

Primitive central idempotents of finite group rings of symmetric and alternating groups in characteristic 2

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This article contains the primitive central idempotents of the group rings \mathbb{F}_2S_n and \mathbb{F}_4A_n for $n \leq 54$. The computational results were achieved with GAP [1] and programs written in SYMMETRICA [2] by Axel Kohnert. The theoretical results which made this computation possible can be found in [3] and [4]. The primitive central idempotents of \mathbb{F}_2S_n for $n \leq 50$ are already displayed in [3].

1 The primitive central idempotents of group rings of symmetric groups in characteristic 2

We use the same notation as in [3]. The conjugacy classes of S_n can be indexed by the partitions of n . We write $\mu = 1^{\alpha_1}, \dots, n^{\alpha_n}$ for the partition

$$\mu = (\underbrace{1, \dots, 1}_{\alpha_1}, \underbrace{2, \dots, 2}_{\alpha_2}, \dots)$$

of n . We define

$$W(\mu) := \sum_{i=2}^n i \cdot \alpha_i$$

and call it the *essential weight* of the partition μ . For our purpose it is convenient to ignore the parts equal to 1 in the partition because an element like $(1, 2, 3) \in S_3$ is also an element of bigger symmetric groups. So we write $\mu = 2^{\alpha_2}, \dots, n^{\alpha_n}$ for a partition and the corresponding class C_μ is a class of an arbitrary symmetric group S_n with $n \geq W(\mu)$ depending on the context, i.e. C_2 denotes the conjugacy class of transpositions in every symmetric group S_n , $n \geq 2$. If $\mu = 2^{\alpha_2}, \dots, n^{\alpha_n}$ is a partition we write $\overline{2^{\alpha_2}, \dots, n^{\alpha_n}}$ for the class sum $C_\mu^+ \in \mathbb{F}_2S_m$, where $m \geq W(\mu)$.

According to Theorem 1 of [3] one can easily deduce the primitive central idempotents of \mathbb{F}_2S_n for $n < 54$ from the primitive central idempotents of \mathbb{F}_2S_{54} and \mathbb{F}_2S_{53} . To simplify that task we added tokens of the form $|_{16}$ to indicate where the primitive central idempotent of \mathbb{F}_2S_{16} ends.

$$\begin{aligned}
e_4 = & \overline{3, 7, 11}|_{21} + \overline{5, 7, 11} + \overline{3, 9, 11} + \overline{3, 7, 13}|_{23} + \overline{5, 9, 11} + \overline{5, 7, 13} + \overline{3, 9, 13}|_{25} + \overline{3, 5, 7, 11} + \overline{7, 9, 11} + \overline{5, 9, 13} + \overline{3, 11, 13} + \\
& \overline{3, 7, 17}|_{27} + \overline{3, 5, 9, 11} + \overline{3, 5, 7, 13} + \overline{7, 9, 13} + \overline{5, 11, 13} + \overline{5, 7, 17} + \overline{3, 9, 17}|_{29} + \overline{3, 5, 9, 13} + \overline{5, 9, 17}|_{31} + \overline{5, 7, 9, 11} + \\
& \overline{3, 5, 11, 13} + \overline{3, 5, 7, 17} + \overline{9, 11, 13} + \overline{7, 9, 17} + \overline{3, 13, 17}|_{33} + \overline{5, 7, 9, 13} + \overline{3, 7, 11, 13} + \overline{3, 5, 9, 17} + \overline{3, 5, 7, 9, 11} + \overline{7, 11, 17} + \\
& \overline{5, 13, 17} + \overline{3, 11, 21} + \overline{3, 7, 25}|_{35} + \overline{5, 7, 11, 13} + \overline{3, 9, 11, 13} + \overline{3, 5, 7, 9, 13} + \overline{9, 11, 17} + \overline{7, 13, 17} + \overline{3, 15, 19} + \overline{5, 11, 21} + \\
& \overline{3, 13, 21} + \overline{3, 11, 23} + \overline{5, 7, 25} + \overline{3, 9, 25} + \overline{3, 7, 27}|_{37} + \overline{5, 7, 9, 17} + \overline{3, 5, 13, 17} + \overline{5, 15, 19} + \overline{3, 17, 19} + \overline{5, 13, 21} + \overline{3, 15, 21} + \\
& \overline{5, 11, 23} + \overline{3, 13, 23} + \overline{5, 9, 25} + \overline{3, 11, 25} + \overline{5, 7, 27} + \overline{3, 9, 27} + \overline{3, 7, 29}|_{39} + \overline{7, 9, 11, 13} + \overline{5, 7, 11, 17} + \overline{3, 7, 13, 17} + \\
& \overline{3, 5, 11, 21} + \overline{3, 5, 7, 25} + \overline{3, 5, 9, 11, 13} + \overline{3, 5, 7, 9, 17} + \overline{11, 13, 17} + \overline{5, 17, 19} + \overline{9, 11, 21} + \overline{5, 15, 21} + \overline{5, 13, 23} + \overline{7, 9, 25} + \\
& \overline{5, 11, 25} + \overline{5, 9, 27} + \overline{5, 7, 29} + \overline{3, 9, 29}|_{41} + \overline{5, 9, 11, 17} + \overline{3, 9, 13, 17} + \overline{3, 5, 15, 19} + \overline{3, 5, 13, 21} + \overline{3, 5, 11, 23} + \overline{3, 5, 9, 25} + \\
& \overline{3, 5, 7, 27} + \overline{9, 15, 19} + \overline{9, 13, 21} + \overline{3, 19, 21} + \overline{9, 11, 23} + \overline{3, 17, 23} + \overline{3, 15, 25} + \overline{7, 9, 27} + \overline{3, 13, 27} + \overline{5, 9, 29} + \overline{3, 11, 29} + \\
& \overline{3, 7, 33}|_{43} + \overline{5, 9, 13, 17} + \overline{3, 5, 17, 19} + \overline{3, 5, 15, 21} + \overline{3, 5, 13, 23} + \overline{3, 5, 11, 25} + \overline{3, 5, 9, 27} + \overline{3, 5, 7, 29} + \overline{9, 17, 19} + \overline{9, 15, 21} + \\
& \overline{5, 19, 21} + \overline{9, 13, 23} + \overline{5, 17, 23} + \overline{9, 11, 25} + \overline{5, 15, 25} + \overline{5, 13, 27} + \overline{7, 9, 29} + \overline{5, 11, 29} + \overline{5, 7, 33} + \overline{3, 9, 33}|_{45} + \overline{7, 9, 13, 17} + \\
& \overline{5, 11, 13, 17} + \overline{5, 9, 11, 21} + \overline{5, 7, 9, 25} + \overline{3, 5, 9, 29} + \overline{3, 5, 9, 13, 17} + \overline{5, 9, 33}|_{47} + \overline{3, 5, 7, 9, 11, 13} + \overline{7, 11, 13, 17} + \overline{5, 9, 15, 19} + \\
& \overline{5, 9, 13, 21} + \overline{3, 11, 13, 21} + \overline{3, 5, 19, 21} + \overline{5, 9, 11, 23} + \overline{3, 5, 17, 23} + \overline{3, 7, 13, 25} + \overline{3, 5, 15, 25} + \overline{5, 7, 9, 27} + \overline{3, 5, 13, 27} + \\
& \overline{3, 5, 11, 29} + \overline{3, 5, 7, 33} + \overline{5, 7, 9, 11, 17} + \overline{3, 5, 9, 11, 21} + \overline{11, 17, 21} + \overline{9, 19, 21} + \overline{9, 17, 23} + \overline{3, 5, 7, 9, 25} + \overline{9, 15, 25} + \overline{7, 17, 25} + \\
& \overline{9, 13, 27} + \overline{9, 11, 29} + \overline{3, 17, 29} + \overline{7, 9, 33} + \overline{3, 13, 33}|_{49} + \overline{9, 11, 13, 17} + \overline{3, 13, 15, 19} + \overline{5, 9, 17, 19} + \overline{5, 11, 13, 21} + \overline{5, 9, 15, 21} + \\
& \overline{3, 5, 11, 23} + \overline{5, 9, 11, 13, 23} + \overline{5, 9, 11, 25} + \overline{5, 7, 13, 25} + \overline{3, 9, 13, 25} + \overline{3, 9, 13, 25} + \overline{3, 7, 13, 27} + \overline{5, 7, 9, 29} + \overline{3, 7, 11, 29} + \overline{3, 5, 9, 33} + \\
& \overline{5, 7, 9, 13, 17} + \overline{3, 7, 11, 13, 17} + \overline{3, 5, 9, 15, 19} + \overline{15, 17, 19} + \overline{3, 7, 9, 11, 21} + \overline{3, 5, 9, 13, 21} + \overline{13, 17, 21} + \overline{3, 5, 9, 11, 23} + \\
& \overline{11, 17, 23} + \overline{3, 5, 7, 11, 25} + \overline{9, 17, 25} + \overline{3, 23, 25} + \overline{3, 5, 7, 9, 27} + \overline{7, 17, 27} + \overline{3, 21, 27} + \overline{5, 17, 29} + \overline{3, 19, 29} + \overline{7, 11, 33} + \\
& \overline{5, 13, 33} + \overline{3, 15, 33}|_{51} + \overline{7, 11, 15, 19} + \overline{5, 13, 15, 19} + \overline{3, 13, 17, 19} + \overline{3, 13, 15, 21} + \overline{5, 11, 13, 23} + \overline{5, 9, 13, 25} + \overline{3, 11, 13, 25} + \\
& \overline{5, 7, 13, 27} + \overline{3, 9, 13, 27} + \overline{5, 7, 11, 29} + \overline{3, 9, 11, 29} + \overline{5, 7, 11, 13, 17} + \overline{3, 9, 11, 13, 17} + \overline{3, 7, 11, 15, 17} + \overline{3, 7, 11, 13, 19} + \\
& \overline{3, 7, 9, 15, 19} + \overline{3, 5, 11, 15, 19} + \overline{3, 5, 9, 17, 19} + \overline{5, 7, 9, 11, 21} + \overline{3, 7, 9, 13, 21} + \overline{3, 5, 9, 15, 21} + \overline{15, 17, 21} + \overline{5, 9, 13, 23} + \\
& \overline{13, 17, 23} + \overline{3, 5, 7, 13, 25} + \overline{11, 17, 25} + \overline{5, 23, 25} + \overline{9, 17, 27} + \overline{5, 21, 27} + \overline{3, 5, 7, 9, 29} + \overline{7, 17, 29} + \overline{5, 19, 29} + \overline{9, 11, 33} + \\
& \overline{7, 13, 33} + \overline{5, 15, 33} + \overline{3, 17, 33} \\
e_5 = & \overline{5, 9, 13, 17}|_{45} + \overline{7, 9, 13, 17} + \overline{5, 11, 13, 17} + \overline{5, 9, 15, 17} + \overline{5, 9, 13, 19} + \overline{3, 5, 9, 13, 17}|_{47} + \overline{7, 11, 13, 17} + \overline{7, 9, 15, 17} + \\
& \overline{5, 11, 15, 17} + \overline{7, 9, 13, 19} + \overline{5, 11, 13, 19} + \overline{5, 9, 15, 19} + \overline{5, 9, 13, 21} + \overline{3, 7, 9, 13, 17} + \overline{3, 5, 11, 13, 17} + \overline{3, 5, 9, 15, 17} + \\
& \overline{3, 5, 9, 13, 19}|_{49} + \overline{9, 11, 13, 17} + \overline{7, 11, 15, 17} + \overline{5, 13, 15, 17} + \overline{7, 11, 13, 19} + \overline{7, 9, 15, 19} + \overline{5, 11, 15, 19} + \overline{5, 9, 17, 19} + \\
& \overline{7, 9, 13, 21} + \overline{5, 11, 13, 21} + \overline{5, 9, 15, 21} + \overline{5, 7, 9, 13, 17} + \overline{3, 7, 11, 13, 17} + \overline{3, 7, 9, 15, 17} + \overline{3, 5, 11, 15, 17} + \overline{3, 7, 9, 13, 19} + \\
& \overline{3, 5, 11, 13, 19} + \overline{3, 5, 9, 15, 19} + \overline{3, 5, 9, 13, 21}|_{51} + \overline{9, 11, 15, 17} + \overline{7, 13, 15, 17} + \overline{9, 11, 13, 19} + \overline{7, 11, 15, 19} + \overline{5, 13, 15, 19} + \\
& \overline{7, 9, 17, 19} + \overline{5, 11, 17, 19} + \overline{7, 11, 13, 21} + \overline{7, 9, 15, 21} + \overline{5, 11, 15, 21} + \overline{5, 9, 13, 25} + \overline{5, 7, 11, 13, 17} + \overline{3, 9, 11, 13, 17} + \\
& \overline{5, 7, 9, 15, 17} + \overline{3, 7, 11, 15, 17} + \overline{3, 5, 13, 15, 17} + \overline{5, 7, 9, 13, 19} + \overline{3, 7, 11, 13, 19} + \overline{3, 7, 9, 15, 19} + \overline{3, 5, 11, 15, 19} + \\
& \overline{3, 5, 9, 17, 19} + \overline{3, 7, 9, 13, 21} + \overline{3, 5, 11, 13, 21} + \overline{3, 5, 9, 15, 21}
\end{aligned}$$

Primitive central idempotents of $\mathbb{F}_2 S_n$ for n even and $n \leq 54$:

$$\begin{aligned}
e_1 = & \overline{1}|_2 + \overline{5}|_6 + \overline{7} + \overline{3, 5}|_8 + \overline{9}|_{10} + \overline{15} + \overline{7, 9} + \overline{5, 11} + \overline{3, 13}|_{16} + \overline{17}|_{18} + \overline{5, 9, 13}|_{28} + \overline{7, 9, 13} + \overline{5, 11, 13} + \overline{5, 9, 15} + \\
& \overline{3, 5, 9, 13}|_{30} + \overline{7, 11, 13} + \overline{7, 9, 15} + \overline{5, 11, 15} + \overline{5, 9, 17} + \overline{31} + \overline{3, 7, 9, 13} + \overline{3, 5, 11, 13} + \overline{3, 5, 9, 15} + \overline{15, 17} + \overline{13, 19} + \\
& \overline{11, 21} + \overline{9, 23} + \overline{7, 25} + \overline{5, 27} + \overline{3, 29}|_{32} + \overline{9, 11, 13} + \overline{7, 11, 15} + \overline{5, 13, 15} + \overline{7, 9, 17} + \overline{5, 11, 17} + \overline{33} + \overline{5, 7, 9, 13} + \overline{3, 7, 11, 13} + \\
& \overline{3, 7, 9, 15} + \overline{3, 5, 11, 15} + \overline{3, 5, 9, 17}|_{34} + \overline{9, 11, 15} + \overline{7, 13, 15} + \overline{7, 11, 17} + \overline{5, 13, 17} + \overline{5, 7, 11, 13} + \overline{3, 9, 11, 13} + \overline{5, 7, 9, 15} + \\
& \overline{3, 5, 13, 15} + \overline{3, 7, 9, 17} + \overline{3, 5, 11, 17}|_{36} + \overline{3, 5, 7, 9, 13} + \overline{9, 11, 17} + \overline{7, 13, 17} + \overline{5, 13, 19} + \overline{5, 9, 23} + \overline{5, 7, 9, 17} + \overline{3, 5, 13, 17}|_{38} + \\
& \overline{3, 5, 7, 11, 13} + \overline{3, 5, 7, 9, 15} + \overline{11, 13, 15} + \overline{7, 13, 19} + \overline{5, 15, 19} + \overline{5, 13, 21} + \overline{7, 9, 23} + \overline{5, 11, 23} + \overline{5, 9, 25} + \overline{7, 9, 11, 13} + \\
& \overline{5, 9, 11, 15} + \overline{3, 9, 13, 15} + \overline{3, 5, 13, 19} + \overline{3, 5, 9, 23}|_{40} + \overline{3, 5, 9, 11, 13} + \overline{3, 5, 7, 9, 17} + \overline{11, 13, 17} + \overline{9, 13, 19} + \overline{7, 15, 19} + \\
& \overline{5, 17, 19} + \overline{7, 13, 21} + \overline{5, 15, 21} + \overline{7, 11, 23} + \overline{5, 13, 23} + \overline{7, 9, 25} + \overline{5, 11, 25} + \overline{5, 9, 11, 17} + \overline{3, 9, 13, 17} + \overline{3, 7, 15, 17} + \\
& \overline{3, 7, 13, 19} + \overline{3, 5, 15, 19} + \overline{3, 7, 11, 21} + \overline{3, 5, 13, 21} + \overline{3, 7, 9, 23} + \overline{3, 5, 11, 23} + \overline{3, 5, 9, 25}|_{42} + \overline{9, 15, 19} + \overline{7, 17, 19} + \overline{9, 13, 21} + \\
& \overline{7, 15, 21} + \overline{9, 11, 23} + \overline{7, 13, 23} + \overline{7, 11, 25} + \overline{5, 9, 29} + \overline{5, 7, 15, 17} + \overline{3, 9, 15, 17} + \overline{5, 7, 13, 19} + \overline{3, 9, 13, 19} + \overline{3, 5, 17, 19} + \\
& \overline{5, 7, 11, 21} + \overline{3, 9, 11, 21} + \overline{3, 5, 15, 21} + \overline{5, 7, 9, 23} + \overline{3, 5, 13, 23} + \overline{3, 7, 9, 25} + \overline{3, 5, 11, 25}|_{44} + \overline{9, 17, 19} + \overline{9, 15, 21} + \\
& \overline{5, 19, 21} + \overline{9, 13, 23} + \overline{5, 17, 23} + \overline{9, 11, 25} + \overline{5, 15, 25} + \overline{5, 13, 27} + \overline{7, 9, 29} + \overline{5, 11, 29} + \overline{5, 9, 31} + \overline{5, 9, 15, 17} + \overline{5, 9, 13, 19} + \\
& \overline{5, 9, 11, 21} + \overline{5, 7, 9, 25} + \overline{3, 5, 9, 29}|_{46} + \overline{5, 7, 9, 11, 15} + \overline{3, 7, 9, 13, 15} + \overline{3, 5, 11, 13, 15} + \overline{3, 5, 7, 15, 17} + \overline{3, 5, 7, 13, 19} + \\
& \overline{13, 15, 19} + \overline{3, 5, 7, 11, 21} + \overline{7, 19, 21} + \overline{3, 5, 7, 9, 23} + \overline{9, 15, 23} + \overline{7, 17, 23} + \overline{7, 15, 25} + \overline{7, 13, 27} + \overline{5, 15, 27} + \overline{7, 11, 29} + \\
& \overline{7, 9, 31} + \overline{5, 11, 31} + \overline{5, 9, 33} + \overline{3, 5, 7, 9, 11, 13} + \overline{7, 9, 15, 17} + \overline{3, 13, 15, 17} + \overline{5, 11, 13, 19} + \overline{7, 9, 11, 21} + \overline{3, 11, 13, 21} + \\
& \overline{3, 5, 19, 21} + \overline{5, 9, 11, 23} + \overline{3, 5, 17, 23} + \overline{3, 7, 13, 25} + \overline{3, 5, 15, 25} + \overline{3, 5, 13, 27} + \overline{3, 7, 9, 29} + \overline{3, 5, 11, 29} + \overline{3, 5, 9, 31}|_{48} + \\
& \overline{5, 7, 9, 11, 17} + \overline{3, 7, 9, 13, 17} + \overline{3, 5, 11, 13, 17} + \overline{3, 5, 9, 15, 17} + \overline{3, 5, 9, 13, 19} + \overline{13, 17, 19} + \overline{3, 5, 9, 11, 21} + \overline{13, 15, 21} + \\
& \overline{9, 19, 21} + \overline{11, 15, 23} + \overline{3, 5, 7, 9, 25} + \overline{9, 13, 27} + \overline{7, 15, 27} + \overline{5, 17, 27} + \overline{9, 11, 29} + \overline{5, 15, 29} + \overline{7, 11, 31} + \overline{5, 13, 31} + \overline{7, 9, 33} + \\
& \overline{5, 11, 33} + \overline{7, 11, 15, 17} + \overline{5, 13, 15, 17} + \overline{7, 11, 13, 19} + \overline{7, 9, 15, 19} + \overline{5, 11, 15, 19} + \overline{7, 9, 13, 21} + \overline{3, 11, 15, 21} + \overline{5, 9, 13, 23} + \\
& \overline{3, 11, 13, 23} + \overline{3, 9, 15, 23} + \overline{5, 9, 11, 25} + \overline{5, 7, 13, 25} + \overline{3, 9, 13, 25} + \overline{3, 7, 15, 25} + \overline{3, 7, 13, 27} + \overline{3, 5, 15, 27} + \overline{5, 7, 9, 29} + \\
& \overline{3, 9, 17, 23} + \overline{3, 11, 13, 25} + \overline{5, 7, 15, 25} + \overline{3, 7, 17, 25} + \overline{5, 7, 13, 27} + \overline{3, 9, 13, 27} + \overline{3, 5, 17, 27} + \overline{5, 7, 11, 29} + \overline{3, 9, 11, 29} + \\
& \overline{3, 5, 15, 29} + \overline{5, 7, 9, 31} + \overline{3, 5, 13, 31} + \overline{3, 7, 9, 33} + \overline{3, 5, 11, 33}|_{52} + \overline{5, 7, 9, 15, 17} + \overline{3, 5, 13, 15, 17} + \overline{5, 7, 9, 13, 19} + \\
& \overline{5, 7, 9, 11, 21} + \overline{3, 5, 11, 13, 21} + \overline{15, 17, 21} + \overline{3, 5, 9, 13, 23} + \overline{13, 17, 23} + \overline{3, 5, 7, 13, 25} + \overline{11, 17, 25} + \overline{5, 23, 25} + \overline{9, 17, 27} + \\
& \overline{5, 21, 27} + \overline{3, 5, 7, 9, 29} + \overline{7, 17, 29} + \overline{3, 5, 19, 29} + \overline{5, 17, 31} + \overline{9, 11, 33} + \overline{7, 13, 33} + \overline{5, 13, 17, 19} + \overline{5, 11, 17, 21} + \overline{5, 9, 17, 23} + \\
& \overline{5, 7, 17, 25} + \overline{3, 5, 17, 29} + \overline{5, 7, 9, 33} + \overline{3, 5, 13, 33}
\end{aligned}$$

$$\begin{aligned}
e_2 = & \overline{5|_6+7+3,5|_8+9+3,7|_{10+5,7+3,9|_{12+5,9|_{14+3,5,7+15+5,11|_{16+3,5,9+17+7,11+5,13+3,15|_{18+9,11+7,13+} \\
& \overline{5,15+3,17|_{20+5,7,9+3,5,13+5,17|_{22+5,7,11+3,7,13+3,5,15+3,5,7,9+11,13+9,15|_{24+5,9,11+3,9,13+} \\
& \overline{3,5,17+9,17|_{26+5,9,13|_{28+7,9,13+5,11,13+5,9,15+3,5,9,13|_{30+7,11,13+3,13,15+5,9,17+31+5,7,9,11+} \\
& \overline{3,5,9,15+13,19+9,23+5,27|_{32+9,11,13+7,11,15+5,13,15+3,13,17+33+5,7,9,13+3,7,11,13+3,7,9,15+} \\
& \overline{3,5,11,15+3,5,9,17+15,19+13,21+11,23+9,25+7,27+5,29+3,31|_{34+9,11,15+7,13,15+7,11,17+5,13,17+} \\
& \overline{5,7,11,13+3,9,11,13+5,7,9,15+3,5,13,15+3,7,9,17+3,5,11,17+17,19+15,21+13,23+11,25+9,27+7,29+} \\
& \overline{5,31+3,33|_{36+3,5,7,9,13+9,11,17+7,13,17+5,15,17+5,11,21+5,7,25+3,5,29+5,7,9,17+3,5,13,17+} \\
& \overline{5,33|_{38+3,5,7,11,13+3,5,7,9,15+11,13,15+7,15,17+5,15,19+7,11,21+5,13,21+5,11,23+5,9,25+} \\
& \overline{5,7,27+3,7,29+3,5,31+7,9,11,13+5,9,11,15+3,9,13,15+3,5,15,17+3,5,11,21+19,21+17,23+3,5,7,25+} \\
& \overline{15,25+13,27+11,29+9,31|_{40+3,5,9,11,13+3,5,7,9,17+11,13,17+9,15,17+7,15,19+5,17,19+9,11,21+} \\
& \overline{7,13,21+5,15,21+7,11,23+5,13,23+5,11,25+5,9,27+3,9,29+3,5,33+5,9,11,17+3,9,13,17+3,7,15,17+} \\
& \overline{3,7,13,19+3,5,15,19+3,7,11,21+3,5,13,21+3,7,9,23+3,5,11,23+3,5,9,25+9,33|_{42+9,15,19+7,17,19+} \\
& \overline{9,13,21+7,15,21+9,11,23+7,13,23+7,11,25+5,9,29+5,7,15,17+3,9,15,17+5,7,13,19+3,9,13,19+} \\
& \overline{3,5,17,19+5,7,11,21+3,9,11,21+3,5,15,21+5,7,9,23+3,5,13,23+3,7,9,25+3,5,11,25|_{44+9,17,19+} \\
& \overline{9,15,21+5,19,21+9,13,23+5,17,23+9,11,25+5,15,25+5,13,27+7,9,29+5,11,29+5,9,31+5,9,15,17+} \\
& \overline{5,9,13,19+5,9,11,21+5,7,9,25+3,5,9,29|_{46+5,7,9,11,15+3,7,9,13,15+3,5,11,13,15+3,5,7,15,17+} \\
& \overline{3,5,7,13,19+3,5,7,11,21+11,15,21+7,19,21+3,5,7,9,23+7,17,23+7,13,27+7,11,29+3,15,29+3,13,31+} \\
& \overline{5,9,33+3,5,7,9,11,13+5,11,15,17+7,9,13,19+3,5,19,21+3,9,13,23+3,5,17,23+5,7,11,25+3,5,15,25+} \\
& \overline{23,25+5,7,9,27+21,27+19,29+3,5,9,31+17,31|_{48+5,7,9,11,17+3,7,9,13,17+3,5,11,13,17+3,5,9,15,17+} \\
& \overline{3,5,9,13,19+3,5,9,11,21+13,15,21+11,17,21+9,19,21+11,15,23+9,17,23+3,5,7,9,25+7,17,25+9,13,27+} \\
& \overline{7,15,27+9,11,29+5,15,29+3,17,29+7,11,31+5,13,31+3,13,33+7,11,15,17+5,13,15,17+7,11,13,19+} \\
& \overline{7,9,15,19+5,11,15,19+7,9,13,21+3,11,15,21+5,9,13,23+3,11,13,23+3,9,15,23+5,9,11,25+5,7,13,25+} \\
& \overline{3,9,13,25+3,7,15,25+3,7,13,27+3,5,15,27+5,7,9,29+3,7,11,29+3,7,9,31+3,5,11,31+3,5,9,33+17,33|_{50+} \\
& \overline{13,17,21+13,15,23+11,17,23+11,15,25+9,17,25+9,15,27+7,17,27+7,15,29+5,17,29+9,11,31+7,13,31+} \\
& \overline{7,11,33+5,13,33+9,11,15,17+7,13,15,17+9,11,13,19+5,13,15,19+7,9,17,19+5,11,17,19+7,11,13,21+} \\
& \overline{7,9,15,21+3,13,15,21+3,11,17,21+5,11,13,23+5,9,15,23+3,9,17,23+3,11,13,25+5,7,15,25+3,7,17,25+} \\
& \overline{5,7,13,27+3,9,13,27+3,5,17,27+5,7,11,29+3,9,11,29+3,5,15,29+5,7,9,31+3,5,13,31+3,7,9,33+} \\
& \overline{3,5,11,33|_{52+5,7,9,15,17+3,5,13,15,17+5,7,9,13,19+5,7,9,11,21+3,5,11,13,21+5,17,21+3,5,9,13,23+} \\
& \overline{13,17,23+3,5,7,13,25+11,17,25+5,23,25+9,17,27+5,21,27+3,5,7,9,29+7,17,29+5,19,29+5,17,31+} \\
& \overline{9,11,33+7,13,33+5,13,17,19+5,11,17,21+5,9,17,23+5,7,17,25+3,5,17,29+5,7,9,33+3,5,13,33} \\
e_3 = & \overline{3,7|_10+5,7+3,9|_{12+5,9|_{14+3,5,7+7,9+3,13|_{16+3,5,9+7,11+5,13+3,15|_{18+9,11+7,13+5,15+3,17|_{20+} \\
& \overline{5,7,9+3,5,13+5,17|_{22+5,7,11+3,7,13+3,5,15+3,5,7,9+11,13+9,15|_{24+5,9,11+3,9,13+3,5,17+9,17|_{26+} \\
& \overline{5,9,13|_{28+7,9,13+5,11,13+5,9,15+3,5,9,13|_{30+7,11,13+3,13,15+5,9,17+5,7,9,11+3,5,9,15+15,17+} \\
& \overline{11,21+7,25+3,29|_{32+9,11,13+7,11,15+5,13,15+3,13,17+5,7,9,13+3,7,11,13+3,7,9,15+3,5,11,15+} \\
& \overline{3,5,9,17+15,19+13,21+11,23+9,25+7,27+5,29+3,31|_{34+9,11,15+7,13,15+7,11,17+5,13,17+5,7,11,13+} \\
& \overline{3,9,11,13+5,7,9,15+3,5,13,15+3,7,9,17+3,5,11,17+17,19+15,21+13,23+11,25+9,27+7,29+5,31+} \\
& \overline{3,33|_{36+3,5,7,9,13+9,11,17+7,13,17+5,15,17+5,11,21+5,7,25+3,5,29+5,7,9,17+3,5,13,17+5,33|_{38+} \\
& \overline{3,5,7,11,13+3,5,7,9,15+11,13,15+7,15,17+5,15,19+7,11,21+5,13,21+5,11,23+5,9,25+5,7,27+} \\
& \overline{3,7,29+3,5,31+7,9,11,13+5,9,11,15+3,9,13,15+3,5,15,17+3,5,11,21+19,21+17,23+3,5,7,25+15,25+} \\
& \overline{13,27+11,29+9,31|_{40+3,5,9,11,13+3,5,7,9,17+11,13,17+9,15,17+7,15,19+5,17,19+9,11,21+7,13,21+} \\
& \overline{5,15,21+7,11,23+5,13,23+5,11,25+5,9,27+3,9,29+3,5,33+5,9,11,17+3,9,13,17+3,7,15,17+3,7,13,19+} \\
& \overline{3,5,15,19+3,7,11,21+3,5,13,21+3,7,9,23+3,5,11,23+3,5,9,25+9,33|_{42+9,15,19+7,17,19+9,13,21+} \\
& \overline{7,15,21+9,11,23+7,13,23+7,11,25+5,9,29+5,7,15,17+3,9,15,17+5,7,13,19+3,9,13,19+3,5,17,19+} \\
& \overline{5,7,11,21+3,9,11,21+3,5,15,21+5,7,9,23+3,5,13,23+3,7,9,25+3,5,11,25|_{44+9,17,19+9,15,21+5,19,21+} \\
& \overline{9,13,23+5,17,23+9,11,25+5,15,25+5,13,27+7,9,29+5,11,29+5,9,31+5,9,15,17+5,9,13,19+5,9,11,21+} \\
& \overline{5,7,9,25+3,5,9,29|_{46+5,7,9,11,15+3,7,9,13,15+3,5,11,13,15+3,5,7,15,17+3,5,7,13,19+3,5,7,11,21+} \\
& \overline{11,15,21+7,19,21+3,5,7,9,23+7,17,23+7,13,27+7,11,29+3,15,29+3,13,31+5,9,33+3,5,7,9,11,13+} \\
& \overline{5,11,15,17+7,9,13,19+3,5,19,21+3,9,13,23+3,5,17,23+5,7,11,25+3,5,15,25+23,25+5,7,9,27+} \\
& \overline{21,27+19,29+3,5,9,31+17,31|_{48+5,7,9,11,17+3,7,9,13,17+3,5,11,13,17+3,5,9,15,17+3,5,9,13,19+} \\
& \overline{3,5,9,11,21+13,15,21+11,17,21+9,19,21+11,15,23+9,17,23+3,5,7,9,25+7,17,25+9,13,27+7,15,27+} \\
& \overline{9,11,29+5,15,29+3,17,29+7,11,31+5,13,31+3,13,33+7,11,15,17+5,13,15,17+7,11,13,19+7,9,15,19+} \\
& \overline{5,11,15,19+7,9,13,21+3,11,15,21+5,9,13,23+3,11,13,23+3,9,15,23+5,9,11,25+5,7,13,25+3,9,13,25+} \\
& \overline{3,7,15,25+3,7,13,27+3,5,15,27+5,7,9,29+3,7,11,29+3,7,9,31+3,5,11,31+3,5,9,33+17,33|_{50+13,17,21+} \\
& \overline{13,15,23+11,17,23+11,15,25+9,17,25+9,15,27+7,17,27+7,15,29+5,17,29+9,11,31+7,13,31+7,11,33+} \\
& \overline{5,13,33+9,11,15,17+7,13,15,17+9,11,13,19+5,13,15,19+7,9,17,19+5,11,17,19+7,11,13,21+7,9,15,21+} \\
& \overline{3,13,15,21+3,11,17,21+5,11,13,23+5,9,15,23+3,9,17,23+3,11,13,25+5,7,15,25+3,7,17,25+5,7,13,27+} \\
& \overline{3,9,13,27+3,5,17,27+5,7,11,29+3,9,11,29+3,5,15,29+5,7,9,31+3,5,13,31+3,7,9,33+3,5,11,33|_{52+} \\
& \overline{5,7,9,15,17+3,5,13,15,17+5,7,9,13,19+5,7,9,11,21+3,5,11,13,21+15,17,21+3,5,9,13,23+13,17,23+} \\
& \overline{3,5,7,13,25+11,17,25+5,23,25+9,17,27+5,21,27+3,5,7,9,29+7,17,29+5,19,29+5,17,31+9,11,33+} \\
& \overline{7,13,33+5,13,17,19+5,11,17,21+5,9,17,23+5,7,17,25+3,5,17,29+5,7,9,33+3,5,13,33}
\end{aligned}$$

$$\begin{aligned}
e_4 = & \overline{5, 9, 13}|_{28} + \overline{7, 9, 13} + \overline{5, 11, 13} + \overline{5, 9, 15} + \overline{3, 5, 9, 13}|_{30} + \overline{7, 11, 13} + \overline{7, 9, 15} + \overline{5, 11, 15} + \overline{5, 9, 17} + \overline{3, 7, 9, 13} + \overline{3, 5, 11, 13} + \\
& \overline{3, 5, 9, 15}|_{32} + \overline{9, 11, 13} + \overline{7, 11, 15} + \overline{5, 13, 15} + \overline{7, 9, 17} + \overline{5, 11, 17} + \overline{5, 7, 9, 13} + \overline{3, 7, 11, 13} + \overline{3, 7, 9, 15} + \overline{3, 5, 11, 15} + \\
& \overline{3, 5, 9, 17}|_{34} + \overline{9, 11, 15} + \overline{7, 13, 15} + \overline{7, 11, 17} + \overline{5, 13, 17} + \overline{5, 7, 11, 13} + \overline{3, 9, 11, 13} + \overline{5, 7, 9, 15} + \overline{3, 7, 11, 15} + \overline{3, 5, 13, 15} + \\
& \overline{3, 7, 9, 17} + \overline{3, 5, 11, 17}|_{36} + \overline{3, 5, 7, 9, 13} + \overline{9, 11, 17} + \overline{7, 13, 17} + \overline{5, 13, 19} + \overline{5, 9, 23} + \overline{5, 7, 11, 15} + \overline{3, 9, 11, 15} + \overline{3, 7, 13, 15} + \\
& \overline{5, 7, 9, 17} + \overline{3, 7, 11, 17} + \overline{3, 5, 13, 17}|_{38} + \overline{3, 5, 7, 11, 13} + \overline{3, 5, 7, 9, 15} + \overline{11, 13, 15} + \overline{7, 13, 19} + \overline{5, 15, 19} + \overline{5, 13, 21} + \\
& \overline{7, 9, 23} + \overline{5, 11, 23} + \overline{5, 9, 25} + \overline{7, 9, 11, 13} + \overline{5, 7, 13, 15} + \overline{5, 7, 11, 17} + \overline{3, 9, 11, 17} + \overline{3, 7, 13, 17} + \overline{3, 5, 13, 19} + \overline{3, 5, 9, 23}|_{40} + \\
& \overline{3, 5, 9, 11, 13} + \overline{3, 5, 7, 11, 15} + \overline{3, 5, 7, 9, 17} + \overline{11, 13, 17} + \overline{9, 13, 19} + \overline{7, 15, 19} + \overline{5, 17, 19} + \overline{7, 13, 21} + \overline{5, 15, 21} + \overline{7, 11, 23} + \\
& \overline{5, 13, 23} + \overline{7, 9, 25} + \overline{5, 11, 25} + \overline{7, 9, 11, 15} + \overline{5, 9, 13, 15} + \overline{3, 11, 13, 15} + \overline{5, 7, 13, 17} + \overline{3, 7, 13, 19} + \overline{3, 5, 15, 19} + \overline{3, 5, 13, 21} + \\
& \overline{3, 7, 9, 23} + \overline{3, 5, 11, 23} + \overline{3, 5, 9, 25}|_{42} + \overline{3, 5, 9, 11, 15} + \overline{3, 5, 7, 13, 15} + \overline{3, 5, 7, 11, 17} + \overline{9, 15, 19} + \overline{7, 17, 19} + \overline{9, 13, 21} + \\
& \overline{7, 15, 21} + \overline{9, 11, 23} + \overline{7, 13, 23} + \overline{7, 11, 25} + \overline{5, 9, 29} + \overline{7, 9, 13, 15} + \overline{5, 11, 13, 15} + \overline{7, 9, 11, 17} + \overline{5, 9, 13, 17} + \overline{3, 11, 13, 17} + \\
& \overline{5, 7, 13, 19} + \overline{3, 9, 13, 19} + \overline{3, 7, 15, 19} + \overline{3, 5, 17, 19} + \overline{3, 7, 13, 21} + \overline{3, 5, 15, 21} + \overline{5, 7, 9, 23} + \overline{3, 7, 11, 23} + \overline{3, 5, 13, 23} + \\
& \overline{3, 7, 9, 25} + \overline{3, 5, 11, 25}|_{44} + \overline{3, 5, 9, 13, 15} + \overline{3, 5, 9, 11, 17} + \overline{3, 5, 7, 13, 17} + \overline{9, 17, 19} + \overline{9, 15, 21} + \overline{5, 19, 21} + \overline{9, 13, 23} + \\
& \overline{5, 17, 23} + \overline{9, 11, 25} + \overline{5, 15, 25} + \overline{5, 13, 27} + \overline{7, 9, 29} + \overline{5, 11, 29} + \overline{5, 9, 31} + \overline{7, 9, 13, 17} + \overline{5, 11, 13, 17} + \overline{5, 9, 13, 19} + \overline{5, 7, 15, 19} + \\
& \overline{3, 9, 15, 19} + \overline{3, 7, 17, 19} + \overline{5, 7, 13, 21} + \overline{3, 9, 13, 21} + \overline{3, 7, 15, 21} + \overline{5, 7, 11, 23} + \overline{3, 9, 11, 23} + \overline{3, 7, 13, 23} + \overline{5, 7, 9, 25} + \\
& \overline{3, 7, 11, 25} + \overline{3, 5, 9, 29}|_{46} + \overline{3, 7, 9, 13, 15} + \overline{3, 5, 9, 13, 17} + \overline{3, 5, 7, 13, 19} + \overline{13, 15, 19} + \overline{7, 19, 21} + \overline{3, 5, 7, 9, 23} + \overline{9, 15, 23} + \\
& \overline{7, 17, 23} + \overline{7, 15, 25} + \overline{7, 13, 27} + \overline{5, 15, 27} + \overline{7, 11, 29} + \overline{7, 9, 31} + \overline{5, 11, 31} + \overline{5, 9, 33} + \overline{3, 5, 7, 9, 11, 13} + \overline{9, 11, 13, 15} + \\
& \overline{5, 11, 13, 19} + \overline{5, 9, 15, 19} + \overline{5, 7, 17, 19} + \overline{3, 9, 17, 19} + \overline{5, 9, 13, 21} + \overline{5, 7, 15, 21} + \overline{3, 9, 15, 21} + \overline{3, 5, 19, 21} + \overline{5, 7, 13, 23} + \\
& \overline{3, 9, 13, 23} + \overline{3, 5, 17, 23} + \overline{5, 7, 11, 25} + \overline{3, 9, 11, 25} + \overline{3, 5, 15, 25} + \overline{3, 5, 13, 27} + \overline{3, 7, 9, 29} + \overline{3, 5, 11, 29} + \overline{3, 13, 15, 19} + \overline{5, 9, 17, 19} + \\
& \overline{5, 7, 9, 13, 15} + \overline{3, 7, 11, 13, 15} + \overline{3, 7, 9, 13, 17} + \overline{3, 5, 9, 13, 19} + \overline{3, 5, 7, 15, 19} + \overline{13, 17, 19} + \overline{3, 5, 7, 15, 19} + \overline{13, 17, 19} + \overline{3, 9, 11, 13} + \overline{21} + \\
& \overline{9, 19, 21} + \overline{3, 5, 7, 11, 23} + \overline{11, 15, 23} + \overline{3, 5, 7, 9, 25} + \overline{9, 13, 27} + \overline{7, 15, 27} + \overline{5, 17, 27} + \overline{9, 11, 29} + \overline{5, 15, 29} + \overline{7, 11, 31} + \\
& \overline{5, 13, 31} + \overline{7, 9, 33} + \overline{5, 11, 33} + \overline{3, 5, 7, 9, 11, 15} + \overline{9, 11, 13, 17} + \overline{7, 11, 13, 19} + \overline{5, 11, 15, 19} + \overline{3, 13, 15, 19} + \overline{5, 9, 17, 19} + \\
& \overline{5, 11, 13, 21} + \overline{3, 7, 11, 13, 15} + \overline{3, 7, 9, 13, 17} + \overline{3, 5, 9, 13, 19} + \overline{3, 5, 7, 15, 21} + \overline{3, 7, 17, 23} + \overline{3, 7, 15, 27} + \overline{3, 5, 17, 27} + \overline{3, 5, 15, 29} + \\
& \overline{5, 7, 9, 29} + \overline{3, 7, 11, 29} + \overline{3, 7, 9, 31} + \overline{3, 5, 11, 31} + \overline{3, 5, 9, 33}|_{50} + \overline{5, 7, 11, 13, 15} + \overline{3, 9, 11, 13, 15} + \overline{5, 7, 9, 13, 17} + \\
& \overline{3, 7, 11, 13, 17} + \overline{3, 5, 9, 15, 19} + \overline{3, 5, 7, 17, 19} + \overline{3, 5, 9, 13, 21} + \overline{3, 5, 7, 15, 21} + \overline{13, 17, 21} + \overline{3, 5, 9, 11, 23} + \overline{3, 5, 7, 13, 23} + \\
& \overline{13, 15, 23} + \overline{11, 17, 23} + \overline{3, 5, 7, 11, 25} + \overline{11, 15, 25} + \overline{9, 17, 25} + \overline{9, 15, 27} + \overline{7, 17, 27} + \overline{7, 15, 27} + \overline{9, 11, 29} + \overline{5, 9, 11, 29} + \overline{9, 11, 31} + \\
& \overline{7, 13, 31} + \overline{7, 11, 33} + \overline{5, 13, 33} + \overline{3, 5, 7, 9, 13, 15} + \overline{3, 5, 7, 9, 11, 17} + \overline{9, 11, 13, 19} + \overline{7, 11, 15, 19} + \overline{5, 11, 17, 19} + \overline{3, 13, 17, 19} + \\
& \overline{7, 11, 13, 21} + \overline{5, 11, 15, 21} + \overline{3, 13, 15, 21} + \overline{5, 7, 19, 21} + \overline{3, 9, 19, 21} + \overline{7, 9, 13, 23} + \overline{5, 9, 15, 23} + \overline{3, 11, 15, 23} + \overline{3, 11, 15, 23} + \overline{5, 7, 17, 23} + \\
& \overline{7, 9, 11, 25} + \overline{5, 9, 13, 25} + \overline{5, 7, 15, 25} + \overline{5, 7, 13, 27} + \overline{3, 9, 13, 27} + \overline{3, 9, 15, 27} + \overline{3, 7, 15, 27} + \overline{3, 5, 17, 27} + \overline{3, 5, 15, 29} + \overline{5, 7, 17, 27} + \\
& \overline{3, 5, 15, 29} + \overline{5, 7, 9, 31} + \overline{3, 7, 11, 31} + \overline{3, 5, 13, 31} + \overline{3, 7, 9, 33} + \overline{3, 5, 11, 33}|_{52} + \overline{5, 7, 11, 13, 17} + \overline{3, 9, 11, 13, 17} + \\
& \overline{5, 7, 9, 13, 19} + \overline{3, 5, 9, 17, 19} + \overline{3, 5, 9, 15, 21} + \overline{15, 17, 21} + \overline{13, 17, 23} + \overline{3, 5, 9, 11, 25} + \overline{11, 17, 25} + \overline{5, 23, 25} + \overline{9, 17, 27} + \\
& \overline{5, 21, 27} + \overline{3, 5, 7, 9, 29} + \overline{7, 17, 29} + \overline{5, 19, 29} + \overline{5, 17, 31} + \overline{9, 11, 33} + \overline{3, 5, 7, 9, 13, 17} + \overline{9, 11, 13, 17} + \overline{9, 11, 15, 19} + \overline{7, 13, 15, 19} + \\
& \overline{7, 11, 17, 19} + \overline{9, 11, 13, 21} + \overline{7, 11, 15, 21} + \overline{3, 13, 17, 21} + \overline{5, 9, 19, 21} + \overline{5, 11, 15, 23} + \overline{3, 13, 15, 23} + \overline{3, 11, 17, 23} + \\
& \overline{7, 9, 13, 25} + \overline{5, 11, 13, 25} + \overline{3, 11, 15, 25} + \overline{3, 9, 17, 25} + \overline{5, 7, 15, 27} + \overline{3, 9, 15, 27} + \overline{3, 7, 17, 27} + \overline{3, 7, 15, 29} + \overline{3, 5, 17, 29} + \\
& \overline{5, 7, 11, 31} + \overline{3, 9, 11, 31} + \overline{3, 7, 13, 31} + \overline{5, 7, 9, 33} + \overline{3, 7, 11, 33} + \overline{3, 5, 13, 33} \\
e_5 = & \overline{3, 7, 11, 15}|_{36} + \overline{5, 7, 11, 15} + \overline{3, 9, 11, 15} + \overline{3, 7, 13, 15} + \overline{3, 7, 11, 17}|_{38} + \overline{5, 9, 11, 15} + \overline{5, 7, 13, 15} + \overline{3, 9, 13, 15} + \overline{5, 7, 11, 17} + \\
& \overline{3, 9, 11, 17} + \overline{3, 7, 13, 17}|_{40} + \overline{3, 5, 7, 11, 15} + \overline{7, 9, 11, 15} + \overline{5, 9, 13, 15} + \overline{3, 11, 13, 15} + \overline{5, 9, 11, 17} + \overline{5, 7, 13, 17} + \overline{3, 9, 13, 17} + \\
& \overline{3, 7, 15, 17} + \overline{3, 7, 11, 21}|_{42} + \overline{3, 5, 9, 11, 15} + \overline{3, 5, 7, 13, 15} + \overline{3, 5, 7, 11, 17} + \overline{7, 9, 13, 15} + \overline{5, 11, 13, 15} + \overline{7, 9, 11, 17} + \\
& \overline{5, 9, 13, 17} + \overline{3, 11, 13, 17} + \overline{5, 7, 15, 17} + \overline{3, 9, 15, 17} + \overline{3, 7, 15, 19} + \overline{5, 7, 11, 21} + \overline{3, 9, 11, 21} + \overline{3, 7, 13, 21} + \overline{3, 7, 11, 23}|_{44} + \\
& \overline{3, 5, 9, 13, 15} + \overline{3, 5, 9, 11, 17} + \overline{3, 5, 7, 13, 17} + \overline{7, 9, 13, 17} + \overline{5, 11, 13, 17} + \overline{5, 9, 15, 17} + \overline{5, 7, 15, 19} + \overline{3, 9, 15, 19} + \overline{3, 5, 15, 19} + \\
& \overline{3, 7, 15, 19} + \overline{5, 9, 11, 21} + \overline{5, 7, 13, 21} + \overline{3, 9, 13, 21} + \overline{3, 7, 15, 21} + \overline{5, 7, 11, 23} + \overline{3, 9, 11, 23} + \overline{3, 7, 13, 23} + \overline{3, 7, 11, 25}|_{46} + \\
& \overline{5, 7, 9, 11, 15} + \overline{3, 5, 11, 13, 15} + \overline{3, 5, 9, 13, 17} + \overline{3, 5, 7, 15, 17} + \overline{3, 5, 7, 11, 21} + \overline{9, 11, 13, 15} + \overline{7, 9, 15, 17} + \overline{3, 13, 15, 17} + \\
& \overline{5, 9, 15, 19} + \overline{5, 7, 17, 19} + \overline{3, 9, 17, 19} + \overline{7, 9, 11, 21} + \overline{5, 9, 13, 21} + \overline{3, 11, 13, 21} + \overline{5, 7, 15, 21} + \overline{3, 9, 15, 21} + \overline{5, 9, 11, 23} + \\
& \overline{5, 7, 13, 23} + \overline{3, 9, 13, 23} + \overline{5, 7, 11, 25} + \overline{3, 9, 11, 25} + \overline{3, 7, 13, 25}|_{48} + \overline{5, 7, 9, 13, 15} + \overline{3, 7, 11, 13, 15} + \overline{5, 7, 9, 11, 17} + \\
& \overline{3, 5, 11, 13, 17} + \overline{3, 5, 9, 15, 17} + \overline{3, 5, 7, 15, 19} + \overline{3, 5, 9, 11, 21} + \overline{3, 5, 7, 13, 21} + \overline{3, 5, 7, 11, 23} + \overline{3, 5, 7, 9, 11, 15} + \\
& \overline{9, 11, 13, 17} + \overline{7, 11, 15, 17} + \overline{5, 13, 15, 17} + \overline{7, 9, 15, 19} + \overline{3, 13, 15, 19} + \overline{5, 9, 17, 19} + \overline{7, 9, 13, 21} + \overline{5, 11, 13, 21} + \overline{5, 9, 15, 21} + \\
& \overline{3, 11, 15, 21} + \overline{3, 7, 19, 21} + \overline{7, 9, 11, 23} + \overline{5, 9, 13, 23} + \overline{3, 11, 13, 23} + \overline{3, 7, 17, 23} + \overline{5, 9, 11, 25} + \overline{7, 13, 25} + \overline{3, 9, 13, 25}|_{50} + \\
& \overline{5, 7, 11, 13, 15} + \overline{3, 9, 11, 13, 15} + \overline{5, 7, 9, 13, 17} + \overline{3, 7, 11, 13, 17} + \overline{3, 5, 9, 15, 19} + \overline{3, 5, 7, 17, 19} + \overline{3, 5, 9, 13, 21} + \\
& \overline{3, 5, 7, 15, 21} + \overline{3, 5, 9, 11, 23} + \overline{3, 5, 7, 13, 23} + \overline{3, 5, 7, 11, 25} + \overline{3, 5, 7, 9, 13, 15} + \overline{3, 5, 7, 9, 11, 17} + \overline{9, 11, 15, 17} + \\
& \overline{7, 13, 15, 17} + \overline{7, 11, 15, 19} + \overline{5, 13, 15, 19} + \overline{7, 9, 17, 19} + \overline{3, 13, 17, 19} + \overline{7, 9, 15, 21} + \overline{5, 11, 15, 21} + \overline{7, 11, 17, 21} + \overline{5, 7, 9, 13, 17} + \\
& \overline{3, 9, 19, 21} + \overline{7, 9, 13, 23} + \overline{5, 11, 13, 23} + \overline{3, 11, 15, 23} + \overline{5, 7, 17, 23} + \overline{3, 9, 17, 23} + \overline{7, 9, 11, 25} + \overline{5, 9, 13, 25} + \overline{3, 11, 13, 25} + \\
& \overline{3, 7, 17, 25} + \overline{3, 7, 15, 27} + \overline{3, 7, 11, 31}|_{52} + \overline{5, 7, 11, 13, 17} + \overline{3, 9, 11, 13, 17} + \overline{5, 7, 9, 15, 17} + \overline{3, 5, 13, 15, 17} + \overline{3, 5, 9, 17, 19} + \\
& \overline{5, 7, 9, 11, 21} + \overline{3, 5, 11, 13, 21} + \overline{3, 5, 9, 15, 21} + \overline{3, 5, 9, 13, 23} + \overline{3, 5, 9, 11, 25} + \overline{3, 5, 7, 13, 25} + \overline{3, 5, 7, 9, 13, 17} + \\
& \overline{9, 11, 15, 19} + \overline{7, 13, 15, 19} + \overline{7, 11, 17, 19} + \overline{5, 13, 17, 19} + \overline{9, 11, 13, 21} + \overline{7, 11, 15, 21} + \overline{5, 11, 17, 21} + \overline{3, 13, 17, 21} + \\
& \overline{5, 9, 19, 21} + \overline{5, 11, 15, 23} + \overline{3, 13, 15, 23} + \overline{5, 9, 17, 23} + \overline{3, 11, 17, 23} + \overline{7, 9, 13, 25} + \overline{5, 11, 13, 25} + \overline{3, 11, 15, 25} + \overline{5, 7, 17, 25} + \\
& \overline{3, 9, 17, 25} + \overline{5, 7, 15, 27} + \overline{3, 9, 15, 27} + \overline{3, 7, 17, 27} + \overline{3, 7, 15, 29} + \overline{5, 7, 11, 31} + \overline{3, 9, 11, 31} + \overline{3, 7, 13, 31} + \overline{3, 7, 11, 33}
\end{aligned}$$

2 The primitive central idempotents of group rings of alternating groups in characteristic 2

The theoretical results and the idempotents can already be found in [3]. We display them here only for the sake of completeness.

For alternating groups \mathbb{F}_4 is always a splitting field. The primitive central idempotents of $\mathbb{F}_4 A_n$ are the primitive central idempotents of $\mathbb{F}_2 S_n$ except for one case: If $n = \frac{m(m+1)}{2}$ then there is an idempotent $e = C^+$ of $\mathbb{F}_2 S_n$, where C is the conjugacy class corresponding to the partition $(2m-1, 2m-5, 2m-9, \dots)$ of n . This idempotent splits in two primitive central idempotents of $\mathbb{F}_4 A_n$. We computed these two idempotents. If a class C of S_n splits in two conjugacy classes of A_n then we write C_- and C_+ for the A_n -classes. ζ denotes a generator of \mathbb{F}_4 over \mathbb{F}_2 . To save space we only write f_1 , the second idempotent f_2 can easily be computed via $f_2 = f_1 + \overline{2m-1, 2m-5, 2m-9, \dots}_+ + \overline{2m-1, 2m-5, 2m-9, \dots}_-$.

n	f_1
3	$\overline{1 + \zeta^2 \cdot 3_+ + \zeta \cdot 3_-}$
6	$\overline{3 + \zeta^2 \cdot 5_+ + \zeta \cdot 5_- + 3^2}$
10	$\overline{7 + 3 \cdot 5 + 3^3 + 5^2 + \zeta^2 \cdot 3 \cdot 7_+ + \zeta \cdot 3 \cdot 7_-}$
15	$\overline{5 \cdot 7 + 3 \cdot 9 + 3^3 \cdot 5 + 7^2 + \zeta \cdot 5 \cdot 9_+ + \zeta^2 \cdot 5 \cdot 9_- + 5^3 + 3 \cdot 5 \cdot 7_+ + 3 \cdot 5 \cdot 7_- + 3^2 \cdot 9}$
21	$\overline{7 \cdot 11 + 3 \cdot 7 \cdot 9 + 3 \cdot 5 \cdot 11 + 3 \cdot 5^2 \cdot 7 + 3^3 \cdot 11 + 3^3 \cdot 5 \cdot 7 + 7^3 + 5 \cdot 7 \cdot 9_+ + 5 \cdot 7 \cdot 9_- + 3 \cdot 9^2 + 5^2 \cdot 11 + 3 \cdot 7 \cdot 11_+}$
28	$\overline{5 \cdot 9 \cdot 11 + 5 \cdot 7 \cdot 13 + 3 \cdot 9 \cdot 13 + 3 \cdot 5^3 \cdot 9 + 3^2 \cdot 5 \cdot 7 \cdot 9 + 9^3 + 7 \cdot 9 \cdot 11_+ + 7 \cdot 9 \cdot 11_- + 5 \cdot 11^2 + 3^3 \cdot 5 \cdot 13 + 7^2 \cdot 13 + 5 \cdot 9 \cdot 13_+ + 5 \cdot 7^2 \cdot 9 + 3 \cdot 5 \cdot 9 \cdot 11_+ + 3 \cdot 5 \cdot 9 \cdot 11_- + 5^3 \cdot 13 + 3 \cdot 5 \cdot 7 \cdot 13_+ + 3 \cdot 5 \cdot 7 \cdot 13_- + 3^2 \cdot 9 \cdot 13}$
36	$\overline{7 \cdot 11 \cdot 15 + 3 \cdot 7 \cdot 11 \cdot 13 + 3 \cdot 7 \cdot 9 \cdot 15 + 3 \cdot 5 \cdot 11 \cdot 15 + 3 \cdot 7^3 \cdot 11 + 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11_+ + 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11_- + 3 \cdot 5^2 \cdot 7 \cdot 15 + 3^3 \cdot 11 \cdot 15 + 3 \cdot 5^3 \cdot 7 \cdot 11 + 3^3 \cdot 7 \cdot 9 \cdot 11 + 7 \cdot 9^2 \cdot 11 + 3 \cdot 11^3 + 5 \cdot 7 \cdot 11 \cdot 13_+ + 5 \cdot 7 \cdot 11 \cdot 13_- + 3 \cdot 9 \cdot 11 \cdot 13_+ + 3 \cdot 9 \cdot 11 \cdot 13_- + 3 \cdot 7 \cdot 13^2 + 3^3 \cdot 5 \cdot 7 \cdot 15 + 7^3 \cdot 15 + 5 \cdot 7 \cdot 9 \cdot 15_+ + 5 \cdot 7 \cdot 9 \cdot 15_- + 3 \cdot 9^2 \cdot 15 + 5^2 \cdot 11 \cdot 15 + 3 \cdot 7 \cdot 11 \cdot 15_+}$
45	$\overline{5 \cdot 9 \cdot 13 \cdot 15 + 5 \cdot 9 \cdot 11 \cdot 17 + 5 \cdot 7 \cdot 13 \cdot 17 + 3 \cdot 9 \cdot 13 \cdot 17 + 5^3 \cdot 7 \cdot 9 \cdot 13 + 3 \cdot 5 \cdot 7^2 \cdot 9 \cdot 13 + 3^2 \cdot 5 \cdot 9 \cdot 11 \cdot 13 + 9 \cdot 11^2 \cdot 13 + 5 \cdot 13^3 + 7 \cdot 9 \cdot 13 \cdot 15_+ + 7 \cdot 9 \cdot 13 \cdot 15_- + 5 \cdot 11 \cdot 13 \cdot 15_+ + 5 \cdot 11 \cdot 13 \cdot 15_- + 5 \cdot 9 \cdot 15^2 + 3 \cdot 5^3 \cdot 9 \cdot 17 + 3^2 \cdot 5 \cdot 7 \cdot 9 \cdot 17 + 9^3 \cdot 17 + 7 \cdot 9 \cdot 11 \cdot 17_+ + 7 \cdot 9 \cdot 11 \cdot 17_- + 5 \cdot 11^2 \cdot 17 + 3^3 \cdot 5 \cdot 13 \cdot 17 + 7^2 \cdot 13 \cdot 17 + 5 \cdot 9 \cdot 13 \cdot 17_+ + 5 \cdot 9^3 \cdot 13 + 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13_+ + 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13_- + 3 \cdot 5 \cdot 9 \cdot 13 \cdot 15_+ + 3 \cdot 5 \cdot 9 \cdot 13 \cdot 15_- + 5 \cdot 7^2 \cdot 9 \cdot 17 + 3 \cdot 5 \cdot 9 \cdot 11 \cdot 17_+ + 3 \cdot 5 \cdot 9 \cdot 11 \cdot 17_- + 5^3 \cdot 13 \cdot 17 + 3 \cdot 5 \cdot 7 \cdot 13 \cdot 17_+ + 3 \cdot 5 \cdot 7 \cdot 13 \cdot 17_- + 3^2 \cdot 9 \cdot 13 \cdot 17}$
55	$\overline{7 \cdot 11 \cdot 15 \cdot 19 + 3 \cdot 7 \cdot 11 \cdot 15 \cdot 17 + 3 \cdot 7 \cdot 11 \cdot 13 \cdot 19 + 3 \cdot 7 \cdot 9 \cdot 15 \cdot 19 + 3 \cdot 5 \cdot 11 \cdot 15 \cdot 19 + 3 \cdot 7 \cdot 9^2 \cdot 11 \cdot 15 + 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 15_+ + 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 15_- + 3 \cdot 7^3 \cdot 11 \cdot 19 + 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 19_+ + 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 19_- + 3 \cdot 5^2 \cdot 7 \cdot 15 \cdot 19 + 3^3 \cdot 11 \cdot 15 \cdot 19 + 3 \cdot 5 \cdot 7^3 \cdot 11 \cdot 15 + 3 \cdot 5^2 \cdot 7 \cdot 9 \cdot 11 \cdot 15 + 7 \cdot 11^3 \cdot 15 + 3^3 \cdot 7 \cdot 11 \cdot 13 \cdot 15 + 7 \cdot 9 \cdot 11 \cdot 13 \cdot 15_+ + 7 \cdot 9 \cdot 11 \cdot 13 \cdot 15_- + 3 \cdot 11 \cdot 13^2 \cdot 15 + 3 \cdot 7 \cdot 15^3 + 5 \cdot 7 \cdot 11 \cdot 15 \cdot 17_+ + 5 \cdot 7 \cdot 11 \cdot 15 \cdot 17_- + 3 \cdot 9 \cdot 11 \cdot 15 \cdot 17_+ + 3 \cdot 9 \cdot 11 \cdot 15 \cdot 17_- + 3 \cdot 7 \cdot 13 \cdot 15 \cdot 17_+ + 3 \cdot 7 \cdot 13 \cdot 15 \cdot 17_- + 3 \cdot 7 \cdot 11 \cdot 17^2 + 3 \cdot 5^3 \cdot 7 \cdot 11 \cdot 19 + 3^3 \cdot 7 \cdot 9 \cdot 11 \cdot 19 + 7 \cdot 9^2 \cdot 11 \cdot 19 + 3 \cdot 11^3 \cdot 19 + 5 \cdot 7 \cdot 11 \cdot 13 \cdot 19_+ + 5 \cdot 7 \cdot 11 \cdot 13 \cdot 19_- + 3 \cdot 9 \cdot 11 \cdot 13 \cdot 19_+ + 3 \cdot 9 \cdot 11 \cdot 13 \cdot 19_- + 3 \cdot 7 \cdot 13^2 \cdot 19 + 3^3 \cdot 5 \cdot 7 \cdot 15 \cdot 19 + 7^3 \cdot 15 \cdot 19 + 5 \cdot 7 \cdot 9 \cdot 15 \cdot 19_+ + 5 \cdot 7 \cdot 9 \cdot 15 \cdot 19_- + 3 \cdot 9^2 \cdot 15 \cdot 19 + 5^2 \cdot 11 \cdot 15 \cdot 19 + \zeta^2 \cdot 3 \cdot 7 \cdot 11 \cdot 15 \cdot 19_+ + \zeta \cdot 3 \cdot 7 \cdot 11 \cdot 15 \cdot 19_-}$

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