Essays on the Role of Preferences in International Trade Theory

Dissertation
zur Erlangung des Grades eines Doktors der Wirtschaftswissenschaft
der Rechts- und Wirtschaftswissenschaftlichen Fakultät
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Für meinen Vater
und meine Mutter

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Abstract

The main research focus of this thesis lies on the role of preferences in international trade theory, with a particular emphasis on how preferences determine the production structure, shape the trade pattern and influence the welfare effects of trade in open economies. The articles in this thesis contain three modeling approaches, which discuss the central topic of preferences in trade theory from different perspectives: (i) a specific form of parametric "price-independent generalized-linear" (PIGL) preferences with price distortion on the labor market, (ii) more general parametric PIGL preferences with a perfectly competitive labor market and (iii) a specific form of parametric PIGL preferences with search frictions and a labor market imperfection. This allows a broad discussion on how preferences affect the equilibrium outcomes in closed and open economies depending on the modeling approach.

After a short introduction in Chapter 1, Chapter 2 relies on a subclass of parametric PIGL preferences and includes rent sharing to capture feedback effects of trade on income. This generates a two-way linkage between income and trade. We set up a home-market model with two sectors, producing differentiated goods and a homogeneous outside good, and labor as the only factor input. Assuming that households differ in their effective labor supply, this leads to differences in their level of labor income. We show that the country featuring a higher ex ante level and/or dispersion of per-capita income has a larger home market and becomes net-exporter of differentiated goods in the open economy. Due to a price distortion on the labor market, the trade pattern is an important factor of welfare in the open economy. The country that increases its market share and net-exports differentiated goods benefits from trade, whereas the other country can lose.

Chapter 3 displays a generalization of Chapter 2 with regard to the choice of preferences, as it relies on a more general form of parametric PIGL preferences, giving rise to an integrability problem. Introducing differentiated intermediate goods that are costlessly assembled to a nontradable, homogeneous luxury good in the model variant of Chapter 2 with perfectly competitive labor markets, allows us to solve the integrability problem for two homogeneous final goods. In the open economy, all other things equal, this makes the country with the relatively higher demand for the luxury good and thus the larger domestic market for differentiated intermediates a net-exporter of intermediate goods. However, with the same market clearing wage paid in the two sectors, the welfare effects of trade are always positive for both trading partners, irrespective of the trade structure.

Chapter 4 contains a two-country model featuring the same parametric PIGL preferences as in Chapter 2. Adding search frictions and firm-level wage bargaining, Chapter 4 elaborates on the role of preferences for employment and welfare effects of trade. We introduce a home-market model with a homogeneous goods sector, producing under perfect competition, and a differentiated goods sector, distorted on the labor market. In the open economy, the larger country specializes on the production of differentiated goods and net-exports these goods, at the cost of a higher economy-wide rate of unemployment. The welfare effects of trade depend on the preference structure, such that the large country is likely to benefit from trade if preferences are homothetic, whereas losses from trade are possible if preferences are quasilinear. The opposite is true in the smaller country.

The thesis concludes with a summary in Chapter 5.

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Chapter 1

Introduction

Ever since the publication of the Linder (1961) hypothesis, the research field of international trade has substantially raised attention for demand-side factors as explanations of patterns of trade. In a first informal account, Linder (1961) has established that countries which are similar in their per-capita income purchase similar bundles of products causing large (intra-industry) trade flows in these products. The similarity in market size as a key determinant of international trade is the focus of the new trade theory, founded by Krugman (1979, 1980), who motivates intra-industry trade due to love-of-variety preferences. Krugman's (1979; 1980) seminal contributions mark the first formal models highlighting the role of preferences in international trade theory, thereby explaining mutual exchange of goods within industries between similar countries. Markusen (1986) has developed a first theoretical model to explain the impact of per-capita income on the structure of international trade in a setting with intra- and inter-industry trade. Thereafter, departing from the standard assumption of homothetic taste in theoretical and empirical models of trade, which makes these models analytically challenging, became more prevalent.[1] The three articles in this thesis capture demand-side factors for explaining trade by using nonhomothetic preferences. The discussion of "price-independent generalized-linear" (PIGL) preferences (cf. Muellbauer, 1975, 1976) in a trade context marks an important contribution, since thereby not only the level of per-capita income but also the dispersion of per-capita income can be taken into account in one model framework.[2] In the remainder of the introduction, the contents of Chapters 2-4 are briefly summarized, whereas a detailed overview of the respective literature and a thorough discussion about the contributions of the different models are postponed to the respective chapter.[3]

Chapter 2 is entitled "Nonhomothetic Preferences and Rent Sharing in an Open Economy".[4] We develop a framework for studying how differences in the level and/or dispersion

[1]Recent contributions relying on nonhomothetic preferences are for instance Fajgelbaum et al. (2011), Fieler (2011), Bertoletti and Etro (2017) and Foellmi et al. (2018).

[2]Evidence in favor of income dispersion as an important factor for explaining international trade flows has been reported, for instance, by Flam and Helpman (1987), Francois and Kaplan (1996), Dalgin et al. (2008), Fajgelbaum et al. (2011) and Bernasconi (2013).

[3]This cumulative dissertation contains three separate chapters, whose contents originate from autonomous manuscripts. Therefore, notations are adopted from the respective manuscripts and might differ between the chapters. Chapters 2 and 4 build on joint work with Hartmut Egger, whereas Chapter 3 is based on a single-authored work.

[4]This chapter is based on Egger and Habermeyer (2019). When working on this article, we have benefited from comments by Timo Boppart, Carsten Eckel, Sergey Kichko, John Morrow, Peter Neary, Marc

of per-capita income affect trade structure and welfare in a two-country model. Thereby, we embed nonhomothetic preferences into a home-market model with two sectors of production and one input factor. Relying on a subclass of parametric PIGL preferences for which a closed form representation of direct utility exists, we avoid an integrability problem. We associate the homogeneous outside good with a necessity and differentiated goods with luxuries, and we assume that heterogeneity of income arises due to heterogeneity of households in their effective labor supply. We then show that, in line with other models featuring home-market effects, countries have a trade surplus in the good for which they have relatively higher domestic demand, making the country with a higher level and/or dispersion of per-capita income a net-exporter of luxuries. The structure of trade is irrelevant for welfare in the open economy if both sectors pay the same wage. If, however, the sector producing luxuries pays a wage premium due to rent sharing at the firm level, there are feedback effects of trade on the level and dispersion of per-capita income, which can lead to losses from trade in the country net-exporting necessities.[5] In an extension of our model, we show that our results remain intact when we allow for positive assortative matching of workers featuring high effective labor supply with jobs offering high wages in the sector of luxuries. In a second extension, we show that the assumption of nonhomothetic preferences seems less important when supply-side differences are the main motive for inter-industry trade.

In Chapter 3 – "PIGL Preferences, Income Differences and International Trade" – we rely on a more general class of parametric PIGL preferences as compared to Chapter 2, for which an explicit solution for the direct utility function does in general not exist. This gives rise to an integrability problem, since it is a priori not clear that the demand functions derived from indirect utility are in fact the solution to a well-defined utility maximization problem. So far, this integrability problem has not been solved for a continuum of differentiated goods. We modify the problem by introducing differentiated intermediate goods along the lines of Ethier (1982b), which are assembled to a homogeneous luxury good. This allows us to solve the integrability problem for two final homogeneous goods by following insights from Boppart (2014). In order to highlight the impact of the form of preferences on trade pattern and welfare, Chapter 3 abstracts from a price distortion on the labor market und thus from feedback effects of trade. We employ the more general form of parametric PIGL preferences in a two-country home-market model of international trade. The economy is populated by heterogeneous households, who differ in their efficiency units of labor which leads to income differences. Labor is the only input factor for the production of homogeneous necessities and differentiated intermediate goods, which are used for the production of a costlessly assembled, homogeneous luxury good. Associating trade with the exchange of necessities and intermediates, we show that the

Muendler and Federico Trionfetti. We are grateful to participants of the TRISTAN Workshop at the University of Bayreuth, the Göttingen Workshop on International Economics, the Research Workshop of the *Bavarian Graduate Program in Economics*, the European Trade Study Group, the Midwest International Trade Meeting, the European Economic Association, and Research Seminars at Aix Marseille University, the Universities of Bayreuth, Hagen, Munich, and Nuremberg for helpful comments and suggestions.

[5] Recent examples dealing with firm-level wage setting in models of international trade are, for instance, Davidson et al. (2008), Egger and Kreickemeier (2009, 2012), Helpman et al. (2010) and Helpman and Itskhoki (2010).

country with the higher level and/or dispersion of per-capita income exhibits a larger domestic demand for the luxury good and net-exports differentiated intermediate goods, which is in line with the well-established model of the home-market effect. In the absence of feedback effects of trade, both countries gain from trade, whereas the magnitude of the welfare gains may differ. Thus, we can show that the fundamental insights of Chapter 2 extend to a generalization of parametric PIGL preferences.

The main purpose of Chapter 4 is to investigate "How Preferences Shape the Welfare and Employment Effects of Trade", when allowing for involuntary unemployment. In economic research, it is well-established that the form of labor market imperfection influences the welfare and employment effects of trade.[6] Chapter 4 goes one step further and studies the relevance of consumer preferences for the effects of trade on unemployment and welfare in the presence of a labor market distortion. We set up a trade model with two countries, two sectors, and one production factor, which features a home-market effect due to the existence of trade costs. We consider search frictions and firm-level wage bargaining in the sector producing differentiated goods and a perfectly competitive labor market in the sector producing a homogeneous good. Consumers have "price-independent generalized-linear" preferences over the two types of goods, covering homothetic and quasilinear preferences as two limiting cases. We show that trade between two countries that differ in their population size leads to an expansion of the differentiated goods sector and a contraction of the homogeneous good sector in the larger economy. This induces the larger country to net-export differentiated goods at the cost of a higher economy-wide rate of unemployment in the open economy (with the effects reversed for the smaller country). The welfare effects of trade depend on the preference structure. Looking at the two limiting cases, we show that the large country is likely to benefit from trade if preferences are homothetic, whereas losses from trade are possible if preferences are quasilinear. The opposite is true in the smaller country. This reveals an important role of preferences for the welfare effects of trade in the presence of labor market imperfection, a result we further elaborate on in two extensions, in which we consider more general preferences and differences of countries in their per-capita income levels.

Finally, Chapter 5 summarizes the most important results and presents concluding remarks.

[6]Notable examples are Brecher (1974), Davis (1998a), Egger and Kreickemeier (2009), Helpman et al. (2010) and Helpman and Itskhoki (2010).

Chapter 2

Nonhomothetic Preferences and Rent Sharing in an Open Economy

> "*Trade operates with [... a] fundamental bias in favor of richer and progressive regions against the other regions [... so] that even the handicrafts and industries existing earlier in the other regions are thwarted.*"
>
> — Myrdal (1957, p. 28)

2.1 Introduction

Comparative advantage has been widely acknowledged as the engine of international trade and an important source of welfare gain since David Ricardo's book "*On the Principles of Political Economy and Taxation*" more than two centuries ago. It took almost one and a half centuries before the dominance of this supply-side view has been broken by Linder's (1961) hypothesis that demand-side factors are also important for explaining international trade patterns. Providing a first, informal account of a new trade theory that emphasizes mutual exchange of goods within industries between similar countries, the first fully-fledged model of intra-industry trade is due to Krugman (1979, 1980). Krugman's new trade theory highlights similarity in market size as a key determinant of international trade, whereas Markusen (1986) and Flam and Helpman (1987) show that the level and dispersion of income constitute further demand-side factors when deviating from the assumption of homothetic preferences. This makes two variables, whose changes to international trade have been the target of economic research for a long time, determinants of the existence of trade. The last two decades have seen a revived interest in models featuring nonhomothetic preferences, as they promise a better description of real world trade flows (cf. Fajgelbaum et al., 2011; Markusen, 2013). We use them here to study under which conditions Myrdal's widely shared concern that trade widens the gap between rich and poor countries is justified and show that it is not only the difference in the initial level but also in the dispersion of per-capita income that matters for the welfare effects of trade.

For this purpose, we introduce a class of nonhomothetic preferences that are simple enough to warrant analytical tractability of a model featuring trade between two countries, which differ in the level and/or dispersion of per-capita income, and allow to dissect the welfare effects of this asymmetry into an exogenous component, determining the trade pattern in the open economy, and an endogenous component, capturing the feedback effects of trade. We consider a two-sector economy that adopts important features of the home-market model proposed by Helpman and Krugman (1985). There is one sector with monopolistic competition producing differentiated varieties and another sector producing a homogeneous good under perfect competition, with both sectors using labor as the only input of production. Due to the assumption of nonhomothetic preferences, we can give the output produced by the two sectors an intuitive interpretation from consumer theory. The differentiated goods are luxuries and the homogeneous good is a necessity, as suggested by Francois and Kaplan (1996).[1] Since the expenditure share for luxuries increases in income, the assumption of nonhomothetic preferences makes the level and dispersion of per-capita income important determinants of the size of the home market for luxuries and hence also crucial factors of the trade pattern in the open economy. To distinguish ex ante differences in the level and/or dispersion of per-capita income from ex post differences materializing from trade liberalization, we impose two additional assumptions. On the one hand, we assume that households differ in their effective labor supply (as in Fajgelbaum et al., 2011) and, on the other hand, we consider firm-level rent sharing through individual bargaining (as in Helpman and Itskhoki, 2010) to generate sector-specific wages and allow for feedback effects of trade on nominal wage income.

To model nonhomothetic utility, we rely on "price-independent generalized-linear" (PIGL) preferences proposed by Muellbauer (1975, 1976). These preferences are more general than the Gorman class, but still admit a representative consumer, who is characterized by an expenditure level for which the value (expenditure) shares of consumption equal the value shares of the aggregate economy.[2] The existence of a representative consumer makes these preferences particularly suited for aggregating consumer demand over households with heterogeneous income. However, PIGL preferences have the disadvantage that an explicit solution for the direct utility function usually does not exist. This gives rise to an integrability problem as outlined by early contributions of Antonelli (1886) and Samuelson (1950), because it is a priori not clear that the underlying demand system results from a constrained utiliy maximization problem. To overcome this issue, we use a subclass of PIGL preferences, for which a closed form representation of the direct utility function can be determined (see Boppart, 2014). This subclass is still general enough to cover two prominent preference specifications as limiting cases. The first one are homothetic Cobb-Douglas preferences and the second one are nonhomothetic quasilinear preferences. In both cases, preferences have Gorman form with linear Engel curves so that, by assumption, changes in the dispersion of income do not affect market demand. Except for these limiting cases Engel curves are, however, not linear. They are convex for luxuries and concave for necessities. With non-linear Engel curves, the representative

[1] For instance, Rauch (1999) classifies electronic products, automobiles, and motorcycles as differentiated goods and thus luxuries in our context, whereas cotton fabrics, food, and tobacco products are not classified as differentiated and can therefore be associated with necessities in our model.

[2] As put forward by Muellbauer (1975), PIGL preferences are the most general class of preferences that avoid an aggregation problem with heterogeneous households by admitting a well-defined representative consumer. If the thus defined expenditure level corresponds to the mean of expenditures, PIGL preferences have Gorman form.

consumer used for aggregation does not have a normative interpretation. To discuss welfare implications of trade, we therefore must take a stance on distributional justice and we do so by choosing a utilitarian perspective that gives each household the same weight in the social welfare function.[3]

Due to the non-linearity of Engel curves, demand for luxuries is larger in the country that features a higher level and/or higher dispersion of per-capita income, which, following the reasoning from the literature on home-market effects, is the country that has a trade surplus in luxuries in the open economy. Larger differences of countries in their expenditure structure lead to a stronger specialization in production, raising inter- and reducing intra-industry trade. Therefore, the model considered here is consistent with Linder's (1961) hypothesis that more equal per-capita income levels of two economies provide larger scope for (intra-industry) trade in those goods, for which local demand is an important determinant of production.[4] As put forward by Davis (1998b), the home-market effect is more pronounced at lower trade costs, making intra-industry trade less important if the two economies become more integrated. If both sectors pay the same wage, there are gains from trade in our model, which are independent of the trade structure in the open economy and thus the same for the two economies. This changes when employment in the sector of luxuries promises a wage premium, so that the allocation of workers influences the level and dispersion of per-capita income. In this case, the trade pattern becomes a determinant of welfare with two important consequences for our analysis.

First, there are nominal income losses for workers losing their jobs in the production of luxuries, which captures the widespread concern that not all workers equally benefit from globalization. Whereas this insight is not new and has received a lot of media attention through recent publications by Autor et al. (2013) and Dauth et al. (2014), our analysis points to the role of demand-side factors and shows that losers are more likely to be found in countries with a lower initial per-capita income level. However, things can be even worse for the poorer economy. Losing market share in the sector of luxuries can lead to an increase in the consumer price index and hurt all households. Hence the specialization of production, while usually understood as an important channel for generating gains from

[3]One may prefer a prioritarian view on distributional justice that gives higher weight to poorer households (cf. Parfit, 1997). However, since our welfare function features social inequality aversion even when weighting poor and rich households equally, a prioritarian view would not have a large impact on our qualitative results. Furthermore, one may be more interested in changes in real GDP per-capita than changes in welfare. However, determining real GDP per-capita requires the construction of an exact consumer price index. Whereas Feenstra and Reinsdorf (2000) and Hamilton (2001) have made significant progress in determining such an exact price index for a class of nonhomothetic preferences introduced by Deaton and Muellbauer (1980), which deliver an almost ideal demand system (AIDS), their insights are of limited help for our analysis. On the one hand, except for the limiting case of Cobb-Douglas, the preferences considered here do not belong to this class (see Pollak and Wales, 1992, for a discussion). On the other hand, Almås et al. (2018) point out that computing a single consumer price index has the inherent problem of disregarding the fact that households with different income levels differ in their expenditure shares if preferences are nonhomothetic. Hence, choosing a single consumer price index fails the purpose of measuring the cost-of-living of heterogeneous households. To avoid the problems associated with constructing a proper consumer price index, we therefore focus on the effects on welfare instead of real GDP in our analysis.

[4]Empirical evidence in favor of the Linder (1961) hypothesis has been reported, for instance, by Thursby and Thursby (1987), Bergstrand (1989, 1990), and Hallak (2010). Francois and Kaplan (1996), Dalgin et al. (2008), Bernasconi (2013), and Vollmer and Martínez-Zarzoso (2016) show that bilateral trade is not only affected by differences in the level of per-capita income but also by differences of the two trading partners in their distributions of income.

trade, can be a source of welfare loss. Losses from trade can exist in our model only for the country that loses market share in the sector featuring increasing economies to scale. However, in contrast to insights from Graham (1923), Markusen and Melvin (1981), and Ethier (1982a), it is not the existence of economies to scale per se that gives scope for welfare loss. Rather losses from trade are the result of a price distortion in the labor market, which makes our results akin to findings by Brecher (1974) and Davis (1998b) and builds on the fundamental insight from the theory of second best that welfare losses from trade are possible if the market equilibrium in the closed economy has not been socially optimal (cf. Markusen, 1981; Newbery and Stiglitz, 1984). Our analysis shows that welfare losses can result from differences in demand-side factors and exist although the price distortions in the labor market are the same in the two economies.

Second, with non-linear Engel curves the concentration of disposable income becomes a further determinant of the home market for luxuries. A lower dispersion of disposable income can make a country net-importer of luxuries and therefore worse off with trade than under autarky. This insight challenges policy measures put forward by the literature to distribute the gains from trade more equally. On the one hand, it cannot be ruled out that all households lose from trade, leaving no scope for a redistributive policy intervention. On the other hand, a policy intervention that targets ex ante sources or ex post realizations of an unjust distribution, while maintaining gains from trade in the aggregate, may not be feasible. Lowering the dispersion of per-capita income decreases the home market for luxuries with potentially detrimental welfare consequences. Therefore, the analysis in this paper raises doubts that so far discussed policy measures remain promising instruments to increase support for trade liberalization (cf. Davidson and Matusz, 2006; Egger and Fischer, 2018), when accounting for demand-side determinants of trade in a setting with nonhomothetic preferences.

We complement our analysis on the link between trade patterns and welfare by two extensions of our model. In the first extension, we give up the simplifying assumption that workers are assigned to the production of luxuries by a lottery that does not discriminate between different levels of effective labor supply. This is, because in the benchmark model firms producing luxuries have to pay the same job installment costs for each unit of labor input and are therefore indifferent between employing workers with low or high effective labor supply. Assuming instead that firms have to pay the same job installment costs per worker, gives them an incentive for selecting applicants with higher effective labor supply to reduce their employment costs. If screening the pool of applicants is not costless and gives an imprecise signal about the effective labor supply (as in Helpman et al., 2010), the thus modified framework features endogenous fixed and variable production costs in the sector of luxuries and thus an additional margin for adjustments to trade. Despite these complications the results from our analysis are largely unaffected. In a second extension, we analyze whether the choice of preferences is also important for understanding the consequences of supply-side differences for trade structure and welfare, pointing to a determinant of the international exchange of goods that has been put forward by traditional models of trade theory. We consider differences in the price distortion at the labor market as the supply-side asymmetry of the two economies and, to keep things simple, assume that rent sharing only exists in the foreign economy. This gives home a comparative advantage in the production of luxuries, making it a net-exporter of these goods in the open economy. As a consequence, home gains from trade, whereas the welfare effects in foreign are less clear. We show that irrespective of the specific nature of preferences, foreign loses from a small reduction of initially high trade costs if the price distortion in the labor market is high, while it benefits from the decline in trade costs if the price distortion is small. This result is in line with the more general observation that welfare losses from forfeiting market share in the sector exhibiting increasing economies to scale are more likely if trading partners are more dissimilar (cf. Francois and Nelson, 2002).

Emphasizing the role of demand-side factors for explaining trade patterns in a setting with nonhomothetic preferences, we build on work by Markusen (1986, 2013) and Bergstrand (1990) who employ Stone-Geary preferences to explain how differences in per-capita income affect the trade structure in open economies.[5] Simonovska (2015) uses Stone-Geary preferences to explain the positive relationship between (relative) prices of tradable goods and per-capita income. Relying on preferences that produce linear Engel curves, market demand in these settings is independent of the distribution of income and an aggregation problem over heterogeneous households therefore does not exist. The aggregation problem is also avoided by a number of studies using non-Gorman form preferences with symmetric households. An early prominent example in this respect is Stockey (1991), who considers nonhomothetic preferences in a setting with vertically differentiated products to shed light on the trade structure between rich and poor countries and to explain empirical evidence that new, high quality products are first consumed in rich countries and are only at later stages also consumed in poor countries. Fieler (2011) introduces preferences that do not have Gorman form to explain the role of per-capita income for trade structure in a multi-country Ricardian model along the lines of Eaton and Kortum (2002), and she uses this model to show that a technology shock in China has different effects on countries with differing per-capita income levels. Caron et al. (2014) employ nonhomothetic preferences to improve the predictions of the Heckscher-Ohlin Vanek model regarding the factor content of trade and show that their correction is quantitatively important. Matsuyama (2015) introduces nonhomothetic preferences into a home-market model to study the effects of per-capita income differences on trade structure and to analyze how the benefits of technological progress are distributed between the rich and the poor country. Matsuyama (2018) uses the same class of preferences to show how trade liberalization and economic growth affect the patterns of structural change, innovation, and trade in the presence of Engel's Law.[6] Whereas these models do not provide

[5]Bergstrand (1989) shows how the gravity equation has to be adjusted in order to account for differences in per-capita income along with differences in factor endowments as key determinants of bilateral trade. Hunter (1991) provides early empirical evidence that accounting for per-capita income differences may explain missing trade in empirical work based on Heckscher-Ohlin models.

[6]Both Fieler (2011) and Caron et al. (2014) build on a generalized CES preference structure, in which the demand elasticities of income and prices are constant and proportional (as suggested by Pigou's Law). Matsuyama (2015, 2018) considers an even more general class of isoelastically nonhomothetic CES preferences, which allow to decouple the effects generated by income elasticity differences and those generated by price elasticity differences. As put forward by Bertoletti and Etro (2018) and Fally (2018), the CES preferences used by Matsuyama (2015, 2018) lead, similar to the Gorman-Pollak form preferences considered by Bertoletti and Etro (2017), to a "generalized separable" demand system, which has the nice property that other prices enter the demand functions through a common price index (see Pollak, 1972).

new insights for the aggregation of consumer demand over heterogeneous households, the preferences are useful for aggregating consumer demand over heterogeneous goods, and hence for solving a problem that is relevant for quantitative studies.

A final group of studies avoids problems from aggregating consumer demand over households with heterogeneous income levels by making the consumption decision a binary choice. For instance, Matsuyama (2000) imposes nonhomothetic '0-1' preferences into a Ricardian model of North-South trade with a continuum of goods and shows that acknowledging the nonhomotheticity of preferences changes the insights from an otherwise identical Dornbusch et al. (1977) model regarding the role of technological advancement, population growth, and income redistribution in the South on the terms-of-trade and welfare in the two economies.[7] Fajgelbaum et al. (2011) build on the preference structure proposed by Flam and Helpman (1987) and assume that households purchase one unit of a vertically differentiated good and allocate the rest of their expenditures on the consumption of a homogeneous outside good. Assuming that quality of the differentiated good and quantity of the homogeneous good are complements makes their preferences nonhomothetic, because the impact of income on indirect utility depends on the chosen quality of the differentiated good. To allow for monopolistic competition between firms producing horizontally differentiated varieties of the same quality level, Fajgelbaum et al. (2011) augment their discrete choice mechanism with a stochastic utility term (similar to McFadden, 1978), and they use this framework to provide a reasoning for the empirical observation that richer countries export goods of higher quality (see Hallak, 2010). Using PIGL preferences, we aggregate demand of heterogeneous households relying on a representative consumer and complement previous work on how differences in the level and/or dispersion of per-capita income shape trade in an open economy, by emphasizing the intensive margin through differences in the consumption level of luxuries.

Employing a mechanism of rent sharing, our model also contributes to a sizable literature dealing with firm-level wage setting in models of international trade. Recent examples to this literature include Davidson et al. (2008), Egger and Kreickemeier (2009, 2012), Helpman et al. (2010), Felbermayr et al. (2011), and Amiti and Davis (2012). Relying on individual bargaining between firms and a continuum of workers in a home-market model with two sectors of production makes the analysis in this paper akin to Helpman and Itskhoki (2010). In contrast to them, we consider homogeneous producers, because firm heterogeneity of the Melitz (2003)-type would complicate the analysis but not affect our results. Furthermore, we assume that rent sharing only exists in one sector, acknowledging the rich evidence on (persistent) inter-industry pay gaps (see Krueger and Summers, 1988; Blanchflower et al., 1996; Katz and Autor, 1999). Associating the sector featuring rent sharing with the sector producing luxuries captures the widespread view that employer characteristics are important determinants of these pay gaps (see Dickens and Katz, 1987;

Neary et al. (2017) introduce a demand system for which the elasticity of marginal revenue with respect to total revenue is constant. While having no direct link to other demand systems, it has the interesting property to be dual to the demand system derived from PIGL preferences in Neary and Mrázová (2017).

[7] Foellmi et al. (2018) consider a model with hierarchical '0-1' preferences and consumption indivisibilities to shed light on the role of per-capita income differences for explaining 'export zeros' observed in the world trade matrix. Wheras similar to Matsuyama (2000), their preferences allow for aggregation of consumer demand over heterogeneous households, they do not elaborate on income dispersion within countries.

Abowd et al., 2012). Finally, we abstract from search frictions and assume that workers who do not find a job in firms producing luxuries are employed in the production of necessities at the market clearing wage (see Bastos and Kreickemeier, 2009). We make this assumption, because we are not interested in employment effects per se, but want to shed light on how the reallocation of labor between sectors offering different wages alters the welfare effects of trade in a setting with nonhomothetic preferences.

The remainder of the paper is organized as follows. In Section 2.2, we set up the basic structure of our model and discuss the closed economy equilibrium. In Section 2.3, we study trade between two countries that are symmetric in all respects, except for the level and/or dispersion of per-capita income. There, we also discuss how differences in the level and/or dispersion of per-capita income affect trade structure and welfare in the open economy. In Section 2.4, we consider two extensions, in which we allow for positive assortative matching of workers featuring high effective labor supply with firms in the sector of luxuries and shed light on the differences between demand- and supply-side asymmetries. Section 2.5 concludes with a summary of our results.

2.2 The closed economy

We consider a static economy that is populated by a continuous set $\mathcal{H}$ of single-person households with Lebesgue measure H. In their role as workers, households inelastically supply labor input for the production of goods. Effective labor supply is household-specific and distributed over interval $[\underline{\lambda}, \overline{\lambda}]$ according to a continuously differentiable cumulative distribution function $L(\lambda)$. Ex ante differences in λ are an important factor of ex post differences in household income and consumption expenditures. Assuming that preferences do not have Gorman form, both the level and dispersion of income are decisive for the aggregate demand for two types of goods: *necessities*, n, which are homogeneous, and *luxuries*, ℓ, which are differentiated. However, the link between effective labor supply and household income is exacerbated by a price distortion in the labor market that makes wages sector-specific.

2.2.1 Preferences and household consumption

To establish a link between the distribution of household expenditures and aggregate demand, we consider price-independent generalized-linear (so-called "PIGL") preferences introduced by Muellbauer (1975, 1976), which can be represented by an indirect utility function of the following form

$$v(\mathbf{P}, e^i) = \frac{1}{\varepsilon}\left[\frac{e^i}{a(\mathbf{P})}\right]^{\varepsilon} + b(\mathbf{P}), \tag{2.1}$$

where $\mathbf{P}$ is a price vector, e^i is expenditure of household i and ε is a constant. The preferences specified in Eq. (2.1) do not entail an aggregation problem, because they allow to define a representative expenditure level such that a household with this expenditure level has the same value (expenditure) shares of consumption as the aggregate economy.[8] We consider a subclass of PIGL preferences and assume that households have preferences over two goods, which are represented by an indirect utility function of the following form:

$$v(P_n, P_\ell, e^i) = \frac{1}{\varepsilon}\left(\frac{e^i}{P_\ell}\right)^{\varepsilon} - \frac{\beta}{\varepsilon}\left(\frac{P_n}{P_\ell}\right)^{\varepsilon} - \frac{1-\beta}{\varepsilon}, \tag{2.2}$$

where P_n, P_ℓ are prices for goods n and ℓ, respectively, and $\varepsilon, \beta \in (0,1)$ is assumed. As explained by Boppart (2014) and formally shown in the Appendix, in contrast to more general forms of PIGL preferences, Eq. (2.2) has a closed form representation of the direct utility function, which proves to be useful for the computation of a proper price index if one of the goods is a composite of differentiated varieties that are sold under imperfect competition (see below). In the limiting cases of ε, the preferences in Eq. (2.2) correspond to two specifications widely used in the literature. If $\varepsilon \to 0$ preferences are Cobb-Douglas and therefore homothetic, delivering an indirect utility function of $v(P_n, P_\ell, e^i) = \ln\left[\frac{e^i}{P_n^\beta P_\ell^{1-\beta}}\right]$. If $\varepsilon \to 1$, preferences are quasilinear and therefore nonhomothetic, delivering an indirect utility function of $v(P_n, P_\ell, e^i) = \frac{e^i}{P_\ell} - \beta\frac{P_n}{P_\ell} - 1 + \beta$.

Applying Roy's identity to the indirect utility in Eq. (2.2), we can derive Marshallian demand functions for X_n^i and X_ℓ^i according to

$$X_n^i = \beta\left(\frac{e^i}{P_n}\right)^{1-\varepsilon} \quad \text{and} \quad X_\ell^i = \frac{e^i}{P_\ell}\left[1 - \beta\left(\frac{e^i}{P_n}\right)^{-\varepsilon}\right], \tag{2.3}$$

respectively. The Engel curve of good n is concave, making this good a necessity with its value share of consumption decreasing in the expenditure level. In contrast, the Engel curve for good ℓ is convex, making this good a luxury with its value share of consumption increasing in the expenditure level. In the limiting cases of $\varepsilon \to 0$ and $\varepsilon \to 1$ preferences have Gorman form and Engel curves are therefore linear in the expenditure level. To ensure that both goods are purchased by household i, it must be true that $e^i/P_n > \beta^{1/\varepsilon}$ and we impose a parameter constraint below that establishes this result.

That Engel curves for necessities and luxuries are differently shaped is the result of assuming that the respective goods enter the utility function asymmetrically. This asymmetry is justified in our model, because we assume that necessities are homogeneous, whereas luxuries are differentiated and can be aggregated to the composite discussed above according to

$$X_\ell^i = \left[\int_{\omega\in\Omega} x_\ell^i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}, \tag{2.4}$$

where $\sigma > 1$ is the constant elasticity of substitution between the differentiated varieties

[8] The term of generalized linearity has been introduced by Muellbauer (1975) to emphasize that the preferences are more general than the Gorman class which features consumption levels that are linear in expenditures, thereby making the value shares of consumption independent of the overall expenditure level. This property does not extend to other preference classes. However, generalized linear preferences accord with the weaker condition that the *ratio* of marginal value shares of any two goods are independent of the overall expenditure level. The notion of price independency is used by Muellbauer (1975) to express that the representative expenditure level, for which an individual household chooses the same value shares of consumption as the aggregate economy, is the same for all permissible prices.

$x_\ell^i(\omega)$ from set Ω. The price corresponding to the composite X_ℓ^i is an index of the prices of differentiated varieties, $p_\ell(\omega)$, and it is defined by the condition that $P_\ell X_\ell^i$ is equal to the household's overall expenditures for luxuries, $\int_{\omega\in\Omega} p_\ell(\omega)x_\ell^i(\omega)d\omega$. As formally shown in the Appendix, the respective price index features constant elasticity and is given by $P_\ell \equiv [\int_{\omega\in\Omega} p_\ell(\omega)^{1-\sigma}d\omega]^{\frac{1}{1-\sigma}}$. Using Roy's identity, we can then derive household demand for a single variety of the luxury good, ω, according to

$$x_\ell^i(\omega) = \frac{e^i}{P_\ell}\left(\frac{p_\ell(\omega)}{P_\ell}\right)^{-\sigma}\left[1-\beta\left(\frac{e^i}{P_n}\right)^{-\varepsilon}\right]. \tag{2.5}$$

Aggregating over all households, gives market demand functions

$$X_n = \int_{i\in\mathcal{H}} X_n^i di = \beta\frac{H\bar{e}}{P_n}\left(\frac{\bar{e}}{P_n}\right)^{-\varepsilon}\psi, \tag{2.6}$$

$$x_\ell(\omega) = \int_{i\in\mathcal{H}} x_\ell^i(\omega)di = \frac{H\bar{e}}{P_\ell}\left(\frac{p_\ell(\omega)}{P_\ell}\right)^{-\sigma}\left[1-\beta\left(\frac{\bar{e}}{P_n}\right)^{-\varepsilon}\psi\right], \tag{2.7}$$

where $\bar{e} \equiv H^{-1}\int_{i\in\mathcal{H}} e^i di$ is the average expenditure level of households and $\psi \equiv H^{-1}\int_{i\in\mathcal{H}} \left(e^i/\bar{e}\right)^{1-\varepsilon} di$ is a dispersion index that is defined on the unit interval and captures how the distribution of household expenditures affects the value shares of consumption. Since the Engel curve for necessities is concave, a more egalitarian distribution of expenditures, captured by a higher value of ψ, increases aggregate demand for necessities. The opposite is true for luxuries, which feature convex Engel curves. The dispersion index reaches a maximum level of one if the distribution of expenditures is egalitarian. An outcome with $\psi = 1$ is also reached if the distribution of household expenditure is irrelevant for aggregate demand because Engel curves are linear, as in the limiting cases of $\varepsilon \to 0$ and $\varepsilon \to 1$.

2.2.2 Technology and the firms' problem

The technology to produce necessities is linear in labor input and we assume that one unit of labor produces one unit of output. Firms producing necessities enter the market at zero cost, hire labor input at a common wage rate w, and sell their output under perfect competition. This establishes $w = P_n$. Production of luxuries requires the creation of workplaces at the costs of one unit of necessities for each labor input employed. One unit of labor input used in a workplace produces one unit of output. To start production firms must develop a blueprint, which comes at the cost of f units of necessities and gives them access to a unique variety, which they can sell under monopolistic competition.

Workers are free to move between sectors up to the point where all workplaces in the sector of luxuries are filled. Then, in each workplace workers and firms form a bilateral monopoly and they distribute the production surplus under Stole and Zwiebel (1996) bargaining.[9] Hiring and wage setting in the sector of luxuries can be understood as a

[9] It has been recently pointed out by Bruegemann et al. (2018) that the microeconomic foundation of the Stole and Zwiebel (1996) bargaining protocol does not give wage and profit profiles that coincide with the Shapley values. However, relying on a Rolodex Game instead of the non-cooperative game put forward by Stole and Zwiebel, one can restore equivalence of the bargaining outcome with the Shapley values.

two-stage process and solved through backward induction. Looking first at the bargaining problem, we can note that its solution is characterized by two conditions: a splitting rule, determining how the production surplus achieved by an agreement is distributed between the bargaining parties; and an aggregation rule, describing how infra-marginal production surpluses add up to the firm's overall surplus from multilateral bargaining with all of its workers. The bargaining problem considered here is exacerbated by the heterogeneity of workers in their effective labor supply. To facilitate the analysis we assume for now that the number of different worker types employed by the firm is discrete and given by J, where firm index ω is suppressed because the hiring and bargaining problem is the same for all producers.

The mass of employees of type j is N_j and the firm's overall surplus from multilateral bargaining with a mass of $N \equiv \sum_{j=1}^{J} N_j$ workers is given by

$$\pi = \int_0^N \mu(\nu|N)\hat{r}(\nu\mathbf{s})d\nu, \tag{2.8}$$

where s_j is calculated as a product of the type-specific effective labor supply λ_j and the pre-determined fraction of employed workers of type j, N_j/N, while $\mathbf{s}$ is the set of resulting s_j-values: $\mathbf{s} \equiv \{s_1, ..., s_J\}$. Furthermore, $\hat{r}(\nu\mathbf{s}) = D^{\frac{1}{\sigma}}Q(\nu\mathbf{s})^{1-\frac{1}{\sigma}}$ are revenues achieved for employment level ν, $D \equiv H\bar{e}(1-\beta(\bar{e}/P_n)^{-\varepsilon}\psi)/P_\ell^{1-\sigma}$ is a common demand shifter, $Q(\cdot)$ is a function determining how the different types of labor are aggregated in the production process,[10] and

$$\mu(\nu|N) \equiv \frac{\eta}{\nu}\left(\frac{\nu}{N}\right)^\eta \tag{2.9}$$

is a probability measure that depends on the firm's *relative* bargaining power $\eta > 0$ and determines the fraction of infra-marginal production surplus the firm can acquire in its wage negotiations with workers. Solving the integral in Eq. (2.8) gives

$$\pi = \frac{\eta\sigma}{\eta\sigma+\sigma-1}D^{\frac{1}{\sigma}}\left(\sum_{j=1}^{J}\lambda_j N_j\right)^{1-\frac{1}{\sigma}} = \frac{\eta\sigma}{\eta\sigma+\sigma-1}\hat{r}(N\mathbf{s}), \tag{2.10}$$

where the first equality sign uses the assumption that the labor input of different worker types is perfectly substitutable, so that $Q(\nu\mathbf{s}) = \nu\sum_{j=1}^{J}\lambda_j N_j/N$.

Since workers forfeit their chance to move to the other sector when accepting the job offer of a firm producing luxuries, they give up their outside income opportunities from employment elsewhere. Therefore, the splitting rule determining how to distribute the production surplus between the firm and its workers can be expressed as

$$\frac{\partial\pi}{\partial N_j} = \eta\hat{w}_\ell^j \tag{2.11}$$

where $\hat{w}_\ell^j$ is labor income of a worker with effective labor supply λ_j. Eqs. (2.10) and (2.11) establish the intuitive result that the wage per unit of labor input, $\hat{w}_\ell^j/\lambda_j$, is the same for

[10] Under non-increasing returns to scale at the firm level, we have $Q'(\cdot) > 0$, $Q''(\cdot) \leq 0$ and thus $\int_0^N \hat{r}(\nu\mathbf{s}) - \hat{r}(N\mathbf{s})d\nu \geq 0$.

all workers, irrespective of their effective labor supply: $\hat{w}^j_\ell/\lambda_j \equiv w_\ell$. Taking stock, we can summarize the solution to the firm's bargaining problem by the two equations

$$\pi = \kappa r, \qquad \frac{\partial \pi}{\partial q_\ell} = \frac{\sigma - 1}{\sigma}\frac{\kappa r}{q_\ell} = \eta w_\ell, \tag{2.12}$$

where $q_\ell \equiv \sum_{j=1}^{J} \lambda_j N_j$ denotes total labor input of the firm, $r \equiv D^{\frac{1}{\sigma}} q_\ell^{1-\frac{1}{\sigma}}$ gives revenues as a function of labor input, q_ℓ (instead of the number of employed workers N), and $\kappa \equiv \eta\sigma/(\eta\sigma + \sigma - 1) < 1$ is the constant fraction of revenues accrued by the firm in the wage bargaining with workers, which is increasing in the firm's relative bargaining power η.[11]

Equipped with Eq. (2.12), we can now determine the solution to the firm's hiring problem. Recollecting from above that firms have to invest f units of necessities to start production and one unit of necessities to install workplace capacity for each labor input, this solution is found by maximizing $\Pi \equiv \pi - P_n q_\ell - P_n f$ with respect to q_ℓ. Since firms face the same cost for each unit of labor input, they are indifferent between all applicants and hire a workforce whose composition mirrors the economy-wide distribution of effective labor supply.[12] The first-order condition for the firm's profit-maximizing q_ℓ choice is given by

$$\frac{d\Pi}{dq_\ell} = \frac{\sigma - 1}{\sigma}\frac{\kappa r}{q_\ell} - P_n = 0. \tag{2.13}$$

Substituting Eq. (2.12) and accounting for the definition of profits, then gives the outcome of hiring and wage-setting for firms producing luxuries:

$$w_\ell = \alpha P_n, \qquad \Pi = \frac{\kappa r}{\sigma} - P_n f, \tag{2.14}$$

where $\alpha \equiv \eta^{-1}$ gives the relative bargaining power of workers in the wage negotiation with the firm. Eq. (2.14) has been derived under the assumption that firms producing luxuries can attract the intended mass of applicants at a wage rate αP_n. This requires that employment at these firms promises a wage at least as high as w in order to convince workers to accept the job offer. Hence, the wage paid in the sector of necessities establishes a participation constraint for workers seeking employment in the sector of luxuries, so that $\alpha \geq 1$ is needed to ensure that at least some of the workplaces installed by firms producing luxuries are filled. If $\alpha > 1$, jobs in the sector of luxuries promise a wage premium, and hence every household prefers working there. This outcome, which we consider in the subsequent analysis, can only be consistent with diversified production in both sectors, if

[11] To determine the solution of wage bargaining for a continuous set of labor types, we can first consider a symmetric J-division of the support of effective labor supply $[\underline{\lambda}, \overline{\lambda}]$ and denote the density of effective labor supplies on the respective subdivisions by $\ell(\lambda_j)$. This establishes the Riemann sum: $\sum_1^J \lambda_j \ell(\lambda_j)\Delta\lambda_j$, with $\Delta\lambda_j \equiv \lambda_j - \lambda_{j-1}$. Taking the limit, then gives $\lim_{J\to\infty} \sum_{j=1}^{J} \lambda_j \ell(\lambda_j)\Delta\lambda_j = \int_{\underline{\lambda}}^{\overline{\lambda}} \lambda dL(\lambda)$, where $\ell(\lambda) = L'(\lambda) = dL(\lambda)/d\lambda$ has been considered.

[12] The assumption that firms have to pay the same workplace installment costs for each unit of labor input facilitates our analysis. Under the alternative assumption that firms install the same workplace capacity for each employee, irrespective of her effective labor supply, our model would generate an incentive for screening the applicants in order to improve the average composition of production workers (see Helpman et al., 2010). We discuss this case in an extension of our model.

some of the workers are not hired by luxury producers and therefore are forced to move to the production of necessities. Of course, these workers want to underbid w_ℓ. However, underbidding cannot be successful if wage offers at stage one are not contractible. Without a binding contract, successful applicants will rationally deviate from their initial offer to accept a wage discount and opt for the highest wage they can achieve in the bargaining with the firm, exploiting the protection from the bilateral monopoly that is established after the workplaces have been filled. Hence, our model generates wage differences due to a market imperfection that is rooted in information asymmetry and the irreversibility of the firm's hiring decision. Since firms can freely enter the sector of luxuries, they must make zero profits in equilibrium, which establishes the zero-profit condition $\kappa r = \sigma P_n f$, according to Eq. (2.14).

2.2.3 The general equilibrium

Household consumption expenditures are equal to labor income and heterogeneous for two reasons: due to ex ante (and thus exogenous) differences in effective labor supply; and due to differences in the wages paid by the sectors producing necessities and luxuries. The worker achieving the lowest income has an ability level of $\underline{\lambda}$ and is employed in the sector producing necessities, yielding an expenditure level of $\underline{\lambda} w$. Since the sector of necessities pays a wage of $w = P_n$, the minimum permissible expenditure level necessary for consuming necessities as well as luxuries then establishes a threshold level for effective labor supply, $\beta^{1/\varepsilon}$, that must be passed in order to ensure that even the poorest households attribute some of their expenditures to the consumption of luxuries. To exclude corner solutions and to focus on changes along the intensive margin of consumption, we assume throughout our analysis that $\underline{\lambda} > \beta^{1/\varepsilon}$.

Nominal per-capita (labor) income is equal to average household expenditures and given by

$$\bar{e} = w(1 - h_\ell) \int_{\underline{\lambda}}^{\overline{\lambda}} \lambda dL(\lambda) + w_\ell h_\ell \int_{\underline{\lambda}}^{\overline{\lambda}} \lambda dL(\lambda) = w\Lambda \left[1 + h_\ell(\alpha - 1)\right], \tag{2.15}$$

where $\Lambda \equiv \int_{\underline{\lambda}}^{\overline{\lambda}} \lambda dL(\lambda)$ is the average effective labor supply of households and h_ℓ is the fraction of workers employed in the production of luxuries, receiving wage premium α. The dispersion index, measuring how the distribution of household expenditures affects aggregate consumer demand, can be computed according to

$$\psi = \left(\frac{w}{\bar{e}}\right)^{1-\varepsilon} \left[1 + h_\ell \left(\alpha^{1-\varepsilon} - 1\right)\right] \int_{\underline{\lambda}}^{\overline{\lambda}} \lambda^{1-\varepsilon} dL(\lambda) = \frac{1 + h_\ell \left(\alpha^{1-\varepsilon} - 1\right)}{\left[1 + h_\ell(\alpha - 1)\right]^{1-\varepsilon}} \psi_\lambda, \tag{2.16}$$

where $\psi_\lambda \equiv \int_{\underline{\lambda}}^{\overline{\lambda}} (\lambda/\Lambda)^{1-\varepsilon} dL(\lambda)$ is a measure of the dispersion of effective labor supply. In the limiting case of $\alpha = 1$, producers of luxuries pay the market clearing wage $w_\ell = w$, implying that the dispersion of labor income in Eq. (2.16) is pinned down and fully determined by the exogenous dispersion of effective labor supply, ψ_λ. If $\alpha > 1$, firms producing luxuries pay a wage premium, which amplifies the dispersion of labor income: $\psi < \psi_\lambda$. The value of the dispersion index depends in a nonmonotonic way on the share of workers employed for producing luxuries, h_ℓ.[13]

To determine the fraction of workers receiving a wage premium, h_ℓ, we can combine two preliminary results from our analysis. First, as a consequence of constant markup pricing the wage bill paid by firms is a constant fraction $\frac{\sigma-1}{\sigma}\kappa = \frac{\sigma-1}{\sigma+\alpha(\sigma-1)}$ of their revenues. This generates a positive link between the share of workers and the mass of firms producing luxuries:

$$h_\ell H\Lambda w = \frac{\sigma-1}{\sigma}\kappa M r. \tag{2.17}$$

Since we have assumed that workers who do not find employment in the production of luxuries can move to the sector of necessities at zero cost, there is no involuntary unemployment in our model, and the fraction of workers employed in the production of necessities is therefore given by $1-h_\ell$. The second preliminary result is the goods market clearing condition for luxuries, which can be derived from Eq. (2.7) according to

$$H\Lambda w\left[1-\beta\Lambda^{-\varepsilon}\psi_\lambda\right] + h_\ell H\Lambda w\frac{\sigma}{\sigma-1}\frac{B}{\kappa} = Mr, \tag{2.18}$$

where $B \equiv \frac{\sigma-1}{\sigma}\kappa\left[(\alpha-1)-(\alpha^{1-\varepsilon}-1)\beta\Lambda^{-\varepsilon}\psi_\lambda\right]$, with $\lim_{\alpha\to 1} B = 0$, $dB/d\alpha > 0$, and $\lim_{\alpha\to\infty} B = 1$, captures how the existence of a wage premium in the sector of luxuries augments economy-wide expenditures for luxuries. In the limiting case of $\alpha = 1$ labor income does not depend on the allocation of workers and the market size for luxuries is therefore pinned down by the level and dispersion of effective labor supply. This makes M in Eq. (2.18) independent of h_ℓ. For $\alpha > 1$, employment in the sector of luxuries promises a wage premium, so that a higher fraction of workers allocated to the production of luxuries provides a positive effect on the market size and thus a stimulus for firm entry in this sector. In this case, the goods market clearing condition in Eq. (2.18) establishes a positive link between h_ℓ and M. Substituting zero-profit condition $\sigma P_n f = \kappa r$, Eqs. (2.17) and (2.18) can be combined to get explicit solutions for the mass of firms and the fraction of workers producing luxuries:

$$M = \frac{\kappa}{1-B}\frac{H\Lambda\left[1-\beta\Lambda^{-\varepsilon}\psi_\lambda\right]}{\sigma f}, \qquad h_\ell = \frac{\sigma-1}{\sigma}\frac{\kappa}{1-B}[1-\beta\Lambda^{-\varepsilon}\psi_\lambda]. \tag{2.19}$$

Both a higher average level of effective labor supply, (a higher Λ), and a higher dispersion of this supply, (a lower ψ_λ), cause an increase in M and h_ℓ, because wealthier households attribute a higher fraction of their expenditures to luxuries if preferences do not have Gorman form. Furthermore, noting $\frac{\kappa}{1-B} = \{1+\frac{\sigma-1}{\sigma}[1+(\alpha^{1-\varepsilon}-1)\beta\Lambda^{-\varepsilon}\psi_\lambda]\}^{-1} < 1$, it follows from Eq. (2.19) that a higher wage premium α reduces both the mass of firms and the fraction of workers producing luxuries. This is intuitive, because a higher α reflects a

[13] In the limiting case $h_\ell = 0$, there is no one employed in the sector of luxuries making wage premium α irrelevant and establishing $\psi = \psi_\lambda$. In the limiting case of $h_\ell = 1$ all workers are employed in the production of luxuries and receive the wage premium, again resulting in $\psi = \psi_\lambda$. Dispersion index ψ is u-shaped and reaches a minimum at

$$h_\ell^{min} \equiv \frac{(1-\varepsilon)(\alpha-1)-(\alpha^{1-\varepsilon}-1)}{\varepsilon(\alpha-1)(\alpha^{1-\varepsilon}-1)} \in (0,1).$$

stronger bargaining power of workers and is therefore associated with a lower fraction of revenues accrued by firms in their wage negotiations, making production of luxuries less attractive for them. However, the finding that a lower fraction of workers is employed in the sector producing luxuries does not imply that less labor income is generated there. The increase in wage premium α that is responsible for the fall in h_ℓ implies a wage stimulus for those workers who continue to produce luxuries. Differentiating $h_\ell(\alpha - 1)$ reveals that a higher wage premium α leads to higher per-capita income $\overline{e}$ and therefore increases economy-wide expenditures for luxuries, $H\Lambda w[1 - \beta\Lambda^{-\varepsilon}\psi_\lambda]/(1 - B)$. The positive market size effect from higher labor income is counteracted and dominated by a labor cost increase for firms producing luxuries, so that higher economy-wide expenditures are consistent with lower levels of M and h_ℓ. Using Eq. (2.19) and the constant markup-pricing rule $p_\ell = \frac{\sigma}{\sigma-1}\frac{w}{\kappa}$ in the definition of the price index of luxuries, we can compute

$$P_\ell = \frac{\sigma}{\sigma - 1}\frac{w}{\kappa}\left[\frac{\kappa}{1 - B}\frac{H\Lambda\left(1 - \beta\Lambda^{-\varepsilon}\psi_\lambda\right)}{\sigma f}\right]^{\frac{1}{1-\sigma}}. \tag{2.20}$$

A higher wage premium α exerts two reinforcing effects on price index P_ℓ. It increases labor costs and therefore the price charged by firms producing luxuries, p_ℓ, and it induces firm exit and thus reduces the mass of available varieties, M, thereby further increasing P_ℓ.

2.2.4 Welfare in the closed economy

We postulate a Bergson-Samuelson social welfare function that is equal to the average indirect utility of households. Accounting for Eq. (2.2), social welfare is then given by[14]

$$V(P_n, P_\ell, \overline{e}, \hat{\psi}) \equiv \frac{1}{\varepsilon}\left(\frac{P_n}{P_\ell}\right)^{\varepsilon}\left[\left(\frac{\overline{e}}{P_n}\right)^{\varepsilon}\hat{\psi} - \beta\right] - \frac{1-\beta}{\varepsilon}, \tag{2.21}$$

where $\hat{\psi} \equiv H^{-1}\int_{i\in\mathcal{H}}\left(e^i/\overline{e}\right)^{\varepsilon} di$ is an inverse measure of income dispersion that is defined on the unit interval and linked to the dispersion of effective labor supply, $\hat{\psi}_\lambda \equiv \int_{\underline{\lambda}}^{\overline{\lambda}}(\lambda/\Lambda)^{\varepsilon} dL(\lambda)$, according to $\hat{\psi} = \hat{\psi}_\lambda[1 + h_\ell(\alpha^{\varepsilon} - 1)]/[1 + h_\ell(\alpha - 1)]^{\varepsilon}$. The two dispersion indices ψ and $\hat{\psi}$ are closely related but nonetheless different, except for the limiting case of $\varepsilon = 1/2$. For a welfare analysis, it is useful to distinguish direct effects through changes in the average level and dispersion of nominal income from indirect effects through adjustments in the price index of luxuries caused by these changes. Furthermore, to facilitate our analysis and to distinguish the different effects that price distortions in the product and labor market have in our setting, we first look at the limiting case of $\alpha = 1$, which yields $\overline{e} = \Lambda w$, $\psi = \psi_\lambda$, and $\hat{\psi} = \hat{\psi}_\lambda$. For this limiting case, a higher nominal level of per-capita

[14] Giving equal weight to all households, we take a utilitarian perspective. Social welfare under this perspective differs from the indirect utility of the household with a representative expenditure level. The (price-invariant) representative level of expenditure is defined by Muellbauer (1975) to ensure that a household with this expenditure level has the same value shares of consumption as the aggregate economy and it is given by $e_r \equiv \overline{e}\psi^{-1/\varepsilon}$, according to Eqs. (2.3) and (2.6). Substituting into Eq. (2.2) establishes $v(P_n, P_\ell, e_r) = \frac{1}{\varepsilon}\left(\frac{P_n}{P_\ell}\right)^{\varepsilon}\left[\left(\frac{\overline{e}}{P_n}\right)^{\varepsilon}\psi^{-1} - \beta\right] - \frac{1-\beta}{\varepsilon}$ and, due to $\psi, \hat{\psi} < 1$, a utility level that is larger than social welfare in Eq. (2.21). Since the representative consumer from Muellbauer (1975, 1976) lacks a normative interpretation, it cannot be used for welfare analysis.

income is rooted in a higher average level of effective labor supply and has unambiguously positive welfare effects, because it allows for additional purchases of necessities and luxuries at given prices and, at the same time, lowers the price index of luxuries through firm entry.

In contrast, the welfare effect of a higher nominal income dispersion, which is due to a higher dispersion of effective labor supply, is a priori not clear. On the one hand, the welfare function in Eq (2.21) features a social aversion to income inequality, and a transfer from a wealthier household to a poorer one that does not change their income ranking reduces inequality and therefore increases social welfare (cf. Dalton, 1920).[15] On the other hand, the incentive to harmonize income is counteracted by a distortion of the resource allocation that exists because households devote part of their expenditures to necessities, which makes, all other things equal, the mass of firms entering inefficiently small from a social planner's point of view. As pointed out by Dhingra and Morrow (2016) this allocational inefficiency exists because the markups charged in the two industries differ. Introducing a transfer from poor to rich people would increase demand for luxuries and therefore provide a (partial) remedy for the misallocation of resources, leading to a fall in the price index of luxuries.

To gain further insights into the relative strength of the two counteracting effects, we can evaluate the social welfare function $V(\cdot)$ at $\varepsilon = 1/2$, which establishes $\psi = \hat{\psi}$. Noting further that $\alpha = 1$ yields $\bar{e} = \Lambda P_n$ and $\psi = \psi_\lambda$, the welfare effects of lower income dispersion (a higher ψ_λ) are then given by

$$\frac{dV(P_n, P_\ell, \Lambda P_n, \psi_\lambda)}{d\psi_\lambda} \equiv \sqrt{\frac{P_n}{P_\ell}\frac{\Lambda}{(\sigma-1)^2}}\left[2\sigma - 1 - \frac{1-\left(\beta/\sqrt{\Lambda}\right)^2}{1-(\beta/\sqrt{\Lambda})\psi_\lambda}\right]. \tag{2.22}$$

From Eq. (2.22), positive welfare effects of lower income dispersion are more likely ceteris paribus if σ is large.[16] This is intuitive, because higher levels of σ reduce the price markup charged by monopolistically competitive firms producing luxuries, which lowers the problem of resource misallocation due to distorted market entry. Also, lower income dispersion increases welfare if β is sufficiently small. In the limiting case of $\beta \to 0$ the model degenerates to a one-sector economy, in which only luxuries are produced, making aggregate demand independent of the distribution of income.

If $\alpha > 1$ the two counteracting effects described above are augmented by rent sharing between firms and workers in the sector producing luxuries. As outlined above, the existence of a wage premium makes entry less attractive and increases the price index of luxuries, with a negative indirect effect on social welfare. This indirect effect is counteracted by a direct effect on social welfare, which exists, because rent sharing leads to an

[15] Dispersion index $\hat{\psi}$ is a negative monotonic transformation of the well-known Atkinson (1970) index. In our setting, the evaluation of income inequality is, however, not the result of giving worse-off households higher weights in the welfare function, as suggested by a prioritarian view on distributional justice (cf. Parfit, 1997). Rather, social inequality aversion is the result of non-Gorman form preferences under a utilitarian perspective.

[16] In the limiting case of $\sigma \to 1$, Eq. (2.22) yields $dV(\cdot)/d\psi_\lambda >, =, < 0$ if $\beta >, =, < \Lambda^{1/2}\int_{\underline{\lambda}}^{\overline{\lambda}}(\lambda/\Lambda)^{1/2}dL(\lambda)$. For $\varepsilon = 1/2$, condition $\underline{\lambda} > \beta^{1/\varepsilon}$ gives $\beta < \int_{\underline{\lambda}}^{\overline{\lambda}}\lambda^{1/2}dL(\lambda)$ and thus $dV(\cdot)/d\psi_\lambda < 0$ if $\sigma \to 1$. In contrast, $dV(\cdot)/d\psi_\lambda > 0$ holds for sufficiently high levels of σ.

increase in the average level and dispersion of nominal income. In the limiting case of Cobb-Douglas preferences, it is the first effect that dominates. Since the mass of firms choosing to produce luxuries is already below the social optimum without rent sharing (cf. Benassy, 1996), a further decrease in the mass of firms producing luxuries due to an increase in α is detrimental for social welfare. In the limiting case of quasilinear preferences, firm entry is socially optimal without rent sharing. Since the increase in labor income triggered by rent sharing leads to an equally strong increase in the expenditures for luxuries, the direct and indirect effect cancel, leaving social welfare unaffected. Finally, if preferences do not have Gorman form and Engel curves are therefore nonlinear, we cannot rule out that social welfare is higher with than without rent sharing (see the Appendix).

We complete the discussion of the closed economy by elaborating on a crucial difference between direct and indirect effects regarding the consequences that changes in nominal income have on individual households. Whereas the direct effect of such changes is household-specific, the indirect effect due to adjustments of the price index of luxuries is the same for all households, provided that $\underline{\lambda} > \beta^{1/\varepsilon}$ induces even the consumer with the lowest income to purchase luxuries. This is a result of indirect utility in Eq. (2.2) being an isoelastic function of price index P_ℓ and it has important consequences for the welfare effects of trade in our setting. Households can only be differently affected by trade if moving to the open economy exerts asymmetric effects on nominal income, which is only possible in turn if rent sharing leads to a wage premium in the sector of luxuries.

2.3 The open economy

In the open economy, we consider trade between two countries that are symmetric in all respects, except for the average level and/or dispersion of effective labor supply.[17] Trade in necessities is free of costs, and hence wage w is the same in the two economies, provided that production remains diversified in both locations. We discuss the parameter domain supporting diversification below. Trade in luxuries is subject to iceberg trade costs, implying that $t^{\frac{1}{\sigma-1}} > 1$ units of the good must be shipped in order for one unit to arrive in the foreign country.

Total domestic plus export revenues of firms in the two countries are linked by the zero-profit conditions, $\kappa r = \sigma P_n f$, $\kappa r^* = \sigma P_n f$, where an asterisk is used to indicate variables of the foreign economy. Since production costs are the same in the two countries, the zero-profit conditions link the differences in the price indices for luxuries to differences in the expenditures for these goods according to

$$\rho\zeta = \left(\frac{P_\ell}{P_\ell^*}\right)^{\sigma-1}, \tag{2.23}$$

[17] We set $H\Lambda = H^*\Lambda^*$, because the effects of differences in total labor endowments are well understood from Helpman and Krugman (1985) and because they are similar for homothetic and nonhomothetic preferences.

with

$$\rho \equiv \frac{1-\beta\,(\Lambda^*)^{-\varepsilon}\,\psi_\lambda^*}{1-\beta\,(\Lambda)^{-\varepsilon}\,\psi_\lambda} \quad \text{and} \quad \zeta \equiv \frac{1+\frac{\sigma}{\sigma-1}\frac{1}{\kappa}h_\ell^* B^*/[1-\beta(\Lambda^*)^{-\varepsilon}\psi_\lambda^*]}{1+\frac{\sigma}{\sigma-1}\frac{1}{\kappa}h_\ell B/[1-\beta\Lambda^{-\varepsilon}\psi_\lambda]}. \tag{2.24}$$

B^* and P_ℓ^* are defined in analogy to the respective variables at home. Parameter $\rho \neq 1$ reflects relative differences of the two countries in their expenditures for luxuries that are due to ex ante differences in the average level and/or dispersion of effective labor supply, whereas ζ captures a magnification ($\zeta > 1$) or diminution ($\zeta < 1$) of these differences due to endogenous reallocations of labor and thus changes in the nominal income of households if $\alpha > 1$. The combined term $\rho\zeta$ captures foreign's relative market size for luxuries and, as explained in detail below, it is larger (smaller) than one if $\Lambda^* > (<)\Lambda$ and/or $\psi_\lambda > (<)\psi_\lambda^*$.

To determine factor allocation and production structure in a diversification equilibrium, we can rely on the insight from the closed economy, that the constant markup pricing rule establishes a positive link between the fraction of workers and the mass of local firms producing luxuries. For home, the respective link is given by Eq. (2.17), whereas for foreign an analogous link can be derived according to

$$h_\ell^* H\Lambda w = \frac{\sigma-1}{\sigma}\kappa M^* r^*, \tag{2.25}$$

where $H\Lambda = H^*\Lambda^*$ has been considered. A second link between the fraction of workers and the mass of local firms producing luxuries is obtained from the market clearing conditions of luxuries, which for home and foreign are given by

$$H\Lambda w\left[1-\beta\Lambda^{-\varepsilon}\psi_\lambda\right] + h_\ell H\Lambda w\frac{\sigma}{\sigma-1}\frac{1}{\kappa}B = Mr\frac{t}{1+t} + M^* r^*\frac{1}{1+t}, \tag{2.26}$$

$$H\Lambda w\left[1-\beta(\Lambda^*)^{-\varepsilon}\psi_\lambda^*\right] + h_\ell^* H\Lambda w\frac{\sigma}{\sigma-1}\frac{1}{\kappa}B^* = M^* r^*\frac{t}{1+t} + Mr\frac{1}{1+t}, \tag{2.27}$$

respectively, where $t/(1+t)$ is the share of revenues that is due to domestic sales and $1/(1+t)$ is the share of revenues that is due to exports.

Combining Eqs. (2.17), (2.25)-(2.27) and acknowledging $r = r^*$, we can solve for three functional relationships between the three endogenous variables h_ℓ, h_ℓ^*, and $\mu \equiv M^*/M$ in general equilibrium. The first relationship is obtained from substituting h_ℓ and h_ℓ^* from Eqs. (2.17) and (2.25) into Eqs. (2.26) and (2.27), respectively, dividing the two resulting expressions, and solving for μ:

$$\mu = \frac{\rho[t-B(1+t)]-1}{t-B^*(1+t)-\rho} = \frac{1}{\underline{\rho}(t)}\frac{\rho-\underline{\rho}(t)}{\overline{\rho}(t)-\rho} \equiv \tilde{\mu}(\rho), \tag{2.28}$$

with $\underline{\rho}(t) = [t-B(1+t)]^{-1}$ and $\overline{\rho}(t) = t-B^*(1+t)$. The link between ρ and μ established by Eq. (2.28) is positive, $d\mu/d\rho > 0$, and depicted by the upper left panel of Figure 2.1. The equilibrium value of μ in Eq. (2.28) is independent of the realizations of h_ℓ, h_ℓ^*, provided that these realizations support diversified production in the two economies. The permissible range of ρ supporting production of luxuries in both countries is given by interval $\left(\underline{\rho}(t), \overline{\rho}(t)\right)$. Noting that $\lim_{t\to\infty}\underline{\rho}(t) = 0$ and $\lim_{t\to\infty}\overline{\rho}(t) = \infty$, we can conclude that an interval of permissible levels of ρ exists if transport costs are sufficiently high. The parameter range supporting production of necessities in both countries is discussed below.

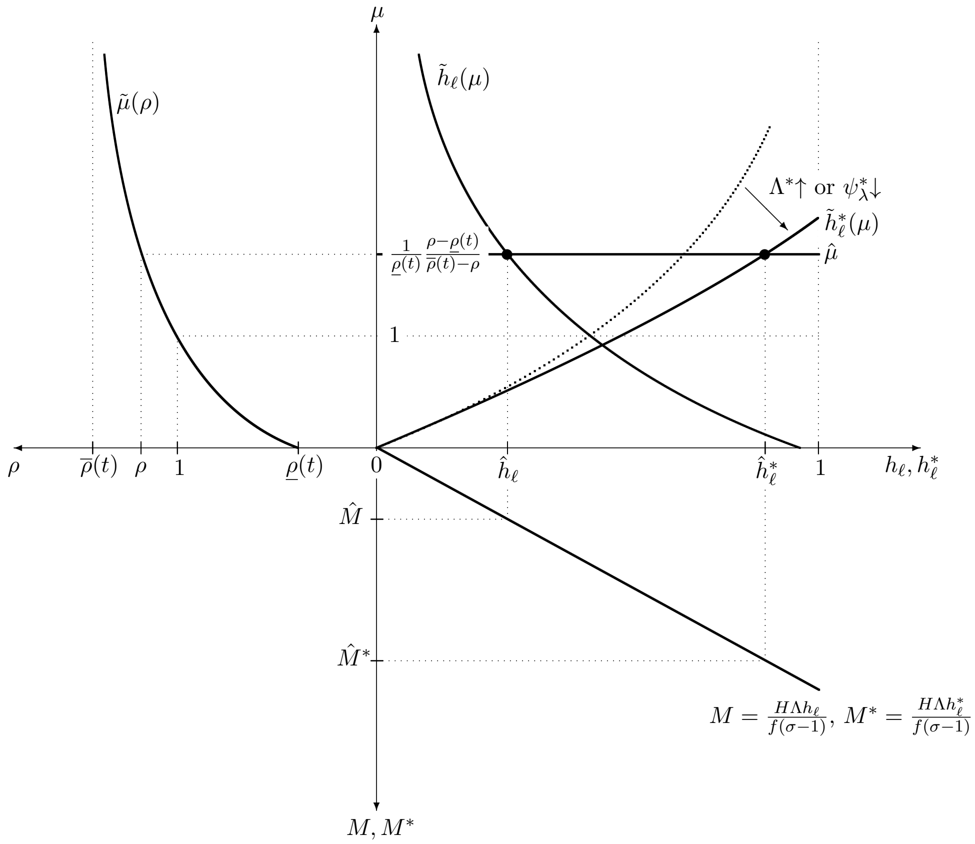

Figure 2.1: Equilibrium in the open economy for $\rho > 1$

Furthermore, substituting Mr from Eq. (2.17) into Eq. (2.26) and substituting M^*r^* from Eq. (2.25) into Eq. (2.27), we can solve for

$$h_\ell = \frac{\sigma-1}{\sigma}\kappa\frac{1-\beta\Lambda^{-\varepsilon}\psi_\lambda}{[\mu+t]/[1+t]-B} \equiv \tilde{h}_\ell(\mu), \tag{2.29}$$

$$h^*_\ell = \frac{\sigma-1}{\sigma}\kappa\frac{1-\beta(\Lambda^*)^{-\varepsilon}\psi^*_\lambda}{[1+\mu t]/[\mu(1+t)]-B^*} \equiv \tilde{h}^*_\ell(\mu), \tag{2.30}$$

respectively. Eq. (2.29) establishes a negative link between the fraction of workers producing luxuries in home, h_ℓ, and firm ratio μ: $dh_\ell/d\mu < 0$. This is intuitive, because a higher fraction of firms located abroad implies that fewer workers are employed for the production of luxuries at home. Eq. (2.30) establishes a positive link between the fraction of workers producing luxuries abroad, h^*_ℓ, and firm ratio μ: $dh^*_\ell/d\mu > 0$. A higher fraction of firms located abroad implies that more workers are hired for the production of luxuries, there. The functional relationships between firm ratio μ and the fraction of local employment in the sector of luxuries are depicted in the upper right panel of Figure 2.1. In the lower right panel of Figure 2.1, we add an additional locus that shows how changes in labor allocation reflected by changes in h_ℓ and h^*_ℓ are related to changes in the mass of firms producing luxuries in the two economies, M and M^*. The locus is obtained from substituting zero-profit condition $\kappa r = \sigma P_n f$ and $w = P_n$ into Eq. (2.17), and the functional relationship between M and h_ℓ established by this equation is the same as the functional relationship between M^* and h_ℓ^* established by Eq. (2.25), provided that total labor endowments do not differ in the two economies.

The open economy equilibrium is characterized by the intersection of the negatively sloped locus $\tilde{h}_\ell(\mu)$ and the positively sloped locus $\tilde{h}_\ell^*(\mu)$ with the horizontal μ-line in the upper right panel of Figure 2.1. In the case of symmetric countries with $\rho = 1$, $\tilde{h}_\ell(\mu)$ and $\tilde{h}_\ell^*(\mu)$ intersect at $\mu = 1$, implying that the fraction of workers and the number of firms producing luxuries is the same in the two economies. An increase in the average level or dispersion of effective labor supply abroad ($\Lambda^* > \Lambda$ or $\psi_\lambda^* < \psi_\lambda$) leads to a higher level of ρ, because foreign expenditures for luxuries increase relative to domestic ones. At the same time, the $\tilde{h}_\ell^*(\mu)$-locus rotates clockwise, because a higher demand for luxuries requires for a given firm ratio μ higher labor input in order to produce the quantity of luxuries necessary for market clearing. Because a larger fraction of firms chooses to enter the now bigger foreign market after the increase in ρ, there is a second-round effect on the fraction of workers producing luxuries, which causes a further expansion in the foreign labor input and a decline in the domestic labor input used for the production of luxuries. This second-round adjustment is captured by movements along the $\tilde{h}_\ell(\mu)$-locus and the rotated $\tilde{h}_\ell^*(\mu)$-locus in the upper right panel of Figure 2.1. From the lower right panel, we furthermore see that the decrease in the fraction of workers induces a decrease in the mass of firms producing luxuries at home, whereas the increase in the fraction of workers leads to an increase in the mass of firms producing luxuries abroad.

From Figure 2.1, we also see that considering a permissible value of ρ is not sufficient to guarantee diversification of production. A positive production level of necessities in both economies requires in addition that labor allocation respects $h_\ell, h_\ell^* < 1$. The formal conditions that guarantee positive production levels of necessities at home and abroad for permissible values of ρ can be derived from substituting Eq. (2.28) into Eqs. (2.29) and (2.30), and they are given by

$$\kappa \frac{\sigma - 1}{\sigma} \left[1 - \beta \Lambda^{-\varepsilon} \psi_\lambda\right] < 1 - B + \frac{\rho(1 - B) - (1 - B^*)}{\overline{\rho}(t) - \rho}, \tag{2.31}$$

$$\kappa \frac{\sigma - 1}{\sigma} \left[1 - \beta (\Lambda^*)^{-\varepsilon} \psi_\lambda^*\right] < 1 - B^* + \frac{(1 - B^*) - \rho(1 - B)}{\rho / \underline{\rho}(t) - 1}, \tag{2.32}$$

respectively (see the Appendix). From the analysis of the closed economy, we know that these two conditions are fulfilled under autarky, which corresponds to the limiting case of $t \to \infty$. Noting further that $\underline{\rho}'(t) < 0$ and $\overline{\rho}'(t) > 0$, we can safely conclude that a diversification equilibrium exists if trade costs are not too small. The impact of higher trade costs on the open economy equilibrium is illustrated in Figure 2.2.

Differentiating firm ratio μ with respect to trade cost parameter t gives

$$\frac{d\mu}{dt} = -\frac{(1 + \rho)\left[\rho\,(1 - B) - (1 - B^*)\right]}{\left[\rho - \overline{\rho}(t)\right]^2} \tag{2.33}$$

and thus $d\mu/dt >, =, < 0$ if $1 >=, < \mu$ or, equivalently, $1 >, =, < \rho$. This accords with the important insight that the home-market effect is stronger at lower trade costs (see Davis,

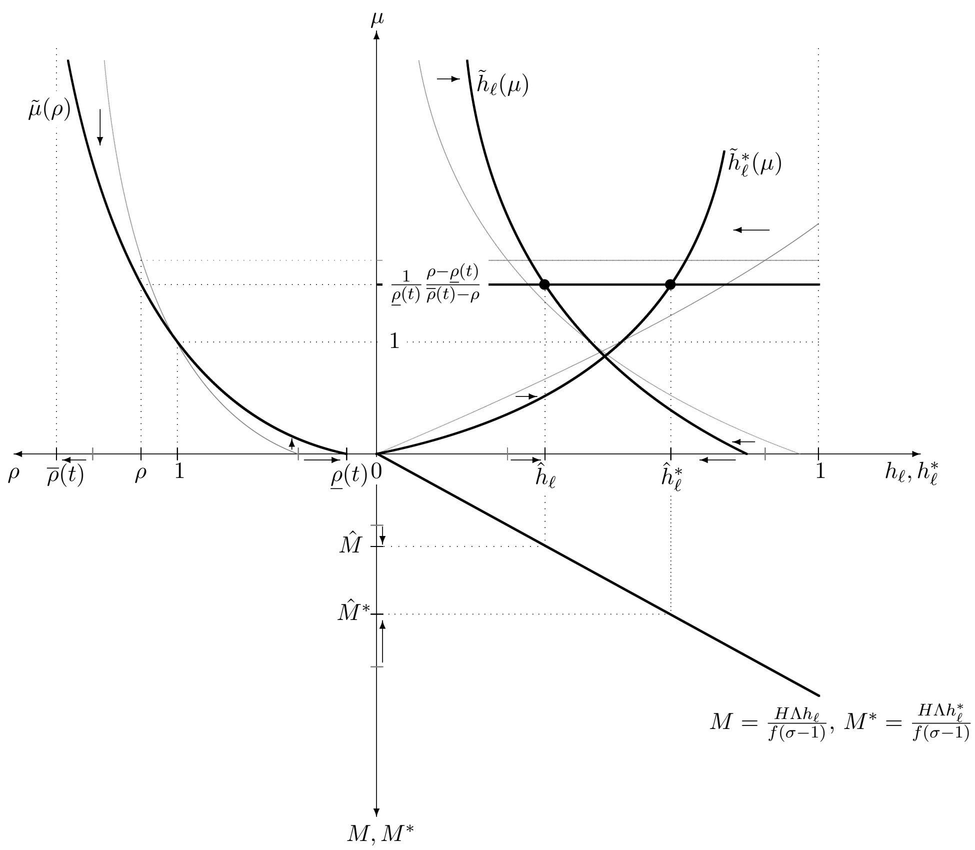

Figure 2.2: Comparative-static effects of an increase in trade cost parameter t

1998b). In the upper left panel of Figure 2.2, we see that the change in firm allocation caused by an increase in the trade cost parameter t leads to a counter-clockwise rotation of $\tilde{\mu}(\rho)$ and to an expansion of the permissible range of expenditure ratio ρ. For the given level of $\rho > 1$, firm ratio μ decreases if trade cost parameter t increases.

Regarding the impact of higher trade costs on the fraction of workers employed in the production of luxuries, we can first determine the direct effect for a given level of μ. From Eqs. (2.29) and (2.30), we can compute $\partial h_\ell/\partial t >,=,< 0$ and $0 >,=,< \partial h_\ell^*/\partial t$ if $\mu >,=,< 1$, with the signs of the derivatives explained by the home-market effect in our model. The direct effect of an increase in t on h_ℓ and h_ℓ^* – captured by a rotation of loci $\tilde{h}_\ell(\mu)$ and $\tilde{h}_\ell^*(\mu)$ in the upper right panel of Figure 2.2 – is reinforced by an indirect effect through adjustments in firm allocation – captured by a movement along the now rotated loci. For the considered case of $\rho > 1$, higher trade costs unambiguously lead to an increase in the share of workers used for producing luxuries at home and to a decrease in the respective share abroad. The observation that employment for the production of luxuries is reduced in the country that uses a larger fraction of its workforce to produce these goods and the observation that the range of permissible levels of ρ has increased lend support to our previous insight that higher trade costs make an outcome with diversified production more likely in the open economy. In the lower right panel of Figure 2.2, we see that for $\rho > 1$ an increase in the fraction of workers producing luxuries in home stimulates firm entry, there. In the foreign country, the decline in the fraction of workers causes a decline in the mass of firms producing luxuries. Firm allocation and production structure have been derived under the assumption that household preferences do not have Gorman form. In the limiting case of Cobb-Douglas preferences, we have $\rho = 1$ and thus $h_\ell = h_\ell^*$, $M = M^*$, irrespective of existing differences between the two countries in their average level and/or dispersion of effective labor supply. In the limiting case of quasilinear preferences $\rho \neq 1$ and thus $h_\ell \neq h_\ell^*$, $M \neq M^*$ require differences of the two countries in their average effective labor supply, whereas differences in the dispersion of effective labor supply do not generate asymmetries in the local markets for luxuries, leading to $\rho = 1$.

The foreign to domestic firm ratio μ is instrumental for the trade pattern in the open economy. Employing the zero-profit condition, we can determine home's total exports and imports of luxuries. Accounting for $r^* = r$, we have

$$EX_\ell = M \frac{1}{1+t} \frac{\sigma P_n f}{\kappa}, \tag{2.34}$$

$$IM_\ell = \mu M \frac{1}{1+t} \frac{\sigma P_n f}{\kappa}, \tag{2.35}$$

respectively, which shows that home is a net-importer (net-exporter) of luxuries if $\mu > (<) 1$. Acknowledging the link between μ and ρ from above, we can therefore conclude that differences in the average level and/or dispersion of effective labor supply are important determinants of the trade structure between two economies if preferences do not have Gorman form. Further insights on the trade structure in the open economy can be obtained by looking at the Grubel-Lloyd index, which is a measure for the share of intra-industry trade and is defined as

$$GLI = 1 - \sum_k \frac{|EX_k - IM_k|}{\sum_k (EX_k + IM_k)},$$

where $k \in \{n, \ell\}$ is an industry index. To pin down the extent of trade in necessities, we assume that households in the case of indifference purchase the domestic product. Then, $\rho > 1$ establishes $IM_n = 0$ and $EX_n = IM_\ell - EX_\ell$, where the latter follows form the balance of payments condition. As a consequence, we have $\sum_k (EX_k + IM_k) = 2IM_\ell$. In contrast, $\rho < 1$ establishes $EX_n = 0$ and $IM_n = EX_\ell - IM_\ell$, leading to $\sum_k (EX_k + IM_k) = 2EX_\ell$. Substituting into the Grubel-Lloyd index, we obtain

$$GLI = \begin{cases} \frac{EX_\ell}{IM_\ell} = \frac{1}{\mu} & \text{if} \quad \rho > 1 \\ 1 & \text{if} \quad \rho = 1 \\ \frac{IM_\ell}{EX_\ell} = \mu & \text{if} \quad \rho < 1 \end{cases} . \tag{2.36}$$

The main insights regarding the role of ρ and t for the trade structure in our model are summarized by the following proposition.

Proposition 1 *Suppose that trade costs are sufficiently high to support a diversification equilibrium. Then, the country with the higher average level and/or dispersion of effective labor supply has a larger home market for luxuries and is a net-exporter of these goods in the open economy. The share of intra-industry trade, measured by the Grubel-Lloyd index, increases in the similarity of countries in terms of their expenditure shares. If the home* *market for luxuries differs between the two economies, the share of intra-industry trade increases monotonically in trade cost t.*

Proof. Proposition 1 follows from substituting Eq. (2.28) into Eqs. (2.34)-(2.36) and accounting for the impact of changes in t and ρ on μ displayed in Figures 2.1 and 2.2. ■

The results in Proposition 1 are closely related to the key finding of Helpman and Krugman (1985) that in a model similar to ours the country with a larger endowment of labor has a larger market for differentiated goods and is therefore a net-exporter of these goods in the open economy. Flam and Helpman (1987) and Fajgelbaum et al. (2011) point out that if preferences do not have Gorman form, a larger home market may be the result of a higher average level and/or dispersion of effective labor supply. From our model we can conclude that the link between exogenous differences in the average level and/or dispersion of effective labor supply and the endogenous differences in expenditure shares are more involved if rent sharing between firms and workers makes wages sector-specific. The reason is that a reallocation of workers to the production of luxuries increases per-capita income in the country net-exporting luxuries. This magnifies pre-existing differences in market size from the closed economy. Since the dispersion of labor income is nonmonotonic in the fraction of workers producing luxuries, the reallocation of labor thus described can, however, reduce income dispersion in the open economy, working against the market size stimulus from higher per-capita labor income. Yet, the possible decline in income dispersion cannot dominate, because it is triggered by an increase in the fraction of workers producing luxuries and thus associated with a higher relative mass of local producers (see Figure 2.1), which further increases the pre-existing trade surplus in luxuries.

The trade structure effects in Proposition 1 are well in line with the Linder (1961) hypothesis, which postulates that manufacturing trade is higher between countries featuring more similar per-capita income levels. Whereas the Linder (1961) hypothesis is sometimes used as a rationale for explaining higher levels of *overall* trade between countries that are more similar in terms of per-capita income (Foellmi et al., 2018), this conclusion is not immediate in a two-sector model. It is well understood from previous work that a higher similarity in per-capita income increases intra-industry trade (see Markusen, 1986; Bergstrand, 1990), but the positive trade stimulus is counteracted by a decline in inter-industry trade (cf. Hunter, 1991). To assess, which of these two effects dominates, we can note from above that total intra- plus inter-industry trade is given by $2EX_\ell$ if $\rho \leq 1$ and by $2IM_\ell$ if $\rho > 1$. Noting from Figure 2.1 that h_ℓ, M decrease while h_ℓ^*, M^* increase in Λ^*, it follows from Eqs. (2.34) and (2.35) that total intra- plus inter-industry trade is lower for $\Lambda^* = \Lambda$ (and thus $\rho = 1$) than for $\Lambda^* < \Lambda$ (and thus $\rho < 1$), contradicting the idea that countries with more similar levels of per-capita income trade more in an open economy (see the Appendix for further details). Of course, as well understood from other studies, higher trade costs also reduce total intra- plus inter-industry trade.

To complete the discussion in this section, we finally determine the effects of trade on welfare. For this purpose, we first look at the price index of luxuries. For home, the price index is given by $P_\ell = p_\ell[M(1+\mu/t)]^{\frac{1}{1-\sigma}}$, where $M(1+\mu/t)$ gives the mass of available luxuries discounted for the price premium paid on imported varieties due to the existence of iceberg trade costs. Using Eqs. (2.17), (2.19), (2.20), and (2.29), we can express the price index as follows

$$P_\ell = P_\ell^a \left(\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}} \left[\frac{1-B}{1-B(1+t)/(\mu+t)}\right]^{\frac{1}{1-\sigma}}, \tag{2.37}$$

where superscript a is used to indicate an autarky variable. If $\alpha = 1$, firms producing luxuries do not pay a wage premium. This yields $B = 0$ and thus $P_\ell = P_\ell^a(1+1/t)^{\frac{1}{1-\sigma}}$. In this case, higher trade costs lower the mass of available consumer goods and increase price index P_ℓ, which is to the detriment of social welfare. Lacking feedback effects of trade on the level and dispersion of nominal per-capita income, a model variant featuring equal wages in the sector of necessities and luxuries therefore leads to the intuitive result of gains from trade in both economies, irrespective of whether preferences have Gorman form or not. More specifically, for the limiting case of $\alpha = 1$ social welfare is given by

$$V(P_n, P_\ell, \Lambda P_n, \psi_\lambda) = \left[V_a(P_n^a, P_\ell^a, \Lambda P_n^a, \psi_\lambda) + \frac{1-\beta}{\varepsilon}\right]\left(\frac{1+t}{t}\right)^{\frac{\varepsilon}{\sigma-1}} - \frac{1-\beta}{\varepsilon}, \tag{2.38}$$

and thus the same for the two countries and independent of the trade structure in the open economy.

If $\alpha > 1$, a reallocation of workers between the two sectors produces an endogenous adjustment in market size captured by ζ which adds to the home-market effect due to exogenous differences in the average level and/or dispersion of effective labor supply. The now larger home-market effect leads to additional firm entry in the country net-exporting luxuries and to firm exit in the other economy. Since locally produced varieties are not subject to trade costs and since the convexity of the Engel curve implies that the total mass of domestic plus foreign producers of luxuries increases, price index P_ℓ falls with trade in the country net-exporting this good. This can be seen from Eq. (2.37), where $\mu < 1$ implies $1 - B(1+t)/(\mu+t) < 1 - B$ and thus $P_\ell < P_\ell^a$ in home. The country net-exporting luxuries is unambiguously better off in the open economy, according to Eq. (2.21), because the fall in the price index of luxuries is accompanied by an increase in nominal income, as more workers are used for the production of luxuries. This nominal income effect is captured by an increase in composite term $(\overline{e}/P_n)^\varepsilon \hat{\psi} = \Lambda^\varepsilon \hat{\psi}_\lambda[1+h_\ell(\alpha^\varepsilon - 1)]$.

The effect of a wage premium $\alpha > 1$ on the price index in the country net-importing luxuries is a priori not clear. For a given total mass of producers, firm exit at home and firm entry abroad lead to an increase in the price index of luxuries, because imports are subject to trade costs. This effect is counteracted by an increase in the total mass of domestic plus foreign producers, which, all other things equal, lowers the price index of luxuries. In the Appendix, we show that the first effect can dominate for high trade costs if market size differences are sufficiently pronounced. In this case, all domestic workers are worse off in the open economy due to an increase in the price index of luxuries, whereas those workers losing their job in the sector of luxuries and finding a new job in the sector of necessities moreover experience a fall in nominal income. This additional source of welfare loss is captured by a decline in the composite term $(\overline{e}/P_n)^\varepsilon \hat{\psi}$ in Eq. (2.21). We summarize the impact of trade on welfare in the following proposition.

Proposition 2 *Suppose that trade costs are sufficiently high to support a diversification equilibrium. Then, if a wage premium does not exist* ($\alpha = 1$)*, there are gains from trade of equal size in both countries and these gains decrease monotonically in trade cost* t*. If* $\alpha > 1$ *causes a wage premium in the sector of luxuries, gains from trade are guaranteed for the country net-exporting luxuries, whereas losses from trade are possible for the country net-importing luxuries.*

Proof. Analysis in the text and formal proof in the Appendix. ■

Proposition 2 points to the notable result that in the case of $\alpha = 1$ welfare effects of trade are independent of the trade structure in the open economy and thus the same for both countries. Thereby, gains from trade are a priori not clear in our setting, because the market outcome in the closed economy is not socially optimal and because we know from the literature of second best that in this case lifting a constraint may aggravate the distortion and lead to welfare loss (see, for instance, Markusen, 1981; Newbery and Stiglitz, 1984, for two prominent contributions in the context of trade). More specifically, our model features increasing economies to scale in only one sector, and it is well known from studies by Graham (1923), Markusen and Melvin (1981), Ethier (1982a), and Francois and Nelson (2002) that in such environments there is a chance that one country loses from trade (even though Grossman and Rossi-Hansberg, 2010, call such outcomes "pathological"). The analysis above reveals that concerns about losses from trade are not justified in our setting if employment in the two sectors promises the same labor return. Since the engine for gains from trade is a decline in the price index of luxuries and since lower trade costs lead to a fall in the price index, gains from trade exist for $\alpha = 1$, irrespective of the specific nature of preferences.

For $\alpha > 1$, our model features a second source of inefficiency originating from a price distortion in the labor market, which makes wages sector-specific. Adding this distortion, trade can generate losers in the country forfeiting market share in the sector of luxuries, because some workers previously employed in this sector will experience a wage decline. This provides a demand-side explanation for anti-globalization attitudes of workers observed in many industrialized economies over the last decades. However, the insight that the price index of luxuries can increase in response to trade liberalization is even more disconcerting, because it implies that all workers in the country losing market share in the production of luxuries may be worse off with than without trade. If all workers lose, the normative results from our analysis do not depend on the specific choice of a utilitarian welfare function. However, this does not mean that preferences are irrelevant. Since preferences determine how the average level and/or dispersion of effective labor supply influence household expenditures, they affect the trade structure in the open economy and are crucial for the existence of gains and losses from trade. Since the demand-side differences considered here are only relevant for expenditures in a setting with nonhomothetic preferences, the home markets for luxuries do not differ in the limiting case of Cobb-Douglas preferences, making the production structure symmetric and all trade intra-industry. This prevents losses from trade due to an unfavorable reallocation of labor in the open economy.

Giving up the assumption of homothetic preferences may therefore change the rather optimistic view shared by many economists that trade, while not necessarily benefitting all households equally, at least increases economy-wide welfare. Proposition 2 points out that losses from trade are a threat for the poorer country, augmenting pre-existing differences in the well-being of the two economies. This provides a rationale for the view shared by many opponents of globalization that the international distribution of trade surplus is unjust. However, our analysis also reveals that losses from trade are not confined to poorer countries but can extend to countries with a more egalitarian distribution of endowments. This suggests that trade can be a peril for countries in Northern and Central Europe, where the idea of offering equal opportunities plays a particularly important role (cf. Dunnzlaff et al., 2011). Also, the results from our analysis provide a challenge to the idea that redistribution – be it ex ante, through equalization of endowments, or ex post, through equalization of outcome – can be a successful instrument to increase support for trade liberalization (cf. Davidson and Matusz, 2006; Egger and Fischer, 2018). Even if the trade reform generates aggregate gains, policy intervention that aims at distributing theses gains more equally will influence the trade structure with unintended welfare consequences. However, such insights should not be misunderstood as an argument against free trade. Losses from trade are the result of pre-existing price distortions in the product and labor market and not per se a consequence of falling trade costs. Our analysis points out that abolishing such distortions may be a more important measure to achieve support for a trade reform than the compensation of losers.

2.4 Extensions

In this section, we consider two extensions of our benchmark model. In the first one, we allow for positive assortative matching, implying that workers with higher effective labor supply end up in the sector of luxuries. Assuming that firms must invest into a screening technology to gather (imperfect) information upon the type of applicants, the thus modified setting produces endogenous fixed and variable production costs, thereby opening an additional adjustment margin to a fall in trade costs. In the second extension, we consider differences of the two economies in the wage premium paid by luxury producers and analyze to what extent the predictions of our model change if we consider supply-side reasons for comparative advantage instead of demand-side reasons for the home-market effect as a motive for inter-industry trade.

2.4.1 Screening and assortative matching

In the analysis above, firms in the luxury sector are indifferent between hiring workers with high or low effective labor supply, because the same workplace capacity is needed for each employed unit of labor input. Whereas this assumption facilitates the analysis, it differs from the usual approach which associates employment with installment of a workplace at a cost that is independent of the worker's effective labor supply. With this alternative specification, firms producing luxuries prefer employing workers with higher effective labor supply. However, following Helpman et al. (2010, 2017) we assume that firms cannot freely observe the effective labor supply of workers prior to their employment and therefore have to screen the pool of applicants to gather information upon their λ-level. Screening is costly and provides an imperfect signal about the effective labor supply of applicants. More specifically, firms detect whether applicants are above or below a threshold, λ_u, and the costs of screening, $P_n F(\lambda_u)$, increase in this threshold with constant elasticity $\varphi > 0$: $F(\lambda_u) = \lambda_u^{\varphi}$. Installing a workplace has costs P_n and if the *average* worker provides labor input $\Lambda_u \equiv [1 - L(\lambda_u)]^{-1} \int_{\lambda_u}^{\overline{\lambda}} \lambda dL(\lambda)$, q_ℓ/Λ_u workplaces are needed to employ q_ℓ units of labor input. Profits of the firm then correspond to $\Pi = \pi - P_n q_\ell \Lambda_u^{-1} - P_n F(\lambda_u) - P_n f$.

Adopting from Helpman et al. (2010) the assumption that effective labor supply is Pareto distributed, with $\underline{\lambda} > 0$, $\overline{\lambda} \to \infty$, and $L(\lambda) = 1 - (\lambda/\underline{\lambda})^{-g}$, $g > 1$, we have $\Lambda_u = \frac{g}{g-1}\lambda_u$, and the first-order conditions for the optimal choice of q_ℓ and λ_u are given by

$$\frac{\partial \Pi}{\partial q_\ell} = \frac{\sigma - 1}{\sigma}\frac{\kappa r}{q_\ell} - \frac{P_n}{\Lambda_u} = 0, \qquad \frac{\partial \Pi}{\partial \lambda_u} = \frac{P_n q_\ell}{\Lambda_u}\frac{1}{\lambda_u} - \varphi P_n \frac{F(\lambda_u)}{\lambda_u} = 0. \tag{2.13$'$}$$

As formally shown in the Appendix, under the sufficient condition of $\varphi > \sigma - 1$, the Hesse matrix of the maximization problem (evaluated at the solutions for the first-order conditions) is negative semi-definite. This implies that if an interior solution exists, it must be a maximum. Furthermore, combining the first-order conditions in (2.13′) with the zero-profit condition $\Pi = 0$, we can solve for

$$\lambda_u = \left(\frac{f(\sigma - 1)}{\varphi - \sigma + 1}\right)^{\frac{1}{\varphi}} \equiv \hat{\lambda}_u, \qquad \Lambda_u = \frac{g}{g-1}\left(\frac{f(\sigma - 1)}{\varphi - \sigma + 1}\right)^{\frac{1}{\varphi}} \equiv \hat{\Lambda}_u.$$

A solution with a positive level of screening, $\hat{\lambda}_u > \underline{\lambda}$, requires $f(\sigma - 1)/(\varphi - \sigma + 1) > \underline{\lambda}^{\varphi}$. Accounting for the bargaining solution in Eq. (2.12), we can summarize the outcome of the firm's maximization problem as follows

$$w_\ell = \frac{\alpha}{\hat{\Lambda}_u} P_n, \qquad \Pi = \frac{\kappa r}{\Phi} - P_n f, \tag{2.14$'$}$$

with $\Phi \equiv \sigma\varphi/(\varphi - \sigma + 1) > \sigma$. Thereby, $\hat{\alpha} \equiv \alpha/\hat{\Lambda}_u > 1$ is needed to ensure that firms producing luxuries pay a wage premium and therefore make a job application attractive for workers.

To solve for the general equilibrium outcome, we can proceed as in the benchmark model and determine the mass of firms and the fraction of workers producing luxuries, M and h_ℓ, respectively. Acknowledging that $q_\ell/\hat{\Lambda}_u$ gives the number of workers per firm, we can derive a first relationship between h_ℓ and M from the first-order condition in Eq. (2.13′) as follows (see the Appendix):

$$h_\ell H w = \frac{\sigma - 1}{\sigma}\kappa M r. \tag{2.17$'$}$$

Eq. (2.17′) reflects constant markup pricing and shows once again that firms pay a constant fraction of their revenues as a wage bill to their workforce. Because in the modified setting considered here firms pay the same workplace installment costs for each worker and because in total these installment costs are equal to their wage payments, the wage bill received in the sector of luxuries is independent of the now higher effective labor supply of the workforce. Of course, the share of workers employed in the sector of luxuries, h_ℓ, cannot be larger than the share of workers with an effective labor supply above the threshold $\hat{\lambda}_u$, $(\hat{\lambda}_u/\underline{\lambda})^{-g}$. To avoid a corner solution, we discuss below a necessary parameter constraint and assume for now excess supply of workers with effective labor endowment $\lambda > \hat{\lambda}_u$. This implies that the share of workers with effective labor supply above the threshold $\hat{\lambda}_u$ finding employment in the sector of luxuries is smaller than one: $\gamma_h \equiv h_\ell(\hat{\lambda}_u/\underline{\lambda})^g < 1$.

To determine a second relationship between h_ℓ and M, we employ goods market clearing in the sector of luxuries, $P_\ell X_\ell = Mr$, which is derived in the Appendix and given by

$$H\Lambda w\left[1-\beta\Lambda^{-\varepsilon}\psi_\lambda\right]+h_\ell Hw\frac{\sigma}{\sigma-1}\frac{1}{\kappa}\hat{B}=Mr, \tag{2.18$'$}$$

with $\hat{B} \equiv \frac{\sigma-1}{\sigma}\kappa\hat{\Lambda}_u\left[(\hat{\alpha}-1)-(\hat{\alpha}^{1-\varepsilon}-1)\beta\hat{\Lambda}_u^{-\varepsilon}\psi_\lambda\right]$ and $\Lambda = \frac{g}{g-1}\underline{\lambda}$, $\psi_\lambda = \left(\frac{g}{g-1}\right)^\varepsilon\frac{g-1}{g+\varepsilon-1}$ due to our assumption that the distribution of effective labor supply is Pareto. Combining Eqs. (2.17′) and (2.18′) and making use of zero-profit condition $\kappa r = \Phi P_n f$, we can derive explicit solutions for M and h_ℓ:

$$M=\frac{\kappa}{1-\hat{B}}\frac{H\Lambda\left[1-\beta\Lambda^{-\varepsilon}\psi_\lambda\right]}{\Phi f},\qquad h_\ell=\frac{\sigma-1}{\sigma}\frac{\kappa}{1-\hat{B}}\Lambda[1-\beta\Lambda^{-\varepsilon}\psi_\lambda]. \tag{2.19$'$}$$

A higher average effective labor supply increases demand for luxuries through two channels. On the one hand, there is a common income effect that increases expenditures for both goods. On the other hand, there is a further demand stimulus for luxuries, which is specific to nonhomothetic preferences, because the now richer households devote a larger fraction of expenditures to the consumption of luxuries. Both of these effects also exist in the benchmark model. However, there, the common income effect was neutralized by an increase in the costs of employing the additional amount of labor needed to fulfill the increased demand for luxuries. This is different in the model variant considered here. Because workplace installment costs per worker are not affected by a common increase in effective labor supply, the common income effect is not neutralized. From Eq. (2.19′), we can moreover infer that an interior solution with $\gamma_h = h_\ell(\hat{\lambda}_u/\underline{\lambda})^g < 1$ requires

$$\kappa\frac{\sigma-1}{\sigma}\Lambda[1-\beta\Lambda^{-\varepsilon}\psi_\lambda] < (1-\hat{B})\left[\left(\frac{f(\sigma-1)}{\varphi-\sigma+1}\right)^{\frac{1}{\varphi}}\frac{1}{\underline{\lambda}}\right]^{-g}.$$

Despite the complications arising from endogenous fixed costs and despite the additional parameter constraint needed to achieve an interior equilibrium, the main insights from the benchmark model remain intact. Since the results for the open economy can be derived for the more sophisticated model variant considered here by following the derivation steps from the main text, we do not repeat the analysis and leave the formal details to the interested reader.

2.4.2 Supply-side differences due to country-specific wage premia

We now consider the role of supply-side differences of the two countries and assume that a wage premium is paid by luxury producers only in the foreign economy. Accordingly, we set $\alpha^* > \alpha = 1$, while we make countries symmetric in all other respects, including the average level and dispersion of effective labor supply. Similar to the benchmark model, we can apply the zero-profit conditions for home and foreign to link differences in the price indices for luxuries to market size differences. As formally shown in the Appendix, this gives

$$\rho\zeta = \frac{t\xi^\sigma - 1}{t - \xi^\sigma}\left(\frac{P_\ell}{P_\ell^*}\right)^{\sigma-1}, \tag{2.23'}$$

where ρ, ζ are defined as above, and $\rho = 1$ holds, because the two countries do not differ in the average level and dispersion of effective labor supply, while $\zeta > 1$ follows, because only the foreign market offers a wage premium for workers employed in the production of luxuries. On the one hand, this indicates that the local market for luxuries is larger in the country featuring a price distortion in the labor market, which is foreign in our case. On the other hand, this country has a comparative disadvantage in producing luxuries, which is reflected by $\xi \equiv \frac{p_\ell^*}{p_\ell} = \frac{\kappa}{\kappa^*} = \frac{\sigma+(\sigma-1)\alpha^*}{2\sigma-1} > 1$. From Eq. (2.23′) we see that $t > \xi^\sigma$ is a necessary (not sufficient) condition for some production of luxuries to remain in the foreign economy. With this result at hand, we can determine the share of revenues achieved in the domestic market (d) and the export market (x). This gives for home and foreign

$$\frac{r_d}{r} = t\frac{t-\xi^\sigma}{t^2-1}, \quad \frac{r_x}{r} = \frac{t\xi^\sigma - 1}{t^2-1} \quad \text{and} \quad \frac{r_d^*}{r^*} = t\frac{t-\xi^{-\sigma}}{t^2-1}, \quad \frac{r_x^*}{r^*} = \frac{t\xi^{-\sigma}-1}{t^2-1},$$

respectively. An increase in the wage premium in the foreign country increases ξ and thus the comparative advantage of home in the production of luxuries. However, it also increases the market for luxuries in foreign and induces firms from both countries to increase their revenues there. As a consequence, a higher ξ lowers r_d/r and increases r_x/r, with the effect mirrored in the foreign country by a decrease in r_x^*/r^* and an increase in r_d^*/r^*.

The assumption of asymmetric production costs does not affect constant markup pricing and the induced result that the wage bill paid by luxury producers is a constant fraction of their revenues. This establishes a positive link between the fraction of workers and the number of firms producing luxuries that is well understood from Eqs. (2.17) and (2.25). However, asymmetric production costs change the market clearing conditions for luxuries in the open economy to

$$H\Lambda w\left[1 - \beta\Lambda^{-\varepsilon}\psi_\lambda\right] = Mrt\frac{t-\xi^\sigma}{t^2-1} + M^*r^*\frac{t\xi^{-\sigma}-1}{t^2-1}, \tag{2.26'}$$

$$H\Lambda w\left[1 - \beta\Lambda^{-\varepsilon}\psi_\lambda\right] + h_\ell^* H\Lambda w\frac{\sigma}{\sigma-1}\frac{1}{\kappa^*}B^* = M^*r^*t\frac{t-\xi^{-\sigma}}{t^2-1} + Mr\frac{t\xi^\sigma-1}{t^2-1}. \tag{2.27'}$$

Combining markup pricing with the two market clearing conditions, we can solve for firm ratio $\mu = M^*/M$, according to

$$\mu = \frac{1}{\xi}\frac{t^2 - 2t\xi^{\sigma} + 1}{t^2 - 2t\xi^{-\sigma} + 1 - (t^2-1)B^*}, \tag{2.28$'$}$$

with

$$\frac{d\mu}{dt} = \frac{2\left[t(1-B^*) - \xi^{-\sigma}\right]}{t^2 - 2t\xi^{-\sigma} + 1 - (t^2-1)B^*}\left[\frac{1}{\xi}\frac{t-\xi^{\sigma}}{t(1-B^*) - \xi^{-\sigma}} - \mu\right]. \tag{2.33$'$}$$

In the Appendix, we show that $d\mu/dt > 0$, while $\lim_{t\to\infty}\mu = \frac{2\sigma-1}{2\sigma-1+(\sigma-1)[(\alpha^*)^{1-\varepsilon}-1]\beta\Lambda^{-\varepsilon}\psi_\lambda} < 1$. This implies that $\mu < 1$ extends to all possible trade costs and that the country with a comparative disadvantage in the production of luxuries hosts fewer luxury goods producers in the open economy. Furthermore, from Eq. (2.28$'$), we see that a positive production level of luxuries in foreign is given by $t > \xi^{\sigma} + \sqrt{\xi^{2\sigma}-1}$, whereas the parameter domain supporting a positive production level of necessities in home can be determined by combining Eq. (2.28$'$) with Eqs. (2.17), (2.26$'$), and zero-profit condition $\kappa r = \sigma P_n f$, and it is given by

$$\kappa\frac{\sigma-1}{\sigma}[1-\beta\Lambda^{-\varepsilon}\psi_\lambda] < \frac{t+\mu\xi^{1-\sigma}}{t+1}\frac{t-\xi^{\sigma}}{t-1}.$$

Similar to the benchmark model the foreign to domestic firm ratio μ is decisive for the trade structure in the open economy. Making use of the zero-profit condition, we can compute home's total value of exports and imports of luxuries according to

$$EX_\ell = Mr_x = M\frac{t\xi^{\sigma}-1}{t^2-1}\frac{\sigma P_n f}{\kappa}, \tag{2.34$'$}$$

$$IM_\ell = M^* r_x^* = M\mu\frac{t\xi^{-\sigma}-1}{t^2-1}\frac{\sigma P_n f}{\kappa^*}, \tag{2.35$'$}$$

respectively. Combining Eqs. (2.34$'$) and (2.35$'$) then implies $IM_\ell = \mu\xi^{1-\sigma}\frac{t-\xi^{\sigma}}{t\xi^{\sigma}-1}EX_\ell$, and, acknowledging $GLI = \mu\xi^{1-\sigma}\frac{t-\xi^{\sigma}}{t\xi^{\sigma}-1} < 1$, we can therefore safely conclude that home exports luxuries, because it has a comparative advantage in producing these goods, which dominates the home-market effect in our setting. The share of intra-industry-trade increases in trade cost parameter t and it decreases in foreign's wage premium α^*, whereas total (intra- plus inter-industry) trade increases in α^* and decreases in t. These effects are well understood from the benchmark model.

We complete the discussion in this section with a brief look at the welfare effects of trade. In home, luxury producers do not pay a wage premium, and hence changes in the allocation of workers do not affect the level and dispersion of nominal income. This implies that all welfare effects of trade are due to changes in the price index of luxuries, which can be expressed as $P_\ell = p_\ell\left[M\frac{t+\mu\xi^{1-\sigma}}{t}\right]^{\frac{1}{1-\sigma}}$. Making use of market clearing condition (2.26$'$), zero-profit condition $\kappa r = \sigma P_n f$, and the solution for firm ratio μ in Eq. (2.28$'$), we can compute $dP_\ell/dt > 0$. This implies that home benefits from trade, because it specializes according to the law of comparative advantage in those goods, whose production features increasing economies to scale and whose exchange is subject to trade costs. In foreign, things are less clear, because there are two potentially counteracting effects. There are adjustments in the share of workers producing luxuries, which affect welfare through changes in nominal income due to the wage premium paid in the sector of luxuries. At already high trade costs, a further increase in t increases the fraction of workers producing luxuries in foreign, with positive effects on nominal income and welfare. This effect is supplemented by a change in the price index of luxuries, which in the foreign economy is given by $P_\ell^* = p_\ell^* \left[M^* \frac{1+t\mu\xi^{1-\sigma}}{t\mu\xi^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}$. Whereas general effects of trade on price index P_ℓ^* are difficult to determine, we show in the Appendix that at an initially high level of t, the price index increases in trade costs if the price distortion in the labor market is small (ξ close to one), whereas the opposite is true if the price distortion is large (ξ close to infinity). This indicates that the country losing market share of those goods, whose production features increasing economies to scale, can be worse off in the open than in the closed economy if supply-side differences are sufficiently pronounced, whereas gains from trade are guaranteed for both trading partners if supply-side differences are small. This conclusion holds for arbitrary levels of ε, and hence does not depend on the specific nature of preferences.

2.5 Conclusion

We have developed a two-country model, in which ex ante differences in the average level and dispersion of effective labor supply are important exogenous sources of demand-side asymmetries, because households have nonhomothetic preferences. The assumption of nonhomothetic preferences produces non-linear Engel curves and makes in a textbook model of the home-market effect, with two output sectors and labor as the only factor input, the differentiated good a luxury and the outside good a necessity. Assuming that production of luxuries promises a wage premium due to firm-level rent sharing and assuming that the export of luxuries is subject to trade costs, we show that the country featuring a higher average level and/or dispersion of effective labor supply has a larger home market and therefore becomes net-exporter of luxuries in the open economy. Due to a price distortion in the labor market, the trade pattern is an important factor of welfare in the open economy. The country that increases its market share and net-exports luxuries benefits, whereas the country that lowers its market share and net-imports luxuries can lose from trade. This implies that trade can increase pre-existing welfare differences and hurt countries with a lower average level and/or lower dispersion of effective labor supply.

In an extension of our model, we consider screening and assortative matching of workers featuring high effective labor supply with firms producing luxuries. This modified framework generates endogenous fixed and variable costs of production and therefore accounts for an additional adjustment margin through which trade can affect welfare in an open economy. Whereas the additional adjustment margin makes the analysis more complicated, the main insights from the benchmark model regarding the link of trade pattern and welfare remain intact. In a second extension, we consider asymmetries of countries in the wage premia paid by luxury goods producers. This modification generates a supply-side asymmetry and sheds light on the role of comparative advantage for trade structure and welfare in the open economy. Accordingly, the country featuring the stronger price distortion in the labor market becomes net-importer of luxuries and, depending on the strength of the distortion, can win or lose from trade, irrespective of the specific nature of preferences. This suggests that the choice of preferences is particularly relevant when demand-side differences are key for the trade pattern in the open economy, whereas the choice of preferences seems less important if supply-side asymmetries are decisive.

This paper provides a first step to introduce PIGL preferences into models of international trade. Relying on these preferences, we have shown that problems associated with the aggregation of heterogeneous households can be avoided and new insights on how demand-side factors affect trade and welfare can be gained even if one chooses to leave the Gorman class. In the specific application considered here, we have shed light on the interaction between supply-side distortions and demand-side asymmetries for the relationship of trade and welfare in open economies. Thereby, we have left other interesting topics aside. For instance, we have not considered unemployment and thus have excluded one important variable, whose adjustment to trade has been subject to a controversial debate over the last few decades. Furthermore, while briefly discussing limitations to redistributing the gains from trade under nonhomothetic preferences, we have not addressed in detail the costs and benefits of tax-transfer systems in open economies. Whereas extending our model in both directions is a worthwhile task for future research, doing so is clearly beyond the scope of this paper.

2.6 Appendix

A closed form representation of the direct utility function

Applying Roy's identity to Eq. (2.2) gives the Marshallian demand functions in Eq. (2.3). These demand functions can be used to solve for

$$\frac{P_n}{P_\ell} = \frac{X_\ell^i}{\left(\frac{X_n^i}{\beta}\right)^{\frac{1}{1-\varepsilon}} - X_n^i} \quad \text{and} \quad \frac{e^i}{P_\ell} = \frac{P_n}{P_\ell}\left(\frac{X_n^i}{\beta}\right)^{\frac{1}{1-\varepsilon}}. \tag{2.39}$$

Substitution into Eq. (2.2), then gives the direct utility function

$$u(X_n^i, X_\ell^i) = \frac{1}{\varepsilon}\left(X_\ell^i\right)^\varepsilon \frac{\left(\frac{X_n^i}{\beta}\right)^{\frac{\varepsilon}{1-\varepsilon}} - \beta}{\left[\left(\frac{X_n^i}{\beta}\right)^{\frac{1}{1-\varepsilon}} - X_n^i\right]^\varepsilon} - \frac{1-\beta}{\varepsilon}, \tag{2.40}$$

which is well defined only if the consumption level of luxuries is strictly positive and it has a value of $-(1-\beta)/\varepsilon$ for all levels of purchased necessities if $X_\ell^i = 0$. This completes the proof.

Derivation of price index P_ℓ

Acknowledging Eq. (2.4), households choose X_n^i, $x_\ell^i(\omega)$ to maximize utility Eq. (2.40), subject to their budget constraint $P_n X_n^i + \int_{\omega\in\Omega} p_\ell(\omega) x_\ell^i(\omega) \leq e^i$. The first-order conditions for the respective Lagrangian problem yield

$$\frac{x_\ell^i(\omega)^{-\frac{1}{\sigma}}}{(X_\ell^i)^{\frac{\sigma-1}{\sigma}}}\left[\left(\frac{X_n^i}{\beta}\right)^{\frac{1}{1-\varepsilon}} - X_n^i\right] = \frac{p_\ell(\omega)}{P_n}. \tag{2.41}$$

This establishes for any two varieties of luxuries ω and $\hat{\omega}$ a link of their consumption expenditures according to $p_\ell(\omega)x_\ell^i(\omega) = x_\ell^i(\hat{\omega})p_\ell(\hat{\omega})\left[\frac{p_\ell(\omega)}{p_\ell(\hat{\omega})}\right]^{1-\sigma}$. Integrating over ω, then gives

$$\int_{\omega\in\Omega} p_\ell(\omega)x_\ell^i(\omega)d\omega = x_\ell^i(\hat{\omega})p_\ell(\hat{\omega})^\sigma \int_{\omega\in\Omega} p_\ell(\omega)^{1-\sigma}d\omega. \tag{2.42}$$

Using the latter together with $X_n^i = \beta\left(\frac{e^i}{P_n}\right)^{1-\varepsilon}$ from Eq. (2.3) in the binding budget constraint, we obtain

$$e^i\left[1 - \beta\left(\frac{e^i}{P_n}\right)^{-\varepsilon}\right] = x_\ell^i(\hat{\omega})p_\ell(\hat{\omega})^\sigma \int_{\omega\in\Omega} p_\ell(\omega)^{1-\sigma}d\omega. \tag{2.43}$$

Evaluating (2.41) at $\hat{\omega}$ and substituting for $x_\ell^i(\hat{\omega})p_\ell(\hat{\omega})^\sigma$, Eq. (2.43) can be solved for

$$e^i\left[1 - \beta\left(\frac{e^i}{P_n}\right)^{-\varepsilon}\right] = X_\ell^i\left[\int_{\omega\in\Omega} p_\ell(\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}}, \tag{2.44}$$

making $P_\ell \equiv [\int_{\omega\in\Omega} p_\ell(\omega)^{1-\sigma} d\omega]^{\frac{1}{1-\sigma}}$ a valid price index for the composite X_ℓ^i, because total expenditures of household i devoted to luxuries are given by $P_\ell X_\ell^i$. This completes the proof.

Rent sharing and social welfare

Starting point is the welfare function in Eq. (2.21). Accounting for $(\frac{\bar{e}}{P_n})^\varepsilon \hat{\psi} = \Lambda^\varepsilon[1+h_\ell(\alpha^\varepsilon - 1)]\hat{\psi}_\lambda$, substituting the share of production workers h_ℓ from Eq. (2.19), the price index of luxuries from Eq. (2.20), and acknowledging $\kappa = \frac{\sigma}{\sigma+\alpha(\sigma-1)}$ as well as $\frac{\kappa}{1-B} = F(\alpha)^{-1}$, with $F(\alpha) \equiv 1 + \frac{\sigma-1}{\sigma}[1 + (\alpha^{1-\varepsilon} - 1)\beta\Lambda^{-\varepsilon}\psi_\lambda]$, we can express welfare in the closed economy as a function of wage premium α: $V(P_n, P_\ell, \bar{e}, \hat{\psi}) = \left(\frac{H\Lambda(1-\beta\Lambda^{-\varepsilon}\psi_\lambda)}{\sigma f}\right)^{\frac{\varepsilon}{\sigma-1}} \hat{V}(\alpha) - \frac{1-\beta}{\varepsilon}$, with

$$\hat{V}(\alpha) \equiv \frac{1}{\varepsilon}\left(\alpha + \frac{\sigma}{\sigma-1}\right)^{-\varepsilon} F(\alpha)^{-\frac{\varepsilon}{\sigma-1}} \left\{\Lambda^\varepsilon \hat{\psi}_\lambda \left[1 + \frac{\sigma-1}{\sigma}\left(1 - \beta\Lambda^{-\varepsilon}\psi_\lambda\right)\frac{\alpha^\varepsilon - 1}{F(\alpha)}\right] - \beta\right\}. \tag{2.45}$$

Differentiation with respect to α gives

$$\begin{aligned}\hat{V}'(\alpha) = \frac{\varepsilon\hat{V}(\alpha)}{\alpha + \frac{\sigma}{\sigma-1}} \Bigg\{ &-1 - \frac{1-\varepsilon}{\sigma}\frac{\left(\alpha + \frac{\sigma}{\sigma-1}\right)\alpha^{-\varepsilon}\beta\Lambda^{-\varepsilon}\psi_\lambda}{F(\alpha)} \\ &+ \frac{\left(\alpha + \frac{\sigma}{\sigma-1}\right)\Lambda^\varepsilon\hat{\psi}_\lambda \frac{\sigma-1}{\sigma}\left(1 - \beta\Lambda^{-\varepsilon}\psi_\lambda\right)}{\Lambda^\varepsilon\hat{\psi}_\lambda\left[1 + \frac{\sigma-1}{\sigma}\left(1 - \beta\Lambda^{-\varepsilon}\psi_\lambda\right)\frac{\alpha^\varepsilon - 1}{F(\alpha)}\right] - \beta}\left[\frac{\alpha^{\varepsilon-1}}{F(\alpha)} - \frac{1-\varepsilon}{\varepsilon}\frac{\alpha^\varepsilon - 1}{F(\alpha)}\frac{\frac{\sigma-1}{\sigma}\alpha^{-\varepsilon}\beta\Lambda^{-\varepsilon}\psi_\lambda}{F(\alpha)}\right]\Bigg\}.\end{aligned}$$

Accounting for $\lim_{\varepsilon\to 0} F(\alpha) = \frac{2\sigma-1}{\sigma} + \frac{\sigma-1}{\sigma}(\alpha - 1)\beta$, $\lim_{\varepsilon\to 0}\frac{1-\varepsilon}{\varepsilon}(\alpha^\varepsilon - 1) = \ln\alpha$, and $\lim_{\varepsilon\to 0}\varepsilon\hat{V}(\alpha) = 1 - \beta$, we compute

$$\begin{aligned}\lim_{\varepsilon\to 0}\hat{V}'(\alpha) = \frac{1-\beta}{\alpha + \frac{\sigma}{\sigma-1}}\Bigg\{ &-1 - \frac{\beta}{\sigma}\frac{\alpha + \frac{\sigma}{\sigma-1}}{\frac{2\sigma-1}{\sigma} + \frac{\sigma-1}{\sigma}(\alpha-1)\beta} + \frac{\frac{\sigma-1}{\sigma} + \frac{1}{\alpha}}{\frac{2\sigma-1}{\sigma} + \frac{\sigma-1}{\sigma}(\alpha-1)\beta} \\ &- \frac{\sigma-1}{\sigma}\frac{\beta\ln\alpha\left(\frac{\sigma-1}{\sigma}\alpha + 1\right)}{\left[\frac{2\sigma-1}{\sigma} + \frac{\sigma-1}{\sigma}(\alpha-1)\beta\right]^2}\Bigg\},\end{aligned} \tag{2.46}$$

where $\frac{\sigma-1}{\sigma} + \frac{1}{\alpha} < \frac{2\sigma-1}{\sigma} + \frac{\sigma-1}{\sigma}(\alpha-1)\beta$ proves that $\lim_{\varepsilon\to 0}\hat{V}'(\alpha) < 0$. This confirms that a higher wage premium in the sector producing luxuries lowers welfare in the Cobb-Douglas case. Furthermore, accounting for $\lim_{\varepsilon\to 1} F(\alpha) = \frac{2\sigma-1}{\sigma}$, we can compute $\lim_{\varepsilon\to 1}\hat{V}(\alpha) = (\Lambda - \beta)\frac{\sigma-1}{\sigma}(\frac{\sigma}{2\sigma-1})^{\frac{\sigma}{\sigma-1}}$ and thus $\lim_{\varepsilon\to 1}\hat{V}'(\alpha) = 0$. Finally, setting $\alpha = 1$ establishes

$$\hat{V}'(1) = \frac{\varepsilon\hat{V}(1)(\sigma-1)\beta}{2\sigma-1}\left[\frac{1 - \beta\Lambda^{-\varepsilon}\psi_\lambda}{\Lambda^\varepsilon\hat{\psi}_\lambda - \beta} - \frac{\sigma-\varepsilon}{\sigma-1}\Lambda^{-\varepsilon}\psi_\lambda\right],$$

which can be positive for sufficiently high levels of σ and negative for sufficiently low ones. This completes the proof.

Derivation and discussion of constraints (2.31) and (2.32)

From Eq. (2.29), the constraint for a positive production level of necessities at home, $h_\ell < 1$, can be rewritten as

$$\kappa\frac{\sigma-1}{\sigma}\left[1-\beta\Lambda^{-\varepsilon}\psi_\lambda\right] < \frac{\mu+t}{1+t} - B. \tag{2.47}$$

Acknowledging Eq. (2.28), we can compute

$$\frac{\mu+t}{1+t} = 1 + \frac{\rho(1-B)-(1-B^*)}{[t-B^*(1+t)]-\rho} = 1 + \frac{\rho(1-B)-(1-B^*)}{\overline{\rho}(t)-\rho}, \tag{2.48}$$

where the second equality sign follows from the definition of $\overline{\rho}(t)$. Substituting Eq. (2.48) into (2.47) then gives

$$\kappa\frac{\sigma-1}{\sigma}\left[1-\beta\Lambda^{-\varepsilon}\psi_\lambda\right] < 1 - B + \frac{\rho(1-B)-(1-B^*)}{\overline{\rho}(t)-\rho} \equiv \hat{g}_0(t), \tag{2.49}$$

which is fulfilled if $\rho \geq 1$. To see this, note that with $H\Lambda = H^*\Lambda^*$, $\rho > 1$ implies $\rho(1-B) > 1-B^*$, so that $\hat{g}_0(t) > 1-B$. Noting that $1-B > \kappa\frac{\sigma-1}{\sigma}[1-\beta\Lambda^{-\varepsilon}\psi_\lambda]$, this is sufficient for a positive production level of necessities at home. In contrast, $\rho < 1$ and thus $\rho(1-B) < 1-B^*$ imply $\hat{g}_0(t) < 1-B$. However, since $\hat{g}_0'(t) > 0$ and $\lim_{t\to\infty}\hat{g}_0(t) = 1-B$ hold in this case, we can safely conclude that the condition in (2.49) is fulfilled for sufficiently high t.

In a next step, we combine Eq. (2.30) with the constraint for a positive production level of necessities abroad, $h_\ell^* < 1$, and obtain

$$\kappa\frac{\sigma-1}{\sigma}\left[1-\beta(\Lambda^*)^{-\varepsilon}\psi_\lambda^*\right] < \frac{1+\mu t}{\mu(1+t)} - B^*. \tag{2.50}$$

Acknowledging Eq. (2.28), we can compute

$$\frac{1+\mu t}{\mu(1+t)} = 1 + \frac{(1-B^*)-\rho(1-B)]}{\rho[t-B(1+t)]-1} = 1 + \frac{[(1-B^*)-\rho(1-B)]}{\rho/\underline{\rho}(t)-1}, \tag{2.51}$$

where the second equality sign follows from the definition of $\underline{\rho}(t)$. Substituting Eq. (2.51) into (2.50), we get

$$\kappa\frac{\sigma-1}{\sigma}\left[1-\beta(\Lambda^*)^{-\varepsilon}\psi_\lambda^*\right] < 1 - B^* + \frac{(1-B^*)-\rho(1-B)}{\rho/\underline{\rho}(t)-1} \equiv \hat{g}_1(t). \tag{2.52}$$

For $\rho \leq 1$ and thus $1-B^* \geq \rho(1-B)$, we have $\hat{g}_1(t) \geq 1-B^*$, which noting that $1-B^* > \kappa\frac{\sigma-1}{\sigma}[1-\beta(\Lambda^*)^{-\varepsilon}\psi_\lambda^*]$ is sufficient for (2.52). In contrast, we have $\hat{g}_1(t) < 1-B^*$ if $\rho > 1$ and thus $1-B^* < \rho(1-B)$, and in this case it is a priori not clear that (2.52) holds. However, acknowledging that $\rho > 1$ gives $\hat{g}_1'(t) > 0$ while $\lim_{t\to\infty}\hat{g}_1(t) = 1-B^*$ holds for any ρ, it follows that (2.52) must be fulfilled for a sufficiently high level of t. This completes the proof.

Relative market size differences and overall trade

Consider first $\rho \leq 1$, which implies that home is a net-exporter of luxuries, according to Proposition 1. As shown in the main text, total exports plus imports of home are then given by $2EX_\ell$, which equals exports plus imports of the foreign economy due to balanced trade. Accounting for $EX_\ell = Mr/(1+t)$, Eqs. (2.17) and (2.29) establish

$$EX_\ell = \frac{H\Lambda w[1-\beta\Lambda^{-\varepsilon}\psi_\lambda]}{(\mu+t)-B(1+t)} = H\Lambda w[1-\beta\Lambda^{-\varepsilon}\psi_\lambda]\frac{\underline{\rho}(t)\,[\overline{\rho}(t)-\rho]}{\overline{\rho}(t)-\underline{\rho}(t)}, \tag{2.53}$$

where the second equality sign follows from substituting Eq. (2.28) for μ and acknowledging the definitions of $\underline{\rho}(t)$, $\overline{\rho}(t)$. In a similar vein, we can note that $\rho > 1$ makes home a net-importer of luxuries, with total exports and imports given by $2IM_\ell$. Accounting for $IM_\ell = M\mu r/(1+t)$, Eqs. (2.17) and (2.29) establish

$$IM_\ell = \frac{H\Lambda w[1-\beta\Lambda^{-\varepsilon}\psi_\lambda]\mu}{(\mu+t)-B(1+t)} = H\Lambda w[1-\beta\Lambda^{-\varepsilon}\psi_\lambda]\frac{\rho-\underline{\rho}(t)}{\overline{\rho}(t)-\underline{\rho}(t)}, \tag{2.54}$$

where the second equality sign follows from substituting Eq. (2.28) for μ and acknowledging the definitions of $\underline{\rho}(t)$, $\overline{\rho}(t)$. Noting that $dEX_\ell/d\rho < 0$ while $dIM_\ell/d\rho > 0$ completes the proof.

Properties of price index (2.37) and proof of Proposition 2

Substituting Eq. (2.28) for μ into Eq. (2.37), gives $P_\ell = P_\ell^a F(t)^{\frac{1}{1-\sigma}}$, with

$$F(t) \equiv \frac{1+t}{t}\frac{1-B}{1-B\frac{1+t}{\mu+t}} = (1-B)\frac{1+t}{t}\frac{\overline{\rho}(t)-\rho B-(1-B^*)}{\overline{\rho}(t)(1-B)-(1-B^*)}. \tag{2.55}$$

Differentiation with respect to t gives $F'(t) = -\hat{F}(t)(1-B)/t^2$, with

$$\hat{F}(t) \equiv 1 + B\frac{\overline{\rho}(t)-\rho}{\overline{\rho}(t)(1-B)-(1-B^*)} - B(1-B^*)t(1+t)\frac{\rho(1-B)-(1-B^*)}{[\overline{\rho}(t)(1-B)-(1-B^*)]^2} \tag{2.56}$$

and $\overline{\rho}(t)(1-B) > 1-B^*$ from the parameter constraint in (2.31). Then, noting that $\rho >, =, < 1$ establishes $\rho(1-B) >, =, < 1-B^*$ it is immediate that $\hat{F}(t) > 0$ and thus $F'(t) < 0$ if $\rho \leq 1$. Furthermore, accounting for $\lim_{t\to\infty} \hat{F}(t) = [1/(1-B)]^2[1-\rho B(1-B)/(1-B^*)]$, we can safely conclude that $\lim_{t\to\infty} \hat{F}(t) = 0$ defines a unique $\hat{\rho} = (1-B^*)/B(1-B) > 1$, such that $\lim_{t\to\infty} \hat{F}(t) > (<)0$ if $\rho < (>)\hat{\rho}$. Then, provided that $\rho > \hat{\rho}$, $F'(t) > 0$ holds for sufficiently high t. Noting finally that $\lim_{t\to\infty} F(t) = 1$, we can safely conclude that $F(t) > 1$ and thus $P_\ell < P_\ell^a$ if $\rho \leq 1$, whereas $F(t) < 1$ and thus $P_\ell > P_\ell^a$ is guaranteed for high levels of t if $\rho > \hat{\rho}$. This completes the proof.

Formal details for the analysis in Section 2.4.1

We begin with a brief discussion of the second-order conditions for the profit-maximization problem of firms producing luxuries. The second-order conditions for an interior maximum require that evaluated at the optimum the Hessian matrix

$$\mathbf{H} = \begin{pmatrix} \frac{\partial^2\Pi}{\partial q_\ell^2} & \frac{\partial^2\Pi}{\partial q_\ell \partial\lambda_u} \\ \frac{\partial^2\Pi}{\partial\lambda_u\partial q_\ell} & \frac{\partial^2\Pi}{\partial\lambda_u^2} \end{pmatrix} \tag{2.57}$$

is negative semi-definite, implying that for any column vector $\mathbf{h} \equiv (h_1, h_2) \neq 0$, $\mathbf{h}^t\mathbf{H}\mathbf{h} \leq 0$. From Eq. (2.13′), we have

$$\begin{aligned} \frac{\partial^2\Pi}{\partial q_\ell^2} &= -\frac{\sigma-1}{\sigma}\frac{1}{\sigma}\frac{\kappa r}{q_\ell^2}, \qquad \frac{\partial^2\Pi}{\partial q_\ell\partial\lambda_u} = \frac{\partial^2\Pi}{\partial\lambda_u\partial q_\ell} = \frac{P_n}{\Lambda_u}\frac{1}{\lambda_u}, \\ \frac{\partial^2\Pi}{\partial\lambda_u^2} &= -2\frac{P_n}{\Lambda_u}\frac{q_\ell}{\lambda_u^2} - P_n\varphi(\varphi-1)\frac{F(\lambda_u)}{\lambda_u^2}. \end{aligned}$$

Evaluating these second derivatives at the first-order conditions $\partial\Pi/\partial q_\ell = 0$, $\partial\Pi/\partial\lambda_u = 0$, we can compute

$$\mathbf{h}^t\mathbf{H}\mathbf{h} = -\frac{P_n}{\Lambda_u}\frac{1}{\sigma q_\ell}\left[h_1^2 - 2h_1h_2\frac{\sigma q_\ell}{\lambda_u} + h_2^2\sigma(1+\varphi)\left(\frac{q_\ell}{\lambda_u}\right)^2\right]. \tag{2.58}$$

Accounting for $\varphi > \sigma - 1$, we obtain

$$\mathbf{h}^t\mathbf{H}\mathbf{h} \leq -\frac{P_n}{\Lambda_u}\frac{1}{\sigma q_\ell}\left[h_1^2 - 2h_1h_2\frac{\sigma q_\ell}{\lambda_u} + h_2^2\left(\frac{\sigma q_\ell}{\lambda_u}\right)^2\right] = -\frac{P_n}{\Lambda_u}\frac{1}{\sigma q_\ell}\left(h_1 - h_2\frac{\sigma q_\ell}{\lambda_u}\right)^2, \tag{2.59}$$

which confirms that the Hessian matrix is negative semi-definite.

In a next step, we derive Eqs. (2.17′) and (2.18′). For this purpose, we first acknowledge that denoting by h_ℓ the share of workers finding employment in the sector of luxuries and denoting by $\gamma_h = h_\ell(\hat\lambda_u/\underline{\lambda})^g$ the share of workers with effective labor supply $\lambda > \hat\lambda_u$, we can write total employment of workers in the sector of luxuries as $Mq_\ell = H\gamma_h \int_{\hat\lambda_u}^{\overline{\lambda}} \lambda dL(\lambda) = Hh_\ell\hat\Lambda_u$, where the second equality sign makes use of the Pareto assumption and $\hat\Lambda_u = \frac{g}{g-1}\hat\lambda_u$. From Eq. (2.13′), we then obtain $\frac{\sigma-1}{\sigma}\Lambda_u\kappa Mr = Hh_\ell\hat\Lambda_u w$, where $w = P_n$ has been used. Accounting for $\alpha = \eta^{-1}$, $\hat\alpha = \alpha\hat\Lambda_u^{-1}$, finally establishes Eq. (2.17′).

In a final step, we derive Eq. (2.18′). For this purpose, we first compute

$$\int_{i\in\mathcal{H}} e^i di = Hw\int_{\underline{\lambda}}^{\overline{\lambda}} \lambda dL(\lambda) + Hw\gamma_h(\hat\alpha - 1)\int_{\hat\lambda_u}^{\overline{\lambda}} \lambda dL(\lambda). \tag{2.60}$$

Making use of the Pareto assumption, we get $\int_{\underline{\lambda}}^{\overline{\lambda}} \lambda dL(\lambda) = \frac{g}{g-1}\underline{\lambda} = \Lambda$ and $\int_{\hat\lambda_u}^{\overline{\lambda}} \lambda dL(\lambda) = (\frac{\hat\lambda_u}{\underline{\lambda}})^{-g}\frac{g}{g-1}\hat\lambda_u = (\frac{\hat\lambda_u}{\underline{\lambda}})^{-g}\hat\Lambda_u$. Accounting for $\gamma_h = h_\ell(\frac{\hat\lambda_u}{\underline{\lambda}})^g$, we then obtain

$$\int_{i\in\mathcal{H}} e^i di = Hw\Lambda + Hwh_\ell(\hat\alpha - 1)\hat\Lambda_u. \tag{2.61}$$

We further compute

$$\int_{i\in\mathcal{H}} e^i\left(\frac{e^i}{P_n}\right)^{-\varepsilon} di = Hw\int_{\underline{\lambda}}^{\overline{\lambda}} \lambda^{1-\varepsilon}dL(\lambda) + Hw\gamma_h\left(\hat\alpha^{1-\varepsilon} - 1\right)\int_{\hat\lambda_u}^{\overline{\lambda}} \lambda^{1-\varepsilon}dL(\lambda), \tag{2.62}$$

where $w = P_n$ has been used. Applying the Pareto assumption, we can determine $\int_{\underline{\lambda}}^{\overline{\lambda}} \lambda^{1-\varepsilon} dL(\lambda) = \frac{g}{g+\varepsilon-1}\underline{\lambda}^{1-\varepsilon} = \Lambda^{1-\varepsilon}\psi_\lambda$, with ψ_λ defined in the main text, and $\int_{\hat{\lambda}_u}^{\overline{\lambda}} \lambda^{1-\varepsilon} dL(\lambda) = (\frac{\hat{\lambda}_u}{\underline{\lambda}})^{-g}\hat{\Lambda}_u^{1-\varepsilon}\psi_\lambda$. Substitution into Eq. (2.62), then gives

$$\int_{i\in\mathcal{H}} e^i \left(\frac{e^i}{P_n}\right)^{-\varepsilon} di = Hw\Lambda^{1-\varepsilon}\psi_\lambda + Hwh_\ell \left(\hat{\alpha}^{1-\varepsilon} - 1\right)\hat{\Lambda}_u^{1-\varepsilon}\psi_\lambda. \tag{2.63}$$

Substituting Eqs. (2.61) and (2.63) into $Mr = \int_{i\in\mathcal{H}} e^i[1 - \beta(\frac{e^i}{P_n})^{-\varepsilon}]di$, we obtain Eq. (2.18′). This completes the proof.

Formal details for the analysis in Section 2.4.2

We first show the derivation details for Eq. (2.23′). For this purpose, we introduce the auxiliary variables $E_\ell \equiv H\Lambda w\,[1 - \beta\Lambda^{-\varepsilon}\psi_\lambda]$ and $E_\ell^* \equiv H\Lambda w\,[1 - \beta\Lambda^{-\varepsilon}\psi_\lambda] + h_\ell^* H\Lambda w \frac{\sigma}{\sigma-1}\frac{1}{\kappa^*}B^*$ to denote economy-wide expenditures for luxury goods at home and abroad. Then, we can determine total revenues of home and foreign firms producing luxuries according to

$$r = E_\ell \left(\frac{p_\ell}{P_\ell}\right)^{1-\sigma} + \frac{E_\ell^*}{t}\left(\frac{p_\ell}{P_\ell^*}\right)^{1-\sigma}, \qquad r^* = E_\ell^* \left(\frac{p_\ell^*}{P_\ell^*}\right)^{1-\sigma} + \frac{E_\ell}{t}\left(\frac{p_\ell^*}{P_\ell}\right)^{1-\sigma}, \tag{2.64}$$

where $\kappa = \frac{\sigma}{2\sigma-1} > \frac{\sigma}{\sigma+(\sigma-1)\alpha^*} = \kappa^*$ and thus $p_\ell = \frac{\sigma}{\sigma-1}\frac{w}{\kappa} < \frac{\sigma}{\sigma-1}\frac{w}{\kappa^*} = p_\ell^*$. Using the zero-profit conditions $\kappa r = \sigma P_n f$, $\kappa^* r^* = \sigma P_n f$ and accounting for $\xi = \frac{p_\ell^*}{p_\ell} = \frac{\kappa}{\kappa^*}$, we derive

$$\frac{E_\ell^*}{(P_\ell^*)^{1-\sigma}} = \frac{t\xi^\sigma - 1}{t - \xi^\sigma}\frac{E_\ell}{P_\ell^{1-\sigma}},$$

which, substituting for E_ℓ and E_ℓ^*, can be reformulated to Eq. (2.23′).

To determine the sign of $d\mu/dt$ in Eq. (2.33′), we can make use of three insights. First, from Eq. (2.28′) it follows that $\mu > 0$ holds if and only if $t > \xi^\sigma + \sqrt{\xi^{2\sigma} - 1} \equiv \underline{t}$. Second, we can show that $t(1 - B^*) - \xi^{-\sigma} > 0$. To see this, note that $t > \underline{t}$ establishes $t > \xi^\sigma$ and thus $t(1 - B^*) - \xi^{-\sigma} > \xi^\sigma(1 - B^*) - \xi^{-\sigma} \equiv g(\alpha^*)$. Since both $\xi = \frac{\sigma+(\sigma-1)\alpha^*}{2\sigma-1}$ and $\xi(1 - B^*) = 1 + \frac{\sigma-1}{2\sigma-1}[(\alpha^*)^{1-\varepsilon} - 1]\beta\Lambda^{-\varepsilon}\psi_\lambda$ increase in α^*, we have $g'(\alpha^*) > 0$, so that $g(1) = 0$ gives $g(\alpha^*) > 0$ and thus $t(1 - B^*) - \xi^{-\sigma} > 0$ for all $\alpha^* > 1$. Third, defining $G(t) \equiv \frac{t-\xi^\sigma}{t(1-B^*)-\xi^{-\sigma}}$, we can compute $G'(t) >,=,< 0$ if $\xi^\sigma(1-B^*) - \xi^{-\sigma} >,=,< 0$. Hence, $G'(t) > 0$ follows from $g(\alpha^*) > 0$. This establishes $\frac{d^2\mu}{dt^2}\big|_{\frac{d\mu}{dt}=0} > 0$, according to Eq. (2.33′). We can thus conclude that if μ had an extremum in t, it would be a minimum. Evaluated at $t = \underline{t}$, we have $\frac{1}{\xi}\frac{t-\xi^\sigma}{t(1-B^*)-\xi^{-\sigma}} - \mu = \frac{1}{\xi}\frac{\underline{t}-\xi^\sigma}{\underline{t}(1-B^*)-\xi^{-\sigma}} > 0$, implying $d\mu/dt > 0$ for small values of t higher than $\underline{t}$. This is inconsistent with a minimum of μ and proves that $d\mu/dt > 0$ for all possible t.[18]

In a final step, we determine the welfare effects of trade and begin with the analysis of home. Substituting zero-profit condition $\kappa r = \sigma P_n f$ into Eq. (2.26′) and accounting for

[18] To determine the effects of supply-side asymmetry on the Grubel-Lloyd index, we also need to know the sign of $d\mu/d\alpha^*$. Rearranging terms in Eq. (2.28′), we can rewrite the ratio of foreign to domestic firms as $\mu = \frac{t^2-2t\xi^\sigma+1}{(t^2-1)\xi(1-B^*)-2t\xi^{1-\sigma}+2\xi}$. Noting $d\xi/d\alpha^* > 0$ and $d\xi(1 - B^*)/d\alpha^* > 0$, we can safely conclude that $d\mu/d\alpha^* < 0$. This is sufficient for $GLI = \mu\xi^{1-\sigma}\frac{t-\xi^\sigma}{t\xi^\sigma-1}$ to decrease in α^*.

$\frac{r^*}{r} = \frac{\kappa}{\kappa^*} = \xi$, we can compute

$$\kappa \frac{\sigma - 1}{\sigma} \frac{H\Lambda \left(1 - \beta\Lambda^{-\varepsilon}\psi_\lambda\right)}{(\sigma - 1)f} \frac{t - 1}{t - \xi^\sigma} = \frac{t + \mu\xi^{1-\sigma}}{1 + t} M. \tag{2.65}$$

Differentiating the left-hand side with respect to t, we find that $\frac{t+\mu\xi^{1-\sigma}}{1+t}M$ decreases in trade costs. Since $(1 + t)/t$ also decreases in t, it follows from $P_\ell = p_\ell \left[M \frac{t+\mu\xi^{1-\sigma}}{1+t} \frac{1+t}{t}\right]^{\frac{1}{1-\sigma}}$ that home benefits from trade.[19] To determine the welfare effects of trade for foreign, we first determine two auxiliary results. Starting from the market clearing condition in Eq. (2.26′) and accounting for $\frac{r}{r^*} = \xi^{-1}$ and $\kappa^* r^* = \sigma P_n f$, we can compute

$$H\Lambda w \left(1 - \beta\Lambda^{-\varepsilon}\psi_\lambda\right) = (t\xi^{-\sigma} - 1) \left[\frac{t^2}{t^2 - 1} \left(1 + \frac{1}{t\mu\xi^{1-\sigma}}\right) - 1\right] M^* \frac{\sigma P_n f}{\kappa^*}$$

and thus

$$M^* = \kappa^* \frac{\sigma - 1}{\sigma} \frac{H\Lambda \left(1 - \beta\Lambda^{-\varepsilon}\psi_\lambda\right)}{(\sigma - 1)f} \frac{t - 1}{t\xi^{-\sigma} - 1} \frac{(t + 1)\mu\xi^{1-\sigma}}{\mu\xi^{1-\sigma} + t}. \tag{2.66}$$

Then, differentiating $G_0(t) \equiv \frac{t-1}{t\xi^{-\sigma}-1} \frac{(t+1)\mu\xi^{1-\sigma}}{\mu\xi^{1-\sigma}+t}$, we compute $G_0'(t) = \frac{G_0(t)}{(t-1)(t+\mu\xi^{1-\sigma})} g_0(t)$, with

$$g_0(t) \equiv -(1 - \xi^{-\sigma}) \frac{t + \mu\xi^{1-\sigma}}{t\xi^{-\sigma} - 1} - (1 - \mu\xi^{1-\sigma}) \frac{t - 1}{t + 1} + \frac{d\mu}{dt} \frac{t(t - 1)}{\mu}. \tag{2.67}$$

Due to $\lim_{t\to\infty} \frac{t+\mu\xi^{1-\sigma}}{t\xi^{-\sigma}-1} = \xi^\sigma$, $\lim_{t\to\infty} \frac{d\mu}{dt} \frac{t(t-1)}{\mu} = 2\frac{(1-B^*)\xi^\sigma - \xi^{-\sigma}}{1-B^*}$, and $\lim_{t\to\infty} \mu\xi^{1-\sigma} = \frac{\xi^{-\sigma}}{1-B^*}$, we can compute $\lim_{t\to\infty} g_0(t) = \frac{(1-B^*)\xi^\sigma - \xi^{-\sigma}}{1-B^*}$, which is zero if $\alpha^* = 1$ and is positive if $\alpha^* > 1$.

From Eq. (2.66) we furthermore obtain

$$M^* \left[1 + \frac{1}{t\mu\xi^{1-\sigma}}\right] = \kappa^* \frac{\sigma - 1}{\sigma} \frac{H\Lambda \left(1 - \beta\Lambda^{-\varepsilon}\psi_\lambda\right)}{(\sigma - 1)f} \frac{t - 1}{t\xi^{-\sigma} - 1} \frac{t + 1}{t} \frac{t\mu\xi^{1-\sigma} + 1}{t + \mu\xi^{1-\sigma}}. \tag{2.68}$$

Differentiating $G_1(t) \equiv \frac{t-1}{t\xi^{-\sigma}-1} \frac{t+1}{t} \frac{t\mu\xi^{1-\sigma}+1}{t+\mu\xi^{1-\sigma}}$ gives $G_1'(t) = \frac{G_1(t)}{(t-1)(t+\mu\xi^{1-\sigma})} g_1(t)$, with

$$\begin{aligned} g_1(t) \equiv {} & -(1 - \xi^{-\sigma}) \frac{t + \mu\xi^{1-\sigma}}{t\xi^{-\sigma} - 1} - \frac{(t + \mu\xi^{1-\sigma})(t - 1)}{t(t + 1)} \\ & - \frac{t - 1}{t\mu\xi^{1-\sigma} + 1} \left[1 - (\mu\xi^{1-\sigma})^2 - \frac{d\mu}{dt}(t^2 - 1)\xi^{1-\sigma}\right]. \end{aligned}$$

Accounting for $\lim_{t\to\infty} \frac{t+\mu\xi^{1-\sigma}}{t\xi^{-\sigma}-1} = \xi^\sigma$, $\lim_{t\to\infty} \frac{t-1}{t\mu\xi^{1-\sigma}+1} = \xi^\sigma(1 - B^*)$, $\lim_{t\to\infty} \mu\xi^{1-\sigma} = \frac{\xi^{-\sigma}}{1-B^*}$, and $\lim_{t\to\infty} \frac{d\mu}{dt}(t^2 - 1)\xi^{1-\sigma} = 2\frac{\xi^\sigma(1-B^*) - \xi^{-\sigma}}{1-B^*} \frac{\xi^{-\sigma}}{1-B^*}$, we compute $\lim_{t\to\infty} g_1(t) = \frac{B^*(1-B^*)\xi^\sigma - \xi^{-\sigma}}{1-B^*} \equiv \tilde{g}_1(\alpha^*)$. Accounting for $\tilde{g}_1(1) = -1$, $\lim_{\alpha^*\to\infty} \tilde{g}_1(\alpha^*) = \infty$, and $\tilde{g}_1'(\alpha^*) >$

[19] Rearranging Eq. (2.65) we find that $M\frac{t-\xi^\sigma}{t^2-1}$ is inversely proportional to $\mu\xi^{1-\sigma} + t$ and therefore decreases in t. Noting from Eq. (2.34′) that home's total exports of luxuries can be expressed as $EX_\ell = M\frac{t-\xi^\sigma}{t^2-1} \frac{t\xi^\sigma-1}{t-\xi^\sigma} \frac{\sigma P_n f}{\kappa}$, we can safely conclude that total (intra- plus inter-industry) trade decreases in t. Noting from Footnote 18 that $\mu\xi^{1-\sigma}$ decreases in α^*, we can further conclude that $M\frac{t-\xi^\sigma}{t^2-1}$ increases in the foreign wage premium and this is sufficient for total (intra- plus inter-industry trade) to increase in supply-side dissimilarity.

0, we can safely conclude that there exists a critical $\underline{\alpha}^* > 1$, such that $\lim_{t\to\infty} g_1(t) >, =, < 0$ if $\alpha^* >, =, < \underline{\alpha}^*$. From $P_\ell^* = p_\ell^* \left[M^* \frac{1+t\mu\xi^{1-\sigma}}{t\mu\xi^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}$, it then follows that for initially high trade costs, the foreign price index of luxuries decreases in t if α^* and therefore ξ are sufficiently high.

Differentiating the welfare function $V(P_n, P_\ell^*, \overline{e}^*, \hat{\psi})$ from Eq. (2.21), we finally obtain

$$\frac{dV(\cdot)}{dt} = \frac{V(\cdot) + (1-\beta)/\varepsilon}{(t-1)(t+\mu\xi^{1-\sigma})} \left[\frac{\varepsilon}{\sigma - 1} g_1(t) + \frac{\Lambda^\varepsilon \hat{\psi}_\lambda h_\ell^*[(\alpha^*)^\varepsilon - 1]}{\Lambda^\varepsilon \hat{\psi}_\lambda \{1 + h_\ell^*[(\alpha^*)^\varepsilon - 1)]\} - \beta} g_0(t)\right], \quad (2.69)$$

where $\left(\frac{\overline{e}^*}{P_n}\right)^\varepsilon \hat{\psi} = \Lambda^\varepsilon \hat{\psi}_\lambda \{1 + h_\ell^*[(\alpha^*)^\varepsilon - 1]\}$ and $h_\ell^* = M^* \frac{(\sigma-1)f}{H\Lambda}$ have been considered. Accounting for $\lim_{t\to\infty} g_0(t) > 0$ and $\lim_{t\to\infty} g_1(t) = \frac{B^*(1-B^*)\xi^\sigma - \xi^{-\sigma}}{1-B^*} > 0$ if $\alpha^* > \underline{\alpha}^*$, we can thus safely conclude that the foreign country loses from a small reduction of initially high trade costs if the price distortion in the labor market is sufficiently high. In the limiting case of $\alpha^* = 1$, we have $\lim_{t\to\infty} g_0(t) = 0$ and $\lim_{t\to\infty} g_1(t) = -1$, so that in this case the foreign country unambiguously benefits from a small reduction of initially high trade costs. This completes the proof.

Chapter 3

PIGL Preferences, Income Differences and International Trade

3.1 Introduction

Empirical research in the field of international trade has shown, that relaxing the assumption of useful but rather restrictive homothetic preferences and instead making use of nonhomothetic preferences seems reasonable. Demand-side as opposed to supply-side driven explanations for trade, have gained in importance during the last decades ever since the introduction of the Linder (1961) hypothesis.[1] Papers like Hunter and Markusen (1988), Bergstrand (1989), Hunter (1991), Hallak (2010) and Fieler (2011) show empirical relevance of per-capita income differences as a determinant of international trade flows. In addition, there is also empirical research highlighting the role of income dispersion for explaining international trade flows (cf. Francois and Kaplan, 1996; Dalgin et al., 2008; Choi et al., 2009; Bernasconi, 2013). Egger and Habermeyer (2019) provide an extensive overview of various theoretical contributions dealing with the role of nonhomothetic preferences in international trade.[2] The authors distinguish different types of preferences used in previous work and discuss prevailing drawbacks. As per-capita income but also income dispersion hold a relevant position in explaining the intensive margin of consumption and thus the structure of international trade, Egger and Habermeyer (2019) introduce a new class of preferences in the trade context, namely "price-independent generalized-linear" (PIGL) preferences (cf. Muellbauer, 1975, 1976), which are capable of capturing both factors of influence. These preferences have the convenient feature that aggregation of individual demand is possible, even though households are assumed to be heterogeneous in their income level. The authors set up a two-country framework along the lines of Helpman and Krugman's (1985) home-market model with two sectors of production and labor

[1] A prominent example for a supply-side approach is the Ricardian model and its numerous extensions. For instance, Dornbusch et al. (1977) use the Ricardian framework to consider a continuum of goods, whereas Eaton and Kortum (2002) extend the analysis to the case of many countries.

[2] Notable examples are Markusen (1986, 2013), Matsuyama (2000), Fajgelbaum et al. (2011) and Foellmi et al. (2018).

as the only input factor. Heterogeneous households consume homogeneous necessities produced under perfect competition and differentiated luxuries produced under monopolistic competition. Egger and Habermeyer (2019) show that the country with a higher level and/or dispersion of per-capita income has the larger home market for luxuries and becomes a net-exporter of these goods. Provided that both sectors pay the same market clearing wage, the structure of trade is irrelevant for welfare in the open economy. In this paper, we show that these fundamental insights extend to a more general class of PIGL preferences.[3]

Starting from an indirect utility function representing parametric PIGL preferences, Egger and Habermeyer (2019) face the fundamental problem that a closed form representation for the direct utility function does in general not exist. This is problematic, because it is a priori not clear that the demand functions derived from indirect utility are in fact the solution to a well-defined utility maximization problem. In consumer theory, this is well-known under the term "integrability problem", whose first formal account is usually attributed to Antonelli (1886), and which has been brought back to the surface of economic research by Samuelson (1950). Focusing on direct utility functions, Samuelson (1950) complements Antonelli's (1886) conditions for mathematical integrability of demand functions with a condition that ensures economic integrability (see Hurwicz, 1971).[4] Hurwicz and Uzawa (1971), further elaborate on the integrability problem outlined by Antonelli (1886) and Samuelson (1950) and provide sufficient conditions for solving this problem. In a recent paper, Fally (2018) draws on these sufficient conditions outlined by Hurwicz and Uzawa (1971) for solving the integrability problem to show that the conjecture formulated by Gorman (1995) on the permissible functional form Gorman-Pollak demand function is correct. In fact, Fally (2018) shows that the respective demand functions can only have two forms, which he summarizes under the term generalized separable demand systems.[5] Although not directly referring to the integrability problem, Boppart (2014) shows that the conditions formulated by Hurwicz and Uzawa (1971) are fulfilled in the case of parametric PIGL preferences with homogeneous goods.[6] However, this does not mean that the solution of the integrability problem extends to a model variant with a continuum of differentiated luxury goods studied by Egger and Habermeyer (2019), who avoid the integrability problem by focussing on a subclass of parametric PIGL preferences,

[3]Since Chapters 2 and 3 originate from autonomous manuscripts, notations might differ between those chapters.

[4]As discussed in Hurwicz (1971) and Hurwicz and Uzawa (1971), demand systems can be distinguished by local and infinitesimal properties or global and finite properties. The former contains assumptions of substitution matrices about symmetry and negative semi-definiteness, as introduced by Antonelli (1886) and Samuelson (1950). The latter includes, for instance, the strong axiom of revealed preferences put forward by Houthakker (1950).

[5]In a subsequent contribution, Fally (2019) uses the demand system to extend the analysis of Arkolakis et al. (2017). In their seminal paper, Arkolakis et al. (2017) depart from CES preferences and compare the welfare gains from trade liberalization with constant and variable markups. Bertoletti and Etro (2018) use Gorman-Pollak demand functions in a model of monopolistic competition, distinguishing homogeneous and heterogeneous firms.

[6]Boppart (2014) incorporates PIGL preferences in a neoclassical growth model and shows that demand-driven structural change can be consistent with balanced growth (and thus the Kaldor facts) if preferences have parametric PIGL form. In his model, Boppart (2014) distinguishes a substitution and an income effect, with non-Gorman form preferences needed to generate an income effect from changes in the level and dispersion of per-capita income on the expenditure shares of consumers.

for which a direct utility function can be derived. Since restricting their analysis to PIGL preferences with a direct representation of the utility function limits the suitability of the Egger and Habermeyer (2019) model for explaining real world trade patterns, it is the purpose of this paper to extend their model to one that allows for more general parametric PIGL preferences.

For this purpose, we modify the assumptions of Egger and Habermeyer (2019) and assume, similar to Boppart (2014), homogeneous luxuries, which are nontradable and are in turn assembled from differentiated, tradable intermediates as in Ethier (1982b).[7] Associating differentiated goods with intermediates allows us to rely on insights from Boppart (2014) to solve the integrability problem, while keeping the main ingredients of the trade model outlined by Egger and Habermeyer (2019). Whereas the resulting model is richer and the analysis of the open economy turns out to be more complicated than in Egger and Habermeyer (2019), the fundamental insight from their analysis that, in the absence of labor market imperfection, the level and dispersion of per-capita income are crucial determinants for international trade flows, but irrelevant for welfare gains from trade, remains intact. This suggests that the preference assumptions imposed by Egger and Habermeyer (2019) for tractability reasons are less restrictive than they appear at a first glance.

Krugman (1980) and Helpman and Krugman (1985) have provided the scientific foundation of models, containing increasing returns to scale, monopolistic competition and trade costs, which give rise to a home-market effect. Its most general interpretation states that "a country whose share of world demand for a good is larger than average will have – ceteris paribus – a more than proportionally larger-than-average share of world production of that good" (Crozet and Trionfetti, 2008, p. 309) and will export that good on average. The home-market effect has strong empirical support. Davis and Weinstein (1999) introduce a model of economic geography to test the existence of home-market effects on a regional level using Japanese data, whereas Davis and Weinstein (2003) focus on the international level by employing a data set of OECD countries. Head and Ries (2001) find support of the home-market effect by using firm-level data for U.S. and Canadian manufacturing. However, details of home-market models, such as the assumption of a freely traded outside good produced under constant returns to scale and sold under perfect competition, seem to be controversial. On the one hand, Davis (1998b) shows that the home-market effect vanishes when trade cost of equal size are introduced for the outside good. On the other hand, Davis (1998b) also hints on the importance of the relative size of trade cost for the existence of a home-market effect. Crozet and Trionfetti (2008) build on the insights of Davis (1998b) and show that, in general, the home-market effect remains valid, however, in mitigated form. This confirms the choice of neglecting trade costs for the outside good as a simplifying assumption for the sake of algebraic convenience.[8] We contribute to this large strand of literature as in our analysis the home-market effect orig-

[7] Ethier (1982b) is one of the first approaches that associates the theory of intermediate inputs with scale economies and imperfect competition in final goods production.

[8] There exist numerous other topics in international trade applying a home-market model structure. For instance, Head et al. (2002) test the role of the market structure for the robustness of the home-market effect, whereas Helpman and Itskhoki (2010) consider a labor market imperfection within the home-market framework.

inates from the demand-side. We consider heterogeneous households, differing in their income levels, who face non-Gorman preferences. This forms country-specific aggregate demand structures, which are decisive for the respective production and trade pattern.[9]

The remainder of the paper is organized as follows. Section 3.2 outlines the main building blocs of our model, shows the solution to the integrability problem, derives the market equilibrium in the closed economy and sheds light on how changes in the level and dispersion of per-capita income affect welfare. Section 3.3 presents a two-country trade model with countries differing only in the level and/or dispersion of per-capita income. After having solved the open economy equilibrium, we derive the trade pattern and resulting welfare effects of trade. Section 3.4 concludes.

3.2 The closed economy

We introduce a static model with nonhomothetic preferences over two types of final goods. Homogeneous necessities (n) and a homogeneous luxury good (ℓ) are both produced under perfect competition. The luxury good is a CES aggregate of differentiated intermediate goods (ω) which are produced under monopolistic competition. The economy is populated by a continuous set $\mathcal{H}$ of single-person households with Lebesgue measure H. The inelastic effective labor supply λ is household-specific and distributed over interval $\left[\underline{\lambda}, \overline{\lambda}\right]$ according to a continuously differentiable cdf $L(\lambda)$. Thus, workers differ in their efficiency units of labor they provide to firms, leading to heterogeneous levels of income. Labor is free to move between the sectors of necessities and intermediate goods, whereas the final luxury good is costlessly assembled from the available intermediates. Assuming non-Gorman form preferences, the level and/or dispersion of per-capita income are instrumental for the aggregate demand of the two final goods.

3.2.1 Preferences and demand

The model features "price-independent generalized-linear" (PIGL) preferences as introduced by Muellbauer (1975, 1976). This is the most general class of preferences that allows for a (positive) representative consumer and consequently avoids an aggregation problem from individual to economy-wide demand. Egger and Habermeyer (2019) put those non-Gorman form preferences in an international trade context considering a labor market imperfection. The authors use a subgroup of parametric PIGL preferences that features an explicit solution for direct utility and hence, avoids an integrability problem and makes the computation of a price index in a model of monopolistic competition possible. Unlike Egger and Habermeyer (2019), in the underlying paper we refer to a more general form of parametric PIGL preferences, put forward by Boppart (2014), with the

[9]Our analysis stands in contrast with the seminal contribution of Krugman (1980), who considers a home-market effect arising from exogenous differences in the preferences of two economies. In this sense, our model is for example closer related to Fajgelbaum et al. (2011), who use a home-market model to show that richer countries export goods of higher quality.

following functional form of the indirect utility function

$$v(P_n, P_\ell, e^i) = \frac{1}{\varepsilon}\left(\frac{e^i}{P_\ell}\right)^\varepsilon - \frac{\beta}{\gamma}\left(\frac{P_n}{P_\ell}\right)^\gamma - \frac{1}{\varepsilon} + \frac{\beta}{\gamma}. \tag{3.1}$$

Households receive utility by consuming necessities and the luxury good, with P_n and P_ℓ being the respective prices. The individual expenditure level is given by e^i and ε, γ and β are preference parameters, where $\beta \in (0, 1)$ and $0 \leq \varepsilon < \gamma < 1$ are assumed. Following Boppart (2014), the ranking $\varepsilon < \gamma$ restricts the analysis to the empirically relevant case of an elasticity of substitution between necessities and the luxury good that is strictly smaller than unity (see Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008, for further discussion).[10] The limiting case of $\varepsilon \to 0$ captures homothetic preferences and hence the case usually considered by the trade literature. The corresponding indirect utility function, derived from Eq. (3.1), then reads $v(P_n, P_\ell, e^i) = \ln\left(e^i/P_\ell\right) + (\beta/\gamma)\left[1 - (P_n/P_\ell)^\gamma\right]$.

By means of Roy's identity, Marshallian demand functions for the two final goods can be derived from the indirect utility function in Eq. (3.1):

$$X_n^i = \beta\frac{e^i}{P_n}\left(\frac{e^i}{P_\ell}\right)^{-\varepsilon}\left(\frac{P_n}{P_\ell}\right)^\gamma, \tag{3.2}$$

$$X_\ell^i = \frac{e^i}{P_\ell}\left[1 - \beta\left(\frac{e^i}{P_\ell}\right)^{-\varepsilon}\left(\frac{P_n}{P_\ell}\right)^\gamma\right]. \tag{3.3}$$

The Engel curves for necessities and the luxury good are non-linear in the expenditure level. On the one hand, the expenditure share of consumption devoted to necessities decreases in the expenditure level, making the Engel curve concave. On the other hand, convexity of the Engel curve arises because the expenditure share of consumption devoted to the homogeneous luxury good increases in the expenditure level. Linear Engel curves reflect homothetic preferences and thus the limiting case of $\varepsilon \to 0$. Aggregating Eqs. (3.2) and (3.3) in each case over all households gives aggregate demand functions

$$X_n = \beta\frac{H\bar{e}}{P_n}\left(\frac{\bar{e}}{P_\ell}\right)^{-\varepsilon}\left(\frac{P_n}{P_\ell}\right)^\gamma\psi, \tag{3.4}$$

$$X_\ell = \frac{H\bar{e}}{P_\ell}\left[1 - \beta\left(\frac{\bar{e}}{P_\ell}\right)^{-\varepsilon}\left(\frac{P_n}{P_\ell}\right)^\gamma\psi\right], \tag{3.5}$$

where households' average expenditure level is introduced by $\bar{e} \equiv \frac{1}{H}\int_{i\in\mathcal{H}} e^i di$ and ψ is a measure of economy-wide expenditure dispersion which lies between 0 and 1. Dispersion measure ψ is a weighted mean of individual consumption expenditure divided by the arithmetic mean of expenditures and shown below:

$$\psi \equiv \frac{1}{H}\int_{i\in\mathcal{H}}\left(\frac{e^i}{\bar{e}}\right)^{1-\varepsilon} di. \tag{3.6}$$

[10] The limiting case of $\varepsilon = \gamma$ establishes a subgroup of parametric PIGL preferences for which the direct utility function can be solved. This case is thoroughly discussed in Egger and Habermeyer (2019) and therefore excluded from the analysis here. In the limiting case of $\varepsilon \to 0$ and $\gamma \to 0$, preferences have Cobb-Douglas form and in the limiting case of $\varepsilon \to 1$ and $\gamma \to 1$, preferences are quasilinear.

In the limiting case of homothetic preferences, Eq. (3.6) establishes $\psi = 1$ and thus induces aggregate demand in Eqs. (3.4) and (3.5) to be independent of the expenditure dispersion. For a given $\varepsilon > 0$, a higher (lower) ψ reflects a lower (higher) expenditure dispersion in the economy, which in turn increases (decreases) the demand for necessities and decreases (increases) the demand for the luxury good.

3.2.2 Economic integrability

Before turning to the production side of our model, we show that the indirect utility function for parametric PIGL preferences in Eq. (3.1) leads to a demand system that is integrable and thus results from utility-maximizing households. This refers to the "integrability problem" discussed, for instance, by Antonelli (1886), Samuelson (1950), Hurwicz and Uzawa (1971) and Fally (2018). In the following, we investigate three sufficient conditions, established by Hurwicz and Uzawa (1971), that must be fulfilled to solve the integrability problem. A demand system arises from utility maximization subject to a budget constraint if (i) the whole budget is spent, (ii) the Slutsky matrix is symmetric and (iii) the Slutsky matrix is negative semi-definite. We now check step by step that these conditions are fulfilled, thereby relying on an online appendix of Boppart (2014) who examines these conditions but does not relate them to the integrability problem discussed above.

Check for condition (i): Calculating the individual expenditure shares of consumption devoted to necessities and the luxury good, respectively, we use Eqs. (3.2) and (3.3) to obtain

$$\eta_n^i = \beta \left(\frac{e^i}{P_\ell}\right)^{-\varepsilon} \left(\frac{P_n}{P_\ell}\right)^{\gamma} \quad \text{and} \quad \eta_\ell^i = 1 - \beta \left(\frac{e^i}{P_\ell}\right)^{-\varepsilon} \left(\frac{P_n}{P_\ell}\right)^{\gamma}. \tag{3.7}$$

Eq. (3.7) implies that the two expenditure shares add up to one, so that the whole budget is spent and condition (i) is fulfilled.

Check for condition (ii): To derive the Slutsky matrix, we can first solve Eq. (3.1) for the individual expenditure level. This yields the expenditure function according to

$$e\left(P_n, P_\ell, v(\cdot)\right) = \left\{\varepsilon v(\cdot) + \frac{\varepsilon\beta}{\gamma}\left(\frac{P_n}{P_\ell}\right)^{\gamma} + 1 - \frac{\varepsilon\beta}{\gamma}\right\}^{\frac{1}{\varepsilon}} P_\ell. \tag{3.8}$$

Applying Shephard's lemma gives the respective Hicksian (or compensated) demand function for both final goods

$$X_n^{i,h} = \beta \left(\frac{e(\cdot)}{P_\ell}\right)^{1-\varepsilon} \left(\frac{P_\ell}{P_n}\right)^{1-\gamma}, \tag{3.9}$$

$$X_\ell^{i,h} = \frac{e(\cdot)}{P_\ell}\left[1 - \beta \left(\frac{e(\cdot)}{P_\ell}\right)^{-\varepsilon} \left(\frac{P_n}{P_\ell}\right)^{\gamma}\right]. \tag{3.10}$$

Contrasting the Hicksian demand functions in Eqs. (3.9) and (3.10) with the Marshallian demand functions in Eqs. (3.2) and (3.3) confirms correctness of our results and illustrates the well-known difference that, by means of Eq. (3.8), the former are functions of prices and utility, whereas the latter are functions of prices and expenditure levels.

The Slutsky matrix captures the first partial derivatives of Eqs. (3.9) and (3.10) with respect to prices and can be expressed as

$$\mathbf{S} = \begin{pmatrix} \frac{\partial X_n^{i,h}}{\partial P_n} & \frac{\partial X_n^{i,h}}{\partial P_\ell} \\ \frac{\partial X_\ell^{i,h}}{\partial P_n} & \frac{\partial X_\ell^{i,h}}{\partial P_\ell} \end{pmatrix} = \Xi \begin{pmatrix} \frac{P_\ell}{P_n} & -1 \\ -1 & \frac{P_n}{P_\ell} \end{pmatrix} \tag{3.11}$$

where $\Xi \equiv \beta\,[e(\cdot)/P_\ell]^{1-2\varepsilon}\,(P_n/P_\ell)^\gamma\,(1/P_n)\,\{\beta(1-\varepsilon)\,(P_n/P_\ell)^\gamma - (1-\gamma)\,[e(\cdot)/P_\ell]^\varepsilon\}$. Eq. (3.11) proves symmetry of the Slutsky matrix and shows that condition (ii) is satisfied.

Check for condition (iii): To show that the Slutsky matrix is negative semi-definite, we can check the Eigenvalues of $\mathbf{S}$, which have to be non-positive. The Eigenvalues of $\mathbf{S}$ are given by the solutions of the linear equation system,

$$\begin{pmatrix} \Xi\frac{P_\ell}{P_n} & -\Xi \\ -\Xi & \frac{P_n}{P_\ell}\Xi \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \xi \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}. \tag{3.12}$$

Ignoring trivial solutions with $y_1, y_2 = 0$, the Eigenvalues, ξ, are determined by the condition that

$$\begin{vmatrix} \left(\Xi\frac{P_\ell}{P_n} - \xi\right) & -\Xi \\ -\Xi & \left(\frac{P_n}{P_\ell}\Xi - \xi\right) \end{vmatrix} = 0 \tag{3.13}$$

Solving the determinant establishes the following quadratic equation: $\xi^2 - \xi\Xi\,[(P_\ell/P_n) + (P_n/P_\ell)]$, which has two solutions in ξ, namely $\xi = 0$ and $\xi = \Xi\,[(P_\ell/P_n) + (P_n/P_\ell)]$. Negative semi-definiteness requires $\xi \leq 0$ and thus $\Xi \leq 0$. This is equivalent to

$$e(\cdot)^\varepsilon \geq \beta\frac{1-\varepsilon}{1-\gamma}P_n^\gamma P_\ell^{\varepsilon-\gamma}. \tag{3.14}$$

If the inequality in (3.14) holds, the Slutsky matrix is negative semi-definite and integrability condition (iii) is fulfilled. Below, we introduce a parameter constraint to establish the inequality in (3.14) even for the poorest individuals.

We complete the discussion in this section by noting that positive consumption levels of necessities and the luxury good further require that

$$e(\cdot)^\varepsilon > \beta P_n^\gamma P_\ell^{\varepsilon-\gamma}, \tag{3.15}$$

according to Eqs. (3.9) and (3.10). Given that $\varepsilon < \gamma$ (see above) positive demand for both goods is guaranteed by (3.14), the negative semi-definiteness of the Slutsky matrix.

3.2.3 Technology and production

In this economy there exist two sectors producing final goods, namely homogeneous necessities (n) and a homogeneous luxury good (ℓ), and one intermediate goods sector. The luxury good is an aggregate of differentiated intermediate goods (ω) and it is assembled under perfect competition at zero cost. The labor market is perfectly competitive and there is no involuntary unemployment in the model. Labor is the only input factor for necessities and intermediates and workers are mobile between those two sectors.

Homogeneous necessities are produced under perfect competition. The firms' production function is linear in the effective labor input and fixed cost do not apply. One unit of labor input receives market clearing wage w and produces one unit of output, i.e. $P_n = w$. Differentiated intermediate goods are produced with labor input via identical production functions under monopolistic competition. Having paid fixed cost of production $P_n f$ in efficiency units of labor, each firm supplies a unique intermediate variety ω from the set of available varieties Ω. Labor mobility between the sectors of necessities and intermediates leads to wage equalization. Consequently, each labor unit receives market clearing wage w and produces one unit of a differentiated variety. It follows that firms are indifferent between hiring a worker with a low or high level of effective labor supply λ, because per-unit cost of labor is the same. Intermediate goods are assembled to a homogeneous luxury good, with all intermediates entering the production process symmetrically. To be more precise, we follow Ethier (1982b) and assume

$$X_\ell = \left[\int_{\omega\in\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}, \tag{3.16}$$

where $q(\omega)$ is the quantity of intermediate good ω which is equivalent to the efficiency units of labor used by intermediate firm ω. The constant elasticity of substitution between the differentiated intermediate goods is given by $\sigma > 1$. The final output of the luxury good has constant returns to scale in the quantity of intermediates assuming a fixed number of differentiated intermediates and has increasing returns to scale in the number of differentiated intermediates considering a fixed total quantity of intermediates. As formally shown in the Appendix, maximizing aggregate profits in the sector of the luxury good, $\Pi = P_\ell X_\ell - \int_{\omega\in\Omega} p(\omega)q(\omega)d\omega$, gives an isoelastic demand function of the following form: $q(\omega) = q(\hat{\omega})\left[p(\hat{\omega})/p(\omega)\right]^{\sigma}$, where $p(\omega)$ and $p(\hat{\omega})$ are prices for differentiated varieties ω and $\hat{\omega}$, respectively. Furthermore, in the Appendix we show that the price corresponding to the composite X_ℓ is an index of the prices of differentiated varieties and is given by $P_\ell \equiv \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$, which features constant elasticity.[11] In Eq. (3.17), we show that profit maximization of a firm producing intermediates, with profit function $\pi(\omega) = p(\omega)q(\omega) - P_n q(\omega) - P_n f$, leads to the well-known result that firms set their prices as a constant markup over marginal cost. The optimal price for intermediate ω and the optimal quantity of intermediate ω are given by

$$p(\omega) = \frac{\sigma}{\sigma-1} w \quad \text{and} \quad q(\omega) = (\sigma-1)f. \tag{3.17}$$

Eq. (3.17) holds for all varieties ω, which implies that all intermediates are produced in equal amounts and are sold for the same price. According to this, firm index ω can be suppressed in the further analysis. Firms achieve zero profits in equilibrium which establishes the zero-profit condition $r = \sigma P_n f$, with firms' revenues given by r.[12]

[11] Notice that we abstract from fixed cost for producing the homogeneous luxury good ℓ, so that total cost of producing the luxury equal aggregate revenues of intermediate goods producers.

[12] In a previous version, we have considered firm heterogeneity along the lines of Melitz (2003). However,

3.2.4 Autarkic equilibrium

Provided that there is no redistribution of income, individual expenditure levels are heterogeneous – due to ex ante heterogeneity of workers in their effective labor supplies – and (acknowledging zero profits) they are equal to the workers' respective labor incomes. To exclude corner solutions on the consumer-side, a positive consumption demand for both goods requires that even the poorest workers spend some of their income on the luxury good. Referring to condition (3.14), which establishes negative semi-definiteness of the Slutsky matrix for all consumers (see above) and noting that workers possessing the lowest level of effective labor supply $\underline{\lambda}$ earn expenditure level $\underline{\lambda}w$, we obtain a lower bound for effective labor supply that, at least, needs to be met:

$$\underline{\lambda}^{\varepsilon} \geq \beta \frac{1-\varepsilon}{1-\gamma} \left(\frac{P_n}{P_\ell} \right)^{\gamma-\varepsilon}. \tag{3.18}$$

Since price index P_ℓ is endogenously determined, we discuss further below under which conditions constraint (3.18) is fulfilled.

Per-capita income and the distribution of per-capita income are given by

$$\bar{e} = \Lambda w \quad \text{and} \quad \psi = \int_{\underline{\lambda}}^{\bar{\lambda}} \left(\frac{\lambda}{\Lambda} \right)^{1-\varepsilon} dL(\lambda), \tag{3.19}$$

respectively, where $\Lambda = \int_{\underline{\lambda}}^{\bar{\lambda}} \lambda dL(\lambda)$ is the average effective labor supply of workers. It follows that a higher average effective labor supply induces a higher average income for a given wage rate w. As described above, a higher dispersion measure ψ is associated either with a more egalitarian distribution of income or with a lower impact of a given income dispersion on consumer demand with the limiting case of no impact at all if preferences are homothetic.

The mass of firms producing intermediate goods in general equilibrium is given by m, then the goods market clearing condition for the luxury good reads

$$H\Lambda w \left[1 - \beta \Lambda^{-\varepsilon} \left(\frac{P_n}{P_\ell} \right)^{\gamma-\varepsilon} \psi \right] = mr. \tag{3.20}$$

The term $H\Lambda w$ denotes aggregate consumption expenditure, which equals the aggregate income of households. The mass of firms producing intermediates cannot be explicitly solved for, because the price index for the luxury good P_ℓ also depends on m. This can be seen by using the constant markup pricing rule, given in Eq. (3.17), in the definition of the price index of differentiated intermediates:

$$P_\ell = m^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} w. \tag{3.21}$$

For a given w, the price index P_ℓ decreases in the number of firms producing intermediate goods, $\partial P_\ell / \partial m < 0$, because of tougher competition.

Condition (3.18) contains endogenous variable P_ℓ given in Eq. (3.21), which is de-

since productivity differences do not contribute to the mechanisms of our analysis, we have decided to stay with the simpler model variant of homogeneous producers.

termined by the market equilibrium. In the limiting case of homogeneous households, implying $\underline{\lambda} = \lambda = \Lambda$ and $\psi = 1$, the goods market clearing condition for the luxury good in Eq. (3.20) is given by $H\lambda w \left[1 - \beta\lambda^{-\varepsilon} \left(P_n/P_\ell\right)^{\gamma-\varepsilon}\right] = mr$. Together with parameter ranking $\varepsilon < \gamma$, the market equilibrium then establishes condition (3.18). For a sufficiently egalitarian distribution of effective labor supply λ, we can therefore safely conclude that condition (3.18) is fulfilled.

Furthermore, using Eq. (3.21) to rewrite the goods market clearing condition for the luxury good in Eq. (3.20) as an implicit function of Λ, ψ and m, we obtain

$$\Gamma_1(\Lambda, \psi, m) \equiv m\sigma f - H\Lambda \left[1 - \beta\Lambda^{-\varepsilon} m^{\frac{\varepsilon-\gamma}{1-\sigma}} \left(\frac{\sigma}{\sigma - 1}\right)^{\varepsilon-\gamma} \psi\right] = 0, \tag{3.22}$$

where the zero-profit condition $r = \sigma P_n f$ and $P_n = w$ have been used. As formally shown in the Appendix, we can apply the implicit function theorem to Eq. (3.22) to show that the number of intermediate producers increases in average effective labor supply, $dm/d\Lambda > 0$, whereas it decreases in expenditure dispersion, $dm/d\psi < 0$. The first result emerges, because a higher average income level increases demand for both final goods, and thus causes firm entry in the sector of intermediate goods. To get an intuition for the second result we use our finding from above, that a higher dispersion index ψ decreases the demand for the luxury good, using Eq. (3.5), resulting in firm exit in the sector of differentiated intermediates.

Using Eqs. (3.4), (3.5) and (3.19), aggregate expenditure shares for necessities and the luxury good can be calculated according to $\eta_n = \beta\Lambda^{-\varepsilon}(P_n/P_\ell)^{\gamma-\varepsilon}\psi$ and $\eta_\ell = 1 - \beta\Lambda^{-\varepsilon}(P_n/P_\ell)^{\gamma-\varepsilon}\psi$, respectively. As already established, a higher per-capita income and/or a higher income dispersion increase consumption expenditures for the luxury good (direct *demand* effect) which leads to firm entry in the sector of intermediate goods which in turn lowers the price index for intermediate goods. This lower price index decreases consumption expenditures for the luxury good and therefore has a counteracting indirect *price* effect on aggregate expenditures. With respect to necessities, a higher per-capita income and/or a higher income dispersion decrease consumption expenditures through the direct demand effect and increase consumption expenditures through the indirect price effect. To investigate the net effect of these two counteracting effects, we make use of Eq. (3.22). Relying on the result that m increases in Λ, and decreases in ψ, establishes that the direct effect always dominates and thus both, a higher Λ and a lower ψ lead to lower consumption expenditures for necessities and higher expenditures for the luxury good.

The labor market clearing condition reads $X_n + mq + mf = H\Lambda$. Employment of efficiency units of labor in the sector of necessities is captured by X_n, mq are the efficiency units of labor employed as variable input in the production of intermediate goods and aggregate fixed labor input in this sector is given by mf. Overall this has to equal economy-wide supply of labor, $H\Lambda$. Using Eq. (3.17), the labor market clearing condition can be rewritten as

$$X_n + m\sigma f = H\Lambda \tag{3.23}$$

and pins down output of necessities in general equilibrium. In the Appendix we show that both final goods are produced under autarky and thus a diversification equilibrium exists.

3.2.5 Welfare analysis

Since the representative consumer under PIGL preferences has only a positive but no normative interpretation, we follow Egger and Habermeyer (2019) and assume a utilitarian welfare function that gives equal weight to all households:

$$V(P_n, P_\ell, \overline{e}, \hat{\psi}) \equiv \frac{1}{\varepsilon}\left(\frac{\overline{e}}{P_\ell}\right)^{\varepsilon}\hat{\psi} - \frac{\beta}{\gamma}\left(\frac{P_n}{P_\ell}\right)^{\gamma} - \frac{1}{\varepsilon} + \frac{\beta}{\gamma}, \tag{3.24}$$

where $\hat{\psi} \equiv \int_{\underline{\lambda}}^{\overline{\lambda}} (\lambda/\Lambda)^{\varepsilon}\, dL(\lambda)$ is yet another dispersion index defined on the unit interval, which (except for the limiting case of $\varepsilon = 1/2$) is different from ψ. Dispersion index $\hat{\psi}$ captures an inequality aversion of the social planner, which exists despite the assumption of utilitarian welfare and is the higher the lower is index $\hat{\psi}$.

Intuitively, from Eq. (3.19) a higher average effective labor supply Λ is associated with higher per-capita income and therefore leads to a welfare improvement (see the Appendix for a formal proof). This has two reasons: On the one hand, for given prices workers can purchase more necessities and luxuries with a higher level of income. On the other hand, the price index for the luxury good falls, due to a higher market demand, which leads to firm entry in the sector of intermediate goods.

In contrast to the level of per-capita income, the welfare effect of a change in income dispersion is, however, a priori not clear. The overall effect can be split into two effects working through indices ψ and $\hat{\psi}$, respectively. To determine which of the two effects dominates, we can look at the case of $\varepsilon = 1/2$, for which $\hat{\psi}$ and ψ are of equal size. The social welfare function for this particular parameter configuration is given by

$$V(P_n, P_\ell, \overline{e}, \psi) = 2\left(\frac{\overline{e}}{P_\ell}\right)^{\frac{1}{2}}\psi - \frac{\beta}{\gamma}\left(\frac{P_n}{P_\ell}\right)^{\gamma} - 2 + \frac{\beta}{\gamma}. \tag{3.25}$$

Differentiating Eq. (3.25) with respect to ψ gives

$$\frac{dV(P_n, P_\ell, \overline{e}, \psi)}{d\psi} = \sqrt{\frac{\overline{e}}{P_\ell(\sigma-1)^2}}\left\{2(\sigma-1) + \frac{1}{m}\frac{dm}{d\psi}\left[\psi - \frac{\beta}{\sqrt{\Lambda}}\left(\frac{P_n}{P_\ell}\right)^{\gamma-\frac{1}{2}}\right]\right\}. \tag{3.26}$$

A higher income dispersion causes a direct negative welfare effect because of the social inequality aversion. However, from aggregate demand functions (3.4) and (3.5) we know that a higher income dispersion also leads to higher consumption expenditures for the luxury good. This attracts new firms into the sector of intermediate goods, thus lowers the price index for the luxury good and this in turn triggers a positive effect on social welfare. Which effect dominates is not clear even if $\varepsilon = 1/2$ and depends inter alia on the level of preference parameter γ, according to Eq. (3.26). To gain further insights about possible outcomes, we can note from Egger and Habermeyer (2019), who show that in the case of $\gamma = 1/2$ (and thus $\gamma = \varepsilon$) a negative welfare effect due to a higher income dispersion is more likely if either the elasticity of substitution σ is sufficiently large, leading to sufficiently small markups, or if β is sufficiently small.[13] In the limiting case of $\gamma \to 1$ a higher income dispersion causes a negative welfare effect for all $\sigma > 1$ (see the Appendix for a formal proof). Compared to the welfare effects for $\varepsilon = \gamma = 1/2$, the indirect price index effect is always dominated by the direct social inequality aversion effect when $\varepsilon = 1/2$ and $\gamma \to 1$ is assumed.

To complete the analysis of the closed economy, we assume homothetic preferences and consider the resulting welfare effects with regard to changes in the level and dispersion of per-capita income. Knowing that $\hat{\psi} = 1$ in the limiting case of $\varepsilon \to 0$, we can apply the rule of L'Hôpital to Eq. (3.24) and calculate the limiting case of $\varepsilon \to 0$ to obtain the welfare function under homothetic preferences

$$\bar{V}(P_n, P_\ell, \overline{e}) = \ln\left(\frac{\overline{e}}{P_\ell}\right) + \frac{\beta}{\gamma}\left[1 - \left(\frac{P_n}{P_\ell}\right)^{\gamma}\right]. \tag{3.27}$$

It is straightforward that welfare increases with per-capita income, whereas a change in the dispersion of income entails no welfare effects. This result comes as no surprise since the dispersion of income does not affect the expenditure structure ($\psi = 1$) and because welfare does not exhibit inequality aversion ($\hat{\psi} = 1$) if preferences are homothetic.

As a final point, it is worth noting that firm entry is, in general, inefficient which means that there exists a resource misallocation that is triggered due to the two final goods sectors charging different markups – see Dhingra and Morrow (2016) for further details. In the Appendix, we show for the limiting case of Cobb-Douglas preferences ($\varepsilon \to 0$ and $\gamma \to 0$) that the social planner prefers a higher mass of firms producing intermediate goods than it is determined in the market solution (see Benassy, 1996, for a discussion). Thus, the market outcome is not allocationally efficient in this case.

3.3 The open economy

In the open economy, we consider two countries of equal size, $H\Lambda = H^*\Lambda^*$, which only differ in the level and/or dispersion of effective labor supply and thus per-capita income. Asterisks refer to foreign variables. Trade just occurs with homogeneous necessities and differentiated intermediate goods, whereas the luxury good is not tradable and therefore consumed locally. Necessities can be traded at zero cost which causes domestic and foreign firms to pay the same market clearing wage w per efficiency units of labor, provided that a diversification equilibrium exists. Since we want to exclude corner solutions and aim at a diversification equilibrium, a parameter constraint supporting the production of necessities and intermediate goods in both countries is discussed below. Standard iceberg transportation cost $\tau > 1$ are assumed to exist for trade in intermediate goods, meaning that more than one unit of variety ω must be shipped in order for one unit to arrive abroad.

Total revenues in the sector of intermediate goods of home and foreign are linked through the respective zero-profit conditions, namely $r = \sigma P_n f$ and $r^* = \sigma P_n f$ which

[13] The model variant with $\beta = 0$ incorporates only one final goods sector as can be seen in Eq. (3.4).

yield

$$\frac{1-\beta\left(\frac{P_n}{P_\ell^*}\right)^{\gamma-\varepsilon}\phi^*}{1-\beta\left(\frac{P_n}{P_\ell}\right)^{\gamma-\varepsilon}\phi}=\left(\frac{P_\ell}{P_\ell^*}\right)^{\sigma-1}. \tag{3.28}$$

Throughout the following analysis, $\tau > 1$ and $H\Lambda = H^*\Lambda^*$ are assumed. We introduce $\phi \equiv \Lambda^{-\varepsilon}\psi$ and $\phi^* \equiv (\Lambda^*)^{-\varepsilon}\psi^*$ to simplify notation. Variables ϕ and ϕ^* capture differences in consumer demand due to exogenous differences in the level and/or dispersion of per-capita income in home and foreign, respectively. For instance, if per-capita income and/or income dispersion increase at home (higher Λ and/or lower ψ), then ϕ drops, indicating that economy-wide demand for the luxury good increases in home and vice versa for necessities. These links were already established in the closed economy scenario.

Provided that both countries produce necessities, the market clearing conditions for the luxury good at home and abroad are given by

$$H\Lambda w\left[1-\beta\left(\frac{P_n}{P_\ell}\right)^{\gamma-\varepsilon}\phi\right]=mr\frac{1}{1+\tau^{1-\sigma}}+m^*r^*\frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}}, \tag{3.29}$$

$$H\Lambda w\left[1-\beta\left(\frac{P_n}{P_\ell^*}\right)^{\gamma-\varepsilon}\phi^*\right]=m^*r^*\frac{1}{1+\tau^{1-\sigma}}+mr\frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}}, \tag{3.30}$$

respectively. The CES price indices of the luxury good for the two countries can be calculated as

$$P_\ell = m^{\frac{1}{1-\sigma}}p\left(1+\mu\tau^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad P_\ell^* = m^{\frac{1}{1-\sigma}}p\left(\mu+\tau^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{3.31}$$

To simplify notation, we introduce $\mu \equiv m^*/m$ as the ratio of foreign to domestic intermediate producers. Again, due to the price indices for the luxury good in each country depending on the domestic and foreign number of firms producing intermediate goods, Eqs. (3.29) and (3.30) cannot be explicitly solved for m and m^*. To ensure a unique interior solution with $\mu \in (0,\infty)$ and thus production of intermediate goods in both countries, we set up two implicit relationships between the number of domestic and foreign firms, the exogenous income variables ϕ and ϕ^*, and the transportation cost τ. Using Eq. (3.31) in Eq. (3.28), the first implicit link is established as a condition for equal revenues of intermediate goods producers in the two economies

$$\Gamma_2(m,\mu) \equiv (1-\tau^{1-\sigma})(1-\mu) - \alpha F(m,\mu)^{\frac{\varepsilon-\gamma}{1-\sigma}}(\mu+\tau^{1-\sigma})\left[f(\mu)\phi^*-\phi\right]=0, \tag{3.32}$$

where $F(m,\mu) \equiv m(1+\mu\tau^{1-\sigma}) > 0$ has to hold to assure a positive mass of domestic intermediate goods producers, $m > 0$. Furthermore, $f(\mu) \equiv \left[(\mu+\tau^{1-\sigma})/(1+\mu\tau^{1-\sigma})\right]^{\frac{\sigma-(1+\gamma-\varepsilon)}{1-\sigma}}$ is introduced as a summary variable, which considerably simplifies the analysis of the open economy equilibrium in the main text and the corresponding formal analysis in the Appendix, and the definitions of ϕ and ϕ^* have been used. We define $\alpha \equiv \beta\,(P_n/p)^{\gamma-\varepsilon} > 0$. Noting that $p = [\sigma/(\sigma-1)]w$ is the price of each intermediate good as given in Eq. (3.17), then together with $P_n = w$ we get $\alpha = \beta\,[(\sigma-1)/\sigma]^{\gamma-\varepsilon}$, which is a constant that is of no further interest for the subsequent analysis. The implicit solution for the equilibrium mass of firms producing intermediate goods in home as a function of firm ratio μ is determined by $\Gamma_2(\cdot) = 0$ and reads[14]

$$m = \left(\frac{A(\mu)}{\alpha}\right)^{\frac{1-\sigma}{\varepsilon-\gamma}} \frac{1}{1+\mu\tau^{1-\sigma}}, \tag{3.33}$$

where $A(\mu) \equiv [(1-\tau^{1-\sigma})(1-\mu)] \,/\, [(\mu+\tau^{1-\sigma})(f(\mu)\phi^* - \phi)]$ is yet another summary variable that substantially simplifies the open economy analysis in the main text and the Appendix. The second link is derived by adding Eqs. (3.29) and (3.30), and using Eq. (3.31), which establishes a goods market clearing condition for the luxury good at the global level according to

$$\Gamma_3(m,\mu) \equiv m\sigma f(1+\mu) - H\Lambda \left\{ 2 - \alpha F(m,\mu)^{\frac{\varepsilon-\gamma}{1-\sigma}} \left[\frac{\mu+\tau^{1-\sigma}}{1+\mu\tau^{1-\sigma}} f(\mu)\phi^* + \phi \right] \right\} = 0, \tag{3.34}$$

where $r = r^* = \sigma P_n f$, the definitions of ϕ and ϕ^*, and $F(m,\mu) \equiv m(1+\mu\tau^{1-\sigma})$ have been used. Inserting Eq. (3.33) into Eq. (3.34) allows us to rewrite the goods market clearing condition as an implicit function of just one endogenous variable μ

$$\Gamma_4(\mu) \equiv \left(\frac{A(\mu)}{\alpha}\right)^{\frac{1-\sigma}{\varepsilon-\gamma}} \sigma f \frac{1+\mu}{1+\mu\tau^{1-\sigma}} - H\Lambda \left\{ 2 - A(\mu) \left[\frac{\mu+\tau^{1-\sigma}}{1+\mu\tau^{1-\sigma}} f(\mu)\phi^* + \phi \right] \right\} = 0. \tag{3.35}$$

In the open economy equilibrium, the ratio of foreign to domestic intermediate producers μ is determined by $\Gamma_4(\mu) = 0$ in Eq. (3.35). Under homothetic preferences with $\varepsilon \to 0$, the unique solution of Eq. (3.35) is given by firm ratio $\mu = 1$. This confirms the well-known result that in the case of homothetic preferences, the level and dispersion of per-capita income do not exert an effect on market outcome beyond the effect captured by country size. Assuming identical size of the two trading partners therefore implies an equal number of firms active in the two economies. This can be easily seen for the case of Cobb-Douglas preferences and thus the limiting case captured by $\varepsilon \to 0$ and $\gamma \to 0$. To see this, note that under Cobb-Douglas preferences, we have $\phi = \phi^*$, implying that Eq. (3.32) simplifies to $\hat{\Gamma}_2(m,\mu) \equiv (1-\tau^{1-\sigma})(1-\mu)(1-\beta) = 0$, with an explicit solution at $\mu = 1$. Using this result in Eq. (3.34) yields $m = m^* = (1-\beta)\frac{H\Lambda}{\sigma f}$. The result is less immediate for general homothetic preferences, and we therefore delegate a formal discussion of the more general case to the Appendix.

Turning to the case of non-Gorman preferences with $0 < \varepsilon < \gamma$, it is formally shown in the Appendix that Eq. (3.35) has a unique, interior solution in $\mu \in (0,\infty)$ if trade cost are sufficiently high. The proof for this result is tedious, because six cases can be distinguished for $\phi > \phi^*$. Two of them can be ruled out immediately, because they violate Eq. (3.32). Another one is excluded by Eq. (3.35). Together the Eqs. (3.32) and (3.35) rule out an

[14]Solving Eq. (3.32) for $F(m,\mu)$ gives $F(m,\mu) = \left\{ \left[(1-\tau^{1-\sigma})(1-\mu)\right] / \left[\alpha(\mu+\tau^{1-\sigma})(f(\mu)\phi^* - \phi)\right] \right\}^{\frac{1-\sigma}{\varepsilon-\gamma}}$. Setting this result equal to the definition of $F(m,\mu) \equiv m(1+\mu\tau^{1-\sigma})$ from above, we can compute the implicitly given equilibrium mass of domestic firms producing intermediate goods as written down in Eq. (3.33).

outcome with $\mu \leq 1$ if $\phi > \phi^*$. The three final cases refer to the ranking of $\sigma \lesseqgtr 1+\gamma-\varepsilon$ for $\mu > 1$ and they can be discussed by making use of Figure 3.1, which illustrates $\Gamma_4(\mu)$ from Eq. (3.35) for the three remaining cases $\sigma < 1+\gamma-\varepsilon$ with $\tau < (\phi/\phi^*)^{\frac{1}{-[\sigma-(1+\gamma-\varepsilon)]}}$, $\sigma < 1+\gamma-\varepsilon$ with $\tau \geq (\phi/\phi^*)^{\frac{1}{-[\sigma-(1+\gamma-\varepsilon)]}}$ and $\sigma \geq 1+\gamma-\varepsilon$. In Figure 3.1 we see the crucial difference between the three cases. If $\sigma \geq 1+\gamma-\varepsilon$, $f(\mu)$ decreases in μ, and in this case $f(\mu) < 1$, which is sufficient for $f(\mu)\phi^* < \phi$, holds for all $\mu > 1$. In contrast, if $\sigma < 1+\gamma-\varepsilon$ with $\tau \geq (\phi/\phi^*)^{\frac{1}{-[\sigma-(1+\gamma-\varepsilon)]}}$, $f(\mu)$ increases in μ and in this case $f(\mu)\phi^* < \phi$ requires $\mu < \mu_1$, with

$$\mu_1 \equiv \frac{\left(\frac{\phi}{\phi^*}\right)^{\frac{1-\sigma}{\sigma-(1+\gamma-\varepsilon)}} - \tau^{1-\sigma}}{1 - \left(\frac{\phi}{\phi^*}\right)^{\frac{1-\sigma}{\sigma-(1+\gamma-\varepsilon)}} \tau^{1-\sigma}} > 1. \tag{3.36}$$

However, for case $\sigma < 1+\gamma-\varepsilon$ with $\tau < (\phi/\phi^*)^{\frac{1}{-[\sigma-(1+\gamma-\varepsilon)]}}$, $f(\mu)$ increases in μ, but the constraint on trade cost parameter τ ensures $f(\mu)\phi^* < \phi$ for all $\mu > 1$. Because $\Gamma_4(1) = -2H\Lambda$ and because $\phi > \phi^*$ establishes $\Gamma_4'(\mu) > 0$ if $\mu > 1$, it is immediate that $\Gamma_4(\mu) = 0$ has a unique solution in μ, provided that $\tau \geq (\phi/\phi^*)^{\frac{1}{\gamma-\varepsilon}} \equiv \tilde{\tau}_1$ ensures existence of the interior solution. Following a similar line of reasoning, $\phi < \phi^*$ establishes a unique solution for $\mu < 1$ if $\tau \geq (\phi^*/\phi)^{\frac{1}{\gamma-\varepsilon}} \equiv \tilde{\tau}_2$. Assuming a lower bound on trade cost for both cases $\phi > \phi^*$ and $\phi < \phi^*$, respectively, we can therefore safely conclude that the open economy equilibrium under diversification exists and must be unique.

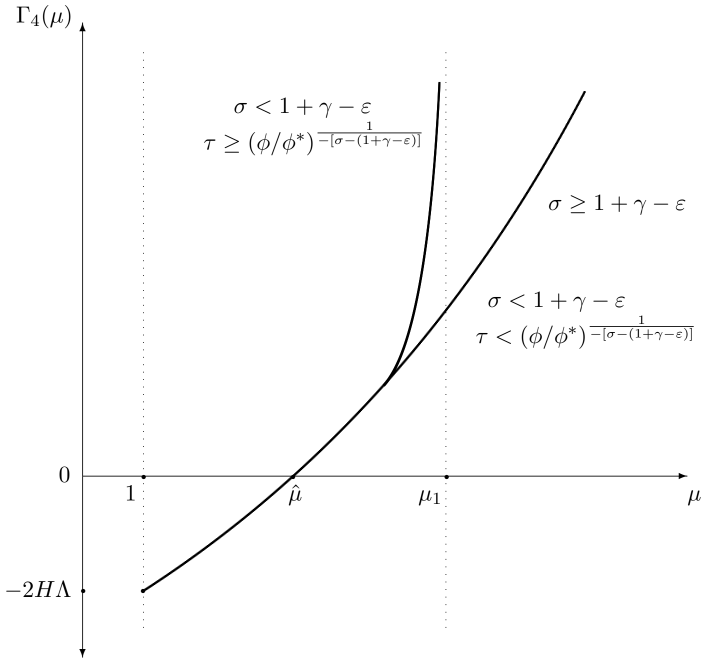

Figure 3.1: Equilibrium in the open economy for $\phi > \phi^*$

The equilibrium in Figure 3.1 has been derived under the caveat that a diversification equilibrium exists in the open economy. To show that this is the case, we must ensure that a positive amount of labor is used for the production of necessities in both economies. As formally shown in the Appendix, using Eq. (3.33) in Eq. (3.23) and accounting for Eq.

(3.35), we can solve for X_n and find that $X_n > 0$ is always fulfilled if $\phi > \phi^*$. Intuitively, this means that the country with the smaller market for luxuries (which is home in the case of $\mu > 1$) always produces necessities. In contrast, if $\phi < \phi^*$ makes home the larger market for luxuries (reflected by $\mu < 1$), a positive local supply of necessities requires

$$\tau > \left[\frac{\mu + \tau^{1-\sigma}}{1+\mu\tau^{1-\sigma}} \frac{f(\mu)\phi^*}{\phi}\right]^{\frac{1}{\sigma-1}}. \tag{3.37}$$

Of course, with μ dependent on τ (3.37) gives only an implicit condition for existence of a diversification equilibrium. Noting that $\frac{\mu+\tau^{1-\sigma}}{1+\mu\tau^{1-\sigma}} f(\mu)$ increases in μ from a minimum level of $\tau^{\varepsilon-\gamma}$ at $\mu = 0$ and noting that it reaches a maximum level of 1 at $\mu = 1$ for $\sigma \geq 1+\gamma-\varepsilon$ as well as $\sigma < 1+\gamma-\varepsilon$. This implies that $\tau > (\phi^*/\phi)^{\frac{1}{\sigma-1}} \equiv \hat{\tau}$ is sufficient for a diversification equilibrium if $\phi < \phi^*$. We can therefore safely conclude that necessities and intermediate goods are produced in both economies if τ is sufficiently large.

Since two lower bounds on trade cost, $\tilde{\tau}_1$ for $\phi > \phi^*$ and $\tilde{\tau}_2$ for $\phi < \phi^*$, were introduced to ensure existence of an interior open economy equilibrium and since one constraint on trade cost, $\hat{\tau}$ for $\phi < \phi^*$, was introduced to ensure a diversification equilibrium, we analyze which constraint is binding depending on the considered case. For $\phi > \phi^*$, a diversification equilibrium exists for all $\tau > 1$. Except for case $\sigma < 1+\gamma-\varepsilon$ with $\tau \geq (\phi/\phi^*)^{\frac{1}{-[\sigma-(1+\gamma-\varepsilon)]}}$, where $\tilde{\tau}_1 < (\phi/\phi^*)^{\frac{1}{-[\sigma-(1+\gamma-\varepsilon)]}}$, for the two remaining cases, $\sigma < 1+\gamma-\varepsilon$ with $\tau < (\phi/\phi^*)^{\frac{1}{-[\sigma-(1+\gamma-\varepsilon)]}}$ and $\sigma \geq 1+\gamma-\varepsilon$, $\tilde{\tau}_1$ is a binding lower bound for trade cost to ensure a unique, interior equilibrium with diversification in the open economy. In contrast, two constraints $\tilde{\tau}_2$ and $\hat{\tau}$ apply to $\phi < \phi^*$, making the distinction in this case more complex.[15] For $\sigma < 1+\gamma-\varepsilon$ and $\sigma = 1+\gamma-\varepsilon$, diversification constraint $\tau > \hat{\tau}$ is binding and ensures a unique, open economy equilibrium with diversification, whereas for $\sigma > 1+\gamma-\varepsilon$, equilibrium constraint $\tau \geq \tilde{\tau}_2$ is sufficient.[16]

Taking stock, the analysis in this section makes clear that under rather mild conditions on trade cost parameter τ we achieve a unique open economy equilibrium with diversification in both economies and $\mu > (<)1$ if $\phi > (<)\phi^*$. This result points out that the country with the higher relative demand for the luxury good (home if $\phi < \phi^*$ and foreign if $\phi > \phi^*$) has the larger market for differentiated intermediates, which means that a relatively larger mass of firms producing intermediates is settled there. These results are akin to the findings in Egger and Habermeyer (2019) and confirm that even though the analysis of the open economy is significantly more complicated in the underlying model, an analytically tractable solution still exists for the open economy even if the parametric PIGL preferences have quite general form.

[15] Since the Appendix does not contain a detailed proof of a unique, interior equilibrium in the open economy for $\varepsilon \neq 0$ if $\phi < \phi^*$, the three final equilibrium cases for $\phi < \phi^*$ are not explicitly specified in this paper. Therefore, details for the discussion about the constraints on trade cost parameter τ for the three remaining cases if $\phi < \phi^*$ are available from the author upon request.

[16] Turning to the consumer-side, condition (3.18), which assures household consumption of necessities and the luxury good, also needs to be fulfilled in the open economy equilibrium. As formally shown in the Appendix, rewriting price ration P_n/P_ℓ as a function of $A(\mu)$ and noting that $A(\mu)$ decreases in τ, we can safely conclude that condition (3.18) must be fulfilled in the open economy for sufficiently high trade cost τ, if it is fulfilled in the closed economy.

3.3.1 Trade structure

As in Egger and Habermeyer (2019), the number of firms producing intermediate goods in foreign relative to home, $\mu \equiv m^*/m$, is decisive for the trade pattern in the open economy. This can be seen by looking at the export and import functions of intermediates in home

$$EX_\omega = m\frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}}\sigma P_n f, \tag{3.38}$$

$$IM_\omega = \mu m\frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}}\sigma P_n f, \tag{3.39}$$

where the zero-profit condition has been used. Eqs. (3.38) and (3.39) indicate that home is a net-exporter (net-importer) of intermediate goods if $\mu < (>)1$. Noting the link between exogenous variables ϕ, ϕ^* and μ established above, we can conclude that a relatively higher level and/or dispersion of per-capita income in home ($\phi < \phi^*$), makes this country a net-exporter of intermediate goods ($\mu < 1$) and the foreign country a net-exporter of necessities. The reverse result holds for $\phi > \phi^*$.

As this model contains inter-industry trade with necessities and intermediates and intra-industry trade only with intermediate goods, we analyze the relative importance of intra-industry trade by using the Grubel-Lloyd index

$$GLI = 1 - \sum_k \frac{|EX_k - IM_k|}{\sum_k (EX_k + IM_k)}, \tag{3.40}$$

where $k \in \{n, \omega\}$ indicates the respective sector. The Grubel-Lloyd index lies in between 0 (only inter-industry trade) and 1 (only intra-industry trade). Assuming that there is no intra-industry trade in necessities and households always buy the domestic necessity when they are indifferent, this yields

$$GLI = \begin{cases} \frac{1}{\mu} & \text{if} \quad \phi > \phi^* \\ 1 & \text{if} \quad \phi = \phi^* \\ \mu & \text{if} \quad \phi < \phi^*. \end{cases} \tag{3.41}$$

If preferences are assumed to be homothetic ($\varepsilon \to 0$), $\phi = \phi^*$ leads to $\mu = 1$. Identical market sizes of the two countries then translate into all trade being driven by only intra-industry exchange. This can be seen by looking at Eqs. (3.38) and (3.39).

Provided that a diversification equilibrium exists and preferences are nonhomothetic ($\varepsilon \neq 0$), the following effects of changes in trade cost parameter τ and market size parameters ϕ and ϕ^* on the trade pattern can be derived from Eqs. (3.35) and (3.41). These effects are summarized in the following proposition.

Proposition 3 *The country with the relatively higher level and/or dispersion of per-capita income has the higher consumption expenditures for the luxury good and is a net-exporter of intermediate goods in the open economy equilibrium. The share of intra-industry trade in differentiated intermediates increases monotonically in trade cost and increases, moreover, in the similarity of countries based on their level and/or dispersion of per-capita income.*

Proof. See the Appendix. ■

The first result in Proposition 3 replicates a home-market effect similar to Helpman and Krugman (1985) who find that the country with the larger endowment of labor (the only input factor) exports the differentiated good because it exhibits higher domestic demand for it. The Helpman and Krugman (1985) two-sector model features homothetic (love-of-variety) preferences over differentiated goods that are produced under increasing returns to scale and a homogeneous good produced with constant returns to scale. As in our setting, the homogeneous good is costlessly traded, whereas trade cost accrue for the differentiated good, leading to the home-market effect explained above. By contrast, the underlying model presents countries that are symmetric in their aggregate labor endowment and thus aggregate income, but differ in their level and/or dispersion of effective labor supply (per-capita income). This leads to differing expenditure shares devoted to necessities and the luxury good because preferences do not have Gorman form. Therefore, the country with the higher level and/or dispersion of per-capita income has the higher domestic demand for the luxury good which leads to the larger domestic market for intermediate goods and this country being a net-exporter of differentiated intermediates. The second result, namely that higher transportation cost increase the importance of intra-industry trade is a well-known result from Davis (1998b). The Appendix contains the derivation details for case $\phi > \phi^*$, which makes home the net-importer of the luxury good ($\mu > 1$). We can show that $d\mu/d\tau^{1-\sigma} > 0$ holds for all cases $\sigma \lesseqgtr 1 + \gamma - \varepsilon$, and therefore μ decreases in trade cost τ. Thus, a rise in trade cost τ causes less specialization, the two countries become more similar in their production structure, and hence, intra-industry trade gains relative importance. It can be summarized that an increase in trade cost weakens the home-market effect and consequently the Grubel-Lloyd index increases. Referring to the Linder (1961) hypothesis serves to explain the last insight in Proposition 3. If consumption expenditures of two countries become more similar, then production patterns become more similar as well, which increases the share of intra-industry trade with intermediate goods. As compared to Linder (1961), where only per-capita income levels are considered, the underlying model additionally includes income dispersion as another demand-side factor in which countries can differ. Accordingly, if countries are more similar in their level and/or dispersion of per-capita income in the underlying model framework, then the nature of their trade flows is relatively more intra-industry. The results from Proposition 3 are well in line with those reported by Egger and Habermeyer (2019). However, Egger and Habermeyer (2019) analyze a more restrictive class of parametric PIGL preferences, where $\varepsilon = \gamma$ is assumed.

Looking at overall inter- plus intra-industry trade, and its response to changes in trade cost, ϕ and ϕ^*, the following proposition can be formulated.

Proposition 4 *Overall trade decreases in trade cost and in the similarity of countries with regard to their level and/or dispersion of per-capita income.*

Proof. See the Appendix. ■

The first result that higher trade cost reduce overall trade is quite intuitive. We assume in the following that $\phi > \phi^*$ makes home a net-importer of intermediate goods.

Then an increase in trade cost parameter τ implies higher import cost for home with regard to intermediate goods. As a consequence, home's market size for differentiated intermediates increases to meet the demand of consumers for the luxury good by using now relatively more domestically produced intermediate goods in the assembly of the homogeneous luxury. Hence, the increase in intra-industry trade caused by higher trade cost is dominated by a decrease in inter-industry trade as in this specific example home's exports of necessities drop and foreign's exports of intermediates decrease. Egger and Habermeyer (2019) already hint at the misleading interpretation of the Linder (1961) hypothesis with regard to overall trade. Opposing to the positive intra-industry trade effect just explained above a counteracting negative effect on inter-industry trade has to be borne in mind when countries converge with regard to their level and/or dispersion of per-capita income. Accordingly, provided that home is the net-exporter of intermediate goods ($\phi < \phi^*$), a decrease in home's relative expenditures for the luxury good (higher ϕ) decreases production of differentiated intermediates in home and vice versa in foreign, although foreign's demand for the luxury good has not changed. This implies lower exports of intermediate goods from home and lower exports of necessities from foreign. In summary, we find that the results from this section concerning the trade structure in the open economy are in support of the findings in Egger and Habermeyer (2019) and extend their results to more general forms of parametric PIGL preferences. The subclass of preferences studied by Egger and Habermeyer (2019) is included as a limiting case of the more general class considered here.

3.3.2 Welfare effects of trade

To complete the discussion about the open economy, we now examine the welfare effects of trade. Initially looking at the price indices of the luxury good in home and abroad, we know from the analysis above that $\phi > (<)\phi^*$ leads to $\mu > (<)1$, which itself induces $P_\ell > (<)P_\ell^*$. This can be seen by using Eq. (3.31). Hence, the country with the larger home market for intermediate goods has the lower price index for the luxury good. Combining Eqs. (3.31) and (3.33) and substituting the result into the social welfare function in Eq. (3.24) gives

$$V(P_n, \bar{e}, \hat{\psi}) = A(\mu)^{\frac{\varepsilon}{\gamma-\varepsilon}} \left\{ \frac{1}{\varepsilon} \left(\frac{\bar{e}}{\tilde{\alpha}} \right)^{\varepsilon} \hat{\psi} - \frac{\beta}{\gamma} A(\mu) \left(\frac{P_n}{\tilde{\alpha}} \right)^{\gamma} \right\} - \frac{1}{\varepsilon} + \frac{\beta}{\gamma}, \tag{3.42}$$

where $\tilde{\alpha} \equiv \beta^{\frac{1}{\gamma-\varepsilon}} P_n > 0$ is introduced to simplify notation. The welfare effects of trade are summarized in the following proposition.

Proposition 5 *Regardless of the prevailing trade structure, welfare gains are reached in both countries due to trade. These gains from trade increase monotonically if trade cost fall.*

Proof. See the Appendix. ■

As the autarky scenario can be considered as limiting case of trade cost approaching infinity, we are able to show that moving to the open economy equilibrium, which then translates into lower trade cost as compared to autarky, increases the number of available differentiated intermediates and lowers the domestic price index P_ℓ. This has a positive welfare effect in home which increase monotonically if trade cost are further reduced, provided that the open economy equilibrium remains diversified. Since the level and dispersion of per-capita income are exogenous and both countries feature the same fixed cost of production leading to the same price for necessities, welfare gains are also obtained in foreign. These welfare effects work through adjustments in the price index of the luxury good and therefore also exist (although in nuanced form) if preferences are homothetic ($\varepsilon \to 0$). Accordingly, there are gains from trade liberalization irrespective of the specific level of preference parameter ε. The result of overall gains from trade under homothetic preferences is well-established in the international trade literature.[17] In this paper, which is a generalization of Egger and Habermeyer (2019) with respect to the underlying preferences, we show that the result of overall gains from trade translates into a model of parametric PIGL preferences that is inconsistent with homothetic taste.

3.4 Conclusion

We have developed a home-market model of international trade between two countries following Egger and Habermeyer (2019) for a fairly general class of parametric PIGL preferences. There are two final goods sectors that produce homogeneous necessities and a homogeneous luxury good, respectively. Production of necessities uses labor as the only input under constant returns to scale, and the output is sold under perfect competition and freely tradable in the open economy. The luxury good is assembled from differentiated intermediates, using a technology that features constant elasticity of substitution between the available intermediates. The luxury good is nontradable, in contrast to the differentiated intermediates, which are produced under increasing returns to scale, using labor as the only input, sold under monopolistic competition and tradable subject to iceberg trade cost. Based on this structure, we solve the integrability problem and show that the demand for necessities and luxuries derived from indirect utility is indeed the solution to a constrained utility maximization problem, even though an explicit solution for the direct utility function does not exist in general for the considered nonhomothetic PIGL preferences.

Nonhomothetic preferences give rise to non-linear Engel curves. This affects the demand structure of heterogeneous workers differing in their efficiency units of labor in the sense that countries with a higher level and/or dispersion of per-capita income have a higher demand for the luxury good. In the open economy, all other things equal, this makes the country with the relatively higher demand for the luxury good and thus the larger domestic market for differentiated intermediates a net-exporter of intermediate goods. However, with the same market clearing wage per efficiency unit of labor paid in the two sectors, the welfare effects of trade are positive for both trading partners although their magnitude might differ. For specific parametric PIGL preferences, Egger and Habermeyer (2019) extend the well-established result of overall gains from trade under homothetic preferences as they show that their model framework also comes up with overall

[17] See for instance the seminal contributions of Krugman (1979, 1980) and Eaton and Kortum (2002).

gains from trade if labor markets are not distorted. In this paper, we go one step further and show that this result is persistent when relying on a generalization of parametric PIGL preferences.

We have shown that solving the integrability problem in a different manner than in Egger and Habermeyer (2019) leads qualitatively to the same results as in their model variant without price distortion in the labor market. In contrast to Egger and Habermeyer (2019), who avoid an integrability problem by relying on a specific subclass of parametric PIGL preferences giving rise to a closed form representation of the direct utility function, in this paper the introduction of differentiated intermediate goods along the lines of Ethier (1982b) allows us to solve the integrability problem for two homogeneous final goods relying on insights from Hurwicz and Uzawa (1971) and Boppart (2014).

Even though the underlying analysis with more general parametric PIGL preferences becomes quite more complicated, these preferences still provide an analytically tractable framework to study the consequences of nonhomothetic preferences in international trade. This framework offers a neat point of departure to address various possible extensions of embedding PIGL preferences in a trade context. Furthermore, solving the integrability problem under the more general form of parametric PIGL preferences for a continuum of goods is a worthwhile task for future research.

3.5 Appendix

Firms profit maximization problem

Using Eq. (3.16), we can express profits of the firms producing the luxury good ℓ as

$$\Pi = P_\ell \left[\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} - \int_{\omega \in \Omega} p(\omega) q(\omega) d\omega. \tag{3.43}$$

Maximizing for $q(\omega)$ and $q(\hat{\omega})$, and rearranging the two first-order conditions establishes

$$P_\ell X_\ell \frac{1}{\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega} q(\omega)^{\frac{\sigma-1}{\sigma}-1} = p(\omega) \quad \text{and} \quad P_\ell X_\ell \frac{1}{\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega} q(\hat{\omega})^{\frac{\sigma-1}{\sigma}-1} = p(\hat{\omega}). \tag{3.44}$$

Combining these two equations, we obtain the isoelastic demand function

$$q(\omega) = q(\hat{\omega}) \left(\frac{p(\hat{\omega})}{p(\omega)} \right)^{\sigma}. \tag{3.45}$$

This completes the proof.

Derivation of price index P_ℓ

Rewriting the isoelastic demand function in Eq. (3.45) and integrating over ω gives

$$\int_{\omega \in \Omega} q(\omega) p(\omega) d\omega = q(\hat{\omega}) p(\hat{\omega})^{\sigma} \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega. \tag{3.46}$$

Solving Eq. (3.44) for $q(\hat{\omega})p(\hat{\omega})^{\sigma}$ and using Eq. (3.16) establishes

$$X_\ell P_\ell^{\sigma} = q(\hat{\omega}) p(\hat{\omega})^{\sigma}. \tag{3.47}$$

Plugging this result into Eq. (3.46) yields

$$\int_{\omega \in \Omega} q(\omega) p(\omega) d\omega = X_\ell P_\ell^{\sigma} \int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega. \tag{3.48}$$

Applying Eq. (3.44) to solve for $\int_{\omega \in \Omega} p(\omega) q(\omega) = P_\ell X_\ell$, then establishes $P_\ell \equiv \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$ as the price index for the CES composite X_ℓ. This completes the proof.

Comparative statics in the closed economy

To investigate the effect of a change in the average effective labor supply on the number of firms producing intermediate goods in the closed economy, we apply the implicit function theorem to Eq. (3.22) and obtain

$$\frac{dm}{d\Lambda} = -\frac{\frac{\partial \Gamma_1(\cdot)}{\partial \Lambda}}{\frac{\partial \Gamma_1(\cdot)}{\partial m}}, \tag{3.49}$$

where $\partial\Gamma_1(\cdot)/\partial\Lambda < 0$ and $\Gamma_1(\cdot)/\partial m > 0$ are immediate and establish $dm/d\Lambda > 0$. Using the implicit function theorem on Eq. (3.22) captures the effect of a change in dispersion index ψ on the mass of intermediate producers according to

$$\frac{dm}{d\psi} = -\frac{\frac{\partial\Gamma_1(\cdot)}{\partial\psi}}{\frac{\partial\Gamma_1(\cdot)}{\partial m}}, \tag{3.50}$$

with $\partial\Gamma_1(\cdot)/\partial\psi > 0$. Thus, we can safely conclude that $dm/d\psi < 0$, which completes the proof.

Proof of diversification under autarky

Diversification in the closed economy is fulfilled if $X_n > 0$ holds. Therefore, we use Eq. (3.23) to establish

$$H\Lambda - m\sigma f > 0. \tag{3.51}$$

Accounting for Eq. (3.20) and the zero-profit condition, $r = \sigma P_n f$, then gives condition

$$H\Lambda^{1-\varepsilon}\beta\left(\frac{P_n}{P_\ell}\right)^{\gamma-\varepsilon}\psi > 0 \tag{3.52}$$

which is always fulfilled. This completes the proof.

Welfare effects of $\Delta\Lambda$

To capture the welfare effects caused by a change in the average effective labor supply and hence per-capita income of workers, we calculate the total derivative of Eq. (3.24) with respect to Λ

$$\frac{dV(\cdot)}{d\Lambda} = \frac{\partial V(\cdot)}{\partial\Lambda} + \frac{\partial V(\cdot)}{\partial P_\ell}\frac{\partial P_\ell}{\partial m}\frac{dm}{d\Lambda}, \tag{3.53}$$

where the direct effect is given by $\partial V(\cdot)/\partial\Lambda > 0$. The indirect effect can be disentangled into $\partial P_\ell/\partial m < 0$ and $dm/d\Lambda > 0$, which are derived in the main text, and

$$\frac{\partial V(\cdot)}{\partial P_\ell} = \frac{1}{P_\ell}\left[\beta\left(\frac{P_n}{P_\ell}\right)^{\gamma} - \left(\frac{\overline{e}}{P_\ell}\right)^{\varepsilon}\hat{\psi}\right]. \tag{3.54}$$

The sign of the bracket term determines the sign of $\partial V(\cdot)/\partial P_\ell$. Substituting the solution for $\overline{e}$ from Eq. (3.19), the definition of $\hat{\psi}$, and accounting for $w = P_n$, we obtain $\partial V(\cdot)/\partial P_\ell \gtreqless 0$ if

$$\beta\left(\frac{P_n}{P_\ell}\right)^{\gamma-\varepsilon} - \int_{\underline{\lambda}}^{\overline{\lambda}} \lambda^{\varepsilon} dL(\lambda) \gtreqless 0. \tag{3.55}$$

In the main text, we show that negative semi-definiteness of the Slutsky matrix (and thus a proper solution to the consumers' utility maximization problem) requires $\underline{\lambda}^{\varepsilon} \geq \beta[(1-\varepsilon)/(1-\gamma)]\,(P_n/P_\ell)^{\gamma-\varepsilon}$ as in condition (3.18). With the parameter ranking given by

$0 \leq \varepsilon < \gamma < 1$ it follows that $(1-\varepsilon)/(1-\gamma) > 1$, which in turn ensures

$$\underline{\lambda}^{\varepsilon} > \beta \left(\frac{P_n}{P_\ell} \right)^{\gamma-\varepsilon}, \tag{3.56}$$

by means of conditions (3.14) and (3.15). Eq. (3.56) implies that Eq. (3.55) is strictly smaller than zero, which then establishes $\partial V(\cdot)/\partial P_\ell < 0$ in Eq. (3.54). The indirect welfare effect is therefore positive, which combined with the positive direct welfare effect thus ensures that social welfare increases in Λ. This completes the proof.

Welfare effects of $\Delta\psi$ for $\gamma \to 1$

Notice that the welfare effect caused by a change in income dispersion in Eq. (3.26) evaluated at $\gamma = 1$ gives

$$\frac{dV(P_n, P_\ell, \bar{e}, \psi)}{d\psi} = \sqrt{\frac{\bar{e}}{P_\ell(\sigma-1)^2}} \left\{ 2(\sigma-1) + \frac{1}{m}\frac{dm}{d\psi} \left[\psi - \frac{\beta}{\sqrt{\Lambda}} \left(\frac{P_n}{P_\ell} \right)^{\frac{1}{2}} \right] \right\}. \tag{3.57}$$

Applying the implicit function theorem to goods market clearing condition in Eq. (3.22) gives Eq. (3.50). Then, using the zero-profit condition and the derivative of the price index with respect to the mass of firms producing intermediates yields

$$\frac{dm}{d\psi} = \frac{-H\Lambda^{\frac{1}{2}}\beta \left(\frac{P_n}{P_\ell} \right)^{\frac{1}{2}}}{\sigma f + H\Lambda^{\frac{1}{2}}\beta \left(\frac{P_n}{P_\ell} \right)^{\frac{1}{2}} \frac{1}{2(\sigma-1)} \frac{1}{m} \psi} < 0, \tag{3.58}$$

which is as well evaluated at $\varepsilon = 1/2$ and $\gamma = 1$. Plugging Eq. (3.58) into Eq. (3.57) and noting that

$$\frac{H\Lambda^{\frac{1}{2}}\beta \left(\frac{P_n}{P_\ell} \right)^{\frac{1}{2}} \psi}{\sigma f 2(\sigma-1)m + H\Lambda^{\frac{1}{2}}\beta \left(\frac{P_n}{P_\ell} \right)^{\frac{1}{2}} \psi} < 1 \tag{3.59}$$

is fulfilled, we can safely conclude that a higher income dispersion, indicated by a lower ψ, always induces negative welfare effects for all $\sigma > 1$, when evaluated at $\varepsilon = 1/2$ and $\gamma \to 1$. This completes the proof.

Proof of inefficient firm entry with Cobb-Douglas preferences

In a first step, we compute the market solution under Cobb-Douglas preferences. For this purpose, we can evaluate Eq. (3.20) at $\varepsilon \to 0$. Then, substituting Eq. (3.21), we compute $(1-\beta)H\Lambda w = mr$. Using the zero-profit condition, $r = \sigma P_n f$, then establishes

$$m = (1-\beta)\frac{H\Lambda}{\sigma f}. \tag{3.60}$$

We now turn to the social planner problem. Setting $\varepsilon = \gamma$ and taking the limit of $\varepsilon \to 0$, we compute for the indirect utility function $v(P_n, P_\ell, e^i) = \ln \left[e^i / \left(P_n^{\beta} P_\ell^{1-\beta} \right) \right]$.

Making use of Eqs. (3.2) and (3.3), we can then compute the direct utility function

$$u(X_n^i, X_\ell^i) = \beta \ln\left(\frac{X_n^i}{\beta}\right) + (1-\beta)\ln\left(\frac{X_\ell^i}{1-\beta}\right), \tag{3.61}$$

for Cobb-Douglas preferences, with $0 < \beta < 1$ being the now constant expenditure share for necessities. A utilitarian social planner then has an objective of the form

$$U(X_n^i, X_\ell^i) = \int_{i\in\mathcal{H}} \left[\beta \ln\left(\frac{X_n^i}{\beta}\right) + (1-\beta)\ln\left(\frac{X_\ell^i}{1-\beta}\right)\right] di. \tag{3.62}$$

Of course, with risk-averse agents (due to the log structure of utility) income distribution also matters for social welfare. However, if the social planner has access to a lump-sum tax-transfer system for redistributing income, he cannot do better than giving all consumers the same level of expenditures. In this case, there is no harm done by considering the alternative (and easier accessible) welfare function

$$\hat{U}(X_n, X_\ell) = \beta \ln\left(\frac{X_n}{\beta}\right) + (1-\beta)\ln\left(\frac{X_\ell}{1-\beta}\right), \tag{3.63}$$

as objective of the social planner. The social planner maximizes Eq. (3.63) subject to a technology constraint and a constraint for factor market clearing. Given that in optimum all firms in the intermediate goods sector, m, produce the same quantity of intermediate good ω from set Ω, and acknowledging Eq. (3.16), the technology constraint is given by

$$X_\ell = m^{\frac{\sigma}{\sigma-1}} q. \tag{3.64}$$

The labor market clearing condition from the main text, $X_n + m(q+f) = H\Lambda$, gives the factor market clearing condition for the social planner and can be rewritten as

$$m = \frac{H\Lambda - X_n}{q+f}. \tag{3.65}$$

Plugging Eqs. (3.64) and (3.65) into Eq. (3.63), we obtain

$$\hat{U}(X_n, q) = \beta \ln\left(\frac{X_n}{\beta}\right) + (1-\beta)\ln\left[\left(\frac{H\Lambda - X_n}{q+f}\right)^{\frac{\sigma}{\sigma-1}} \frac{q}{1-\beta}\right]. \tag{3.66}$$

Maximizing Eq. (3.66) with respect to q and X_n and rewriting the corresponding first-order conditions gives

$$q = (\sigma-1)f \quad \text{and} \quad X_n = \beta\frac{\sigma-1}{\sigma-\beta}H\Lambda, \tag{3.67}$$

respectively. Finally, using both results from Eq. (3.67) in Eq. (3.65) yields the mass of firms producing intermediate goods in social optimum according to

$$m = (1-\beta)\frac{H\Lambda}{f(\sigma-\beta)}, \tag{3.68}$$

which is larger than the market solution in Eq. (3.60). Notice that Eqs. (3.60) and

(3.68) coincide if the model degenerates to a one-sector economy with $\beta = 0$, leading to allocational efficiency. This completes the proof.

Proof of unique, interior equilibrium in the open economy for $\varepsilon = 0$

Assuming homothetic preferences, $\varepsilon = 0$, implies $\phi = \phi^* = 1$. Evaluating $\Gamma_2(\cdot)$ in Eq. (3.32) at $\varepsilon = 0$, we obtain

$$\bar{\Gamma}_2(m,\mu) \equiv (1-\tau^{1-\sigma})(1-\mu) - \alpha m^{\frac{\gamma}{\sigma-1}}(1+\mu\tau^{1-\sigma})^{\frac{\gamma}{\sigma-1}}(\mu+\tau^{1-\sigma})\left[\bar{f}(\mu)-1\right] = 0, \quad (3.69)$$

where $\bar{f}(\mu) \equiv \left[(\mu+\tau^{1-\sigma})/(1+\mu\tau^{1-\sigma})\right]^{\frac{\gamma}{\sigma-1}-1}$, with $\bar{f}(1) = 1$ and $\bar{f}'(\mu) \gtreqless 0$ iff $\gamma \gtreqless \sigma - 1$. According to Eq. (3.69), $\bar{\Gamma}_2(\cdot) = 0$ holds for all m if $\mu = 1$, due to $\bar{f}(1) = 1$. Furthermore, three cases can be distinguished. The case $\gamma = \sigma - 1$ establishes $\bar{f}(\mu) = 1$ for all μ, and hence $\bar{\Gamma}_2(\cdot) = 0$ in Eq. (3.69) is possible if $\mu = 1$. The second case $\gamma > \sigma - 1$ leads to $\bar{f}(\mu) \gtreqless 1$ if $\mu \gtreqless 1$. We can therefore safely conclude that $\mu \gtreqless 1$ implies $0 \gtreqless \bar{\Gamma}_2(\cdot)$, such that $\mu \neq 1$ violates $\bar{\Gamma}_2(\cdot) = 0$ in Eq. (3.69). Considering now that $\mu \neq 1$ were a possible solution for the case $\gamma < \sigma - 1$. Then we can solve $\bar{\Gamma}_2(\cdot) = 0$ in Eq. (3.69) for m and substitute the resulting expression in $\Gamma_3(\cdot)$ in Eq. (3.34), evaluated at $\varepsilon = 0$, to arrive at

$$\bar{\Gamma}_4(\mu) \equiv \left(\frac{\bar{A}(\mu)}{\alpha}\right)^{\frac{\sigma-1}{\gamma}} \sigma f \frac{\mu+\tau^{1-\sigma}}{(1+\mu\tau^{1-\sigma})(1+\tau^{1-\sigma})} - H\Lambda\left[1-\hat{f}(\mu)\bar{A}(\mu)\right] = 0, \qquad (3.70)$$

where $\bar{A}(\mu) \equiv \left[(1-\tau^{1-\sigma})(1-\mu)\right]/\left[(\mu+\tau^{1-\sigma})(\bar{f}(\mu)-1)\right]$, $\hat{f}(\mu) \equiv \frac{\mu+\tau^{1-\sigma}}{1+\mu\tau^{1-\sigma}}\bar{f}(\mu)$ and the reformulation $2-\bar{A}(\mu)[\hat{f}(\mu)+1] = [1-\hat{f}(\mu)\bar{A}(\mu)](1+\tau^{1-\sigma})(1+\mu)/(\mu+\tau^{1-\sigma})$ has been used. Accounting for $\bar{A}(\mu) > 0$, it follows from Eq. (3.70) that $\bar{\Gamma}_4(\mu) = 0$ requires $1-\hat{f}(\mu)\bar{A}(\mu) > 0$. This is equivalent to $1 - \left\{\left[\bar{f}(\mu)(1-\tau^{1-\sigma})(1-\mu)\right]/\left[(1+\mu\tau^{1-\sigma})(\bar{f}(\mu)-1)\right]\right\} > 0$ and ultimately to $[\hat{f}(\mu)-1]/[\bar{f}(\mu)-1] > 0$. However, noting that $\hat{f}(\mu) \gtreqless 1$ if $\mu \gtreqless 1$, while in the case of $\gamma < \sigma - 1$ $\bar{f}(\mu) \gtreqless 1$ if $1 \gtreqless \mu$, we can safely conclude that $1 - \hat{f}(\mu)\bar{A}(\mu) < 0$ if $\mu \neq 1$, which is in violation to $\bar{\Gamma}_4(\mu) = 0$. This completes the proof.

Proof of unique, interior equilibrium in the open economy for $\varepsilon \neq 0$

To investigate existence and uniqueness of the open economy equilibrium for the case of nonhomothetic preferences ($\varepsilon \neq 0$), we assume in the following that $\phi > \phi^*$ holds due to $\Lambda < \Lambda^*$ and/or $\psi > \psi^*$. Furthermore, we focus on the case of $\tau > 1$. The proof for $\phi < \phi^*$ due to $\Lambda > \Lambda^*$ and/or $\psi < \psi^*$ can be derived in analogy.[18]

Focussing on $\phi > \phi^*$, we can distinguish six different cases, where three cases assume $\mu \leq 1$ and three cases assume $\mu > 1$. In a first step, these cases are discussed in detail to investigate whether $\Gamma_2(\cdot) = 0$ in Eq. (3.32) can be fulfilled. If this is the case, then in a second step, we examine whether these outcomes are consistent with $\Gamma_3(\cdot) = 0$ in Eq. (3.34). Hence, we assess whether the remaining outcomes are consistent with $\Gamma_4(\cdot) = 0$ from Eq. (3.35) and therefore characterize candidates for an open economy equilibrium. Before we start with the analysis of the six cases, some general insights about Eq. (3.32) can be established: From the main text we know that $F(m,\mu) > 0$ has to hold to assure

[18] Derivation details for this case are available from the author upon request.

$m > 0$. Furthermore, looking at $f(\mu)$ as defined in the main text, we find that $f(0) = \tau^{\sigma-(1+\gamma-\varepsilon)} \gtreqless 1$ iff $\sigma \gtreqless 1+\gamma-\varepsilon$, $f(1) = 1$, $\lim_{\mu\to\infty} f(\mu) = \tau^{-[\sigma-(1+\gamma-\varepsilon)]} \gtreqless 1$ iff $\sigma \lesseqgtr 1+\gamma-\varepsilon$ and $f'(\mu) \gtreqless 0$ iff $\sigma \lesseqgtr 1+\gamma-\varepsilon$, and thus $f(\mu)$ is a monotone function.

1. $\sigma \leq 1+\gamma-\varepsilon$ and $\mu \leq 1$:
   This outcome can be excluded due to a violation of $\Gamma_2(\cdot) = 0$. This can be seen from Eq. (3.32) since $1-\mu \geq 0$, whereas bracket term $f(\mu)\phi^* - \phi < 0$, due to $\phi > \phi^*$ and $f(\mu) \leq 1$. Thus $\Gamma_2(\cdot) > 0$ is established.

2. $\sigma > 1+\gamma-\varepsilon$, $\tau \leq (\phi/\phi^*)^{\frac{1}{\sigma-(1+\gamma-\varepsilon)}}$, and $\mu \leq 1$:
   On the one hand, we have $1-\mu \geq 0$, due to $\mu \leq 1$. On the other hand, we have $f(\mu) > 1$, due to $f(1) = 1$ and $f'(\mu) < 0$ if $\sigma > 1+\gamma-\varepsilon$, while $f(\mu)\phi^* - \phi \leq 0$ still holds for all $\mu \leq 1$, because $\tau \leq (\phi/\phi^*)^{\frac{1}{\sigma-(1+\gamma-\varepsilon)}}$ establishes $f(0) \leq \phi/\phi^*$. According to Eq. (3.32), we therefore have $\Gamma_2(\cdot) > 0$, which eliminates an outcome with $\mu \leq 1$ in this case.

3. $\sigma > 1+\gamma-\varepsilon$, $\tau > (\phi/\phi^*)^{\frac{1}{\sigma-(1+\gamma-\varepsilon)}}$, and $\mu \leq 1$:
   First, $1-\mu \geq 0$ is established by $\mu \leq 1$. Moreover, $f(\mu) > 1$ holds, due to $f(1) = 1$ and $f'(\mu) < 0$ if $\sigma > 1+\gamma-\varepsilon$ (see above). However, $f(0) > \phi/\phi^*$ now implies that there exists a critical $\mu_0 \equiv \left[(\phi/\phi^*)^{\frac{\sigma-1}{\sigma-(1+\gamma-\varepsilon)}}\tau^{1-\sigma} - 1\right] / \left[\tau^{1-\sigma} - (\phi/\phi^*)^{\frac{\sigma-1}{\sigma-(1+\gamma-\varepsilon)}}\right] < 1$, such that $f(\mu)\phi^* - \phi \leq 0$ for all $\mu \in [\mu_0, 1]$. In this case, $\Gamma_2(\cdot) > 0$ rules out an open economy equilibrium $\mu \in [\mu_0, 1]$. But what about $\mu < \mu_0$? In this case, $f(\mu)\phi^* - \phi > 0$, so that $\Gamma_2(\cdot) = 0$ is possible and the outcome is consistent with $F(m,\mu) > 0$. But is the outcome also consistent with $\Gamma_3(\cdot) = 0$? To answer this question, we can substitute m from Eq. (3.33) into Eq. (3.34), which establishes $\Gamma_4(\mu)$ in Eq. (3.35), with $\Gamma_4(0) > 0$, $\Gamma_4(1) < 0$ and

$$\begin{aligned}\Gamma_4'(\mu) = &\left(\frac{A(\mu)}{\alpha}\right)^{\frac{1-\sigma}{\varepsilon-\gamma}} \sigma f \frac{1-\tau^{1-\sigma}}{(1+\mu\tau^{1-\sigma})^2} + \frac{\varepsilon-\gamma}{1-\sigma} H\Lambda f(\mu) A(\mu)\phi^* \frac{1-(\tau^{1-\sigma})^2}{(1+\mu\tau^{1-\sigma})^2} \\ &+ A'(\mu)\left\{\frac{1-\sigma}{\varepsilon-\gamma}\left(\frac{A(\mu)}{\alpha}\right)^{\frac{1-\sigma}{\varepsilon-\gamma}} \frac{\sigma f}{A(\mu)} \frac{1+\mu}{1+\mu\tau^{1-\sigma}} + H\Lambda\left[\frac{\mu+\tau^{1-\sigma}}{1+\mu\tau^{1-\sigma}} f(\mu)\phi^* + \phi\right]\right\}.\end{aligned} \tag{3.71}$$

   Whereas we cannot pin down the sign of $\Gamma_4'(\mu)$ in general, it is worth noting that the bracket term in the second line of Eq. (3.71) gives the derivative of $\Gamma_4(\mu)$ with respect to $A(\mu)$, which is unambiguously positive. With this insight at hand, we can make further progress by looking at the properties of $A(\mu)$ on μ-interval $(0, \mu_0)$, which can be summarized as follows: $A(0) > 0$, $\lim_{\mu\to\mu_0^-} A(\mu_0) = \infty$ and

$$A'(\mu) = \left[1-(\tau^{1-\sigma})^2\right] \frac{a(\mu)}{(\mu+\tau^{1-\sigma})^2\left[f(\mu)\phi^* - \phi\right]^2}, \tag{3.72}$$

   where $a(\mu) \equiv -\left[f(\mu)\phi^* - \phi\right] + \frac{\sigma-(1+\gamma-\varepsilon)}{\sigma-1} f(\mu)\phi^* \frac{(1-\tau^{1-\sigma})(1-\mu)}{1+\mu\tau^{1-\sigma}}$ with $a'(\mu) = -\frac{\varepsilon-\gamma}{1-\sigma}\frac{\sigma-(1+\gamma-\varepsilon)}{1-\sigma} f(\mu)\phi^* \frac{(1+\tau^{1-\sigma})(1-\tau^{1-\sigma})^2(1-\mu)}{(\mu+\tau^{1-\sigma})(1+\mu\tau^{1-\sigma})^2} > 0$, due to $\mu \in (0, \mu_0)$ and $\sigma > 1+\gamma-\varepsilon$. There are two possible outcomes. Either $A(\mu)$ is monotonically increasing on interval $(0, \mu_0)$ or it is non-monotonic and has at least one minimum. If $A(\mu)$ is increasing, $\Gamma_4'(\mu) > 0$ follows from Eq. (3.71), and in this case $\Gamma_4(0) > 0$ is sufficient for $\Gamma_4(\mu) > 0$ to hold for all $\mu \in (0, \mu_0)$. This is in violation to Eq. (3.35). If, however, $A(\mu)$ is non-monotonic, and $A(\mu)$ therefore has an interior extremum, with $A'(\mu) = 0$ and thus $a(\mu) = 0$, it follows from $a'(\mu) > 0$ that the extremum has indeed to be a minimum. Evaluated at this minimum, we compute

$$A(\mu)\Big|_{a(\mu)=0} = \left[\tilde{f}(\mu)\phi^*\right]^{-1} \frac{\sigma - 1}{\sigma - (1 + \gamma - \varepsilon)}, \tag{3.73}$$

with $\tilde{f}(\mu) \equiv \frac{\mu + \tau^{1-\sigma}}{1 + \mu\tau^{1-\sigma}} f(\mu)$ and $\tilde{f}(0) = \tau^{\varepsilon-\gamma} < 1$, $\tilde{f}(1) = 1$, $\lim_{\mu\to\infty} \tilde{f}(\mu) = \tau^{\gamma-\varepsilon} > 0$, $\tilde{f}'(\mu) > 0$. We use these insights to determine $A(\mu)\Big|_{a(\mu)=0} > 0$, which translates into

$$\begin{aligned}\Gamma_4(\mu)\Big|_{a(\mu)=0} = &\left\{\left[\alpha\tilde{f}(\mu)\phi^*\right]^{-1} \frac{\sigma - 1}{\sigma - (1 + \gamma - \varepsilon)}\right\}^{\frac{1-\sigma}{\varepsilon-\gamma}} \sigma f \frac{1 + \mu}{1 + \mu\tau^{1-\sigma}} \\ &- H\Lambda\left[2 - \frac{\sigma - 1}{\sigma - (1 + \gamma - \varepsilon)} - \frac{\sigma - 1}{\sigma - (1 + \gamma - \varepsilon)} \frac{\phi}{\tilde{f}(\mu)\phi^*}\right],\end{aligned} \tag{3.74}$$

which is unambiguously positive due to $\phi > \phi^*$, $\tilde{f}(\mu) < 1$ and $\sigma > 1 + \gamma - \varepsilon$. Since we know from above that $\partial\Gamma_4(\mu)/\partial A(\mu) > 0$, it is then immediate that $\Gamma_4(\mu) > 0$ extends to all $\mu \in (0, \mu_0)$. This is again in violation to Eq. (3.35). Accordingly, we can rule out existence of an outcome with $\mu \in (0, 1)$ for $\sigma > 1 + \gamma - \varepsilon$ and $\tau > (\phi/\phi^*)^{\frac{1}{\sigma-(1+\gamma-\varepsilon)}}$ by combining Eqs. (3.32) and (3.34).

4. $\sigma < 1 + \gamma - \varepsilon$, $\tau < (\phi/\phi^*)^{\frac{1}{-[\sigma-(1+\gamma-\varepsilon)]}}$ and $\mu > 1$:
From $\mu > 1$, we know that $1 - \mu < 0$. Due to $f(1) = 1$, $f'(\mu) > 0$ and $\lim_{\mu\to\infty} f(\mu) > 1$ if $\sigma < 1 + \gamma - \varepsilon$ this implies $f(\mu)\phi^* - \phi < 0$ for all $\mu > 1$, because $\tau < (\phi/\phi^*)^{\frac{1}{-[\sigma-(1+\gamma-\varepsilon)]}}$ establishes $\lim_{\mu\to\infty} f(\mu) < \phi/\phi^*$. According to Eq. (3.32), $\Gamma_2(\cdot) = 0$ is possible and the outcome consistent with $F(m, \mu) > 0$. In the following, we use $\Gamma_4(\mu)$ in Eq. (3.35) to investigate whether this outcome is also consistent with $\Gamma_3(\cdot) = 0$. We acknowledge $\Gamma_4(1) < 0$ and

$$\lim_{\mu\to\infty} \Gamma_4(\mu) = \left[\frac{-(1 - \tau^{1-\sigma})}{\tau^{-[\sigma-(1+\gamma-\varepsilon)]}\phi^* - \phi}\right]^{\frac{1-\sigma}{\varepsilon-\gamma}} \alpha^{\frac{\sigma-1}{\varepsilon-\gamma}} \sigma f \tau^{\sigma-1} - H\Lambda \frac{(1 + \tau^{1-\sigma})(\tau^{\gamma-\varepsilon}\phi^* - \phi)}{\tau^{-[\sigma-(1+\gamma-\varepsilon)]}\phi^* - \phi}, \tag{3.75}$$

where the sign of $\lim_{\mu\to\infty} \Gamma_4(\mu)$ is not immediate, because $f(\mu)\phi^* - \phi < 0$ for all $\mu > 1$ implies the first term in Eq. (3.75) to be positive, whereas the sign of the second term depends on bracket term $\tau^{\gamma-\varepsilon}\phi^* - \phi$. To make further progress and ensure an interior solution, we assume that $\tau^{\gamma-\varepsilon} \geq \phi/\phi^*$ to establish $\lim_{\mu\to\infty} \Gamma_4(\mu) > 0$. To determine the sign of $\Gamma_4'(\mu)$ in Eq. (3.71), we look at the properties of $A(\mu)$, with $A(1) = 0$ and $\lim_{\mu\to\infty} A(\mu) \to \infty$. Eq. (3.72) together with $a(\mu) > 0$, due to $\mu > 1$ and $\sigma < 1 + \gamma - \varepsilon$, establish $A'(\mu) > 0$. As can be seen in Eq. (3.71), this implies $\Gamma_4'(\mu) > 0$, with $\Gamma_4(1) < 0$ and $\lim_{\mu\to\infty} \Gamma_4(\mu) > 0$ ensured by $\tau^{\gamma-\varepsilon} \geq \phi/\phi^*$, which proves existence of a unique solution for $\Gamma_4(\mu) = 0$ on interval $(1, \infty)$ for

$(\phi/\phi^*)^{\frac{1}{\gamma-\varepsilon}} \leq \tau < (\phi/\phi^*)^{\frac{1}{-[\sigma-(1+\gamma-\varepsilon)]}}$.

5. $\sigma < 1+\gamma-\varepsilon$, $\tau \geq (\phi/\phi^*)^{\frac{1}{-[\sigma-(1+\gamma-\varepsilon)]}}$ and $\mu > 1$:
Noting that $1-\mu < 0$ is established by $\mu > 1$, furthermore $\lim_{\mu\to\infty} f(\mu) > 1$ holds with $f(1) = 1$ and $f'(\mu) > 0$ if $\sigma < 1+\gamma-\varepsilon$ (see above). However, $\lim_{\mu\to\infty} f(\mu) \geq \phi/\phi^*$ now implies that there exists a critical $\mu_1 \equiv \left[(\phi/\phi^*)^{\frac{1-\sigma}{\sigma-(1+\gamma-\varepsilon)}} - \tau^{1-\sigma}\right] / \left[1-(\phi/\phi^*)^{\frac{1-\sigma}{\sigma-(1+\gamma-\varepsilon)}}\tau^{1-\sigma}\right] > 1$, such that $f(\mu)\phi^* - \phi \geq 0$ for all $\mu \in [\mu_1, \infty)$. In this case, $\Gamma_2(\cdot) < 0$ rules out an open economy equilibrium $\mu \in [\mu_1, \infty)$. By contrast, $f(\mu)\phi^* - \phi < 0$ holds for case $\mu < \mu_1$, so that $\Gamma_2(\cdot) = 0$ is possible and the outcome is consistent with $F(m,\mu) > 0$. But is this outcome also consistent with $\Gamma_4(\mu)$ in Eq. (3.35)? First, we look at the properties of $A(\mu)$ on μ-interval $(1, \mu_1)$, which can be summarized as follows: $A(1) = 0$, $\lim_{\mu\to\mu_1^-} A(\mu) \to \infty$ and $A'(\mu)$ as given in Eq. (3.72). With $a(\mu) > 0$, due to $\mu \in (1, \mu_1)$ and $\sigma < 1+\gamma-\varepsilon$, this establishes $A'(\mu) > 0$. From Eq. (3.71), it then follows $\Gamma_4'(\mu) > 0$, with $\Gamma_4(1) < 0$ and $\lim_{\mu\to\mu_1^-} \Gamma_4(\mu) \to \infty$. This proves that the solution to $\Gamma_4(\mu) = 0$ on interval $(1, \mu_1)$ exists and is unique.

6. $\sigma \geq 1+\gamma-\varepsilon$ and $\mu > 1$:
For this case, $\Gamma_2(\cdot) = 0$ in Eq. (3.32) can hold due to $1-\mu < 0$ and $f(1) = 1$, with $f'(\mu) \leq 0$ and $\lim_{\mu\to\infty} f(\mu) \leq 1$ if $\sigma \geq 1+\gamma-\varepsilon$. This requires $F(m,\mu) > 0$ and thus $m > 0$ as given in Eq. (3.33). Turning to Eq. (3.35), we can note that $\Gamma_4(1) < 0$ and $\tau^{\gamma-\varepsilon} \geq \phi/\phi^*$ ensures $\lim_{\mu\to\infty} \Gamma_4(\mu) > 0$ (see above) and thus existence of a solution for $\Gamma_4(\mu) = 0$ on interval $(1, \infty)$, which we denote by $\hat{\mu}$. To prove uniqueness of this solution, we show that $\Gamma_4'(\mu) < 0$ is inconsistent with $\Gamma_4(\mu) \leq 0$, which is sufficient for $\Gamma_4(\mu) > 0$ if $\mu > \hat{\mu}$. Therefore, to make further progress on the sign of $\Gamma_4'(\mu)$ in Eq. (3.71), we look at the properties of $A(\mu)$, which are given by $A(1) = 0$, $\lim_{\mu\to\infty} A(\mu) > 0$ and $A'(\mu)$ in Eq. (3.72). The sign of $A'(\mu)$ is not immediate, because $a(\mu) \lesseqgtr 0$ is possible, due to $f(\mu)\phi^* - \phi < 0$ for all $\mu > 1$ and $\sigma \geq 1+\gamma-\varepsilon$. There are two possible outcomes. Either $A(\mu)$ is monotonically increasing on interval $(1, \infty)$ or it is non-monotonic. If $A(\mu)$ is increasing, $\Gamma_4'(\mu) > 0$ follows from Eq. (3.71) and in this case $\Gamma_4(1) < 0$ and $\lim_{\mu\to\infty} \Gamma_4(\mu) > 0$ are sufficient to show uniqueness of this solution. If, however, $A(\mu)$ is non-monotonic, and $A(\mu)$ therefore has an interior extremum, with $A'(\mu) = 0$ and thus $a(\mu) = 0$, it follows from $a'(\mu) \leq 0$ (due to $\sigma \geq 1+\gamma-\varepsilon$ and $\mu > 1$) that the extremum has to be a maximum. To simplify the further analysis, rewrite Eq. (3.35) by using the reformulation of $2 - A(\mu)[\tilde{f}(\mu)\phi^* + \phi] = [1-\tilde{f}(\mu)A(\mu)\phi^*](1+\tau^{1-\sigma})(1+\mu)/(\mu+\tau^{1-\sigma})$ which gives

$$\tilde{\Gamma}_4(\mu) \equiv \left(\frac{A(\mu)}{\alpha}\right)^{\frac{1-\sigma}{\varepsilon-\gamma}} \sigma f \frac{\mu+\tau^{1-\sigma}}{(1+\mu\tau^{1-\sigma})(1+\tau^{1-\sigma})} - H\Lambda\left[1-\tilde{f}(\mu)A(\mu)\phi^*\right] = 0. \tag{3.76}$$

Since the first term in Eq. (3.76) is unambiguously positive, the second term, more precisely $1-\tilde{f}(\mu)A(\mu)\phi^*$, determines the outcome of $\tilde{\Gamma}_4(\mu)$. Furthermore, rewriting

Eq. (3.72), we obtain

$$A'(\mu) = -\frac{1-(\tau^{1-\sigma})^2}{(\mu+\tau^{1-\sigma})^2\,[f(\mu)\phi^* - \phi]}\left\{1 - \frac{\sigma-(1+\gamma-\varepsilon)}{\sigma-1}\tilde{f}(\mu)A(\mu)\phi^*\right\}, \qquad (3.77)$$

with $f(\mu)\phi^* - \phi < 0$ and $[\sigma-(1+\gamma-\varepsilon)]/(\sigma-1) < 1$. Consequently, since $A'(\mu) < 0$ is a prerequisite for $\Gamma_4'(\mu) < 0$, according to Eq. (3.71), and since $A'(\mu) < 0$ requires $\tilde{\Gamma}_4(\mu) > 0$, we can safely conclude that $\Gamma_4'(\mu) < 0$ would be in violation of $\Gamma_4(\mu) = 0$. This proves that the solution to $\Gamma_4(\mu) = 0$ on interval $(1, \infty)$ – whose existence follows from the assumption that $\tau^{\gamma-\varepsilon} \geq (\phi/\phi^*)$ – is unique.

Putting together, under the mild parameter constraint on trade cost $\tau^{\gamma-\varepsilon} \geq \phi/\phi^*$, we have shown for $\phi > \phi^*$ that $\Gamma_4(\mu) = 0$ from Eq. (3.35) has a unique solution on interval $(1, \infty)$. Following the line of reasoning outlined above, we can further conclude that for $\phi < \phi^*$ $\Gamma_4(\mu) = 0$ has a unique solution on interval $(0, 1)$ if $\tau^{\gamma-\varepsilon} \geq \phi^*/\phi$. This completes the proof.

Derivation of the parameter constraint in (3.37)

Consider first case $\phi > \phi^*$, which implies $\mu > 1$. Solving the labor market clearing condition in Eq. (3.23) for X_n, using the equilibrium mass of domestic firms m from Eq. (3.33) and substituting Eq. (3.35) yields

$$X_n = H\Lambda\left\{1 - \frac{1}{1+\mu}\left[2 - A(\mu)\left[\tilde{f}(\mu)\phi^* + \phi\right]\right]\right\}. \qquad (3.78)$$

Diversification is confirmed if $X_n > 0$ holds, which is equivalent to

$$\mu - 1 + A(\mu)\left[\tilde{f}(\mu)\phi^* + \phi\right] > 0. \qquad (3.79)$$

Acknowledging $\mu > 1$, this verifies that Eq. (3.79) is always fulfilled for home if $\phi > \phi^*$.

For case $\phi < \phi^*$, which leads to $\mu < 1$, we again use Eqs. (3.23) and (3.33) to show that $X_n > 0$ requires

$$H\Lambda - \left(\frac{A(\mu)}{\alpha}\right)^{\frac{1-\sigma}{\varepsilon-\gamma}}\frac{\sigma f}{1+\mu\tau^{1-\sigma}} > 0. \qquad (3.80)$$

Looking at Eq. (3.76) and using the insight that $d\tilde{\Gamma}_4(\mu)/d(H\Lambda) < 0$ if $1 - \tilde{f}(\mu)A(\mu)\phi^* > 0$, we can conclude that $X_n > 0$ is guaranteed in the open economy equilibrium if $\tilde{\Gamma}_4(\mu)$ evaluated at $H\Lambda = [A(\mu)/\alpha]^{\frac{1-\sigma}{\varepsilon-\gamma}}\sigma f/(1+\mu\tau^{1-\sigma})$ is larger than zero. This finally gives (3.37) as a constraint on trade cost parameter τ. This completes the proof.

Proof of positive consumption levels of final goods in the open economy

This proof outlines detailed derivation steps for $\phi > \phi^*$, leading to $\mu > 1$. The analysis for $\phi < \phi^*$ is achieved in analogy.

Combining Eqs. (3.31) and (3.33) gives $P_\ell = [A(\mu)/\beta]^{\frac{1}{\varepsilon-\gamma}}P_n$, which then can be used to rewrite constraint (3.18) as

$$\underline{\lambda}^{\varepsilon} \geq \frac{1-\varepsilon}{1-\gamma} A(\mu). \tag{3.81}$$

Furthermore, we can determine

$$\begin{aligned} \frac{dA(\mu)}{d\tau^{1-\sigma}} \lesseqgtr 0 \quad &\Leftrightarrow \quad \frac{\partial A(\mu)}{\partial \tau^{1-\sigma}} + A'(\mu)\frac{d\mu}{d\tau^{1-\sigma}} \lesseqgtr 0 \\ &\Leftrightarrow \frac{d\mu}{d\tau^{1-\sigma}} \lesseqgtr -\frac{1-\mu^2}{1-(\tau^{1-\sigma})^2}, \end{aligned} \tag{3.82}$$

where Eq. (3.77) and

$$\frac{\partial A(\mu)}{\partial \tau^{1-\sigma}} = -\frac{1-\mu^2}{(\mu+\tau^{1-\sigma})^2\left[f(\mu)\phi^* - \phi\right]}\left\{1 - \frac{\sigma-(1+\gamma-\varepsilon)}{\sigma-1}\tilde{f}(\mu)A(\mu)\phi^*\right\} < 0 \tag{3.83}$$

have been used. Applying the implicit function theorem to Eq. (3.35) establishes

$$0 = \frac{\partial \Gamma_4(\mu)}{\partial \tau^{1-\sigma}} + \frac{\partial \Gamma_4(\mu)}{\partial \mu}\frac{d\mu}{d\tau^{1-\sigma}} + \frac{\partial \Gamma_4(\mu)}{\partial A(\mu)}\frac{dA(\mu)}{d\tau^{1-\sigma}}. \tag{3.84}$$

Combining Eqs. (3.82) and (3.84) then gives

$$\frac{dA(\mu)}{d\tau^{1-\sigma}} \lesseqgtr 0 \quad \Leftrightarrow \quad 0 \lesseqgtr \frac{\partial \Gamma_4(\mu)}{\partial \tau^{1-\sigma}} + \frac{\partial \Gamma_4(\mu)}{\partial \mu}\left[-\frac{1-\mu^2}{1-(\tau^{1-\sigma})^2}\right] + \frac{\partial \Gamma_4(\mu)}{\partial A(\mu)}\frac{dA(\mu)}{d\tau^{1-\sigma}}.$$

Plugging in the results for the three partial derivatives $\partial\Gamma_4(\mu)/\partial\tau^{1-\sigma}$, $\partial\Gamma_4(\mu)/\partial\mu$ and $\partial\Gamma_4(\mu)/\partial A(\mu)$, then rearranging gives

$$\frac{dA(\mu)}{d\tau^{1-\sigma}} \lesseqgtr 0 \quad \Leftrightarrow \quad 0 \lesseqgtr -\left(\frac{A(\mu)}{\alpha}\right)^{\frac{1-\sigma}{\varepsilon-\gamma}} \sigma f \frac{1+\mu}{(1+\mu\tau^{1-\sigma})(1+\tau^{1-\sigma})} + \frac{\partial \Gamma_4(\mu)}{\partial A(\mu)}\frac{dA(\mu)}{d\tau^{1-\sigma}}. \tag{3.85}$$

Using $\partial\Gamma_4(\mu)/\partial A(\mu) > 0$, this shows that assuming $dA(\mu)/d\tau^{1-\sigma} \leq 0$, the right-hand side of Eq. (3.85) is violated. We can safely conclude that Eq. (3.85) to be fulfilled requires $dA(\mu)/d\tau^{1-\sigma} > 0$. This completes the proof.

Proof of Proposition 3

We consider in the subsequent analysis the case $\phi > \phi^*$ leading to $\mu > 1$. The proof for $\phi < \phi^*$ implying $\mu < 1$ can be derived in analogy.

First, we compute the partial derivative of Eq. (3.35) with respect to $\tau^{1-\sigma}$ and get

$$\left(\frac{\partial \Gamma_4(\mu)}{\partial \tau^{1-\sigma}}\right)_{total} = \frac{\partial \Gamma_4(\mu)}{\partial \tau^{1-\sigma}} + \frac{\partial \Gamma_4(\mu)}{\partial A(\mu)}\frac{\partial A(\mu)}{\partial \tau^{1-\sigma}} < 0, \tag{3.86}$$

where $\partial\Gamma_4(\mu)/\partial\tau^{1-\sigma} < 0$, $\partial\Gamma_4(\mu)/\partial A(\mu) > 0$ and $\partial A(\mu)/\partial\tau^{1-\sigma} < 0$ as given in Eq. (3.83). Applying the implicit function theorem to Eq. (3.35) and acknowledging $\Gamma_4'(\mu) > 0$ then establishes $d\mu/d\tau^{1-\sigma} > 0$. With the insight at hand that higher trade cost τ lead to a decrease in $\tau^{1-\sigma}$, we can conclude that the relative number of firms producing intermediate goods in foreign falls in trade cost. The total derivative of the corresponding Grubel-Lloyd index in Eq. (3.41) reads

$$\frac{dGLI}{d\tau^{1-\sigma}} = -\frac{1}{\mu^2}\frac{d\mu}{d\tau^{1-\sigma}} < 0, \tag{3.87}$$

and hence it follows that the Grubel-Lloyd index increases in trade cost. Partially differentiating Eq. (3.35) with respect to ϕ gives

$$\left(\frac{\partial\Gamma_4(\mu)}{\partial\phi}\right)_{total} = \frac{H\Lambda}{f(\mu)\phi^* - \phi}\left\{\tilde{f}(\mu)A(\mu)\phi^*\frac{(1+\tau^{1-\sigma})(1+\mu)}{\mu+\tau^{1-\sigma}} + \frac{1-\sigma}{\varepsilon-\gamma}\left[2 - A(\mu)\left(\tilde{f}(\mu)\phi^* + \phi\right)\right]\right\} < 0. \tag{3.88}$$

Applying the implicit function theorem to Eq. (3.35) and noting $\Gamma_4'(\mu) > 0$, delivers $d\mu/d\phi > 0$. If ϕ drops, the two countries get more similar in their consumption expenditures and hence the relative number of foreign firms μ shrinks, implying that it moves closer to 1. We compute the total derivative of the Grubel-Lloyd index from Eq. (3.41) with respect to ϕ according to

$$\frac{dGLI}{d\phi} = -\frac{1}{\mu^2}\frac{d\mu}{d\phi} < 0, \tag{3.89}$$

which indicates that this index increases in the similarity of both countries with regard to their consumption expenditures. Finally, we derive from Eq. (3.35)

$$\left(\frac{\partial\Gamma_4(\mu)}{\partial\phi^*}\right)_{total} = \frac{\partial\Gamma_4(\mu)}{\partial\phi^*} + \frac{\partial\Gamma_4(\mu)}{\partial A(\mu)}\frac{\partial A(\mu)}{\partial\phi^*} > 0, \tag{3.90}$$

where $\partial\Gamma_4(\mu)/\partial\phi^* > 0$ and $\partial A(\mu)/\partial\phi^* > 0$ have been computed. Applying the implicit function theorem to Eq. (3.35) and accounting for $\Gamma_4'(\mu) > 0$, this implies $d\mu/d\phi^* < 0$. Using Eq. (3.41) gives

$$\frac{dGLI}{d\phi^*} = -\frac{1}{\mu^2}\frac{d\mu}{d\phi^*} > 0, \tag{3.91}$$

and repeats the finding from above, that the Grubel-Lloyd index increases in the similarity of home and foreign based on their level and/or dispersion of per-capita income, here triggered by a higher ϕ^*. This completes the proof.

Proof of Proposition 4

To show Proposition 4, we notice first that the export and import functions of intermediate goods in home, Eqs. (3.38) and (3.39), can be rewritten by using the equilibrium mass of domestic firms producing intermediate goods in Eq. (3.33) according to

$$EX_\omega = \left(\frac{A(\mu)}{\alpha}\right)^{\frac{1-\sigma}{\varepsilon-\gamma}}\sigma P_n f\frac{\tau^{1-\sigma}}{(1+\tau^{1-\sigma})(1+\mu\tau^{1-\sigma})}, \tag{3.92}$$

$$IM_\omega = \left(\frac{A(\mu)}{\alpha}\right)^{\frac{1-\sigma}{\varepsilon-\gamma}}\sigma P_n f\frac{\mu\tau^{1-\sigma}}{(1+\tau^{1-\sigma})(1+\mu\tau^{1-\sigma})}, \tag{3.93}$$

respectively, where $A(\mu)$ is defined as in the main text. Due to the balanced trade assumption, we can compute that overall trade (inter- plus intra-industry) in home is given by $2IM_\omega$ if $\phi > \phi^*$ and by $2EX_\omega$ if $\phi < \phi^*$. We focus in the subsequent analysis on case $\phi > \phi^*$, and hence, we investigate how the import function of differentiated intermediates in Eq. (3.93) responds to changes in trade cost parameter τ, and variables ϕ and ϕ^*. The results for case $\phi < \phi^*$ can be derived in analogy, using Eq. (3.92).

In a first step, we compute the total derivative of Eq. (3.93) with respect to $\tau^{1-\sigma}$ as

$$\frac{dIM_\omega}{d\tau^{1-\sigma}} = \frac{\partial IM_\omega}{\partial \tau^{1-\sigma}} + \frac{\partial IM_\omega}{\partial \mu}\frac{d\mu}{d\tau^{1-\sigma}} + \frac{\partial IM_\omega}{\partial A(\mu)}\frac{dA(\mu)}{d\tau^{1-\sigma}}, \tag{3.94}$$

where $\partial IM_\omega/\partial\mu > 0$, $d\mu/d\tau^{1-\sigma} > 0$ and $\partial IM_\omega/\partial A(\mu) > 0$ are immediate. From above, we furthermore know $dA(\mu)/d\tau^{1-\sigma} > 0$. Nevertheless, the sign of Eq. (3.94) is not immediate since $\partial IM_\omega/\partial\tau^{1-\sigma} \lesseqgtr 0$ has not been determined yet . Making use of the insight from Eq. (3.82), that $dA(\mu)/d\tau^{1-\sigma} > 0$ leads to $d\mu/d\tau^{1-\sigma} > -(1-\mu^2)/[1-(\tau^{1-\sigma})^2]$ in Eq. (3.94), we can compute

$$\frac{dIM_\omega}{d\tau^{1-\sigma}} > \frac{\partial IM_\omega}{\partial \tau^{1-\sigma}} + \frac{\partial IM_\omega}{\partial \mu}\left[-\frac{1-\mu^2}{1-(\tau^{1-\sigma})^2}\right] + \frac{\partial IM_\omega}{\partial A(\mu)}\frac{dA(\mu)}{d\tau^{1-\sigma}}. \tag{3.95}$$

Using the derivatives $\partial IM_\omega/\partial\tau^{1-\sigma}$, $\partial IM_\omega/\partial\mu$, $\partial IM_\omega/\partial A(\mu)$ and noting that $dA(\mu)/d\tau^{1-\sigma} > 0$ shows that the right-hand side of Eq. (3.95) is greater than zero which gives rise to $dIM_\omega/d\tau^{1-\sigma} > 0$.

In a second step, we investigate the total derivative of Eq. (3.93) with respect to ϕ, which reads

$$\frac{dIM_\omega}{d\phi} = \frac{\partial IM_\omega}{\partial \phi} + \frac{\partial IM_\omega}{\partial \mu}\frac{d\mu}{d\phi} + \frac{\partial IM_\omega}{\partial A(\mu)}\frac{dA(\mu)}{d\phi}, \tag{3.96}$$

where $\partial IM_\omega/\partial\phi = 0$, $\partial IM_\omega/\partial\mu > 0$, $d\mu/d\phi > 0$ and $\partial IM_\omega/\partial A(\mu) > 0$. Applying the implicit function theorem to Eq. (3.35) yields

$$0 = \frac{\partial \Gamma_4(\mu)}{\partial \phi} + \frac{\partial \Gamma_4(\mu)}{\partial \mu}\frac{d\mu}{d\phi} + \frac{\partial \Gamma_4(\mu)}{\partial A(\mu)}\frac{dA(\mu)}{d\phi}, \tag{3.97}$$

with $\partial\Gamma_4(\mu)/\partial\phi > 0$, $\partial\Gamma_4(\mu)/\partial\mu > 0$, $\partial\Gamma_4(\mu)/\partial A(\mu) > 0$, implying that $dA(\mu)/d\phi < 0$ is needed for Eq. (3.97) to be fulfilled. Still, the sign of Eq. (3.96) is not clear, because there exist two counteracting effects, more precisely $[\partial IM_\omega/\partial\mu][d\mu/d\phi] > 0$ and $[\partial IM_\omega/\partial A(\mu)][dA(\mu)/d\phi] < 0$. For this reason, we use Eq. (3.76), which is just a reformulation of Eq. (3.35), and partially differentiate with respect to μ to obtain

$$\tilde{\Gamma}_4'(\mu) = H\Lambda\left\{\frac{1-\sigma}{\varepsilon-\gamma}\left[1-\tilde{f}(\mu)A(\mu)\phi^*\right] + \tilde{f}(\mu)A(\mu)\phi^*\right\}$$
$$\left\{\frac{A'(\mu)}{A(\mu)} + \frac{\varepsilon-\gamma}{1-\sigma}\frac{1-(\tau^{1-\sigma})^2}{(1+\mu\tau^{1-\sigma})(\mu+\tau^{1-\sigma})}\right\}. \tag{3.98}$$

Using the insight of $2 - A(\mu)[\tilde{f}(\mu)\phi^* + \phi] = [1-\tilde{f}(\mu)A(\mu)\phi^*](1+\tau^{1-\sigma})(1+\mu)/(\mu+\tau^{1-\sigma})$, introduced above, in Eq. (3.88) to get

$$\left(\frac{\partial\Gamma_4(\mu)}{\partial\phi}\right)_{total} = \frac{H\Lambda}{f(\mu)\phi^* - \phi}\left\{\frac{1-\sigma}{\varepsilon-\gamma}\left[1-\tilde{f}(\mu)A(\mu)\phi^*\right]+\tilde{f}(\mu)A(\mu)\phi^*\right\} < 0. \tag{3.99}$$

Again applying the implicit function theorem to Eq. (3.76), a new expression for $d\mu/d\phi$ is found by incorporating Eqs. (3.98) and (3.99)

$$\frac{d\mu}{d\phi} = -\frac{1}{f(\mu)\phi^*-\phi}\left[\frac{A'(\mu)}{A(\mu)}+\frac{\varepsilon-\gamma}{1-\sigma}\frac{1-(\tau^{1-\sigma})}{(1+\mu\tau^{1-\sigma})(\mu+\tau^{1-\sigma})}\right]^{-1}. \tag{3.100}$$

Looking back at Eq. (3.96), substituting $dA(\mu)/d\phi = \partial A(\mu)/\partial\phi + A'(\mu)(d\mu/d\phi)$, and plugging in the results for the partial derivatives and Eq. (3.100), we obtain

$$\frac{dIM_\omega}{d\phi} = -IM_\omega\frac{\tau^{1-\sigma}}{\mu(\mu+\tau^{1-\sigma})\left[f(\mu)\phi^*-\phi\right]}\left[\frac{A'(\mu)}{A(\mu)}+\frac{\varepsilon-\gamma}{1-\sigma}\frac{1-(\tau^{1-\sigma})}{(1+\mu\tau^{1-\sigma})(\mu+\tau^{1-\sigma})}\right]^{-1}. \tag{3.101}$$

This proves that $dIM_\omega/d\phi > 0$ holds. In a last step, we focus on the total derivative of Eq. (3.93) with respect to ϕ^*. Substituting Eq. (3.93) into Eq. (3.35) shows that both equations are linked to each other according to

$$\Gamma_4(\mu) \equiv IM_\omega\frac{(1+\tau^{1-\sigma})(1+\mu)}{\mu\tau^{1-\sigma}} - 2H\Lambda w + H\Lambda w A(\mu)\left[\tilde{f}(\mu)\phi^*+\phi\right] = 0. \tag{3.102}$$

Defining $T_1(\mu,\phi^*) \equiv IM_\omega(1+\tau^{1-\sigma})(1+\mu)/(\mu\tau^{1-\sigma})$ and $T_2(\mu,\phi^*) \equiv A(\mu)[\tilde{f}(\mu)\phi^*+\phi]$, we can infer from applying the implicit function theorem that $dT_1(\mu,\phi^*)/d\phi^* + dT_2(\mu,\phi^*)/d\phi^* = 0$. By substituting the respective results into

$$\frac{dT_2(\mu,\phi^*)}{d\phi^*} = \left\{A'(\mu)\left[\tilde{f}(\mu)\phi^*+\phi\right]+A(\mu)\tilde{f}'(\mu)\phi^*\right\}\frac{d\mu}{d\phi^*} + \frac{\partial A(\mu)}{\partial\phi^*}\left[\tilde{f}(\mu)\phi^*+\phi\right]+A(\mu)\tilde{f}(\mu), \tag{3.103}$$

we can compute $dT_2(\mu,\phi^*)/d\phi^* > 0$. This implies $dT_1(\mu,\phi^*)/d\phi^* < 0$, which is given by

$$\frac{dT_1(\mu,\phi^*)}{d\phi^*} = \frac{1+\tau^{1-\sigma}}{\mu\tau^{1-\sigma}}\left[(1+\mu)\frac{dIM_\omega}{d\phi^*} - IM_\omega\frac{1}{\mu}\frac{d\mu}{d\phi^*}\right]. \tag{3.104}$$

Hence, $d\mu/d\phi^* < 0$ from above, yields $dIM_\omega/d\phi^* < 0$. These formal results establish Proposition 4, which completes the proof.

Proof of Proposition 5

In the following proof, detailed derivation steps for the case $\phi > \phi^*$, which leads to $\mu > 1$, are discussed. The analysis of the social welfare function for $\phi < \phi^*$ is achieved in analogy.

To obtain the welfare effects in the open economy for $\phi > \phi^*$, we compute the total derivative of the social welfare function in Eq. (3.42) with respect to trade cost parameter

$\tau^{1-\sigma}$ and get

$$\frac{dV(P_n, \overline{e}, \hat{\psi})}{d\tau^{1-\sigma}} = \frac{\partial V(\cdot)}{\partial A(\mu)} \frac{dA(\mu)}{d\tau^{1-\sigma}}, \tag{3.105}$$

where the second term $dA(\mu)/d\tau^{1-\sigma} > 0$ is know from above. The first term reads

$$\frac{\partial V(\cdot)}{\partial A(\mu)} = \frac{1}{\gamma - \varepsilon} \left(\frac{A(\mu)}{\beta}\right)^{\frac{\varepsilon}{\gamma-\varepsilon}} \left\{ \left[\beta \Lambda^{-\varepsilon} \left(\frac{P_n}{P_\ell}\right)^{\gamma-\varepsilon} \right]^{-1} \hat{\psi} - 1 \right\}, \tag{3.106}$$

where $\tilde{\alpha} \equiv \beta^{\frac{1}{\gamma-\varepsilon}} P_n$ as introduced in the main text and $P_\ell = [A(\mu)/\beta]^{\frac{1}{\varepsilon-\gamma}} P_n$, combining Eqs. (3.31) and (3.33), were used. The sign of the bracket term in Eq. (3.106) is decisive for the sign of $\partial V(\cdot)/\partial A(\mu)$. Noting from conditions (3.14), (3.15) and (3.18) that $\underline{\lambda}^\varepsilon > \beta \left(P_n/P_\ell\right)^{\gamma-\varepsilon}$ must hold to make the demand system a solution to a proper utility maximization problem, we can safely conclude that $\hat{\psi} > (\underline{\lambda}/\Lambda)^\varepsilon$ is sufficient for $\partial V(\cdot)/\partial A(\mu) > 0$. Using the definition of $\hat{\psi}$, the respective condition reduces to $\int_{\underline{\lambda}}^{\overline{\lambda}} \lambda^\varepsilon dL(\lambda) > \underline{\lambda}^\varepsilon$, which is always fulfilled. Thus, from Eq. (3.105) we can safely conclude that $dV(\cdot)/d\tau^{1-\sigma} > 0$, which reveals that gains from trade exist, social welfare monotonically increases with trade liberalization and that the welfare result is independent of the trade pattern. This completes the proof.

Chapter 4

How Preferences Shape the Welfare and Employment Effects of Trade

4.1 Introduction

The question of how labor market imperfection shapes the welfare and employment effects of trade has played a prominent role in economic research since Brecher's (1974) seminal work on the role of minimum wages in a Heckscher-Ohlin model. Due to strong public discontent about the negative consequences of globalization for domestic workers, this question has gained momentum over the last 15 years. Building on various forms of labor market imperfection, recent theoretical work has been successful in identifying new, so far unexplored channels through which trade can affect economy-wide unemployment and the distribution of income with important consequences for the expected welfare effects. In this paper, we take a different perspective and show that the effects of trade not only depend on the form of labor market imperfection but also on the type of consumer preferences. That preferences matter and can give a demand-side explanation for international trade flows is well-known from Krugman's (1979; 1980) foundation of a new trade theory. However, the role of preferences for the existence of gains or losses from trade in the presence of labor market distortion has so far not been in the focus of economic research.

To fill this gap, we set up a prototype model of trade featuring a home-market effect, with two countries, two sectors of production, and labor as the only input factor. Similar to Helpman and Krugman (1985), we assume that one sector produces differentiated goods under monopolistic competition, which are subject to trade costs in the open economy. The other sector produces a homogeneous good under perfect competition that can be shipped to the foreign country at zero costs. Our home-market model differs from previous ones in two important respects. On the one hand, we consider price-independent generalized-linear (so-called PIGL) preferences, which have been put forward by Muellbauer (1975, 1976) and refer to the most general class of preferences admitting a representative consumer and thereby avoiding complications from aggregating consumer demand over heterogeneous households. The subclass of parametric PIGL preferences considered here has the advantage of delivering an explicit solution for the direct utility function (see Boppart, 2014), which is particularly useful to avoid an otherwise potentially complicated integrability problem.[1] Whereas the preferences do not have Gorman form in general, they cover homothetic and quasilinear preferences – and thus two widely used examples of Gorman-form preferences – as limiting cases (see Egger and Habermeyer, 2019). On the other hand, we consider search frictions in the sector of differentiated goods and assume that wages in this sector are set by bargaining between the firm and a continuum of workers (cf. Stole and Zwiebel, 1996; and the correction in Bruegemann et al., 2018). The assumption that the labor markets differ in the two sectors implies that higher wages bring along a higher risk of unemployment. This feature of our model is akin to the distinction of good and bad jobs in Acemoglu (2001) and gives the preferences a particular role, since they shape the risk attitude of households and thus the wage compensation demanded by them to accept the possibility of an unfavorable outcome of unemployment when seeking employment in the production of differentiated goods.

We use our framework to study the role of preferences for the effects of trade on unemployment and welfare. As a result of the labor market distortion, wages are higher in the sector of differentiated than in the sector of homogeneous goods. The probability to find employment in the sector of differentiated goods is directly linked to the wage premium paid in this industry by an indifference condition that makes applying for jobs in the two sectors equally attractive for workers prior to the revelation of who is successfully matched with a firm. The exact link between the wage premium and the employment probability established by this indifference condition depends on household preferences. If households have quasilinear preferences, they are risk-neutral and hence for a given wage premium the employment probability in the sector of differentiated goods can be fairly small. Things are different if households have homothetic (log-transformed Cobb-Douglas) preferences which make them risk-averse. In the case of risk-averse households the employment probability must be fairly high for a given wage premium in order to make applying for jobs in the sector of differentiated goods attractive for them. Differentiating quasilinear and homothetic preferences by the risk attitudes of households is not an ad hoc assumption but follows from looking at two limiting cases of the class of parametric PIGL preferences put forward by our analysis. Due to differences in the risk attitudes, a given change in the fraction of workers applying for jobs in the sector of differentiated goods can have quite different effects on nominal income for the two types of preferences. With the employment probability unchanged, a higher fraction of workers seeking employment in the sector of differentiated goods will reduce income if preferences are quasilinear, whereas it increases income under homothetic preferences, provided that the unemployment compensation for those who do not find a job is not too generous.[2]

[1] It is well understood from Samuelson (1950) and Hurwicz and Uzawa (1971) that associating consumer demand derived from indirect utility with the solution of a maximization problem of rational households requires integrability of demand functions. Hurwicz and Uzawa (1971) have worked out sufficient conditions to solve the integrability problem, relying on properties of the Slutsky matrix. In the context of parametric PIGL preferences, Boppart (2014) has shown that these conditions are fulfilled for homogeneous goods, whereas a proof for a continuum of differentiated goods is so far missing. Here, we circumvent the problem by focussing on a subclass of PIGL preferences, which delivers an explicit solution for the direct utility function.

[2] Focussing on the two limiting cases of our parametric PIGL preferences in the main part of our analysis

With this fundamental insight at hand, we then turn to the open economy and consider trade between two countries that are fully symmetric except for their population sizes. In line with the literature on home-market effects, we show that the sector of differentiated goods expands in the larger country and contracts in the smaller country, with the opposite being true for the sector producing the homogeneous good. As a consequence, the larger country will net-export the differentiated good and net-import the homogeneous good in the open economy. With a larger fraction of workers seeking employment in the sector of differentiated goods, the larger country experiences an increase in economy-wide unemployment. This is, because the risk of unemployment for an individual worker seeking employment in the sector of differentiated goods is the same in the closed and the open economy, whereas the fraction of workers prone to this risk has increased in the larger country when trade induces specialization and thus a change in the production pattern. However, the increase in unemployment does not necessarily imply a welfare loss. We can distinguish three effects: First, households in both economies benefit from lower import prices (which in the case of a movement from the closed to the open economy fall from infinity to a finite positive value). Second, provided that an increase in the fraction of workers seeking employment in the sector producing differentiated goods is associated with an increase in nominal income, trade generates an income gain in the larger and an income loss in the smaller country. This is the case if preferences are homothetic and unemployment compensation is not too generous, whereas the opposite is true if preferences are quasilinear. Third, welfare is influenced by a variety effect, which can be decomposed into two partial effects, namely an increase in the fraction of firms producing differentiated goods in the larger country and an increase or decrease in the global mass of firms producing differentiated goods. The combined variety effect is positively linked to the effect of trade on nominal income and can therefore also be positive or negative for either economy.

Taking stock, our model produces the well-known result that lower trade costs exhibit a direct positive welfare effect in both countries by lowering the costs of imports. In contrast, the income and variety effects differ in the two economies and can only be positive for one of them. If preferences are quasilinear, the income and variety effects are to the detriment of the larger economy and it is possible that these negative effects dominate the gains associated with a fall in the costs of imports so that the larger country loses from trade. In this case, the larger country experiences double losses, because, as outlined above, its economy-wide rate of employment decreases as well. Things are different in the smaller country, which due to its specialization on the production of the homogeneous good will experience double gains from trade if preferences are quasilinear. However, if preferences are homothetic and unemployment compensation is not too generous, welfare gains are guaranteed in the larger country, despite an increase in the economy-wide rate of unemployment. At the same time, the smaller country can be worse off in the open economy, despite a decrease in the economy-wide rate of unemployment. This points to an important role of preferences (and more specifically the risk attitudes implied by these

is attractive to separate the role of risk attitudes for the link between production structure and the level of income from additional effects due to changes in the second moment of income distribution, which becomes relevant for the structure of consumer demand and welfare if preferences do not have Gorman form.

preferences) for determining the welfare effects of trade in settings featuring labor market imperfection.

We consider two extensions of our model. In a first one, we analyze the case of non-Gorman preferences, implying that the distribution of income matters for consumer demand and welfare. With differentiated goods being luxuries and the homogeneous good being a necessity from the households' point of view, a larger income dispersion increases demand for differentiated goods and lowers demand for the homogeneous good. Of course, if preferences do not have Gorman form, the representative consumer in our model does not have a normative interpretation, so that the choice of a proper welfare function is a priori not clear. Choosing a utilitarian perspective, we show that welfare exhibits social inequality aversion, implying that an increase in income dispersion lowers social welfare. This effect is counteracted, however, by new entry of firms in the now larger market for differentiated goods, which increases welfare due to the households' love of variety. In the open economy, changes in the dispersion of income imply that a higher level of income is no longer sufficient for gains from trade to materialize in the larger country. This confirms our insight from the benchmark model that the form of preferences is crucial for the welfare consequences of trade.

In a second extension, we account for differences of countries in their per-capita income levels. We generate a priori differences in per-capita income by considering differences of the two countries in the labor endowments of households – while abstracting from differences in the total effective labor supply of the two economies. With this modification at hand, we show that trade does not change the labor allocation in the two economies if preferences are homothetic, leaving the economy-wide rate of unemployment at its autarky level and establishing gains from trade due to a fall in the costs of imported goods. Things are different if preferences are quasilinear. In this case, the richer country net-exports the differentiated good and net-imports the homogeneous good. This leads to an increase in economy-wide unemployment and can lead to an overall welfare loss, because the negative income and variety effects counteract the welfare stimulus from lower costs of imports.

Assessing the effects of trade in a setting that features search frictions in the sector producing differentiated goods, our model contributes to a sizable literature dealing with labor market distortions in open economies. Starting with Brecher (1974), this literature has aimed at improving our understanding about the role of labor market institutions as a determinant of international trade flows and as an important factor influencing the effects of trade on employment and welfare (cf. Davidson et al., 1988; Davis, 1998a; Kreickemeier and Nelson, 2006). Whereas the focus in recent years has shifted towards models featuring heterogeneous firms and only a single sector of production (cf. Egger and Kreickemeier, 2009, 2012; Helpman et al., 2010; Amiti and Davis, 2012), advancements have also been made in trade models with multiple sectors and differences of these sectors in their labor market institutions (cf. Bastos and Kreickemeier, 2009; Egger et al., 2015). Most closely related to our model in this respect is Helpman and Itskhoki (2010) who consider, as we do, a two-sector trade model featuring a home-market effect. However, similar to other existing work, they do not look at the role of preferences for the employment and welfare effects of trade.

Pointing to potential welfare loss from trade, the analysis in this paper adds to an old and well established debate about the conditions, under which such losses can materialize (see Graham, 1923, for an early example and Helpman, 1984, for a thorough literature review). In multi-sector models disadvantageous specialization in the open economy is usually put forward as a key explanation of why trade can be to the detriment of an economy. Whereas the results from our model are well in line with this argument, we deviate from the widespread view that disadvantageous specialization requires external economies of scale in at least one industry. Excluding external economies of scale, we show that losses from trade can also be the result of a labor market distortion and may exist even if a country expands the sector offering 'good jobs' (in the terminology of Acemoglu, 2001). Provided that specialization in the open economy leads to an expansion of a sector prone to unemployment, increasing the number of good jobs can come at the cost of a higher fraction of workers not finding a job at all. This can generate welfare loss, with preferences playing a crucial role for such disadvantageous specialization to materialize in our model.

Postulating that households have PIGL preferences, this paper also contributes to a strand of literature, which points out that important new insights on the motives for trade, its structure, and consequences can be obtained when deviating from the widespread assumption of homothetic utility. Building on the insight of Linder (1961) that demand-side factors are important determinants of international trade flows, Krugman (1979, 1980), Markusen (1986), and Flam and Helpman (1987) have provided first theoretical accounts of the role of preferences for international trade flows. The main insight from this early research is that a substantial fraction of trade remains unexplained when only considering supply-side motives for its existence (see Markusen, 2013). Matsuyama (2000), Fajgelbaum et al. (2011), and Foellmi et al. (2018) have further contributed to the analysis by distinguishing high- and low-quality goods and by adding a discrete choice element to allow for an aggregation of consumer demand over heterogeneous households even if preferences do not have Gorman form.[3] Fieler (2011) and Caron et al. (2014) consider generalized CES preferences, whereas Bertoletti and Etro (2017) and Matsuyama (2015, 2018) consider a class of preferences that establish a "generalized separable" demand system (see Pollak, 1972). These preferences have the particular advantage to allow for aggregation of demand over various industries with differing price elasticities and are therefore well equipped for studying quantitative general equilibrium trade models. Lacking a representative consumer, the preferences are, however, less suited for aggregating consumer demand over households with differing income levels.

The remainder of the paper is organized as follows. In Section 4.2, we discuss the building blocks of our model and in Section 4.3, we analyze the main mechanisms in the closed economy. In Section 4.4, we investigate trade between two countries that differ in their population size and study the effects of trade on production structure, economy-wide employment, and welfare. In Section 4.5, we consider non-Gorman preferences and

[3] Tarasov (2012) considers a model with '0-1' preferences over a continuum of goods to study how price changes in the process of globalization affect welfare of different income groups. He shows that welfare consequences of price adjustments exert asymmetric effects if, due to nonhomothetic preferences, income groups differ in their expenditure shares.

investigate the effects of trade in rich and poor countries. Section 4.6 concludes with a summary of our results.

4.2 The model: basics

4.2.1 Endowment and preferences

We consider a static economy that is populated by a continuum of households with mass H, which in their role as workers inelastically supply $\lambda > 1$ units of labor input for the production of goods. We can interpret λ as worker productivity which is the same for all households. Households have price-independent generalized-linear (so-called PIGL) preferences over two goods, which are represented by a direct utility function of the form

$$\mathcal{U}(X_i, Y_i) = \frac{1}{\varepsilon}(X_i)^{\varepsilon}\left[\left(\frac{Y_i}{\beta}\right)^{\frac{\varepsilon}{1-\varepsilon}} - \beta\right]\left[\left(\frac{Y_i}{\beta}\right)^{\frac{1}{1-\varepsilon}} - Y_i\right]^{-\varepsilon} - \frac{1-\beta}{\varepsilon}, \tag{4.1}$$

where $\varepsilon, \beta \in (0, 1)$ are two constants, Y_i is a homogeneous good, and X_i is a CES aggregate over a continuum of differentiated goods:

$$X_i = \left[\int_{\omega\in\Omega} x_i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}, \tag{4.2}$$

with $x_i(\omega)$ being the consumption level of variety ω and $\sigma > 1$ being the constant elasticity of substitution between the varieties from set Ω. The utility function in Eq. (4.1) is well-defined only if $X_i > 0$. As pointed out by Muellbauer (1975, 1976), PIGL preferences are the most general class of preferences that deliver a representative consumer and therefore avoid an aggregation problem over households with differing levels of income. Whereas PIGL preferences are usually represented by an indirect utility function, Boppart (2014) shows that for a subclass of these preferences an explicit solution for the direct utility function exists. Egger and Habermeyer (2019) discuss the parameter assumptions needed to arrive at the utility function in Eq. (4.1) and explain that this utility function has the particularly nice feature of covering homothetic (log-transformed Cobb-Douglas) preferences and quasilinear preferences by the limiting cases of $\varepsilon \to 0$ and $\varepsilon \to 1$, respectively.

Solving the standard protocol of utility maximization delivers individual demand functions

$$Y_i = \beta\left(\frac{e_i}{P_Y}\right)^{1-\varepsilon} \quad \text{and} \quad x_i(\omega) = \frac{e_i}{P_X}\left(\frac{p(\omega)}{P_X}\right)^{-\sigma}\left[1 - \beta\left(\frac{e_i}{P_Y}\right)^{-\varepsilon}\right], \tag{4.3}$$

respectively, where e_i is the expenditure level of individual i, P_Y is the price of the homogeneous good, $p(\omega)$ is the price of variety ω of the differentiated good, and $P_X \equiv [\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega]^{\frac{1}{1-\sigma}}$ is a CES index over the prices of all these varieties. From Eq. (4.3), we see that the Engel curve of homogeneous good Y_i is concave, making this good a *necessity* with its value share of consumption decreasing in the expenditure level. In contrast, the Engel curves of differentiated goods $x_i(\omega)$ are convex making these goods *luxuries.*

Aggregating over households, gives market demand functions

$$Y = \int_{i\in\mathcal{H}} Y_i di = \beta \frac{H\bar{e}}{P_Y}\left(\frac{\bar{e}}{P_Y}\right)^{-\varepsilon}\psi, \tag{4.4}$$

$$x(\omega) = \int_{i\in\mathcal{H}} x_i(\omega) di = \frac{H\bar{e}}{P_X}\left(\frac{p(\omega)}{P_X}\right)^{-\sigma}\left[1-\beta\left(\frac{\bar{e}}{P_Y}\right)^{-\varepsilon}\psi\right], \tag{4.5}$$

where $\bar{e} \equiv H^{-1}\int_{i\in\mathcal{H}} e_i di$ is the average expenditure level of households and $\psi \equiv H^{-1}\int_{i\in\mathcal{H}} (e_i/\bar{e})^{1-\varepsilon} di$ is a dispersion index that is defined on the unit interval and captures how the distribution of household expenditures affects the value shares of consumption. The dispersion index reaches a maximum level of one if the distribution of expenditures is egalitarian or if the distribution of household expenditure is irrelevant for aggregate demand because Engel curves are linear, which happens in the two limiting cases of homothetic and quasilinear preferences.

4.2.2 Technology and the firms' problem

Firms in the sector of the homogeneous good enter the market at zero cost and hire workers at a common wage rate w per unit of labor input. Workers need one unit of their labor input to produce one unit of the homogeneous good, which is sold under perfect competition. This establishes $w = P_Y$. Firms producing differentiated goods have to develop a blueprint, which comes at the cost of f units of the homogeneous good and gives them access to a unique variety that can be sold under monopolistic competition. To produce their output firms hire workers, who manufacture one unit of the differentiated good with each unit of their labor input. Hiring and wage setting in the sector of differentiated goods is a two-stage problem. At stage one, firms install vacancies at the cost of one unit of the homogeneous good and search for workers filling these vacancies. There are search frictions and the assignment of workers to jobs is solved through random matching (cf. Pissarides, 2000; Helpman and Itskhoki, 2010; Felbermayr and Prat, 2011). For those vacancies successfully filled, firms and workers form a bilateral monopoly at stage two and distribute the production surplus generated in the workplace through Stole and Zwiebel (1996) bargaining.[4] We solve the firm's hiring and wage setting problem through backward induction and begin with stage two.

The bargaining problem at stage two is reminiscent of the multilateral problem in Helpman and Itskhoki (2010), with the difference that we allow for asymmetric bargaining power of workers and firms. The asymmetric bargaining protocol is already discussed by Stole and Zwiebel (1996) and it has been applied to a model similar as ours by Egger and Habermeyer (2019). Our problem is simpler though, because we assume that all workers employed by a firm provide the same level of labor input λ. Following Stole and Zwiebel (1996), we can characterize the solution of the bargaining problem by a splitting rule, which determines how the production surplus achieved by an agreement is distributed between the bargaining parties; and an aggregation rule, describing how infra-marginal

[4]Bruegemann et al. (2018) show that, in contrast to common belief, the Stole and Zwiebel (1996) bargaining protocol does not give wage and profit profiles that coincide with the Shapley values. They suggest using a Rolodex Game instead of the non-cooperative game put forward by Stole and Zwiebel to achieve equivalence of the bargaining outcome with the Shapley values.

production surpluses add up to the firm's total surplus from multilateral bargaining with all of its workers. Bargaining with a mass $l(\omega)$ of workers, firm ω's total bargaining surplus is given by

$$\pi(\omega) = \int_0^{l(\omega)} \kappa[\ell|l(\omega)]\hat{r}(\ell)d\ell, \tag{4.6}$$

where $\hat{r}(\ell) = D^{\frac{1}{\sigma}}(\lambda\ell)^{1-\frac{1}{\sigma}}$ are revenues achieved with employment level ℓ, D is a common demand shifter, and

$$\kappa[\ell|l(\omega)] \equiv \frac{1}{\alpha\ell}\left(\frac{\ell}{l(\omega)}\right)^{\frac{1}{\alpha}} \tag{4.7}$$

is a probability measure that determines the fraction of infra-marginal production surplus the firm can acquire in its wage negotiation with workers. This probability measure declines in the workers' *relative* bargaining power $\alpha > 0$. Solving the integral in Eq. (4.6) gives

$$\pi(\omega) = \frac{\sigma}{\sigma+\alpha(\sigma-1)}D^{\frac{1}{\sigma}}\left[\lambda l(\omega)\right]^{1-\frac{1}{\sigma}} = \frac{\sigma}{\sigma+\alpha(\sigma-1)}r(\omega), \tag{4.8}$$

where the second equality sign uses the definition $r(\omega) \equiv \hat{r}[l(\omega)]$.

If an agreement in the wage negotiation between the firm and a worker is not achieved, the worker becomes unemployed and receives an unemployment compensation of $\gamma\lambda w$, where $\gamma \in (0,1)$ is a common replacement rate. Higher unemployment compensation improves the disagreement income of workers in their wage negotiations and thus the rent accrued by workers in the bargaining with the firm. The influence of unemployment compensation on wages is reflected in the splitting rule determining how to distribute the production surplus between the firm and its workers. This splitting rule is given by

$$\frac{\partial\pi(\omega)}{\partial l(\omega)} = \lambda\frac{w_\kappa(\omega)-\gamma w}{\alpha}, \tag{4.9}$$

where $w_\kappa(\omega)$ is the wage rate for each unit of labor input paid by firm ω. Eqs. (4.8) and (4.9) jointly determine the solution for the firm's bargaining problem at stage two. Thereby, firms accrue a constant fraction $\rho \equiv \sigma/[\sigma+\alpha(\sigma-1)] < 1$ of revenues in the wage bargaining with workers, which is decreasing in the relative bargaining power of workers, α.

Equipped with the solution for the bargaining problem, we can now determine the outcome of the firm's hiring problem. Recollecting from above that firms have to invest f units of the homogeneous good to start production and one unit of the homogeneous good for each vacancy installed, this solution is found by maximizing profits $\Pi(\omega) \equiv \rho r(\omega) - q^{-1}P_Y l(\omega) - P_Y f$ with respect to $l(\omega)$, where $q < 1$ is the probability that a vacancy can be filled, which in the case of random matching is exogenous to the individual firm and the same for all producers. The first-order condition for the firm's profit-maximizing choice of $l(\omega)$ is given by

$$\frac{d\Pi(\omega)}{dl(\omega)} = \frac{\sigma-1}{\sigma}\frac{\rho r(\omega)}{l(\omega)} - \frac{P_Y}{q} = 0. \tag{4.10}$$

Accounting for Eqs. (4.8) and (4.9) then gives the outcome of hiring and wage-setting for firms producing differentiated goods:

$$w_\kappa(\omega) = \frac{\alpha+\gamma\lambda q}{\lambda q}P_Y, \qquad \Pi(\omega) = \frac{\rho r(\omega)}{\sigma} - P_Y f. \tag{4.11}$$

Since all firms producing differentiated goods employ the same technology and pay the same wage, they are symmetric producers. This allows us to drop firm index ω from now on.

4.2.3 Industry-wide outcome in the sector of differentiated goods

Eq. (4.11) has been derived under the assumption that firms producing differentiated goods can attract the intended mass of applicants at a wage rate w_κ. To see under which condition this is the case, we have to determine the labor market outcome in the sector of differentiated goods. For this purpose, we note that the supply of workers in the sector of differentiated goods is given by the product of the mass of households, H, and the fraction of these households seeking employment in the sector of differentiated goods, h. The ratio between the mass of workers seeking employment, hH, and the total mass of vacancies installed, Q, is pinned down by a Cobb-Douglas matching function and given by $hH/Q = m(1-u)^{-1}$, where m is a positive constant that measures matching efficiency, and $1-u$ is the share of workers successfully matched to a firm and thus the employment rate in the sector of differentiated goods. In the Appendix, we provide a microfoundation of this outcome and show that the matching technology considered here can be interpreted as a special case of the matching technology in Helpman and Itskhoki (2010). The probability of filling a vacancy is given by $q = hH(1-u)/Q = m$ and thus independent of the employment rate in our model. Setting $m = \lambda^{-1}$ proves particularly useful for our purposes, because it allows us to get rid of uninteresting constants. This additional simplification generates a negative relationship between matching efficiency and labor productivity, which can be justified by assuming that workers with higher and more specialized abilities are more difficult to place in the labor market.[5]

With this matching technology at hand, we can solve for the employment rate in the sector of differentiated goods, using the indifference condition for production workers, who can either enter the sector of the homogeneous good, which promises an income of w per unit of labor input, or enter the sector of differentiated goods, which promises for each unit of labor input an income $w_\kappa = (\alpha+\gamma)w$ with probability $1-u$ and an unemployment compensation of γw with probability u. Assuming that unemployment compensation is financed by a proportional tax on all types of income, including the transfer payment to the unemployed (see Egger and Kreickemeier, 2012), taxation does not influence the

[5]As briefly discussed in the Appendix, the results from our analysis extend to more general matching technologies, with further derivation details available from the authors upon request.

sector, workers choose for offering their labor input. Considering the utility function in Eq. (4.1) and individual demand functions in Eq. (4.3), we can solve the indifference condition of workers for

$$1-u=\frac{1-\gamma^{\varepsilon}}{(\alpha+\gamma)^{\varepsilon}-\gamma^{\varepsilon}}, \tag{4.12}$$

where $w_{\kappa}=(\alpha+\gamma)w$ and $q=m=\lambda^{-1}$ have been used. Eq. (4.12) reveals that an interior solution with $0<u<1$ requires $\alpha>1-\gamma$, and hence that the sector of differentiated goods offers a wage premium $\tilde{\alpha}\equiv\alpha+\gamma>1$. Provided that such an outcome exists, a higher relative bargaining power of workers, α, increases the wage premium, and therefore the employment rate has to fall in order to restore indifference of workers to enter the two sectors. We can complete the characterization of the industry equilibrium by noting that free entry of firms into the sector of differentiated goods establishes the zero-profit condition $\rho r=\sigma P_Y f$.

4.2.4 Production structure and disposable labor income

We complete the discussion of the main building blocks of our model by elaborating on how changes in the production structure affect the average level and dispersion of *disposable* labor income with a particular focus on the role of preferences for this outcome. Due to our assumption that all types of income are subject to the same income tax, average disposable labor income and thus the average household consumption expenditure is given by

$$\bar{e}=w\lambda\left\{1+h[(1-u)\tilde{\alpha}-1]\right\}. \tag{4.13}$$

Eq. (4.13) points to a trade-off an increase in the fraction of workers producing differentiated goods has on average disposable income. On the one hand, a higher h leads to an increase in the fraction of workers receiving the wage premium offered by luxury producers. On the other hand, it increases the economy-wide rate of unemployment, $U\equiv uh$, and thus the share of labor input not productively used in the economy. In general, $(1-u)\tilde{\alpha}>,=,<1$ is possible, so that allocating more workers to the sector of differentiated goods can have a positive or negative effect on average disposable household income, depending on whether the first or the second effect dominates. Since the effect that changes in the fraction of workers seeking employment in the production of differentiated goods have on average disposable labor income is essential for the welfare effects of trade, it is useful to shed light on the role of preferences for the ranking of $(1-u)\tilde{\alpha}>,=,<1$. The following lemma summarizes this role.

Lemma 1 *If preferences are quasilinear (and thus $\varepsilon=1$), we have $(1-u)\tilde{\alpha}<1$. In all other cases $(1-u)\tilde{\alpha}>,=,<1$ is possible, with $(1-u)\tilde{\alpha}>1$ achieved for sufficiently high levels of α. In the limiting case of homothetic preferences (and thus $\varepsilon=0$), $(1-u)\tilde{\alpha}>1$ extends to all possible $\alpha>1-\gamma$ if $\gamma<\exp[-1]$.*

Proof. Formal proof in the Appendix ■

Whereas production of differentiated goods promises a wage premium if households are successfully matched with firms, applying for jobs in the sector producing differentiated goods comes at the risk of being not successfully matched and experiencing an income loss. The households' risk attitudes and hence the evaluation of the risk of job loss depend on their preferences (or, more specifically, on preference parameter ε). If $\varepsilon = 1$ preferences are quasilinear and households are risk-neutral. In this case, the constraint in Eq. (4.12), which makes workers indifferent between the two sectors, reduces to a condition equalizing the expected disposable income from job search in the two sectors: $(1-u)\tilde{\alpha}w + u\gamma w = w$. Because of their risk neutrality, households accept a relatively low probability of a successful match and thus a relatively high rate of unemployment, when seeking employment in the sector of differentiated goods, leading to $(1-u)\tilde{\alpha} < 1$. Things are different if households are risk-averse due to $\varepsilon < 1$, with the degree of risk aversion maximized in our model if $\varepsilon = 0$ makes preferences homothetic. In this case, households applying for jobs in the sector producing differentiated goods must be compensated for accepting the risk of unemployment. With the wage premium $\tilde{\alpha} > 1$ fixed, risk aversion leads to a fall in the unemployment rate, thereby increasing $(1-u)\tilde{\alpha}$. In the case of homothetic preferences $(1-u)\tilde{\alpha} > 1$ is achieved for all $\tilde{\alpha} > 1$ and thus for all $\alpha > 1-\gamma$, if unemployment compensation is not too generous, i.e if $\gamma < \exp[-1]$.[6] This is the parameter domain we focus on in the subsequent analysis in order to emphasize the important role played by the degree of risk aversion when contrasting the two limiting cases of quasilinear and homothetic preferences.

With the insights regarding the relationship of production structure and average disposable household income (expenditures) at hand, we now turn to the dispersion index of disposable household income, which can be computed according to

$$\psi = \left[\frac{(1-\tau)w\lambda}{\bar{e}}\right]^{1-\varepsilon} \left\{1 + h\left[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1\right]\right\}$$

where $\tau \in (0,1)$ is the common income tax rate that is determined by the condition of a balanced budget of the government:

$$\tau \equiv \frac{hu\gamma}{1 + h\left[(1-u)\tilde{\alpha} + u\gamma - 1\right]}. \tag{4.14}$$

Tax rate τ increases in the fraction of workers seeking employment in the production of differentiated goods, h. This is because a higher h is associated with higher economy-wide unemployment, U, implying that the now fewer employed production workers have to finance the compensation for a larger mass of unemployed. Substituting tax rate τ and $\bar{e}$ into ψ establishes

$$\psi = \frac{1 + h\left[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1\right]}{\left\{1 + h\left[(1-u)\tilde{\alpha} + u\gamma - 1\right]\right\}^{1-\varepsilon}}, \tag{4.15}$$

[6]For a given wage premium $\tilde{\alpha}$, a higher replacement rate γ increases household income in the event of unemployment, and hence unemployment rate u has to increase in order to restore indifference condition (4.12). This provides an intuition for an upper limit of γ needed to ensure $(1-u)\tilde{\alpha} > 1$ for all possible levels of $\tilde{\alpha}$ if preferences are homothetic.

where $\psi = 1$ holds in the case of Gorman form preferences, which are associated with the limiting cases of $\varepsilon \to 0$ and $\varepsilon \to 1$. This points to the important result that higher degrees of risk aversion do not exert a monotonic effect on dispersion index ψ. This is, because the dispersion index does not capture the second moment of income distribution but the impact of income distribution on the structure of consumer demand. With quasilinear or homothetic preferences, aggregate consumer demand does not depend on the distribution of disposable household income – provided that even the households with the lowest income consume both goods. To ensure that this is the case, condition $(1-\tau)\gamma\lambda > \beta^{1/\varepsilon}$ must be fulfilled. This condition depends on the endogenous, yet to be determined, fraction of workers seeking employment in the production of differentiated goods, h, which is different for the closed and the open economy.[7]

4.3 The closed economy

To determine the fraction of workers seeking employment in the sector of differentiated goods, h, we can make use of two important insights from our analysis. The first one is that combining Eqs. (4.8) and (4.9), we can compute $[(\sigma-1)/\sigma]\rho r(\omega)/l(\omega) = \lambda[w_\kappa(\omega) - \gamma w]/\alpha$ and can thus express the wage bill of firms as $\lambda l w_\kappa = \lambda l \gamma w + \rho r \alpha(\sigma-1)/\sigma$. This captures the outcome of wage bargaining (plus constant markup pricing) and, noting that M firms enter and $hH(1-u)$ workers find a job, allows us to determine a positive link between the share of workers seeking employment in the sector of differentiated goods and the mass of firms producing them according to

$$hH\lambda w(1-u) = \frac{\sigma-1}{\sigma}\rho M r, \tag{4.16}$$

where $w_\kappa = (\alpha+\gamma)w$ has been considered. A second relationship between h and M follows from the market clearing condition for differentiated goods, can be derived from Eq. (4.5), and is given by

$$H\lambda w\left(1-\beta\lambda^{-\varepsilon}\right) + H\lambda w B(h) = Mr, \tag{4.17}$$

where $B(h) \equiv h\big[(1-u)\tilde{\alpha}-1\big] + \beta\lambda^{-\varepsilon}\big[1-T(h)\big]$ is derived in the Appendix and captures the additional effect on consumer demand from the labor market distortion and the tax-transfer scheme implemented to compensate the unemployed. Rent-sharing increases *market* income of an endogenous fraction of $h(1-u)$ workers, who find employment in the sector of differentiated goods and therefore benefit from a wage premium $\tilde{\alpha} > 1$. This gives term $h\big[(1-u)\tilde{\alpha}-1\big]$ as a first component of $B(h)$. The second component captures the demand effect through endogenous changes in the dispersion of disposable household income, because workers seeking employment in the sector of differentiated goods can experience an income increase or decrease, depending on their employment status, and because the tax-transfer system makes disposable income more egalitarian. The combined

[7] To derive a sufficient parameter constraint for this condition to hold, we can note from above that τ reaches a maximum at $h = 1$, which we denote by $\overline{\tau}$. Making use of Eqs. (4.12) and (4.14) we compute $\overline{\tau} \equiv \{1+\tilde{\alpha}(1-\gamma^{\varepsilon})/[\gamma(\tilde{\alpha}^{\epsilon}-1)]\}^{-1} < 1$, and hence $(1-\overline{\tau})\gamma\lambda > \beta^{1/\varepsilon}$ gives a sufficient condition in exogenous model parameters for the intended result that all households purchase both types of goods.

dispersion effect is captured by $\beta\lambda^{-\varepsilon}[1-T(h)]$, with

$$T(h) \equiv \left\{1 + h\left[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1\right]\right\}\left(\frac{1+h[(1-u)\tilde{\alpha}-1]}{1+h[(1-u)\tilde{\alpha}+u\gamma-1]}\right)^{1-\varepsilon}. \tag{4.18}$$

In the limiting case of homothetic preferences, we have $\lim_{\varepsilon\to 0} T(h) = 1+h[(1-u)\tilde{\alpha}-1] > 0$ and thus $\lim_{\varepsilon\to 0} B(h) = h[(1-u)\tilde{\alpha}-1](1-\beta)$, whereas in the limiting case of quasilinear preferences, we have $\lim_{\varepsilon\to 1} T(h) = 1$ and thus $\lim_{\varepsilon\to 1} B(h) = h[(1-u)\tilde{\alpha}-1]$. In both scenarios, $B(h)$ captures a pure efficiency effect due to changes in the level of average disposable household income, while changes in the dispersion of income do not exert an additional effect in the case of Gorman form preferences. However, this efficiency effect is not the same for homothetic and quasilinear preferences. As pointed out by Lemma 1, in the case of quasilinear preferences $(1-u)\tilde{\alpha} < 1$ holds for all possible parameter configurations, and hence the demand for differentiated goods is reduced by the labor market distortion, because the negative employment effect dominates the positive wage effect of those successfully matched to firms. In contrast, with homothetic preferences, $(1-u)\tilde{\alpha} > 1$ is achieved for all possible $\alpha > 1-\gamma$ if unemployment compensation is not too generous, establishing $T(h) > 1$. If preferences do not have Gorman form, demand for differentiated goods is furthermore influenced by the dispersion of disposable household income. Whereas this complicates the analysis considerably, the model remains nicely tractable for $\varepsilon = 1/2$. In this case, we have $T(h) < 1$ (using the indifference condition in Eq. (4.12)), so that the combined dispersion effect on demand for differentiated goods is positive. This suggests that the increase in the dispersion of market income dominates the decrease in the dispersion of disposable income due to the tax-transfer system and implies that demand for differentiated goods is further increased by the labor market distortion.

Combining Eqs. (4.16) and (4.17) allows us to solve for the equilibrium fraction of workers seeking employment in the sector of differentiated goods. The respective solution is given by the condition $\Gamma(h) = 0$, with

$$\Gamma(h) \equiv 1 - h\left[1 + \frac{\sigma}{\sigma-1}(1-u) - \gamma(1-u)\right] - \beta\lambda^{-\varepsilon}T(h). \tag{4.19}$$

We show in the Appendix that $\Gamma(h) = 0$ has a unique solution in h. Combining this solution with Eq. (4.16) and the zero-profit condition $\rho r = \sigma P_Y f$ determines the equilibrium mass of firms producing differentiated goods, M. For Gorman form preferences, we get explicit solutions for h and M. For the limiting case of homothetic preferences, we compute

$$h = \frac{\frac{1-\beta}{1-u}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1-\beta}{1-u}\left[(1-u)\tilde{\alpha}-1\right]}, \qquad M = \frac{\frac{(1-\beta)H\lambda}{(\sigma-1)f}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1-\beta}{1-u}\left[(1-u)\tilde{\alpha}-1\right]}, \tag{4.20}$$

whereas in the case of quasilinear preferences, we obtain

$$h = \frac{\frac{\lambda-\beta}{\lambda(1-u)}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1}{1-u}\left[(1-u)\tilde{\alpha}-1\right]}, \qquad M = \frac{\frac{(\lambda-\beta)H}{(\sigma-1)f}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1}{1-u}\left[(1-u)\tilde{\alpha}-1\right]}. \tag{4.21}$$

Higher levels of per-capita labor endowment λ make for a given allocation of workers all households richer and increase the expenditures for differentiated goods. In the case of homothetic preferences the expenditure shares of differentiated goods are independent of λ, so that the now higher demand for differentiated goods is offset by the now higher supply of labor producing them, leaving the fraction of workers seeking employment in the sector of differentiated goods, h, unaffected. Things are different in the case of quasilinear preferences. With expenditure shares for differentiated goods increasing in λ, more workers are needed in the sector of differentiated goods to fulfill the now higher consumer demand for these goods. As a consequence, h has to increase to restore market clearing. Irrespective of the preferences, more firms will enter the now larger market for differentiated goods.

A higher relative bargaining power of workers α can increase or decrease the fraction of workers seeking employment in the sector of differentiated goods. A higher α must lower employment rate $1-u$ to restore indifference of workers between the two sectors. All other things equal, a higher fraction of workers must therefore seek employment in the sector of differentiated goods to fulfill a given demand. This effect can be counteracted if an increase in average disposable household income, due to an increase in $(1-u)\tilde{\alpha}$, induces households to increase their demand for the homogeneous good, causing a reallocation of labor away from the sector of differentiated goods. This second effect needs not to work against the first one, because average disposable household income can fall in α and because with quasilinear preferences income changes leave demand for the homogeneous good unaffected. However, in general it is a priori not clear, which of the two effects dominates, so that $dh/d\alpha$ can be positive or negative. Whereas we cannot rule out positive effects of a stronger labor market distortion on the fraction of workers seeking employment in the sector of differentiated goods, the mass of firms producing them, M, unambiguously decreases in α in the two limiting cases captured by Eqs. (4.20) and (4.21). This is, because a higher wage premium increases the costs of production, and therefore makes entry less attractive for firms. Whereas an increase in average disposable household income would counteract this effect, it does not dominate because the respective demand stimulus is mitigated by an income loss of those workers becoming newly unemployed in the sector of differentiated goods.

Changes in the fraction of workers seeking employment in the sector of differentiated goods and changes in the mass of firms producing them are important determinants of welfare effects. In the two limiting cases of $\varepsilon \to 0$ and $\varepsilon \to 1$ preferences have Gorman form, giving the representative consumer a normative interpretation. This allows us to consider utility of the representative consumer as a proper welfare function, establishing[8]

$$V_{CD}\left(\bar{e}, P_Y, P_X\right) \equiv \ln\left(\frac{\bar{e}}{P_Y^{\beta} P_X^{1-\beta}}\right), \qquad V_{QL}\left(\bar{e}, P_Y, P_X\right) \equiv \frac{\bar{e}}{P_X} - \beta\frac{P_Y}{P_X} - 1 + \beta \tag{4.22}$$

in the case of homothetic (log-transformed Cobb-Douglas) and quasilinear preferences,

[8] The (price-invariant) representative level of expenditures is defined by Muellbauer (1975) as the expenditure level that gives the same expenditure shares for the homogeneous good and differentiated goods as observed for the whole economy. It is given by $e_r = \bar{e}\psi^{-\frac{1}{\varepsilon}}$, and the household with this income level is therefore called representative consumer. With Gorman form preferences, we have $e_r = \bar{e}$, and we can compute the welfare functions in Eq. (4.22) by determining indirect utility of the representative household, using Eqs. (4.1)-(4.3).

respectively. Substituting Eqs. (4.13), (4.20), and (4.21), and accounting for $P_X = \frac{\sigma}{\sigma-1}\frac{w}{\rho}M^{\frac{1}{1-\sigma}}$, we can express welfare as a function of λ and α. Intuitively, welfare increases with per-capita labor endowment λ for two reasons: On the one hand, an increase in per-capita labor endowment makes all households richer. In the case of quasilinear preferences this direct effect is counteracted by an indirect effect, because the reallocation of labor towards the production of differentiated goods leads to an aggregate income loss. However, differentiating $\overline{e}$, it is easily verified that the indirect effect cannot dominate. On the other hand, a higher λ induces more firms to enter the sector of differentiated goods, which leads to a fall in the CES price index P_X. Regarding the effect of a higher wage premium, we show in the Appendix that welfare unambiguously decreases in α if preferences are quasilinear. This is, because a stronger labor market distortion decreases average disposable household income, lowers the mass of firms producing differentiated goods, and increases the prices charged by the remaining firms. All three effects are detrimental for social welfare. If preferences are homothetic, average disposable household income can increase in α, thereby counteracting negative effects from a lower mass of firms and higher prices for differentiated goods. In this case, a stronger labor market distortion can be a stimulus for social welfare.

Whereas specifying a welfare function in the case of Gorman preferences is straightforward, choosing a proper welfare function is less obvious if preferences do not have Gorman form, because the representative consumer does not bear a normative interpretation in this case (see Muellbauer, 1975, 1976). One possibility put forward by Egger and Habermeyer (2019) is to take a utilitarian perspective and we follow this approach in Section 4.5, where we discuss how the results from our analysis change when $\varepsilon \in (0,1)$. This completes the discussion of the closed economy.

4.4 The open economy

In the open economy, we consider trade between two countries that are symmetric in all respects, except for their population size: $H \neq H^*$, where an asterisk is used to indicate foreign variables and to distinguish them from home variables. Trade in the homogeneous good is free of costs, and hence wage w is the same in the two economies, provided that production is diversified in either of the two economies. We discuss the parameter domain supporting diversification below. Trade in differentiated goods is subject to iceberg trade costs, implying that $t^{\frac{1}{\sigma-1}} > 1$ units of the good must be shipped in order for one unit to arrive in the foreign country.

4.4.1 Characterization of the open economy equilibrium

Under diversification, the open economy equilibrium can be characterized by combining the outcome of wage bargaining with the zero-profit conditions and goods market clearing for differentiated goods in the two economies. Following the steps of the closed economy, we find that wage bargaining (plus constant markup pricing) establishes a proportional link between the fraction of workers seeking employment in the sector of differentiated goods, h, and the mass of firms producing them, M. We obtain

$$hH\lambda w(1-u) = \frac{\sigma-1}{\sigma}\rho M r, \qquad h^*H^*\lambda w(1-u) = \frac{\sigma-1}{\sigma}\rho M^* r^* \tag{4.23}$$

for home and foreign, respectively. Contrasting Eqs. (4.16) and (4.23), we see that trade leaves the link between h and M established by wage bargaining unaffected. This result is intuitive because Eqs. (4.8) and (4.9) are the same in the closed and the open economy. Furthermore, firm-level revenues in home and foreign, r and r^*, respectively, are linked by the zero-profit conditions $\rho r = \sigma P_Y f$, $\rho r^* = \sigma P_Y f$. Accordingly, firm-level revenues are the same in the two economies, provided that production is diversified and that trade of the homogeneous good is costless. Market clearing in the sector of differentiated goods gives for home and foreign

$$\begin{aligned} H\lambda w\left(1-\beta\lambda^{-\varepsilon}\right) + H\lambda w B(h) &= \frac{Mrt}{1+t} + \frac{M^*r^*}{1+t}, \\ H^*\lambda w\left(1-\beta\lambda^{-\varepsilon}\right) + H^*\lambda w B(h^*) &= \frac{M^*r^*t}{1+t} + \frac{Mr}{1+t}, \end{aligned} \tag{4.24}$$

respectively.

Combining Eqs. (4.23) and (4.24) and accounting for the zero-profit conditions, we can solve for the equilibrium values of h and h^* in the open economy. These values are determined by a system of two equations

$$h^* = \frac{1}{\eta}\Phi(h), \qquad h = \eta\Phi(h^*), \tag{4.25}$$

with $\eta \equiv H^*/H$,

$$\Phi(x) \equiv x + \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}\Gamma(x), \quad x = h, h^*, \tag{4.26}$$

and $\Gamma(\cdot)$ being defined in Eq. (4.19). The first expression in (4.25) makes use of market clearing for differentiated goods at home and therefore gives the response of h to changes in h^* that is necessary to restore market clearing in home. The second expression in (4.25) makes use of market clearing for differentiated goods abroad and therefore gives the response of h^* to changes in h that is necessary to restore market clearing in foreign.

We illustrate the open economy equilibrium for the case of symmetric countries ($\eta = 1$) in Figure 4.1. There, we depict the two equations in (4.25) in (h, h^*)-space by the two curves $\Phi(h)$ and $\Phi(h^*)$, respectively. The negative slope of the two curves is assumed for now and further discussed below. $\Phi(h)$ has an intercept with the vertical axis at $\Phi(0) = \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}(1-\beta\lambda^{-\varepsilon})$ and this intercept is denoted by $f_1(t)$, with $f_1'(t) > 0$. Due to symmetry of the two trading partners, the intercept of $\Phi(h^*)$ with the horizontal axis is also given by $f_1(t)$. Furthermore, $\Phi(h)$ has an intercept with the horizontal axis if $h + \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}\Gamma(h) = 0$ has a solution in h. For $-\frac{1-u}{1+t}\left[t(\frac{\sigma}{\sigma-1}-\gamma)-\tilde{\alpha}\right] - \beta\lambda^{-\varepsilon}T(1) < 0$ a solution exists and it lies on the unit interval.[9] We denote this solution by $f_2(t)$, with

[9] To see this, we can substitute Eq. (4.19) for $\Gamma(\cdot)$ and evaluate $\Phi(x)$ at $x = 0$ and $x = 1$. This gives $\Phi(0) = \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}(1-\beta\lambda^{-\varepsilon}) > 0$ and $\Phi(1) = \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}\left\{-\frac{1-u}{1+t}\left[t\left(\frac{\sigma}{\sigma-1}-\gamma\right)-\tilde{\alpha}\right]-\beta\lambda^{-\varepsilon}T(1)\right\}$, respectively.

$f_2'(t) < 0$, and it is unique due to our assumption that $\Phi(h)$ has a negative slope.

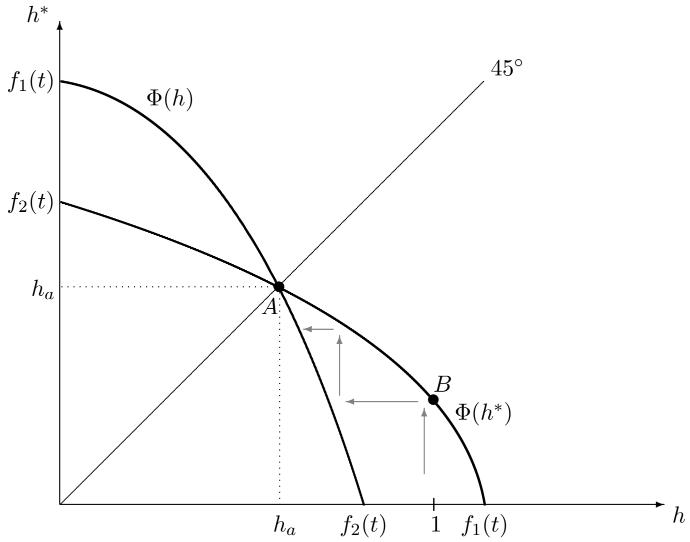

Figure 4.1: Equilibrium in the open economy with symmetric countries

We can now make use of Figure 4.1 to discuss existence, uniqueness, and stability of the open economy equilibrium. Showing existence of the open economy equilibrium is simple for the case of symmetric countries, because we see in Figure 4.1 that the two curves $\Phi(h)$ and $\Phi(h^*)$ intersect in point A at the 45°-line, where the fractions of workers seeking employment in the sector of differentiated goods are the same in the two economies and are given by their autarky levels. This establishes $h = h^* = h_a$, with subscript a used to indicate an autarky variable. To show uniqueness of the intersection point, we can define a critical

$$\underline{t}(x) \equiv -\frac{2\sigma}{\sigma - 1}\frac{1-u}{\rho}\Gamma'(x)^{-1} - 1, \quad x = h, h^* \tag{4.27}$$

such that $\Phi'(h) < -1$ holds if $t > \underline{t}(h)$, whereas $\Phi'(h^*) < -1$ holds if $t > \underline{t}(h^*)$. Noting that $t > \max\{\underline{t}(0), \underline{t}(1)\}$ is sufficient for $\Phi'(h) < -1$ to extend to all $h \in (0,1)$ and for $\Phi'(h^*) < -1$ to extend to all $h^* \in (0,1)$[10], it follows from $t > \max\{\underline{t}(0), \underline{t}(1)\}$ that curve $\Phi(h)$ is steeper than curve $\Phi(h^*)$, proving uniqueness of intersection point A on the unit interval. Finally, stability of the open economy equilibrium in point A follows from its uniqueness and is illustrated by the grey arrows in Figure 4.1. Of course, the analysis so far has been confined to diversification equilibria, and one may suspect that an equilibrium with full specialization of production in one of the two economies also exists, as indicated, for instance, by a point like B. However, this is not true, because the requirement of market clearing rules out such an outcome provided that $f_2(t) < 1$. This follows from the direction of the grey arrows in Figure 4.1.

The open economy equilibrium is no longer symmetric, however, if the two countries

[10]To see this, it is worth noting that the second derivative of $\Phi(h)$ adopts the properties of the second derivative of $\Gamma(h)$: $\Phi''(h) = \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}\Gamma''(h)$. In the Appendix we discuss the properties of $\Gamma(h)$ and show in particular that only two cases are possible, namely either $\Gamma''(h) > 0$ or $\Gamma''(h) < 0$ for all h, ruling out that $\Gamma(h)$ has an extremum at the unit interval. In both cases, we can conclude that $\Phi'(h) < -1$ must hold for all possible $h \in (0,1)$ if $\Phi'(0) < -1$ and $\Phi'(1) < -1$.

differ in their population size. For instance, if the foreign country is larger than the domestic one, we have $\eta > 1$, and in this case the foreign country features a larger market for differentiated goods. This case is illustrated in Figure 4.2. In the closed economy, the additional demand for labor from a larger population size is offset by a larger labor supply, leaving the fraction of workers seeking employment in the sector of differentiated goods unaffected. Accordingly, the autarky equilibrium remains to be given by point A, irrespective of the prevailing differences in population size. Things are different in the open economy. From previous work on home-market effects (cf. Helpman and Krugman, 1985), we know that in a setting as ours "a country whose share of demand for a good is larger than average will have – ceteris paribus – a more than proportionally larger-than average share of world production of that good" (Crozet and Trionfetti, 2008, p.309). Therefore, in the open economy the fraction of workers seeking employment in the production of differentiated goods increases in foreign and decreases in home if $\eta > 1$. In Figure 4.2 the relative increase in foreign market size leads to a counter-clockwise rotation of locus $\Phi(h)$ and locus $\Phi(h^*)$ in their respective intercepts $f_2(t)$. These intercepts are unaffected because they capture the local market clearing conditions in the respective countries if worldwide production of differentiated goods is concentrated there. Accordingly, relative country size differences are irrelevant for the positions of these intercepts. Things are different for intercepts $f_1(t)$, which reflect the local market clearing conditions in the respective countries if no local production is left. In this case, relative country size differences exhibit the largest effect. Figure 4.2 shows a new open economy equilibrium in point $\tilde{A}$ and illustrates that access to trade leads to an expansion of the production of differentiated goods in the country with the initially larger market for these goods and to a contraction of the production of differentiated goods in the other economy.

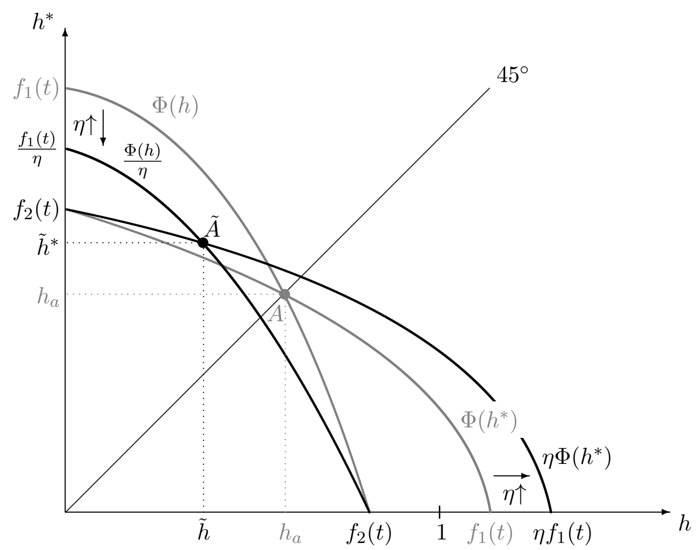

Figure 4.2: Equilibrium in the open economy with asymmetric countries

For a better understanding of how trade affects the allocation of labor, we can determine the effects of marginal changes in trade cost parameter t on h and h^*. These effects are illustrated in Figure 4.3. Starting point is the open economy equilibrium for asymmetric countries depicted by point $\tilde{A}$. Due to our assumption that foreign is larger than home, this equilibrium corresponds to a production pattern with $\tilde{h}^* > \tilde{h}$. The autarky equilibrium is depicted by point A and leads to a symmetric outcome in the two economies regarding the fraction of workers seeking employment in the sector of differentiated goods: $h = h^* = h_a$. An increase in the trade cost parameter from t to t' rotates locus $\frac{1}{\eta}\Phi(h)$ clockwise in point C. To understand this effect, it is worth noting that a clockwise rotation of $\frac{1}{\eta}\Phi(h)$ captures that higher trade costs make the home market more relevant for firms and guard domestic producers in their home market from competition with foreign ones. As a consequence, for higher levels of t an increase in foreign production (reflected by an increase in h^*) induces a smaller production decrease at home (reflected by a less pronounced decline in h) to restore market clearing there. This makes locus $\frac{1}{\eta}\Phi(h)$ steeper. Locus $\frac{1}{\eta}\Phi(h)$ rotates in point C, because in this point the fraction of workers seeking employment in the sector of differentiated goods at home is at its autarky level: $h = h_a$. This establishes $\Gamma(h) = 0$, and we can conclude from Eq. (4.25) that in this case changes in t do not affect h^* for a given level of h. Using the same reasoning, it follows that $\eta\Phi(h^*)$ rotates counter-clockwise in point C^*, implying that higher trade costs bring the fraction of workers seeking employment in the sector of differentiated goods closer to the autarky levels of the two economies. To put it differently, higher trade costs lower the scope for specialization in the open economy.

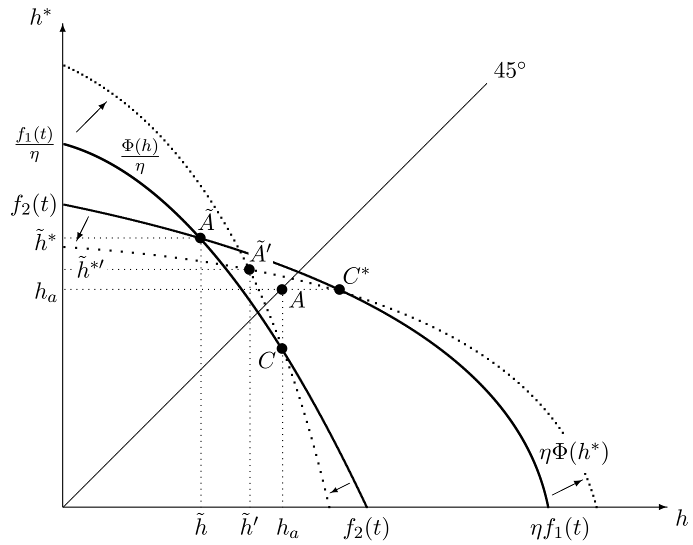

Figure 4.3: Increase in trade cost parameter from t to t'

With the solution for h and h^* at hand, we can make use of the outcome of wage bargaining in (4.23) and the zero-profit conditions $\rho r = \rho r^* = \sigma P_Y f$ to solve for the equilibrium masses of domestic and foreign producers of differentiated goods, M and M^*, respectively. As pointed out above, $h = h^*$ holds under autarky, irrespective of prevailing size differences of the two economies. Whereas the fraction of workers seeking employment in the sector of differentiated goods is the same, the two countries differ in the mass of firms producing differentiated goods in the closed economy. Since the market for differentiated goods is larger in foreign than in home if $\eta > 1$, we have $M^* > M$. Since wage bargaining (plus constant markup pricing) establishes for either country a positive link between the fraction of workers seeking employment in the sector of differentiated goods and the mass of firms producing them, we can conclude from the graphical analysis in Figure 4.2 that trade leads to firm entry in the larger country and to firm exit in the smaller one, thereby augmenting pre-existing differences in the mass of local firms producing differentiated goods. From Figure 4.3, we can further conclude that higher trade costs bring the masses of firms closer to their respective autarky levels, reducing the differences in the local mass of firms producing differentiated goods. This completes the characterization of the open economy equilibrium.

4.4.2 Trade pattern, unemployment and welfare

With the mass of firms determined in the previous section, we can now make use of the zero-profit conditions and compute home's total exports and imports of differentiated goods according to

$$EX_X = M\frac{1}{1+t}\frac{\sigma P_Y f}{\rho}, \qquad IM_X = M^*\frac{1}{1+t}\frac{\sigma P_Y f}{\rho}, \tag{4.28}$$

respectively. This implies that home is a net-importer of differentiated goods, $EX_X < IM_X$, if the foreign to domestic firm ratio $\mu \equiv M^*/M$ is larger than one. This is the case, if foreign is the larger economy, $\eta > 1$, and therefore offers the larger home market for differentiated goods. The opposite is true if home is the larger economy. In this case, $\eta < 1$ establishes $\mu < 1$ and thus $EX_X > IM_X$. This trade structure is well in line with other models featuring a home-market effect (see Helpman and Krugman, 1985). Assuming that households in the case of indifference purchase the domestic product, we have $IM_Y = 0$ and $EX_Y = IM_X - EX_X$ if $\eta > 1$ and therefore $\sum_j(EX_j + IM_j) = 2IM_X$, where $j \in \{X, Y\}$ is an industry index. In contrast, $\eta < 1$ gives $EX_Y = 0$ and $IM_Y = EX_X - IM_X$ and thus $\sum_j(EX_j + IM_j) = 2EX_X$. Also, higher trade costs lower the mass of firms that are active in the larger economy, thereby reducing the volume of trade.

The trade structure in our model is directly linked to the employment effects of trade. From the analysis in the closed economy, we know that only a fraction $1 - u$ of workers seeking employment in the sector of differentiated goods is successfully matched with a firm. Since $1 - u$ is pinned down by the condition that under diversification workers must be indifferent between employment in the production of the homogeneous good or employment in the production of differentiated goods and since this indifference condition is given by Eq. (4.12) and thus the same in the closed and the open economy, the economy-wide rate of unemployment, $U \equiv hu$, can be affected by trade only through adjustments in the fraction of workers seeking employment in the sector producing differentiated goods, h. This establishes the following proposition.

Proposition 6 *In the open economy, the larger country is net-exporter of differentiated goods and suffers from a higher rate of unemployment. An increase in trade costs lowers the export of differentiated goods in the larger and the import of differentiated goods in the smaller economy. The economy-wide rate of unemployment decreases in the larger and increases in the smaller economy.*

Proof. The proposition follows from the analysis above. ■

The link between trade structure and unemployment established in Proposition 6 is a direct consequence of associating employment in the sector of differentiated goods with a higher risk of unemployment. This property of our model is akin to the distinction put forward by Acemoglu (2001) between good jobs offering high wages at the cost of a longer duration of unemployment to wait for the respective offer and bad jobs associated with low wages and a shorter duration of unemployment. The link between unemployment and wages is also well in line with the observation from the US that manufacturing, while offering higher hourly earnings than the average workplace according to data from Bureau of Labor Statistics, is prone to longer durations of unemployment (see Chien and Morris, 2016).

Since the large country is net-exporter of differentiated goods, it experiences an increase in the rate of unemployment in the open economy. However, this does not mean that trade is to the detriment of the larger economy. To see this, we can determine the welfare effects of trade. As pointed out in the analysis of the closed economy, the representative consumer in the case of PIGL preferences does not have a normative interpretation in general, implying that the choice of a proper welfare function is a priori not clear. This is different if preferences have Gorman form, and we therefore focus on the two limiting cases of homothetic and quasilinear preferences for now, while discussing the case of $\varepsilon \in (0,1)$ in Section 4.5.

If households have Gorman form preferences, we can combine Eqs. (4.23) and (4.24) to compute an explicit solution for the ratio of foreign to domestic firms μ as a function of the relative foreign population size η and trade cost parameter t. This gives for homothetic and quasilinear preferences

$$\mu = \frac{\eta\delta(t) - 1}{\delta(t) - \eta}, \qquad \mu = \frac{\eta\hat{\delta}(t) - 1}{\hat{\delta}(t) - \eta}, \tag{4.29}$$

respectively, with

$$\delta(t) \equiv t - \frac{\sigma - 1}{\sigma}\frac{\rho(1+t)}{1-u}\left[(1-u)\tilde{\alpha} - 1\right](1-\beta), \quad \hat{\delta}(t) \equiv t - \frac{\sigma - 1}{\sigma}\frac{\rho(1+t)}{1-u}\left[(1-u)\tilde{\alpha} - 1\right]. \tag{4.30}$$

Furthermore, using the definition of μ in Eq. (4.24) and accounting for the markup pricing rule in Eq. (4.23) and zero-profit condition $\rho r = \sigma P_Y f$, we can determine the fraction of workers seeking employment in the sector of differentiated goods and the mass of firms producing them in home. For the case of homothetic preferences, we compute

$$h = \frac{\frac{1-\beta}{1-u}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha} - 1]}, \qquad M = \frac{\frac{(1-\beta)H\lambda}{(\sigma-1)f}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha} - 1]}, \tag{4.31}$$

whereas for the case of quasilinear preferences, we obtain

$$h = \frac{\frac{\lambda-\beta}{\lambda(1-u)}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha} - 1]}, \qquad M = \frac{\frac{(\lambda-\beta)H}{(\sigma-1)f}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha} - 1]}. \tag{4.32}$$

With Eqs. (4.31) and (4.32) at hand, we can formulate the following proposition.

Proposition 7 *Let us assume that preferences have Gorman form and let us consider an open economy equilibrium with diversified production in both economies. Then, a decline in the trade cost parameter increases welfare in the larger economy, while it can increase or decrease welfare in the smaller economy if* $(1-u)\tilde{\alpha} > 1$. *Things are different if* $(1-u)\tilde{\alpha} < 1$. *In this case, a decline in the trade cost parameter increases welfare in the smaller economy, whereas it can increase or decrease welfare in the larger economy.*

Proof. See the Appendix. ■

To provide an intuition for the welfare effects described in Proposition 7, we can distinguish three channels through which a decline in trade costs impacts welfare in our model. The first one is a fall in the price of differentiated goods imported from the foreign economy. This effect is captured by an increase in $(1+t)/t$ in price index $P_X = \frac{\sigma}{\sigma-1}\frac{w}{\rho}\left(M\frac{\mu+t}{1+t}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}$, and it also exists if countries are symmetric and hence in cases in which $\eta = 1$ and the fraction of workers seeking employment in the sector of differentiated goods as well as the mass of firms producing them remain at their autarky levels. If countries differ in their population size, there are two additional effects. The first one is a disposable income effect, which materializes through changes in $\bar{e} = w\lambda\{1 + h[(1-u)\tilde{\alpha} - 1]\}$ and can be positive or negative. It is positive for the larger country net-exporting differentiated goods if $(1-u)\tilde{\alpha} > 1$, because in this case the wage premium received by workers newly employed by firms producing differentiated goods dominates the income loss of the newly unemployed. The opposite is true if $(1-u)\tilde{\alpha} < 1$. Disposable income effects in the two countries go into opposite directions, because the fraction of workers seeking employment in the sector of differentiated goods increases in the larger and decreases in the smaller economy.

Finally, there exists a variety effect, because existing firms change the location of production (captured by changes in μ for a given total mass of producers, $M + M^*$) and because firms enter or exit the market (captured by changes in the total mass of producers, $M + M^*$, for a given μ). This variety effect materializes through changes in price index $P_X = \frac{\sigma}{\sigma-1}\frac{w}{\rho}\left(M\frac{\mu+t}{1+t}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}$ due to changes in the composite term $M\frac{\mu+t}{1+t}$ and it can be positive or negative. In the larger country, which net-exports differentiated goods, the mass of domestic producers increases. However, the mass of foreign firms decreases and the former dominates the latter only if trade increases average disposable household income, i.e. if $(1-u)\tilde{\alpha} > 1$. In this case, the larger country net-exporting differentiated goods unambiguously benefits from a fall in the trade cost parameter. Things are different if $(1-u)\tilde{\alpha} < 1$. In this case, a negative disposable income effect and a negative variety effect counteract the positive effect of cheaper access to foreign imports, and we show in the Appendix that they can dominate if σ is sufficiently large, because for high levels of σ both the positive price effect for imported goods as well as the negative variety effect are relatively small compared to the negative income effect.

While Proposition 7 is valid for both types of Gorman form preferences, there is a difference regarding the expected trade effects for homothetic and quasilinear preferences.

As pointed out by Lemma 1, quasilinear preferences establish $(1-u)\tilde{\alpha} < 1$ for all possible $\alpha > 1-\gamma$. This is because in the limiting case of $\varepsilon \to 1$ households are risk-neutral and hence they find it attractive to seek employment in the sector of differentiated goods and accept a lower probability of finding a job whenever this causes an increase in their expected income. This leads to a relatively low employment rate in the sector of differentiated goods, implying that the impact of trade on economy-wide unemployment is fairly strong. As a consequence, average disposable labor income falls in the country expanding production of differentiated goods, so that the larger country is at risk of double losses from trade due to an increase in the economy-wide unemployment and a decrease in the representative consumer's welfare level if preferences are quasilinear. Things are different in the case of homothetic (log-transformed Cobb-Douglas) preferences, because households are risk-averse and thus expect a compensation for the possibility of ending up in an unfavorable state of unemployment when applying for jobs in the sector of differentiated goods. For a given wage premium offered by firms producing differentiated goods, this results in a higher employment rate $1-u$, and thus in a moderate increase in unemployment when exporting in the open economy increases the fraction of workers seeking employment in the sector of differentiated goods, h. As put forward by Lemma 1, $(1-u)\tilde{\alpha} > 1$ is guaranteed for all $\alpha > 1-\gamma$ if $\gamma < \exp[-1]$. This implies that if preferences are homothetic and unemployment compensation is not too generous, trade is to the benefit of the larger economy, but may be detrimental for the smaller country.[11] Double losses from trade are not possible in this case.

4.5 Extensions

To complete the analysis in this paper, we discuss two extensions of our model. In the first one, we consider the case of $\varepsilon \in (0,1)$ and analyze to what extent the insight from the two limiting cases of homothetic and quasilinear preferences are informative about the trade effects if preferences do not have Gorman form. In the second extension, we consider differences of countries in the per-capita labor endowment of households and study whether rich or poor countries are more likely to benefit from trade liberalization.

4.5.1 Trade effects if preferences do not have Gorman form

As pointed out above, the representative consumer in our model does not have a normative interpretation if $\varepsilon \in (0,1)$. This makes the choice of a social welfare function somewhat arbitrary. Egger and Habermeyer (2019) suggest to take a utilitarian perspective and to use average household utility as a social welfare function. This establishes

$$V(\overline{e}, P_Y, P_X, \hat{\psi}) \equiv \frac{1}{\varepsilon}\left(\frac{P_Y}{P_X}\right)^{\varepsilon}\left[\left(\frac{\overline{e}}{P_Y}\right)^{\varepsilon}\hat{\psi} - \beta\right] - \frac{1-\beta}{\varepsilon}, \tag{4.33}$$

[11] In many applications to international trade, economists set unemployment compensation equal to 0 (see, e.g., Helpman et al., 2010; Egger and Kreickemeier, 2012). Due to the risk aversion of households with homothetic preferences in our model, the employment probability in the sector of differentiated goods increases to one in the limiting case $\gamma \to 0$, so that trade would not affect economy-wide unemployment and would therefore increase average disposable household income unambiguously.

where $\hat{\psi} \equiv H^{-1} \int_{i\in\mathcal{H}} (e_i/\bar{e})^\varepsilon \, di$ is a dispersion index, which is equal to ψ only if $\varepsilon = 1/2$. Eq. (4.33) is a natural candidate for our welfare analysis and it converges to $V_{CD}(\bar{e}, P_Y, P_X)$ and $V_{QL}(\bar{e}, P_Y, P_X)$ in the limiting cases of $\varepsilon \to 0$ and $\varepsilon \to 1$, respectively. As extensively discussed in Egger and Habermeyer (2019), the welfare function in Eq. (4.33) features social inequality aversion (through $\hat{\psi} < 1$), which, however, is not the consequence of a prioritarian social planner but is rooted in the risk aversion of households imposed by the preferences in Eq. (4.1). Thus, the welfare function in Eq. (4.33) would associate a market outcome with the same level but a higher dispersion of disposable household income with a lower level of welfare, providing scope for achieving a welfare gain through redistribution of income from richer to poorer households.

In comparison to the limiting cases of homothetic and quasilinear preferences studied in the previous section, the assumption of non-Gorman form preferences opens an additional channel through which trade affects welfare in the open economy, namely through changes in the dispersion of disposable household income. Thereby, changes in the dispersion of disposable household income influence welfare through a direct and an indirect effect. The direct effect works through the social income inequality aversion and implies that welfare decreases if trade lowers $\hat{\psi}$. The indirect effect works through changes in firm entry. Because the Engel curves for luxuries are convex, while the Engel curve for necessities is concave, an increase in the dispersion of disposable household income increases consumer demand for differentiated goods and therefore leads to additional firm entry through a decline in ψ. This firm entry lowers price index P_X relative to price P_Y with positive welfare implications, according to Eq. (4.33). To keep things simple, we look at the case of $\varepsilon = 1/2$, implying that the two dispersion measures are equal: $\psi = \hat{\psi}$. In this case, we have $(1-u)\sqrt{\tilde{\alpha}} + u\sqrt{\gamma} = 1$ from Eq. (4.12) and thus

$$\sqrt{\frac{\bar{e}}{P_Y}}\psi = \sqrt{\lambda}T(h) = \sqrt{\lambda\frac{1+h[(1-u)\tilde{\alpha}-1]}{1+h[(1-u)\tilde{\alpha}+u\gamma-1]}}, \tag{4.34}$$

with $T'(h) < 0$. Furthermore, the constraint that even unemployed households consume the differentiated good, $(1-\tau)\lambda\gamma > \beta^2$, establishes $\sqrt{\bar{e}/P_Y}\psi = \sqrt{\lambda}T(h) > \beta$. Combining the market clearing condition in Eq. (4.24) with the zero-profit condition $\rho r = \sigma P_Y f$, we further compute

$$M\frac{\mu+t}{1+t} = \frac{H\lambda\rho}{\sigma f}\left\{1+h\left[(1-u)\tilde{\alpha}-1\right]-\beta\left(\sqrt{\lambda}\right)^{-1}T(h)\right\}. \tag{4.35}$$

Substituting into the price index for differentiated goods, we then obtain the welfare function

$$V(\cdot) = 2\left(\sqrt{\frac{\sigma}{\sigma-1}\frac{1}{\rho}}\right)^{-1}\left(\frac{H\lambda\rho}{\sigma f}\right)^{\frac{1}{2(\sigma-1)}}\hat{V}(h) - 2(1-\beta), \tag{4.36}$$

with

$$\hat{V}(h) \equiv \left\{\left[1+h[(1-u)\tilde{\alpha}-1]-\beta\left(\sqrt{\lambda}\right)^{-1}T(h)\right]\frac{1+t}{t}\right\}^{\frac{1}{2(\sigma-1)}}\left[\sqrt{\lambda}T(h)-\beta\right]. \tag{4.37}$$

Noting from Figure 4.3 that $dh/dt < 0$ if home is a net-exporter of differentiated goods, we can conclude that $(1-u)\tilde{\alpha} > 1$ is no longer sufficient for gains from trade in the larger economy. If σ is sufficiently large, the detrimental impact of trade on the level and dispersion of disposable household income (captured by a lower $\sqrt{\lambda}T(h)-\beta$) may dominate the gains from a lower import price and a positive variety effect. This strengthens our insights from the main text that the specific form of preferences plays a crucial role for the welfare effects of trade in our model.

4.5.2 Trade effects in the case of rich and poor countries

We now consider trade between two countries that differ in the labor endowments of households but feature the same total effective labor supply, $H\lambda = H^*\lambda^*$. Households with a richer labor endowment receive higher disposable income and their country is thus associated with the richer economy. With differences in the households' labor endowments, the outcome of wage bargaining (plus constant markup pricing) and the market clearing conditions for differentiated goods change to

$$hH\lambda w(1-u) = \frac{\sigma-1}{\sigma}\rho M r, \qquad h^*H^*\lambda^* w(1-u) = \frac{\sigma-1}{\sigma}\rho M^* r^* \tag{4.23$'$}$$

and

$$\begin{aligned} H\lambda w(1-\beta\lambda^{-\varepsilon}) + H\lambda w B(h) &= \frac{Mrt}{1+t} + \frac{M^*r^*}{1+t}, \\ H^*\lambda^* w(1-\beta(\lambda^*)^{-\varepsilon}) + H^*\lambda^* w B^*(h^*) &= \frac{M^*r^*t}{1+t} + \frac{Mr}{1+t}, \end{aligned} \tag{4.24$'$}$$

respectively, where $B^*(h^*)$ is defined in analogy to $B(h)$ with λ^* replacing λ. Combining Eqs. (4.23$'$) and (4.24$'$), we compute

$$h^* = \Phi(h), \qquad h = \Phi^*(h^*), \tag{4.25$'$}$$

with $\Phi(h)$ given by Eq. (4.26), $\Phi^*(h^*) \equiv h^* + \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}\Gamma^*(h^*)$, and $\Gamma^*(h^*)$ defined in analogy to $\Gamma(h)$, with λ^* replacing λ.

System (4.25$'$) gives two equations in two unknowns, which can be combined to solve for the equilibrium values of h and h^* in the open economy. For this purpose, we make use of Figure 4.4, where the open economy equilibrium for the case of two symmetric countries is given by point A (similar to Figure 4.1). A richer labor endowment of households in the foreign country ($\lambda^* > \lambda$) increases the home market for differentiated goods there, provided that higher average disposable household income increases demand for differentiated goods, which is the case if $\varepsilon > 0$. Then, the fraction of workers producing differentiated goods is already under autarky higher in foreign than at home, which can be seen from contrasting h_a^* in point A' with h_a in point A. In the open economy equilibrium (point $\tilde{A}$), the difference between h and h^* is further increased, because foreign specializes on the production of differentiated goods in line with the idea of a home-market effect put forward by Helpman and Krugman (1985).[12]

[12]The equilibrium is derived for the case of diversification of production in both economies. With a

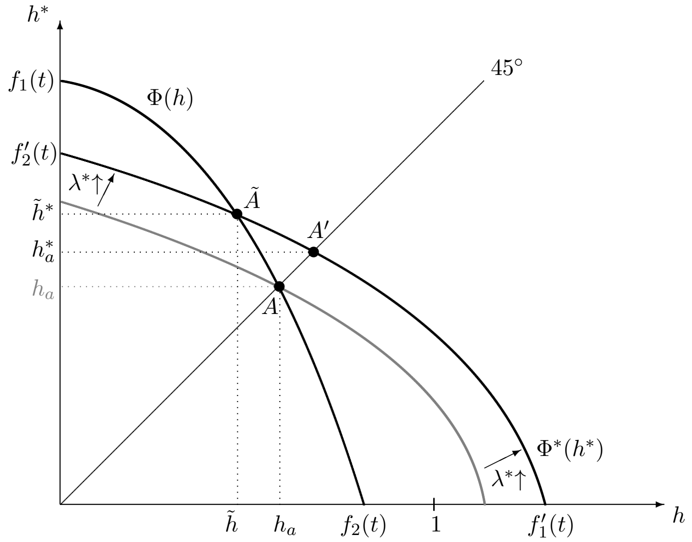

Figure 4.4: Open economy equilibrium if foreign is richer than home ($\lambda^* > \lambda$)

With the equilibrium labor allocation at hand, we can derive the mass of firms producing differentiated goods from the outcome of wage bargaining in Eq. (4.23′) and the zero-profit conditions $\rho r = \rho r^* = \sigma P_Y f$. Provided that $\varepsilon > 0$, the richer country hosts a larger mass of firms producing differentiated goods, and hence becomes net-exporter of these goods in the open economy. Similar to the baseline scenario with country asymmetries rooted in different population sizes, net-exporting differentiated goods comes at the cost of a higher economy-wide unemployment rate. To determine the welfare effects of trade, we proceed as in the main text and focus on the two limiting cases representing Gorman form preferences. From Eq. (4.31), we see that for symmetry of the two countries in aggregate labor supply $H\lambda = H^*\lambda^*$, h and M are the same in the two economies and do not differ from their autarky levels (due to $\mu = 1$) if preferences are homothetic ($\varepsilon = 0$). In this case, trade leaves unemployment unaffected and increases welfare in both economies. With quasilinear preferences ($\varepsilon = 1$), differences in the households' labor endowments generate differences of the two economies in their demand for differentiated goods. This establishes $h^* > h$ and $M^* > M$ if $\lambda^* > \lambda$, implying that the richer country net-exporting differentiated goods not only suffers from an increase in the economy-wide rate of unemployment but may also experience welfare losses from trade if σ is sufficiently large (see the Appendix).

4.6 Conclusion

We have developed a two-country model of trade with differentiated and homogeneous goods using labor as the only production input. The model features a home-market effect due to trade costs of differentiated goods. Whereas the labor market in the homogeneous goods sector is perfectly competitive, there are search frictions and firm-level wage bargaining in the sector of differentiated goods. This generates involuntary unemployment,

reasoning similar to the one in the main text, one can show that such an outcome is guaranteed for sufficiently high trade costs.

whose extent is linked to the fraction of workers seeking employment in the sector of differentiated goods. The exact form of this link depends on consumer preferences, which are assumed to be from the PIGL class and cover homothetic and quasilinear preferences as two limiting cases.

In the open economy, the larger of the two countries specializes on the production of differentiated goods and net-exports these goods. Since seeking employment in the sector of differentiated goods is prone to the risk of unemployment, trade increases the economy-wide rate of unemployment in the larger economy. In the case of quasilinear preferences, trade lowers average disposable household income and exerts a negative variety effect in the larger country, so that social welfare can be reduced there, although the prices of imported goods are reduced. Things are different in the smaller country, which benefits from trade. If preferences are homothetic, trade induces an increase of average disposable household income and generates a positive variety effect in the larger economy, provided that unemployment compensation is not too generous. This adds to the gains from lower import prices, implying that the larger country benefits from trade, despite an increase in the economy-wide rate of unemployment. At the same time, the smaller country can lose from trade, because the negative income and variety effects work against the gains from lower import prices.

In an extension of our analysis, we study non-Gorman preferences and show that in this case changes in the dispersion of income exert an additional impact on welfare, which is missing under homothetic and quasilinear preferences. The impact of changes in the dispersion of income is twofold. On the one hand, a higher income dispersion increases demand for differentiated goods, which are luxuries in our model. This implies that higher income dispersion leads to firm entry and therefore induces indirect welfare gains due to a love-of-variety effect. On the other hand, from a utilitarian perspective welfare exhibits social inequality aversion, so that higher income dispersion reduces welfare through a direct effect. In the open economy, the assumption of non-Gorman preferences implies that an increase in the level of income is no longer sufficient for welfare gains from trade. In a second extension, we consider differences of the two countries in their per-capita labor endowments and show that such differences may lead to welfare loss in the richer economy if preferences are quasilinear. In contrast, welfare gains are guaranteed for both countries if preferences are homothetic, because with homothetic utility per-capita income levels do not matter for aggregate consumer demand, implying that trade does not change the production structure in the open economy.

To improve the exposition of our analysis, we have imposed several simplifying assumptions, which are not crucial for our results. For instance, allowing for differentiated goods in only one sector and associating output of the other sector with a homogeneous good is useful for the analysis of asymmetric countries. However, as long as the wage premium as well as the risk of unemployment are larger in the sector associated with the production of luxuries and as long as the elasticity of substitution between necessities is sufficiently high, the main mechanisms of our model remain valid in a modified setting, in which the differences of the two sectors are less pronounced. Also, allowing for heterogeneous firms in the production of differentiated goods would not alter our results in a qualitative way.

Whereas extensions in these directions are straightforward, we leave a detailed analysis of them to the interested reader.

4.7 Appendix

Microfoundation for the search and matching model

Starting point is the static search and matching model proposed by Helpman and Itskhoki (2010),[13] where the number of matches of workers with firms, L, is determined as a Cobb-Douglas function of the mass of vacancies generated by firms, Q, and the mass of workers seeking employment in the sector of differentiated goods, hH (see Pissarides, 2000, for an extensive discussion of the Cobb-Douglas matching function):

$$L = \hat{m}Q^{\chi}\left(hH\right)^{1-\chi}, \quad 0 < \chi < 1. \tag{4.38}$$

Thereby, parameter $\hat{m}$ is a positive constant that measures the efficiency of the matching process. Establishing a vacancy comes at the cost of one unit of the homogeneous good. Assuming that not all vacancies can be successfully filled, hiring costs per worker can be expressed by $q^{-1}w$, where $q \equiv L/Q < 1$ is the probability to fill a vacancy. Denoting the probability of finding a job by $1-u < 1$ the number of successful matches can be expressed as $L = hH(1-u)$. Substituting into Eq. (4.38), we can write

$$\frac{Q}{hH} = m^{-1}(1-u)^{\frac{1}{\chi}}, \tag{4.39}$$

where $m \equiv \hat{m}^{\frac{1}{\chi}}$. In the main text, we consider the limiting case of $\chi \to 1$ and $m = \lambda^{-1}$, which then establishes Eq. (4.12) from the indifference condition of workers. To see that looking at the limiting case does not change the main insights from our analysis, we can determine employment rate $1-u$ for the more general case of $m < 1$ (needed for $q < 1$) and $\chi < 1$. In this case, the employment rate $1-u$ is implicitly determined by

$$1-u = \frac{1-\gamma^{\varepsilon}}{\left[\frac{\alpha}{m\lambda}(1-u)^{1/\chi-1}+\gamma\right]^{\varepsilon}-\gamma^{\varepsilon}}, \tag{4.40}$$

which delivers $d(1-u)/d\alpha < 0$ and $d(1-u)/d\gamma < 0$ as in the baseline specification. Furthermore, the insight from the main text regarding the ranking of $(1-u)\tilde{\alpha} >, =, < 1$ also extends to the more general case. This completes our discussion of the matching technology.

Proof of Lemma 1

Multiplying Eq. (4.12) by $\tilde{\alpha}$ gives $(1-u)\tilde{\alpha} = \tilde{\alpha}(1-\gamma^{\varepsilon})/(\tilde{\alpha}^{\varepsilon}-\gamma^{\varepsilon})$ and thus $(1-u)\tilde{\alpha}-1 = \tilde{\alpha}[(1-\gamma^{\varepsilon})/(\tilde{\alpha}^{\varepsilon}-\gamma^{\varepsilon})]-1 \equiv \Psi(\tilde{\alpha})$. We compute $\Psi(1)=0$, $\lim_{\tilde{\alpha}\to\infty}\Psi(\tilde{\alpha})=\infty$, and

$$\Psi'(\tilde{\alpha}) = \frac{\Psi(\tilde{\alpha})+1}{\tilde{\alpha}}\left[1-\frac{\varepsilon\tilde{\alpha}^{\varepsilon}}{\tilde{\alpha}^{\varepsilon}-\gamma^{\varepsilon}}\right], \qquad \Psi''(\tilde{\alpha}) = -\frac{\varepsilon\tilde{\alpha}^{\varepsilon}}{\tilde{\alpha}(\tilde{\alpha}^{\varepsilon}-\gamma^{\varepsilon})}\Psi'(\tilde{\alpha}) + \frac{\varepsilon^2\tilde{\alpha}^{\varepsilon}\gamma^{\varepsilon}[\Psi(\tilde{\alpha})+1]}{\tilde{\alpha}^2(\tilde{\alpha}^{\varepsilon}-\gamma^{\varepsilon})^2} \tag{4.41}$$

[13] Helpman and Itskhoki (2010) also discuss an extension of their model to a dynamic setting, and we therefore refer readers interested in such dynamic effects to their paper.

From the derivatives of $\Psi(\tilde{\alpha})$, we can safely conclude that if $\Psi(\tilde{\alpha})$ has an extremum at $\tilde{\alpha} > 1$, this extremum must be unique and a minimum, implying that $\Psi(\tilde{\alpha}) > 0$ holds for sufficiently high levels of α (with $\alpha = \tilde{\alpha} - \gamma$). Furthermore $\Psi'(1) \geq 0$ follows if $\gamma \leq (1-\varepsilon)^{\frac{1}{\varepsilon}} \equiv \underline{\gamma}(\varepsilon)$ and, in this case, $\Psi'(\tilde{\alpha}) > 0$ and thus $\Psi(\tilde{\alpha}) > 0$ holds for all $\tilde{\alpha} > 1$ or, equivalently, for all $\alpha > 1 - \gamma$. Accounting for $\underline{\gamma}'(\varepsilon) < 0$, $\lim_{\varepsilon \to 0} \underline{\gamma}(\varepsilon) = \exp[-1]$, and $\lim_{\varepsilon \to 1} \underline{\gamma}(\varepsilon) = 0$ then establishes Lemma 1.

Derivations details for $B(h)$ and Eqs. (4.17) and (4.18)

From Eq. (4.5) it follows that total expenditures for differentiated goods are equal to

$$\int_{\omega \in \Omega} p(\omega) x(\omega) d\omega = H\overline{e}\left[1 - \beta\left(\frac{\overline{e}}{P_Y}\right)^{-\varepsilon}\psi\right]. \tag{4.42}$$

Substituting Eq. (4.13) for $\overline{e}$ and Eq. (4.15) for ψ, we can express economy-wide demand for differentiated goods as

$$\begin{aligned}\int_{\omega \in \Omega} p(\omega) x(\omega) d\omega &= H\lambda w \left\{1 + h[(1-u)\tilde{\alpha} - 1]\right\} - \beta H \lambda^{1-\varepsilon} w T(h) \\ &= H w \lambda (1 - \beta\lambda^{-\varepsilon}) + H w \lambda B(h),\end{aligned} \tag{4.43}$$

where the first equality sign uses the definition of $T(h)$ in Eq. (4.18), while the second equality sign uses the definition of $B(h)$ in the main text. Setting $\int_{\omega \in \Omega} p(\omega) x(\omega) d\omega = Mr$ finally establishes the market clearing condition in Eq. (4.17). This completes the proof.

Determination of h and M in the closed economy

In the main text, we argue that $\Gamma(h) = 0$ has a unique solution on the unit interval. To see this, we can first note that $\Gamma(0) = 1 - \beta\lambda^{-\varepsilon} > 0$ and that $\Gamma(1) = -(\frac{\sigma}{\sigma-1} - \gamma)(1 - u) - \beta\lambda^{-\varepsilon}T(1) < 0$. Making use of the Intermediate Value Theorem, we can thus safely conclude that $\Gamma(h) = 0$ has a solution in $h \in (0,1)$. As put forward in the main text, in the two limiting cases of $\varepsilon \to 0$ and $\varepsilon \to 1$, we have $T(h) = 1 + h\left[(1-u)\tilde{\alpha} - 1\right]$ and $T(h) = 1$, implying that $\Gamma(h) = 0$ has an explicit and unique solution in $h \in (0,1)$. Things are less obvious if $\varepsilon \in (0,1)$. Twice differentiating $\Gamma(h)$, we obtain

$$\begin{aligned}\Gamma'(h) = &-\left[1 + \left(\frac{\sigma}{\sigma-1} - \gamma\right)(1-u)\right] - \beta\lambda^{-\varepsilon}T(h)\frac{(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1}{1 + h\left[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1\right]} \\ &+ \beta\lambda^{-\varepsilon}T(h)\frac{(1-\varepsilon)u\gamma}{\left\{1 + h\left[(1-u)\tilde{\alpha} - 1\right]\right\}\left\{1 + h\left[(1-u)\tilde{\alpha} + u\gamma - 1\right]\right\}}\end{aligned}$$

and

$$\begin{aligned}\Gamma''(h) = &\beta\lambda^{-\varepsilon}T(h)\frac{(1-\varepsilon)u\gamma}{\left\{1 + h\left[(1-u)\tilde{\alpha} - 1\right]\right\}\left\{1 + h\left[(1-u)\tilde{\alpha} + u\gamma - 1\right]\right\}} \times \\ &\left[\frac{2\left[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1\right]}{1 + h\left[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1\right]} - \frac{(2-\varepsilon)u\gamma + 2[(1-u)\tilde{\alpha} - 1]\{1 + h[(1-u)\tilde{\alpha} + u\gamma - 1]\}}{\left\{1 + h\left[(1-u)\tilde{\alpha} - 1\right]\right\}\left\{1 + h\left[(1-u)\tilde{\alpha} + u\gamma - 1\right]\right\}}\right].\end{aligned}$$

We next show that $\Gamma'(0) < 0$ and $\Gamma'(1) < 0$. For this purpose, we can first note that $\Gamma'(0) = -1 - (\frac{\sigma}{\sigma-1} - \gamma)(1-u) - \beta\lambda^{-\varepsilon}[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1 - (1-\varepsilon)u\gamma]$ and thus $\Gamma'(0) < -(1-\beta\lambda^{-\varepsilon}) - \beta\lambda^{-\varepsilon}[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-\varepsilon)u\gamma]$. Positive expenditures of differentiated goods require $\gamma\lambda(1-\tau) > \beta^{1/\varepsilon}$. Noting that $\tau = 0$ if $h = 0$, we have $\beta < (\gamma\lambda)^{\varepsilon}$ and thus $1-\beta\lambda^{-\varepsilon} > 1-\gamma^{\varepsilon} > 0$. This implies that $(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-(1-\varepsilon)u\gamma > 0$ is sufficient for $\Gamma'(0) < 0$. Second, we can note that

$$\Gamma'(1) = -[1 + \frac{\sigma}{\sigma-1}(1-u) - \gamma(1-u)] - \beta\lambda^{-\varepsilon}\left(\frac{(1-u)\tilde{\alpha}}{(1-u)\tilde{\alpha}+u\gamma}\right)^{1-\varepsilon} Z(\tilde{\alpha}), \tag{4.44}$$

with

$$Z(\tilde{\alpha}) \equiv (1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1 - \frac{(1-\varepsilon)u\gamma\left[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon}\right]}{(1-u)\tilde{\alpha}\left[(1-u)\tilde{\alpha}+u\gamma\right]}. \tag{4.45}$$

If $Z(\tilde{\alpha}) \geq 0$, then $\Gamma'(1) < 0$ is immediate. If $Z(\tilde{\alpha}) < 0$, we can note that $h = 1$ gives $\tau = u\gamma/[(1-u)\tilde{\alpha}+u\gamma]$ and that $\lambda\gamma(1-\tau) > \beta^{1/\varepsilon}$ establishes

$$\beta\lambda^{-\varepsilon}\left(\frac{(1-u)\tilde{\alpha}}{(1-u)\tilde{\alpha}+u\gamma}\right)^{1-\varepsilon} < \gamma^{\varepsilon}\frac{(1-u)\tilde{\alpha}}{(1-u)\tilde{\alpha}+u\gamma}$$

and thus

$$\begin{aligned}\beta\lambda^{-\varepsilon}\left(\frac{(1-u)\tilde{\alpha}}{(1-u)\tilde{\alpha}+u\gamma}\right)^{1-\varepsilon} Z(\tilde{\alpha}) > \gamma^{\varepsilon}\Bigg\{&-1 + (1-u)\tilde{\alpha}\frac{(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}}{(1-u)\tilde{\alpha}+u\gamma} \\ &+\frac{u\gamma}{(1-u)\tilde{\alpha}+u\gamma}\left[1-(1-\varepsilon)\frac{(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}}{(1-u)\tilde{\alpha}+u\gamma}\right]\Bigg\}.\end{aligned}$$

Using Eq. (4.12), we can note that $[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon}]/[(1-u)\tilde{\alpha} + u\gamma] >,=,< 1$ if $f(\tilde{\alpha}) \equiv (1-\gamma^{\varepsilon})\tilde{\alpha}^{1-\varepsilon} + (\tilde{\alpha}^{\varepsilon}-1)\gamma^{1-\varepsilon} - (1-\gamma^{\varepsilon})\tilde{\alpha} - (\tilde{\alpha}^{\varepsilon}-1)\gamma >,=,< 0$. Thereby, we have $f(1) = 0$ and $f'(\tilde{\alpha}) = -(1-\gamma^{\varepsilon})[1-(1-\varepsilon)\tilde{\alpha}^{-\varepsilon}] + \varepsilon\tilde{\alpha}^{\varepsilon-1}[\gamma^{1-\varepsilon}-\gamma]$, $f''(\tilde{\alpha}) = -\varepsilon(1-\varepsilon)[\tilde{\alpha}^{-\varepsilon-1}(1-\gamma^{\varepsilon})+\tilde{\alpha}^{\varepsilon-2}(\gamma^{1-\varepsilon}-\gamma)] < 0$. Hence, if $f(\tilde{\alpha})$ has an extremum, it must be a maximum. Noting further that $f'(1) = -\varepsilon(1-\gamma^{\varepsilon}-\gamma^{1-\varepsilon}+\gamma) < 0$ holds for all permissible levels of γ,[14] it follows that $f(\tilde{\alpha}) < 0$ and thus $[(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}]/[(1-u)\tilde{\alpha}+u\gamma] < 1$ hold for all $\alpha > 1-\gamma$ (and thus $\tilde{\alpha} > 1$). Putting together, we can therefore conclude that

$$\beta\lambda^{-\varepsilon}\left(\frac{(1-u)\tilde{\alpha}}{(1-u)\tilde{\alpha}+u\gamma}\right)^{1-\varepsilon} Z(\tilde{\alpha}) > -\gamma^{\varepsilon}$$

and this is sufficient for $\Gamma'(1) < 0$.

Let us now turn to the second derivative of $\Gamma(h)$, for which we can note that $\Gamma''(h) >,=,< 0$ is equivalent to $F(h) >,=,< 0$, with

$$\begin{aligned}F(h) \equiv 2\left[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-u)\tilde{\alpha}\right]\{1 + h\left[(1-u)\tilde{\alpha}+u\gamma-1\right]\} \\ -(2-\varepsilon)u\gamma\left\{1+h\left[(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-1\right]\right\}.\end{aligned} \tag{4.46}$$

[14]To see this, one can note that $f'(1)$ is increasing in γ and takes a value of zero if $\gamma = 1$.

Then, $F(h) < 0$ and thus $\Gamma''(h) < 0$ holds if $(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-u)\tilde{\alpha} \leq 0$, and in this case $\Gamma'(0) < 0$ is sufficient for $\Gamma'(h) < 0$ to hold for all $h > 0$. To see whether this can be the case, we can note that $(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-u)\tilde{\alpha} >,=,< 0$ is equivalent to $\zeta(\tilde{\alpha}) \equiv (1-\gamma^{\varepsilon})\tilde{\alpha}^{1-\varepsilon} + (\tilde{\alpha}^{\varepsilon} - 1)\gamma^{1-\varepsilon} - (1-\gamma^{\varepsilon})\tilde{\alpha} >,=,< 0$. Then, accounting for $\zeta(1) = 0$, $\zeta'(\tilde{\alpha}) = -(1-\gamma^{\varepsilon})\big[1 - (1-\varepsilon)\tilde{\alpha}^{-\varepsilon}\big] + \varepsilon\tilde{\alpha}^{\varepsilon-1}\gamma^{1-\varepsilon}$, $\zeta''(\tilde{\alpha}) = -\varepsilon(1-\varepsilon)\big[(1-\gamma^{\varepsilon})\tilde{\alpha}^{-\varepsilon-1} + \tilde{\alpha}^{\varepsilon-2}\gamma^{1-\varepsilon}\big] < 0$, and $\lim_{\tilde{\alpha}\to\infty} \zeta(\tilde{\alpha}) = -\infty$, we can conclude that if $\zeta(\tilde{\alpha})$ has an extremum, it must be a maximum and establish $\zeta(\tilde{\alpha}) > 0$. Such a maximum can only exist if $\zeta'(1) > 0$. We have $\zeta'(1) = -\varepsilon(1 - \gamma^{\varepsilon} - \gamma^{1-\varepsilon}) >,=,< 0$ if $0 >,=,< 1 - \gamma^{\varepsilon} - \gamma^{1-\varepsilon}$. This determines a unique $\underline{\gamma} \in (0,1)$, which is implicitly given by $1 - \underline{\gamma}^{\varepsilon} = \underline{\gamma}^{1-\varepsilon}$, such that $\zeta'(1) >,=,< 0$ if $\gamma >,=,< \underline{\gamma}$. This implies that $\gamma \leq \underline{\gamma}$ is sufficient for $(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-u)\tilde{\alpha} \leq 0$ to hold for all $\tilde{\alpha} > 1$. In contrast, if $\gamma > \underline{\gamma}$, there exists a unique $\tilde{\alpha}^0 > 0$, such that $(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-u)\tilde{\alpha} >,=,< 0$ if $\tilde{\alpha}^0 >,=,< \tilde{\alpha}$.

Let us now consider a parameter configuration $(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-u)\tilde{\alpha} > 0$. This requires $1 - \gamma^{\varepsilon} < \gamma^{1-\varepsilon}$. Then, differentiating Eq. (4.46), we see that $F(h)$ is a monotonic function. Furthermore, evaluating $F(h)$ at $h = 0$ and $h = 1$, we obtain $F(0) = 2\big[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-u)\tilde{\alpha} - u\gamma\big] + \varepsilon u\gamma$ and $F(1) = \big\{2\big[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-u)\tilde{\alpha} - u\gamma\big] + \varepsilon u\gamma\big\}(1-u)\tilde{\alpha} + \varepsilon u\gamma\big[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-u)\tilde{\alpha}\big]$, so that $F(0) \geq 0$ is sufficient for $F(1) > 0$. Substituting $(1-u)$ and u from Eq. (4.12), we furthermore obtain

$$(\tilde{\alpha}^{\varepsilon} - \gamma^{\varepsilon})\, F(0) = 2\left[(1-\gamma^{\varepsilon})\left(\tilde{\alpha}^{1-\varepsilon} - \tilde{\alpha}\right) + (\tilde{\alpha}^{\varepsilon} - 1)\left(\gamma^{1-\varepsilon} - \gamma\right)\right] + \varepsilon(\tilde{\alpha}^{\varepsilon} - 1)\gamma \equiv G(\tilde{\alpha}). \tag{4.47}$$

Differentiation of $G(\tilde{\alpha})$ gives $G'(\tilde{\alpha}) = 2\big\{(1-\gamma^{\varepsilon})\big[(1-\varepsilon)\tilde{\alpha}^{-\varepsilon} - 1\big] + \varepsilon\tilde{\alpha}^{\varepsilon-1}(\gamma^{1-\varepsilon} - \gamma)\big\} + \varepsilon^2\tilde{\alpha}^{\varepsilon-1}\gamma$, $G'(1) = -2\varepsilon(1 + \gamma - \gamma^{1-\varepsilon} - \gamma^{\varepsilon}) + \varepsilon^2\gamma$, $\lim_{\tilde{\alpha}\to\infty} G'(\tilde{\alpha}) = -2(1-\gamma^{\varepsilon})$, and $G''(\tilde{\alpha}) = -\varepsilon(1-\varepsilon)\big\{2\big[(1-\gamma^{\varepsilon})\tilde{\alpha}^{-\varepsilon-1} + (\gamma^{1-\varepsilon} - \gamma)\tilde{\alpha}^{\varepsilon-2}\big] + \varepsilon\tilde{\alpha}^{\varepsilon-2}\gamma\big\} < 0$. Two cases can be distinguished.[15] If $2(1 + \gamma - \gamma^{1-\varepsilon} - \gamma^{\varepsilon}) \geq \varepsilon\gamma$, which is the case for sufficiently low values of γ, then $G'(1) \leq 0$, and hence $G'(\tilde{\alpha}) < 0$ holds for all possible $\tilde{\alpha} > 1$. In this case, $G(1) = 0$ is sufficient for $G(\tilde{\alpha}) < 0$ and thus $F(0) < 0$ hold for all $\tilde{\alpha} > 1$. We can therefore conclude that either $F(h) < 0$ for all h or there exists a critical h^0, such that $F(h) >,=,< 0$ if $h >,=,< h^0$. With these considerations, we cannot rule out that $\Gamma(h)$ has multiple extrema. However, $\Gamma(h)$ cannot have more than two interior extrema and if two extrema existed, the first one would have to be a maximum, while the second one would have to be a minimum. This is inconsistent with $\Gamma'(0) < 0$, $\Gamma'(1) < 0$, which requires in the case of two extrema that the first one must be a minimum and the second one must be a maximum. For the same reason, there cannot be a unique extremum, so that it must be true that $\Gamma'(h) < 0$ holds for all $h \in (0,1)$. This is sufficient for a unique interior solution of $\Gamma(h) = 0$. If $2(1 + \gamma - \gamma^{1-\varepsilon} - \gamma^{\varepsilon}) < \varepsilon\gamma$, which is the case for high levels of γ, then $G'(1) > 0$ implies that $G(\tilde{\alpha})$ is positive for low levels of $\tilde{\alpha} > 1$ and negative for high levels of $\tilde{\alpha}$. From $\lim_{\tilde{\alpha}\to\infty} G(\tilde{\alpha}) = -\infty$ and the derivation properties of $G(\tilde{\alpha})$, it follows that there exists a unique $\tilde{\alpha}^1 > 1$, such that $G(\tilde{\alpha}) >,=,< 0$ if $\tilde{\alpha}^1 >,=,< \tilde{\alpha}$. The analysis above extends to the case $2(1 + \gamma - \gamma^{1-\varepsilon} - \gamma^{\varepsilon}) < \varepsilon\gamma$ if $\tilde{\alpha} \geq \tilde{\alpha}^1$, which ensures that the solution of $\Gamma(h) = 0$ on the unit interval is unique. Things are different, however, if $\tilde{\alpha} < \tilde{\alpha}^1$ establishes $G(\tilde{\alpha}) > 0$

[15] From above, we know that $1 - \gamma^{\varepsilon} < \gamma^{1-\varepsilon}$. However, this does not rule out one of these cases.

and thus $F(0) > 0$. However, using the monotonicity of $F(h)$ it follows from $F(1) > 0$ – due to our assumption of $(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-u)\tilde{\alpha} > 0$ – that $\Gamma''(h) > 0$ must hold. This implies that $\Gamma(h)$ has at most one extremum, which would have to be a unique minimum. However, a minimum is in contradiction to $\Gamma'(1) < 0$, so that we can safely conclude that $\Gamma'(h) < 0$ again holds for all $h \in (0,1)$, which is sufficient for the solution of $\Gamma(h) = 0$ to be unique. This completes the proof.

Welfare effects of an increase in α in the closed economy

We first consider the case of homothetic (log-transformed Cobb-Douglas) preferences, so that welfare is given by $V_{CD}(\overline{e}, P_Y, P_X)$ in Eq. (4.22). Substituting $P_X = \frac{\sigma}{\sigma-1}\frac{w}{\rho}M^{\frac{1}{1-\sigma}}$ and $\overline{e} = w\lambda\{1 + h[(1-u)\tilde{\alpha} - 1]\}$, and accounting for h and M from Eq. (4.20), we compute $V_{CD}(\cdot) = \ln\lambda + \frac{1-\beta}{\sigma-1}\ln\left(\frac{(1-\beta)H\lambda}{(\sigma-1)f}\right) + \ln[V_0(\alpha)]$, with

$$V_0(\alpha) = \left(\frac{\sigma}{\sigma-1}\frac{1}{\rho}\right)^{\beta}\left\{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha} - 1]\right\}^{-\frac{\sigma-\beta}{\sigma-1}}.$$

$dV_{CD}(\cdot)/d\lambda > 0$ is immediate. Furthermore, acknowledging $\rho = \frac{\sigma}{\sigma+\alpha(\sigma-1)}$, $\tilde{\alpha} = \alpha + \gamma$ and $1 - u = -\frac{\ln\gamma}{\ln\tilde{\alpha}-\ln\gamma}$, the derivative of $V_0(\alpha)$ can be computed according to

$$V_0'(\alpha) = V_0(\alpha)\left\{\frac{\beta}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}} - \frac{\sigma-\beta}{\sigma-1}\frac{\beta - (1-\beta)\frac{1}{\tilde{\alpha}\ln\gamma}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha} - 1]}\right\}. \tag{4.48}$$

Evaluated at $\alpha = 1-\gamma$ (and thus $\tilde{\alpha} = 1$), we compute $V_0'(1-\gamma) < 0$. For higher levels of α, the marginal effect is however not clear. For instance, setting parameter values $\sigma = 2$, $\beta = 0.8$, and $\gamma = 0.98$, $V_0(\alpha)$ has a local minimum at $\alpha = 6.46$.

Let us now turn to the limiting case of $\varepsilon \to 1$. Accounting for h and M from Eq. (4.21), we can express welfare by $V_{QL}(\cdot) = (\lambda-\beta)^{\frac{\sigma}{\sigma-1}}\left[\frac{H}{(\sigma-1)f}\right]^{\frac{1}{\sigma-1}}\hat{V}_0(\alpha)^{\frac{\sigma}{\sigma-1}} - 1 + \beta$, with

$$\hat{V}_0(\alpha) = \left\{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1}{1-u}[(1-u)\tilde{\alpha} - 1]\right\}^{-1} = \left(\frac{\sigma}{\sigma-1} - \gamma + \frac{\alpha}{1-\gamma}\right)^{-1}. \tag{4.49}$$

Thereby, the second equality sign makes use of the definition of ρ and $1 - u = \frac{1-\gamma}{\alpha}$ from Eq. (4.12). From these computations, we can conclude that $V_{QL}(\cdot)$ increases in λ and decreases in α. This completes the proof.

Proof of Proposition 7

Let us first consider the limiting case of homothetic (log-transformed Cobb-Douglas) preferences, with welfare given by $V_{CD}(\overline{e}, P_Y, P_X)$ in Eq. (4.22). Substituting h and M from Eq. (4.31) into $\overline{e} = w\lambda\{1 + h[(1-u)\tilde{\alpha} - 1]\}$ and $P_X = \frac{\sigma}{\sigma-1}\frac{w}{\rho}\left(M\frac{\mu+t}{1+t}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}$, we can compute

$$\bar{e} = w\lambda \frac{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]}, \tag{4.50}$$

$$P_X = \frac{\sigma}{\sigma-1}\frac{w}{\rho}\left(\frac{(1-\beta)H\lambda\rho}{\sigma f}\right)^{\frac{1}{1-\sigma}} \left(\frac{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}. \tag{4.51}$$

Substituting into $V_{CD}(\bar{e}, P_Y, P_X)$, then gives $V_{CD}(\cdot) = -(1-\beta)\ln\left(\frac{\sigma}{\sigma-1}\frac{1}{\rho}\right) + \ln\lambda + \frac{1-\beta}{\sigma-1}\ln\left(\frac{(1-\beta)H\lambda\rho}{\sigma f}\right) + \ln[V_1(t)]$,

$$V_1(t) = \left(\frac{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]}\right)^{\frac{\sigma-\beta}{\sigma-1}} \left(\frac{1+t}{t}\right)^{\frac{1-\beta}{\sigma-1}}, \tag{4.52}$$

where μ is given by Eq. (4.29). Differentiating $f(t) \equiv \frac{\mu+t}{1+t}$ establishes

$$f'(t) = \frac{1-\mu}{(1+t)^2} + \frac{d\mu}{dt}\frac{1}{1+t} = \frac{1}{1+t}\left[\frac{1-\mu}{1+t} + \frac{1-\eta^2}{[\delta(t)-\eta]^2}\delta'(t)\right]. \tag{4.53}$$

Noting that $\mu >, =, < 1$ if $\eta >, =, < 1$ from Eq. (4.29) and that $\delta'(t) > 0$ from Eq. (4.30), we can safely conclude that $f'(t) >, =, < 0$ if $1 >, =, < \eta$. Furthermore, differentiating $V_1(t)$ gives

$$V_1'(t) = V_1(t)\left[-\frac{\sigma-\beta}{\sigma-1}\frac{\frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]\frac{1+t}{\mu+t}f'(t)}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]} - \frac{1-\beta}{\sigma-1}\frac{1}{t(1+t)}\right]. \tag{4.54}$$

This derivative is unambiguously negative if either $1 > \eta$ (home net-exporting differentiated goods) and $(1-u)\tilde{\alpha} > 1$ or $1 < \eta$ (home net-importing differentiated goods) and $(1-u)\tilde{\alpha} < 1$. In contrast,

$$\lim_{\sigma\to\infty} V_1'(t) = -\frac{(1+\alpha)\frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]f'(t)}{\left\{(1+\alpha)\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]\right\}^2} \tag{4.55}$$

is positive if $1 > \eta$ (home net-exporting differentiated goods) and $(1-u)\tilde{\alpha} < 1$ or if $1 < \eta$ (home net-importing differentiated goods) and $(1-u)\tilde{\alpha} > 1$. This completes the proof of Proposition 7 for the limiting case of $\varepsilon \to 0$.

If preferences are quasilinear, welfare is given by $V_{QL}(\bar{e}, P_Y, P_X)$ in Eq. (4.22). Substituting h and M from Eq. (4.32) into $\bar{e} = w\lambda\{1 + h[(1-u)\tilde{\alpha}-1]\}$ and $P_X = \frac{\sigma}{\sigma-1}\frac{w}{\rho}\left(M\frac{\mu+t}{1+t}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}$, we can compute

$$\frac{\bar{e}}{P_Y} - \beta = \frac{(\lambda-\beta)\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha}-1]}, \tag{4.56}$$

$$P_X = \frac{\sigma}{\sigma-1}\frac{w}{\rho}\left(\frac{H\rho}{\sigma f}\right)^{\frac{1}{1-\sigma}} \left(\frac{(\lambda-\beta)\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha}-1]}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}. \tag{4.57}$$

This allows us to determine $V_{QL}(\cdot) = \left(\frac{\sigma}{\sigma-1}\frac{1}{\rho}\right)^{-1}\left(\frac{H\rho}{\sigma f}\right)^{\frac{1}{\sigma-1}}\hat{V}_1(t) - 1 + \beta$, with

$$\hat{V}_1(t) = \left(\frac{(\lambda-\beta)\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha}-1]}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{1+t}{t}\right)^{\frac{1}{\sigma-1}}. \tag{4.58}$$

Differentiation with respect to t gives

$$\hat{V}_1'(t) = \hat{V}_1(t)\left[-\frac{\sigma}{\sigma-1}\frac{\frac{1}{1-u}[(1-u)\tilde{\alpha}-1]\frac{1+t}{\mu+t}\hat{f}'(t)}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha}-1]} - \frac{1}{\sigma-1}\frac{1}{t(1+t)}\right], \tag{4.59}$$

where $\hat{f}(t) \equiv \frac{\mu+t}{1+t}$ and $\mu = \frac{\eta\hat{\delta}(t)-1}{\hat{\delta}(t)-\eta}$ have been considered. In analogy to the case of homothetic preferences, we find that this derivative is unambiguously negative if $1 < \eta$ (home net-importing differentiated goods) and $(1-u)\tilde{\alpha} < 1$. In contrast, we find that $\lim_{\sigma\to\infty}\hat{V}_1'(t)$ is positive if $1 > \eta$ (home net-exporting differentiated goods) and $(1-u)\tilde{\alpha} < 1$. This completes the proof of Proposition 7 for the limiting case of $\varepsilon \to 1$.

Formal details for the analysis in Section 4.5.2

Let us consider the limiting case of $\varepsilon \to 1$ and focus on an interior solution with $h, h^* \in (0,1)$. Then, accounting for the definition of $\hat{\delta}(t)$ in Eq. (4.30), we can follow the steps from the main text to compute

$$\mu = \frac{\hat{\eta}\hat{\delta}(t)-1}{\hat{\delta}(t)-\hat{\eta}}, \qquad \hat{\eta} \equiv \frac{\lambda^*-\beta}{\lambda^*}\frac{\lambda}{\lambda-\beta}. \tag{4.60}$$

Thereby, $\mu >,=,< 1$ if $\lambda^* >,=,< \lambda$ and thus $\hat{\eta} >,=,< 1$. Noting that h and M are given by (4.32) and following derivation details from above, we can compute $V_{QL}(\cdot) = \left(\frac{\sigma}{\sigma-1}\frac{1}{\rho}\right)^{-1}\left(\frac{H\rho}{\sigma f}\right)^{\frac{1}{\sigma-1}}\hat{V}_1(t) - 1 + \beta$, with $\hat{V}_1(t)$ given by Eq. (4.58). The welfare effects of trade discussed in Section 4.5.2 then follow from the proof of Proposition 7.

Chapter 5

Conclusions

The purpose of this thesis has been to analyze the role of preferences in international trade theory by means of three different modeling approaches. In all three articles, we have concentrated on the class of "price-independent generalized-linear" (PIGL) preferences and have focused on how these preferences determine the production structure, shape the trade pattern and influence the welfare effects of trade in open economies, with the three models differing, however, in their respective focus.

Different from previous research on the home-market effect, Chapter 2 has considered a subclass of parametric PIGL preferences and rent sharing at the firm level. Relying on a subclass of parametric PIGL preferences for which a closed form representation of direct utility exists, we have avoided an integrability problem. Rent sharing has generated sector-specific wages, which are important to generate a two-way linkage between income differences and trade. Assuming that households differ in their effective labor supply, which has established differences in their ex ante level of labor income, demand for the differentiated good has been larger in the country that features a higher level and/or higher dispersion of per-capita income. We have then shown that, in line with the home-market effect, countries have a trade surplus in the good for which they have relatively higher domestic demand. Furthermore, due to the labor market imperfection, the trade pattern has been decisive for the welfare outcome in the open economy, such that there might be losers from globalization.

Chapter 3 has put forward a generalization of parametric PIGL preferences in a home-market model along the lines of Chapter 2. This generalization has come at the cost that a closed form representation of the direct utility function does not exist, giving rise to an integrability problem, which needs to be solved in order to ensure that the demand functions derived from indirect utility are indeed the result of utility maximization of rational households. We have solved this problem by introducing an intermediate goods sector, which produces differentiated goods that are costlessly assembled to a homogeneous final good. Workers have been ex ante heterogeneous in their effective labor supplies, which has led to heterogeneous labor incomes. Due to the non-linearity of Engel curves, demand for the homogeneous luxury good has been larger in the country that features a higher level and/or higher dispersion of per-capita income, translating into a larger home-market for differentiated intermediates. Associating trade with the exchange of the outside good and differentiated intermediate goods, the level and/or dispersion of per-capita income have shaped the production and trade structure in accordance with the well-established model of the home-market effect. In the absence of a price distortion on the labor market, the existence of welfare gains from trade for both trading partners has been independent of the trade structure, however, their magnitude may vary.

Chapter 4 has presented a home-market model with a homogeneous goods sector, producing under perfect competition, and search frictions and firm-level wage bargaining in the monopolistically competitive sector of differentiated goods, while featuring a specific form of parametric PIGL preferences. With a particular emphasis on the limiting cases of homothetic and quasilinear preferences, we have elaborated on how the specific nature of preferences affects the employment and welfare effects of trade. With differences of countries only due to differences in their population size, the findings in the open economy have been in line with the home-market effect discussed in Helpman and Krugman (1985). In our setting, the larger country has featured a higher economy-wide rate of unemployment in the open economy, irrespective of the considered preferences. However, the preference structure has been decisive for the welfare effects of trade. If preferences were homothetic, the large country would likely benefit from trade, whereas the smaller country might lose from trade. Considering quasilinear preferences, the opposite has been true.

Relaxing the assumption of useful but rather restrictive homothetic preferences makes the analysis in this thesis formally demanding. For that reason, we have made use of several other simplifying assumptions – standardized in the trade literature – to keep the different model frameworks tractable and to focus on the main questions of interest. There exist numerous possibilities how to model nonhomotheticity in a theoretical trade context, once relying, for instance, on different demand-side factors and preference structures. Of course, this thesis cannot provide all possible channels through which preferences affect outcomes of models in international trade theory. Nevertheless, this thesis contributes to this large strand of literature by incorporating parametric PIGL preferences, which feature a well-defined representative consumer, to existing trade models. This has the convenient advantage, that an aggregation problem with heterogeneous households does not exist and thus provides a model framework, which can tackle per-capita income and the distribution of per-capita income. Since income inequality and redistribution of income are topics of particular importance in the theoretical and empirical trade literature, our modeling approaches pave the way for further research about the role of PIGL preferences in international trade theory.

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