little error as possible in order to minimise the performance loss due to sampling.

A first approach to achieve this goal is to apply Theorem 3.2 with $h = V$. Using this theorem, from (17) one immediately gets

$$V(\phi_T(T, x_0, u_T)) - V(x) \leq -\alpha(T, x) + O(T^p) \tag{19}$$

for $p = 2, 3, 4$ depending on the choice of u_T ($p = 4$ being feasible if (13) holds for $h = V$).

Although this choice might already yield satisfactory performance, in the special case of the Lyapunov difference it makes sense to go beyond the mere high order approximation of the continuous-time behaviour. Indeed, we can mimic techniques of Lyapunov controller redesign for continuous-time systems for robustification of the system (see [5, 17]). In this approach, the redesigned controller is providing more negativity to the Lyapunov difference than the original controller $u_0(x)$. This typically yields high gain controllers that have the well known $-L_gV$ structure, see, e.g., [31].

In our sampled-data setting this leads to the following problem: Suppose that we have (19). Then we want to design $\tilde{u}_T$ so that

$$V(\phi_T(T, x_0, \tilde{u}_T)) - V(x) \leq -\tilde{\alpha}(T, x) + O(T^{\tilde{p}})$$

holds with

$$\tilde{\alpha}(T, x) > \alpha(T, x) \text{ for } T > 0,\ x \neq 0 \quad \text{and} \quad \tilde{p} \geq p\ .$$

Hence we want to achieve more Lyapunov decrease with the redesigned controller (i.e., $\tilde{\alpha}(T, x) > \alpha(T, x)$) while not decreasing the order in T of the (possibly) positive remainder terms (i.e., $\tilde{p} \geq p$). Thus, it is expected that the redesigned controller will provide faster decrease of the Lyapunov function for sufficiently small sampling periods, which results in a faster response and typically enlarges the domain of attraction.

In the following we sketch how such a redesign can be done based on Theorem 3.1. We start from $u_T = u_0$ which according to Theorem 3.2 realises (19) with $p = 2$. Using the controller structure (9) the first $M + 1$ terms in (10) turn out to have the following form:

$$O\left(T^0\right): \quad L_{g_1}V \cdot u_0 + L_{g_0}V \tag{20}$$

$$O\left(T^1\right): \quad L_{g_1}V \cdot u_1 + p_1(x, u_0) \tag{21}$$

$$\vdots \quad \vdots$$

$$O\left(T^M\right): \quad L_{g_1}V \cdot u_M + p_M(x, u_0, \ldots, u_{M-1}). \tag{22}$$

This special triangular structure allows us to use a recursive redesign. Since u_0 is already given from the continuous-time plant model (7), $p_1(x, u_0)$ is known and thus we can design u_1 from (21). We will choose u_1 such that the $O(T)$ terms in the expansion (10) are more negative than when $u_1 = 0$. Proceeding recursively, at each step $j \in \{2, \ldots, M\}$ we design u_j to make the $O(T^j)$-term more negative based on the knowledge of $p_j(x, u_0, \ldots, u_{j-1})$ since all previous $u_i, i = 0, 1, 2, \ldots, j-1$ have already been designed. One way to choose u_j arises from the observation that any function $u_j(x)$ with $u_j(x)L_{g_1}V(x) \leq 0$ will achieve more decrease of $V(\cdot)$ if we neglect the terms of order $\geq j+1$. For example, one such choice is

$$u_j(x) = -\gamma_j(V(x)) \cdot (L_{g_1}V(x))\,, \tag{23}$$

where the gain $\gamma_j \in \mathcal{K}$ is a design parameter determined in order to be able to dominate the sign indefinite function $p_j(x, u_0, \ldots, u_{j-1})$. For details of this and a comparative discussion of other designs see [23, Section 4.1].

Remark 3.7 *An important point is that whenever $L_{g_1}V(x) \neq 0$, i.e., when $x \in \widetilde{K}$ in Theorem 3.2, then in principle we can dominate the terms $p_j(x, u_0, \ldots, u_{j-1})$ by increasing the gain of u_j. However, due to saturation in actuators which is always present in practice, an arbitrary increase in gain is not feasible.*

4 Optimisation based redesign

All approaches developed so far yield asymptotic estimates, i.e., estimates which hold up to some remainder term of order $O(T^p)$ which will become negligible for sufficiently small T. On the one hand this approach yields considerable theoretical insight into the problem and always ensures the existence of an interval $(0, T^*]$ such that the resulting redesigned controllers u_T perform better than the original non-redesigned controller $u_0(x)$ for $T \in (0, T^*]$. On the other hand, however, the upper bound T^* of this interval is no design parameter in this approach. This may pose a problem, for instance if bandwidth constraints of a network impose a lower bound $T_{\min}$ on the realisable sampling time. Then, in order to obtain a performance improvement we need to ensure $T^* \geq T_{\min}$, which may only be achievable by tuning the design parameters using trial and error.

This explains why in this section we present alternative approaches based on numerical optimisation techniques, which enable us to design u_T for prespecified sampling times T. Other benefits of these methods are that the inter sampling behaviour of the solutions can be explicitly included in the optimisation criterion and that we do not need to impose a control affine structure or scalar input. The price we have to pay for this enlarged flexibility is that we can no longer expect closed analytic expressions for u_T. Instead, the controllers are now given as solutions to — at times computationally involved — optimisation problems, which need efficient numerical solvers in order to allow for an online implementation.

4.1 One-step optimisation

The simplest possible optimisation scheme, investigated in [33], is to minimise the distance between ϕ and ϕ_T individually on each sampling interval $(kT, (k+1)T]$ for $k = 0, 1, 2, \ldots$. In order to encode this in an objective function taking the inter sampling behaviour into account we pick $m \geq 1$ and define for the k-th sampling interval

$$F(x_k, u) := \frac{1}{2}\sum_{i=1}^{m} |\phi_T(iT/m, x_k, u) - \phi(k + iT/m, x_0)|^2$$

where $x_k = \phi_T(kT, x_0, u_T)$. Then, recursively for $k = 0, 1, \ldots$ we let $u_T(x_k)$ be the minimiser of this expression, i.e.,

$$u_T(x_k) = \operatorname*{argmin}_{u \in U} F(x_k, u).$$

This is a standard nonlinear least squares problem which can be solved efficiently with the Levenberg-Marquardt or the Gauss-Newton algorithm, cf. [33, section 7.2]. A decisive factor in the selection of these optimisation algorithms is that both are iterative ones. It turns out that for a satisfactory solution it is often sufficient to use very few iterations, a fact which significantly reduces the computational effort. This simple approach often leads to noticeable improvements over the analytic redesign, see [33, section 7.3 and 7.4] and Section 5, in particular it enlarges the range of sampling periods for which the sampled-data system remains stable. A further improvement is obtained if we initialise our iterative optimisation with the analytically redesigned controller instead of u_0 which helps to keep the number of iterations small.

Closeness of the trajectories $\phi(t, x_0)$ and $\phi_T(t, x_0, u_T)$ for t from compact time intervals (and thus practical asymptotic stability of the sampled-data closed-loop provided ϕ is asymptotically stable [29]) follows inductively from the closeness on the individual intervals.

4.2 Model Predictive Control

The one-step optimisation is easy to implement and to analyse, however, the individual optimisation on each sampling interval is clearly insufficient in order to minimise the deviation over the whole time interval $[0, \infty)$. For this purpose, the model predictive control (MPC) approaches developed in [24, 10, 11] turn out to be more powerful.

In order to measure and minimise the infinite horizon difference between both trajectories in a suitable averaged sense, for piecewise constant control functions $v : \mathbb{R}_0^+ \to \mathbb{U}$ with $v_{[jT,(j+1)T)} \equiv v_j$ we denote the solution of

$$\dot{x}(t) = f(x(t), v(t)), \quad x(0) = \xi_0 \tag{24}$$

by $\phi_T(t, \xi_0, v)$. Then we define the cost functional

$$J(\xi_0, x_0, v) := \sum_{j=0}^{\infty} \int_{jT}^{(j+1)T} l(\phi_T(t, \xi_0, v) - \phi(t, x_0), v_j)dt,$$

where $l : \mathbb{R}^n \times \mathbb{U} \to \mathbb{R}_{\geq 0}$. For instance, for $l(x, u) = |x|^2$ the optimal control minimises the L_2 distance between the two trajectories, but this approach is very flexible and more general choices of l are possible including, e.g., the penalisation of the control effort.

The minimisation of J poses an optimal control problem with infinite horizon which involves solving a Hamilton-Jacobi-Bellman type equation in order to obtain the optimal feedback control which in our nonlinear setting is typically too hard to be solved directly.

In order to circumvent this computational burden we apply a receding horizon technique in which we reduce the horizon to a finite length $M \cdot T$ for some $M \in \mathbb{N}$. This will give us a suboptimal MPC controller whose numerical computation is manageable and results in the problem of minimising

$$\begin{aligned} J_M(\xi_0, x_0, v) := \sum_{j=0}^{M-1} \int_{jT}^{(j+1)T} l(\phi_T(t, \xi_0, v) - \phi(t, x_0), v_j)dt \\ + \; F(\phi_T(MT, \xi_0, v), \phi(MT, x_0)), \end{aligned} \tag{25}$$

where F is a terminal cost estimating the cost-to-go.

In order to obtain the sampled-data feedback law u_T from the minimisation of (25) we proceed in the typical MPC fashion: Solving the minimisation problem yields an optimal control function

$$\hat{u} = \operatorname*{argmin}_{v} J_M(\xi_0, x_0, v) \tag{26}$$

which is defined on $[0, MT]$ and piecewise constant, i.e., $\hat{u}_{[jT,(j+1)T)} \equiv \hat{u}_j \in \mathbb{U}$ for $j \in \{0, \ldots, M-1\}$. We define the feedback value in (ξ_0, x_0) to be the first element of the sequence $\hat{u}_j$, i.e.,

$$u_T(\xi_0, x_0) := \hat{u}_0. \tag{27}$$

This procedure is repeated iteratively by shifting the horizon forward in time by T. According to this procedure $u_T = u_M$ is a static state sampled-data feedback for the coupled system (24, 2).

In contrast to the one-step optimisation a simple iterative proof for the closeness of $\phi_T(t, x, u_T)$ to $\phi(t, x)$ does not work: due to the minimisation over larger horizons it may happen that the difference $\phi_T(T, x, u_T) - \phi(T, x)$ after one sampling interval is large in which case an induction to the next sampling interval fails. Thus, we need to use genuine MPC techniques [7] in order to analyse the closed-loop behaviour. The following theorem gives the respective stability statement, for its proof see [24, Theorem 3.1].

Theorem 4.1 *Suppose the following conditions hold:*

(i) l and F are continuous;

(ii) $\mathbb{U}$ is bounded;

(iii-a) The continuous-time system (2) *is globally asymptotically stable;*

(iii-b) There exists a constant $r_0 > 0$ and a function $\gamma \in \mathcal{K}_\infty$ such that $\forall\, |y| \geq r_0$

$$l(y,u) \geq \max\left\{ \max_{|x|\leq 2|y|} |f(x,u)|, \gamma(|y|) \right\};$$

(iii-c) $f(\cdot,\cdot)$ and $u_0(\cdot)$ in (2) *are locally Lipschitz in their arguments;*

(iv) The value function is such that for some $\bar{\alpha} \in \mathcal{K}_\infty$ we have that $V_M(\xi,x) \leq \bar{\alpha}(|(\xi,x)|)$ for all $M \geq 0$ and all $(\xi,x) \in \mathbb{R}^{2n}$.

Then there exists a function $\beta \in \mathcal{KL}$ such that for each pair of strictly positive real numbers (Δ,δ) there exists a constant $M^* \in \mathbb{Z}_{\geq 1}$ such that for all $(\xi,x) \in B_\Delta$, $M \geq M^*$ and all $t \geq 0$ the solutions of the continuous-time system (2), (24) *satisfy*

$$|(\phi_T(t,\xi_0,u_T) - \phi(t,x_0))| \leq \max\{\beta(|(\xi_0,x_0)|,t),\delta\}. \tag{28}$$

It should be noted that no assumption on the cost-to-go F is made here, implying that asymptotic stability does indeed hold for arbitrary terminal costs provided $M \geq M^*$. However, the choice of F affects the constant M^*. In particular, under suitable Lyapunov function like conditions M^* is typically smaller than in the general case, cf. [7, 24]. Furthermore, note that our stability result does not require $\xi_0 = x_0$, although this is of course a natural choice.

Beyond stability, the main motivation of the MPC setup is to ensure closeness of the trajectories in the averaged sense of minimising the functional J. While within the MPC approach one cannot expect u_T to be optimal for J, under a Lyapunov function like condition on F it is possible to show that u_T is inversely optimal, i.e., optimal for a suitably modified infinite horizon functional $\overline{J}$, see [24, Theorem 4.3]. Alternatively, one may use appropriate controllability assumptions in order to prove suboptimality of u_T with respect to the original cost functional J even for $F \equiv 0$, see [13].

Compared to a direct MPC stabilisation of the sampled-data system, our redesign approach using the continuous-time trajectory as reference has certain performance advantages. For instance, our discrete time controller inherits the transient behaviour of the continuous-time system, while in standard MPC schemes the transient behaviour can only be influenced indirectly through the choice of the cost functional. Although the design of a suitable continuous-time controller requires a considerable amount of a priori work, this may be rewarded by a better performance of the resulting sampled-data closed-loop. Furthermore, several examples show that the numerical optimisation becomes easier when a good reference trajectory is available. We have used a direct discretisation approach followed by the solution of the resulting finite dimensional problem using an SQP method, see [10, 11] for details. With this approach it turns out that for optimising the distance to the continuous-time reference one or two SQP iterations already yield a satisfactory performance.

Finally, concerning the sampling time, we would like to emphasise that whenever the problem is feasible (i.e., whenever the sampled-data system can be held close to the reference for a given lower bound $T_{\min}$ on the sampling time), then the MPC approach will yield a good solution. We consider this as the main advantage of the MPC approach over the other approaches presented in this paper.

5 Examples

In this section we demonstrate how the methods presented in this paper complement each other. For this purpose we describe three different situations which can be categorised by the sampling period T, or equivalently, by the bandwidth of the communications channel. If a large bandwidth is available, then small sampling periods T can be used and the analytical redesign may be the method of choice. Note that in this case the evaluation of the redesigned feedback law is fast since the feedback is given by an analytical formula which will be calculated in advance.

In order to illustrate the improvement of the analytical redesign over the straightforward implementation of the continuous time feedback, we first present a simple academic example

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} x_2^2 \\ 2x_1^2 \end{pmatrix} u \tag{29}$$

with $u_0(x) = -x_1 - 1$. For this system one computes $[g_0, g_1] = g_1(x) \neq 0$, which implies that (14) holds on every compact set K with $\alpha(x) = x_1^3 + x_1^2 - x_1$. Therefore we can calculate the feedback laws from Theorem 3.2 (i), (ii) and (iii), i.e., for order $p = 2$, 3 and 4. The resulting x_1-component of the trajectories for $x_0 = (-1,1)^T$ for the sampling rate $T = 0.1$ can be seen in Figure 1.

These numerical results confirm the theoretically expected approximation order, see [12].

The main drawback of this method is the fact that for practical problems the existence of a feedback of order $p > 3$ can not be expected in general. To show this our next example is a second order version of the Moore-Greitzer jet engine model

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -x_2 - 3x_1^2/2 - x_1^3/2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \tag{30}$$

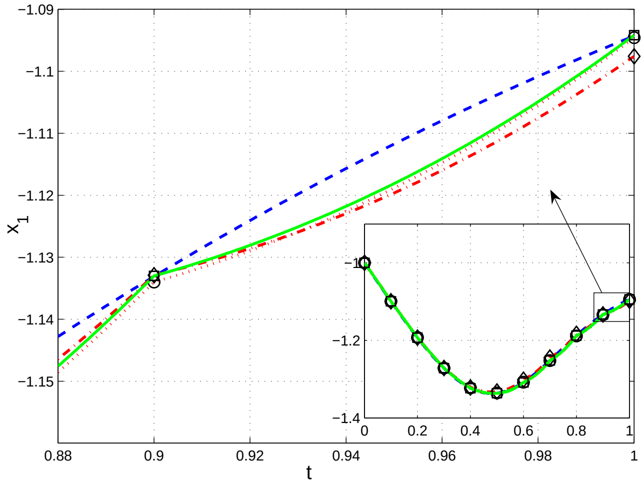

Bild 1: $x_1(t)$ for example (29). Continuous-time solution (- -); sampled-data solutions for $u_T = u_0$ ($\diamond$) and u_T from Theorem 3.2 (ii) ($\circ$) and (iii) ($\square$), $T = 0.1$

with the continuous-time stabilising backstepping feedback law $u_0(x) = -7x_1 + 5x_2$ derived in [18, Section 2.4.3]. Here condition (14) shows that no admissible sampled-data feedback u_T satisfies $\Delta\phi(T, x, u_T) \leq O(T^4)$, cf., [12, Section 4]. Hence, although analytically redesigned feedback laws can work well, cf. [23], due to the fundamental order limitations of the asymptotic approximation, they typically do so only in a rather small region of sampling periods. Thus, if the bandwidth of the network does not support the use of the necessary large sampling rate, they may exhibit poor performance. In this case optimisation based techniques typically perform better, as shown in Figure 2 for $x_0 = (22, 21)^T$ and $T = 0.2$. Here the standard Matlab implementation of the Gauss-Newton method was used.

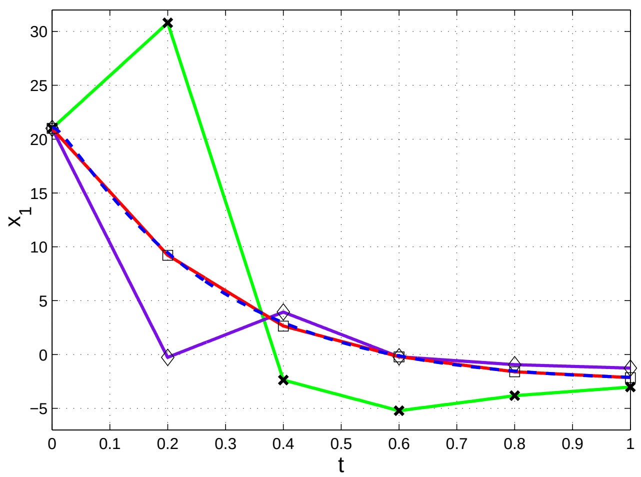

Bild 2: $x_1(t)$ for example (30). Continuous-time solution (- -); sampled-data solutions for $u_T = u_0$ ($\times$) and u_T from one-step optimisation with 1 ($\diamond$) and 2 ($\square$) Gauss-Newton iterations, $T = 0.2$

The third situation we examine in this section is the case of rather large T, i.e., when only very low bandwidth for the communication between controller and plant is available. In order to compare the one-step and the MPC optimisation we consider the arm/rotor/platform model

$$\begin{aligned}
\dot{x}_1 &= x_2 + x_6 x_3 \\
\dot{x}_2 &= -\frac{k_1}{M}x_1 - \frac{b_1}{M}x_2 + x_6 x_4 - \frac{mrb_1}{M^2}x_6 \\
\dot{x}_3 &= -x_6 x_1 + x_4 \\
\dot{x}_4 &= -x_6 x_2 - \frac{k_1}{M}x_3 - \frac{b_1}{M}x_4 + \frac{mrk_1}{M^2} \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= -a_1 x_5 - a_2 x_6 + a_1 x_7 + a_3 x_8 - p_1 x_1 - p_2 x_2 \\
\dot{x}_7 &= x_8 \\
\dot{x}_8 &= a_4 x_5 + a_5 x_6 - a_4 x_7 - (a_5 + a_6)x_8 + \frac{1}{J}u
\end{aligned} \tag{31}$$

and a continuous-time full-state feedback u_0 which was designed for this system via backstepping such that the output

$$\zeta(t) := x_5 - \frac{a_3}{a_1 - a_2 a_3}[x_6 - a_3 x_7]$$

is close to x_5 and tracks a given reference signal ζ_{ref}, see [6, Chapter 7.3.2] for details on the backstepping design and the specification of the parameters. To generate the results given in Figure 3 for the reference function $\zeta_{\text{ref}}(t) := \sin(t)$ we used the SQP based optimisation software NUDOCCCS for both approaches and performed 3 SQP iterations for each optimisation.

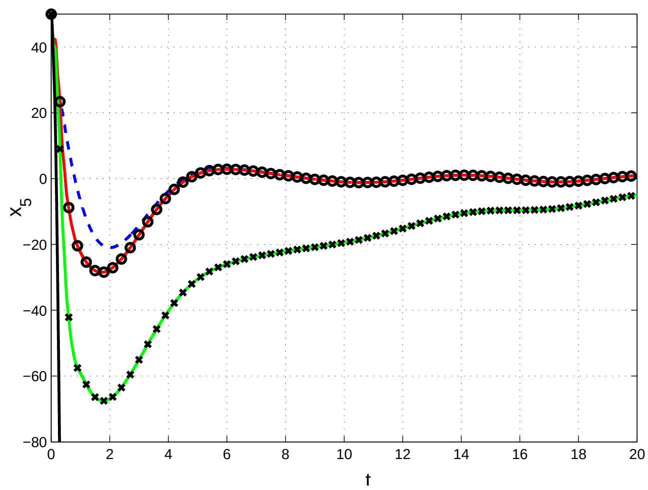

Bild 3: $x_5(t)$ for example (31). Continuous-time solution (- -); sampled-data solutions for $u_T = u_0$ (solid line without markers), u_T from one-step optimisation ($\times$) and MPC ($\circ$), $T = 0.3$ using $M = 4$

Here one observes that the sampled-data implementation of the continuous-time controller (solid line without markers) leads to immediate instability while both one-step and MPC optimisation stabilise the system with the MPC approach showing a better performance due to its ability to take a longer horizon into account. Moreover, due to the fact that a prediction of the system's future states is inherently available in the MPC scheme, this provides a natural way to reconstruct missing information in case of data loss during broadcast. The design and stability analysis of algorithms which use this fact in order to create mechanisms coping with this situation are currently under investigation. Still, we want to emphasize that — in particular if long prediction horizons are used — MPC is computationally more demanding than the other methods, which means that the price we have to pay for the ability to cope with small bandwidth and package dropouts is a higher computational effort in the controller.

6 Conclusion and outlook

In this paper we have presented analytical and numerical sampled-data redesign techniques for nonlinear systems. In particular, we have discussed and illustrated the main advantages and disadvantages of these approaches: while the analytical techniques provide theoretical insight and explicit formulas which are easy to implement and fast to evaluate they have a somewhat limited applicability and performance. On the other hand, the numerical techniques typically show very good performance and allow to explicitly include the sampling time as a design parameter but require fast online optimisation algorithms in order to be implementable in practice.

Current research is focused on two areas: On the theoretical side we want to generalise our results to cover more general NCS instead of sampled-data systems, which are just a particular special case. On the numerical side, we aim at increasing the efficiency of the algorithms, e.g., by trying to combine the conceptual and numerical simplicity of the one-step approach and the generality and flexibility of the MPC approach.

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