

# Skill Estimates – Olympic Beach Volleyball Tournament 2012

Susanne Hoffmeister and Jörg Rambau

**Abstract** This paper provides skill estimates for the players Julius Brink (GER), Jonas Reckermann (GER), Alison Cerutti (BRA) and Emanuel Rego (BRA) who participated in the final of the Olympic beach volleyball tournament 2012 in London. The skill estimates are based on all matches the teams have played during the Olympic tournament. Each ball contact has been analyzed according to a specification of hits and a division of the field. The resulting tables aggregate the skill estimates such that they can be used as input parameters of mathematical models like, for example, an appropriate Markov decision problem.

## 1 Introduction

The need for positional and player depended skill estimates originates from the development of a Markov decision problem for beach volleyball, see [1]. This Markov decision problem has a very large state space that contains among other things the positions of the players and the ball. Since no appropriate data source was publicly available, a project was started to raise a database on our own.

To estimate the skills, we evaluated all matches Julius Brink (GER) and Jonas Reckermann (GER), as well as Alison Cerutti (BRA) and Emanuel Rego (BRA), played in the tournament. These were 7 matches for each team, from which, in total, 1635 ball contacts of Brink-Reckermann and 1857 ball contacts of the Brazilian team were observed. The data collection is based on publicly available video material of the 2012 Olympics beach volleyball tournament.

Software support was used during the process of collecting, recording and evaluating the data. However, each hit and all positions were manually specified while using a new video analysis software tool as a support for the data handling.

The paper is structured in two parts. The first part, Section 2, specifies the hits and as well as the division of the court that are used to evaluate the ball contacts. The second part, Section 3, Section 4 and Section 5, presents the resulting skill estimates

in several tables. There exist skill estimates based on all matches, all matches except the final match, and skill estimates based only on the final match. This subdivision of the data is made such that a mathematical model can be calibrated on the pre-final match data and evaluated on the final match.

## 2 Hit Specification and Aggregation

In general, a hit is a deliberate contact of the player with the ball. We define three possible outcomes of a hit:

- *succ*: The hit is successful and the ball flies in the desired direction.
- *dev*: The hit is successful but the ball deviates from the desired direction.
- *fault*: The hit is not successful, i.e., an execution fault occurs or the ball flies into the net.

Since the data has been collected from a real competition and not from training sessions, we can only guess which point on the court the player targeted with his hit. We made the assumption that a player never aims to hit the ball outside of the court. So, each hit that lands outside the court is counted as a deviation, whereas each hit that lands inside the court is counted as a successful hit without deviation.

Table 1 lists all hitting types that are used to classify the ball contacts together with an abbreviation and a description. In the database, each ball contact is classified by its hitting type and saved together with the following information:

- the hitting player,
- the hitting player's position on the court,
- the ball's position of the court before the hit,
- the targeted point on the court,
- the outcome of the hit.

All positions are recorded as  $(x, y)$  tuples such that deliberately exact position can be specified. The distinction whether a reception or defense was made with or without a move was done in a second step after the discretization of the court in several fields. Furthermore, data-records of receptions and defenses contained also an indicator whether the received respectively defended ball was a hard ball.

Beside the listed hits, we also estimated the blocking skills of each player. In contrast to a hit, the outcome of a block is distinguished in five cases:

- *no block*: The blocking player does not touch the ball.
- *block point*: The block results in a non-defendable ball and yields a point.
- *block fault*: The block results in a point loss, e.g., due to an execution fault.
- *block ok*: The ball's direction is affected by the block and the rally goes on.

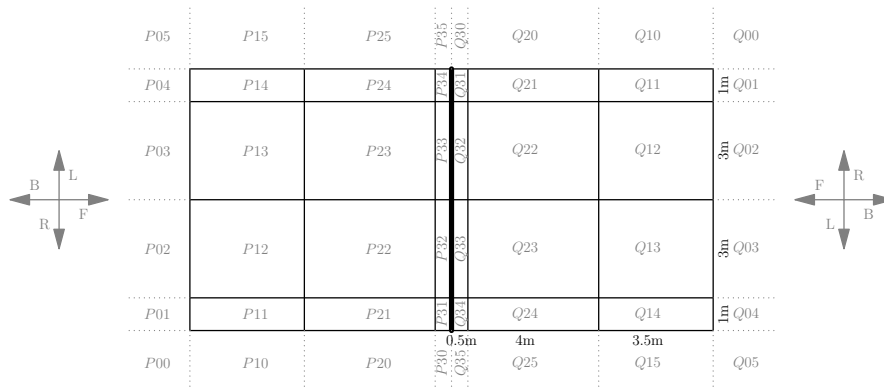
Data records of a block contain the blocking player and the outcome of the block.

After evaluating the recorded positions, a discretization of the beach volleyball court in fields is made. Figure 1 presents the used grid of the beach volleyball court.

**Table 1** Hit specification

<i>tech</i> name	description
<b>Serve</b>	
$S_F$ float serve $S_J$ jump serve	First hit of a rally.
<b>Reception</b>	
$r$ receive $r_m$ receive with move	First hit after a serve.
<b>Setting</b>	
$s$ set	Second hit after successful reception or defence.
<b>Attack-Hit</b>	
$F_{SM}$ smash $F_E$ emergency shot $F_P$ planned shot	Attack hit targets a point on the opponent's court.
<b>Defence</b>	
$d$ defence $d_m$ defence with move	First hit after an attack hit.

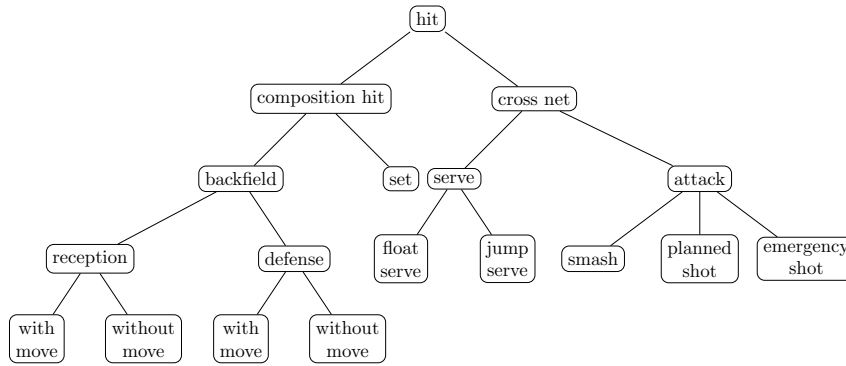
Using this grid, all receptions and defences are evaluated whether the hitting player's position and the ball's positions are in the same field. If this is not the case, the reception respectively defence is marked as a reception with move. The idea behind this differentiation is that a movement towards the ball affects the outcome of the hit.



**Fig. 1** Court grid

Finally, the data-records were evaluated for every player. For example, Table 2 contains the maximum likelihood estimates of Julius Brink’s hitting skills for all types of serves and attack-hits.

We aggregated different player positions and target fields to ranges to get a larger number of observations. The number of observations for a certain combination of player position and target field is stated in the #-column. We found that with a minimum of eleven observations we get sufficiently stable results in our mathematical model. For the presented data, eleven observations are the best trade-off between error through aggregation and error from observing rare events. For hits with less than eleven observations, we included a prior assumption about the probability from the closest more aggregated category that contains at least eleven observations. The categories used on our aggregation schema are presented in Figure 2.



**Fig. 2** Aggregation schema

For all skill estimates, we present two values. The probabilities shown in brackets are the maximum likelihood estimates for the specified hit whereas the other probabilities are the maximum a posteriori probability estimations which include a prior assumption. The prior assumption is included in the estimation as described in [2]. For categories with more than eleven observations, both probabilities are equal. The column *succ* states for each combination the probability that the hit lands in the target field and the column *fault* contains the probability of a technical error. The remaining probability is the probability that the hit was successful but the ball deviated into a neighbor-field of the target field.

Table 3 specifies the estimated probabilities of Julius Brink for defense, receptions, settings and blocks. The estimated probabilities fit the intentions we had when we defined the hits, e.g., receptions have a higher success rate than defense actions and hard balls are harder to defend or receive than normal balls. For the blocking skills, the first three columns after the number of observations describe the possible results of a block that catches the ball, while the last column is the probability

that the block misses the ball. Since Jonas Reckermann is the designated blocking player in the German team, Julius Brink has done nearly no blocks in all these matches (see Table 13 in Section 4, where the substantially larger blocking counts of Jonas Reckermann are presented).

The following sections present the skill estimates of all players of the Olympic final. The database is subdivided into pre-final matches and the final much such that the skills are evaluated three times: for all match, all matches except the final match, and the final match only. In every case, the serving and attacking skills as well as the defense, reception, setting and blocking skills of all players are estimated.

### 3 All Matches

**Table 2** Input data from all matches: Julius Brink – Serves and Attack-Hits

target fields		Q11-Q14				Q21-Q24				Q31-Q34			
performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	
Serve													
$S_F$	P01 - P04	45	0.91 (0.91)	0.00 (0.00)	53	0.91 (0.91)	0.09 (0.09)	-	-	-	-	-	
$S_J$		38	0.92 (0.92)	0.00 (0.00)	17	0.76 (0.76)	0.18 (0.18)	-	-	-	-	-	
Attack-Hit													
$F_{SM}$	out	0	0.88 ( - )	0.02 ( - )	0	0.88 ( - )	0.02 ( - )	-	-	-	-	-	
	P11-P14	0	0.88 ( - )	0.02 ( - )	0	0.88 ( - )	0.02 ( - )	-	-	-	-	-	
	P21-P24	65	0.88 (0.88)	0.03 (0.03)	21	0.90 (0.90)	0.00 (0.00)	-	-	-	-	-	
	P31-P34	9	0.80 (0.78)	0.00 (0.00)	3	0.91 (1.00)	0.01 (0.00)	-	-	-	-	-	
$F_E$	out	0	0.71 ( - )	0.10 ( - )	1	0.74 (1.00)	0.09 (0.00)	-	-	-	-	-	
	P11-P14	0	0.71 ( - )	0.10 ( - )	1	0.74 (1.00)	0.09 (0.00)	-	-	-	-	-	
	P21-P24	8	0.65 (0.63)	0.12 (0.13)	9	0.77 (0.78)	0.11 (0.11)	-	-	-	-	-	
	P31-P34	1	0.65 (0.00)	0.09 (0.00)	1	0.74 (1.00)	0.09 (0.00)	-	-	-	-	-	
$F_P$	out	0	0.96 ( - )	0.04 ( - )	0	0.96 ( - )	0.04 ( - )	0	0.96 ( - )	0.04 ( - )	0	0.96 ( - )	
	P11-P14	0	0.96 ( - )	0.04 ( - )	0	0.96 ( - )	0.04 ( - )	0	0.96 ( - )	0.04 ( - )	0	0.96 ( - )	
	P21-P24	9	0.99 (1.00)	0.01 (0.00)	35	0.97 (0.97)	0.03 (0.03)	0	0.96 ( - )	0.04 ( - )	0	0.96 ( - )	
	P31-P34	2	0.97 (1.00)	0.03 (0.00)	3	0.88 (0.67)	0.12 (0.33)	0	0.96 ( - )	0.04 ( - )	0	0.96 ( - )	

**Table 3** Input data from all matches: Julius Brink – Defence, Reception, Set, Block

attack strength		<i>normal</i>			<i>hard</i>		
performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	
Defence	<i>d</i>	24	0.83 (0.83)	0.08 (0.08)	23	0.57 (0.57)	0.35 (0.35)
	<i>d<sub>m</sub></i>	45	0.69 (0.69)	0.27 (0.27)	29	0.34 (0.34)	0.59 (0.59)
Reception	<i>r</i>	35	0.97 (0.97)	0.03 (0.03)	10	0.81 (0.80)	0.09 (0.10)
	<i>r<sub>m</sub></i>	53	0.94 (0.94)	0.02 (0.02)	6	0.98 (1.00)	0.01 (0.00)
Set	<i>s</i>	157	0.99 (0.99)	0.00 (0.00)	-	-	-
performance		#	<i>direct point over net but no point fault misses ball</i>				
Block	<i>b</i>	6	0.17	0.17	0.17	0.50	

**Table 4** Input data from all matches: Jonas Reckermann – Serves and Attack-Hits

target fields		Q11-Q14		Q21-Q24		Q31-Q34				
performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	
Serve										
$S_F$	P01 - P04	39	0.82 (0.82)	0.00 (0.00)	43	0.93 (0.93)	0.05 (0.05)	-	-	-
		41	0.83 (0.83)	0.02 (0.02)	27	0.74 (0.74)	0.26 (0.26)	-	-	-
$S_J$	P01 - P04	39	0.82 (0.82)	0.00 (0.00)	43	0.93 (0.93)	0.05 (0.05)	-	-	-
		41	0.83 (0.83)	0.02 (0.02)	27	0.74 (0.74)	0.26 (0.26)	-	-	-
Attack-Hit										
$F_{SM}$	out	0	0.95 ( - )	0.02 ( - )	0	0.95 ( - )	0.02 ( - )	-	-	-
	P11-P14	0	0.95 ( - )	0.02 ( - )	0	0.95 ( - )	0.02 ( - )	-	-	-
	P21-P24	68	0.97 (0.97)	0.00 (0.00)	33	0.88 (0.88)	0.06 (0.06)	-	-	-
	P31-P34	9	0.99 (1.00)	0.00 (0.00)	3	0.96 (1.00)	0.01 (0.00)	-	-	-
$F_E$	out	0	0.93 ( - )	0.00 ( - )	1	0.94 (1.00)	0.00 (0.00)	-	-	-
	P11-P14	0	0.93 ( - )	0.00 ( - )	0	0.93 ( - )	0.00 ( - )	-	-	-
	P21-P24	7	0.88 (0.86)	0.00 (0.00)	5	0.96 (1.00)	0.00 (0.00)	-	-	-
	P31-P34	0	0.93 ( - )	0.00 ( - )	1	0.94 (1.00)	0.00 (0.00)	-	-	-
$F_P$	out	0	0.89 ( - )	0.04 ( - )	0	0.89 ( - )	0.04 ( - )	0	0.89 ( - )	0.04 ( - )
	P11-P14	0	0.89 ( - )	0.04 ( - )	0	0.89 ( - )	0.04 ( - )	0	0.89 ( - )	0.04 ( - )
	P21-P24	4	0.75 (0.50)	0.03 (0.00)	33	0.94 (0.94)	0.03 (0.03)	1	0.90 (1.00)	0.04 (0.00)
	P31-P34	0	0.89 ( - )	0.04 ( - )	8	0.88 (0.88)	0.10 (0.13)	0	0.89 ( - )	0.04 ( - )

**Table 5** Input data from all matches: Jonas Reckermann – Defence, Reception, Set

		attack strength		<i>normal</i>		<i>hard</i>	
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Defence	<i>d</i>	28	0.86 (0.86)	0.07 (0.07)	2	0.77 (0.50)	0.05 (0.00)
	<i>d<sub>m</sub></i>	25	0.84 (0.84)	0.08 (0.08)	1	0.73 (0.00)	0.20 (1.00)
Reception	<i>r</i>	34	1.00 (1.00)	0.00 (0.00)	12	0.83 (0.83)	0.08 (0.08)
	<i>r<sub>m</sub></i>	75	0.95 (0.95)	0.03 (0.03)	10	0.81 (0.80)	0.09 (0.10)
Set	<i>s</i>	152	0.97 (0.97)	0.01 (0.01)	-	-	-
		performance	#	<i>direct point over net but no point</i>	<i>fault misses ball</i>		
Block	<i>b</i>	263	0.11	0.12	0.14	0.63	

**Table 6** Input data from all matches: Alison Cerutti – Serves and Attack-Hits

		target fields		P11-P14		P21-P24		P31-P34		
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Serve										
<i>S<sub>F</sub></i>	Q01 - Q04	51	0.86 (0.86)	0.00 (0.00)	48	0.96 (0.96)	0.04 (0.04)	-	-	-
<i>S<sub>J</sub></i>		52	0.73 (0.73)	0.06 (0.06)	19	0.79 (0.79)	0.21 (0.21)	-	-	-
Attack-Hit										
<i>F<sub>SM</sub></i>	out	0	0.88 ( - )	0.06 ( - )	0	0.88 ( - )	0.06 ( - )	-	-	-
	Q11-Q14	0	0.88 ( - )	0.06 ( - )	0	0.88 ( - )	0.06 ( - )	-	-	-
	Q21-Q24	56	0.91 (0.91)	0.04 (0.04)	24	0.83 (0.83)	0.08 (0.08)	-	-	-
	Q31-Q34	9	0.98 (1.00)	0.01 (0.00)	6	0.77 (0.67)	0.21 (0.33)	-	-	-
<i>F<sub>E</sub></i>	out	0	0.92 ( - )	0.00 ( - )	0	0.92 ( - )	0.00 ( - )	-	-	-
	Q11-Q14	1	0.93 (1.00)	0.00 (0.00)	1	0.93 (1.00)	0.00 (0.00)	-	-	-
	Q21-Q24	5	0.87 (0.80)	0.00 (0.00)	5	0.96 (1.00)	0.00 (0.00)	-	-	-
	Q31-Q34	0	0.92 ( - )	0.00 ( - )	1	0.93 (1.00)	0.00 (0.00)	-	-	-
<i>F<sub>P</sub></i>	out	0	0.95 ( - )	0.00 ( - )	0	0.95 ( - )	0.00 ( - )	0	0.95 ( - )	0.00 ( - )
	Q11-Q14	0	0.95 ( - )	0.00 ( - )	0	0.95 ( - )	0.00 ( - )	0	0.95 ( - )	0.00 ( - )
	Q21-Q24	2	0.87 (0.50)	0.00 (0.00)	10	1.00 (1.00)	0.00 (0.00)	0	0.95 ( - )	0.00 ( - )
	Q31-Q34	2	0.96 (1.00)	0.00 (0.00)	5	0.97 (1.00)	0.00 (0.00)	0	0.95 ( - )	0.00 ( - )

**Table 7** Input data from all matches: Alison Cerutti – Serves and Attack-Hits

attack strength		<i>normal</i>			<i>hard</i>		
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Defence	<i>d</i>	36	0.78 (0.78)	0.08 (0.08)	8	0.46 (0.38)	0.21 (0.25)
	<i>d<sub>m</sub></i>	23	0.74 (0.74)	0.22 (0.22)	7	0.41 (0.29)	0.47 (0.57)
Reception	<i>r</i>	32	0.94 (0.94)	0.00 (0.00)	6	0.98 (1.00)	0.00 (0.00)
	<i>r<sub>m</sub></i>	41	0.98 (0.98)	0.00 (0.00)	1	0.87 (0.00)	0.11 (1.00)
Set	<i>s</i>	232	0.98 (0.98)	0.00 (0.00)	-	-	-
	performance	#	<i>direct point over net but no point fault misses ball</i>				
Block	<i>b</i>	303	0.12	0.14	0.14	0.60	

**Table 8** Input data from all matches: Emanuel Rego – Serves and Attack-Hits

target fields		P11-P14		P21-P24			P31-P34				
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	
Serve											
$S_F$	Q01 - Q04	52	0.96 (0.96)	0.02 (0.02)	53	0.87 (0.87)	0.11 (0.11)	-	-	-	
$S_J$		41	0.98 (0.98)	0.00 (0.00)	17	0.76 (0.76)	0.18 (0.18)	-	-	-	
Attack-Hit											
$F_{SM}$	out	0	0.87 ( - )	0.06 ( - )	0	0.87 ( - )	0.06 ( - )	-	-	-	
	Q11-Q14	0	0.87 ( - )	0.06 ( - )	0	0.87 ( - )	0.06 ( - )	-	-	-	
	Q21-Q24	104	0.90 (0.90)	0.03 (0.03)	71	0.80 (0.80)	0.10 (0.10)	-	-	-	
	Q31-Q34	17	0.94 (0.94)	0.00 (0.00)	8	0.78 (0.75)	0.20 (0.25)	-	-	-	
$F_E$	out	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )	0.00 ( - )	-	-	-	
	Q11-Q14	1	1.00 (1.00)	0.00 (0.00)	0	1.00 ( - )	0.00 ( - )	-	-	-	
	Q21-Q24	14	1.00 (1.00)	0.00 (0.00)	5	1.00 (1.00)	0.00 (0.00)	-	-	-	
	Q31-Q34	0	1.00 ( - )	0.00 ( - )	2	1.00 (1.00)	0.00 (0.00)	-	-	-	
$F_P$	out	0	0.94 ( - )	0.00 ( - )	0	0.94 ( - )	0.00 ( - )	0	0.94 ( - )	0.00 ( - )	
	Q11-Q14	0	0.94 ( - )	0.00 ( - )	0	0.94 ( - )	0.00 ( - )	0	0.94 ( - )	0.00 ( - )	
	Q21-Q24	5	0.97 (1.00)	0.00 (0.00)	25	0.92 (0.92)	0.00 (0.00)	1	0.95 (1.00)	0.00 (0.00)	
	Q31-Q34	0	0.94 ( - )	0.00 ( - )	4	0.96 (1.00)	0.00 (0.00)	0	0.94 ( - )	0.00 ( - )	



**Table 9** Input data from all matches: Emanuel Rego – Defence, Reception, Set, Block

attack strength		<i>normal</i>			<i>hard</i>		
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Defence	<i>d</i>	29	0.86 (0.86)	0.07 (0.07)	37	0.24 (0.24)	0.46 (0.46)
	<i>d<sub>m</sub></i>	54	0.65 (0.65)	0.31 (0.31)	29	0.34 (0.34)	0.45 (0.45)
Reception	<i>r</i>	82	0.93 (0.93)	0.05 (0.05)	20	0.65 (0.65)	0.05 (0.05)
	<i>r<sub>m</sub></i>	75	0.93 (0.93)	0.01 (0.01)	3	0.95 (1.00)	0.01 (0.00)
Set	<i>s</i>	107	1.00 (1.00)	0.00 (0.00)	-	-	-
	performance	#	<i>direct point over net but no point fault misses ball</i>				
Block	<i>b</i>	13	0.08	0.38	0.08	0.46	

## 4 All Matches except final match

**Table 10** Input data from all matches except final: Julius Brink – Serves and Attack-Hits

target fields		Q11-Q14			Q21-Q24			Q31-Q34		
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Serve										
$S_F$	P01 - P04	34	0.88 (0.88)	0.00 (0.00)	43	0.88 (0.88)	0.12 (0.12)	-	-	-
$S_J$		34	0.94 (0.94)	0.00 (0.00)	16	0.75 (0.75)	0.19 (0.19)	-	-	-
Attack-Hit										
$F_{SM}$	out	0	0.86 ( - )	0.02 ( - )	0	0.86 ( - )	0.02 ( - )	-	-	-
	P11-P14	0	0.86 ( - )	0.02 ( - )	0	0.86 ( - )	0.02 ( - )	-	-	-
	P21-P24	55	0.85 (0.85)	0.04 (0.04)	17	0.94 (0.94)	0.00 (0.00)	-	-	-
	P31-P34	7	0.77 (0.71)	0.01 (0.00)	2	0.89 (1.00)	0.02 (0.00)	-	-	-
$F_E$	out	0	0.76 ( - )	0.06 ( - )	0	0.76 ( - )	0.06 ( - )	-	-	-
	P11-P14	0	0.76 ( - )	0.06 ( - )	1	0.79 (1.00)	0.05 (0.00)	-	-	-
	P21-P24	7	0.73 (0.71)	0.11 (0.14)	7	0.82 (0.86)	0.02 (0.00)	-	-	-
	P31-P34	1	0.70 (0.00)	0.05 (0.00)	1	0.79 (1.00)	0.05 (0.00)	-	-	-
$F_P$	out	0	0.95 ( - )	0.05 ( - )	0	0.95 ( - )	0.05 ( - )	0	0.95 ( - )	0.05 ( - )
	P11-P14	0	0.95 ( - )	0.05 ( - )	0	0.95 ( - )	0.05 ( - )	0	0.95 ( - )	0.05 ( - )
	P21-P24	8	0.99 (1.00)	0.01 (0.00)	30	0.97 (0.97)	0.03 (0.03)	0	0.95 ( - )	0.05 ( - )
	P31-P34	2	0.96 (1.00)	0.04 (0.00)	3	0.88 (0.67)	0.12 (0.33)	0	0.95 ( - )	0.05 ( - )

**Table 11** Input data from all matches except final: Julius Brink – Defence, Reception, Set, Block

attack strength		<i>normal</i>			<i>hard</i>		
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Defence	<i>d</i>	20	0.85 (0.85)	0.05 (0.05)	14	0.71 (0.71)	0.21 (0.21)
	<i>d<sub>m</sub></i>	29	0.93 (0.93)	0.00 (0.00)	13	0.46 (0.46)	0.38 (0.38)
Reception	<i>r</i>	34	1.00 (1.00)	0.00 (0.00)	9	0.90 (0.89)	0.10 (0.11)
	<i>r<sub>m</sub></i>	42	0.95 (0.95)	0.02 (0.02)	3	0.97 (1.00)	0.02 (0.00)
Set	<i>s</i>	117	0.99 (0.99)	0.00 (0.00)	-	-	-
	performance	#	<i>direct point over net but no point fault misses ball</i>				
Block	<i>b</i>	5	0.20	0.20	0.20	0.40	

**Table 12** Input data from all matches except final: Jonas Reckermann – Serves and Attack-Hits

target fields		Q11-Q14			Q21-Q24			Q31-Q34		
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Serve										
<i>S<sub>F</sub></i>	P01 - P04	25	0.76 (0.76)	0.00 (0.00)	35	0.91 (0.91)	0.06 (0.06)	-	-	-
<i>S<sub>J</sub></i>		38	0.82 (0.82)	0.03 (0.03)	23	0.70 (0.70)	0.30 (0.30)	-	-	-
Attack-Hit										
<i>F<sub>SM</sub></i>	out	0	0.95 ( - )	0.02 ( - )	0	0.95 ( - )	0.02 ( - )	-	-	-
	P11-P14	0	0.95 ( - )	0.02 ( - )	0	0.95 ( - )	0.02 ( - )	-	-	-
	P21-P24	49	1.00 (1.00)	0.00 (0.00)	25	0.84 (0.84)	0.08 (0.08)	-	-	-
	P31-P34	9	0.99 (1.00)	0.00 (0.00)	3	0.97 (1.00)	0.02 (0.00)	-	-	-
<i>F<sub>E</sub></i>	out	0	0.93 ( - )	0.00 ( - )	1	0.94 (1.00)	0.00 (0.00)	-	-	-
	P11-P14	0	0.93 ( - )	0.00 ( - )	0	0.93 ( - )	0.00 ( - )	-	-	-
	P21-P24	7	0.88 (0.86)	0.00 (0.00)	5	0.96 (1.00)	0.00 (0.00)	-	-	-
	P31-P34	0	0.93 ( - )	0.00 ( - )	1	0.94 (1.00)	0.00 (0.00)	-	-	-
<i>F<sub>P</sub></i>	out	0	0.88 ( - )	0.06 ( - )	0	0.88 ( - )	0.06 ( - )	0	0.88 ( - )	0.06 ( - )
	P11-P14	0	0.88 ( - )	0.06 ( - )	0	0.88 ( - )	0.06 ( - )	0	0.88 ( - )	0.06 ( - )
	P21-P24	3	0.82 (0.67)	0.04 (0.00)	23	0.91 (0.91)	0.04 (0.04)	1	0.89 (1.00)	0.05 (0.00)
	P31-P34	0	0.88 ( - )	0.06 ( - )	7	0.87 (0.86)	0.11 (0.14)	0	0.88 ( - )	0.06 ( - )

**Table 13** Input data from all matches except final: Jonas Reckermann – Defence, Reception, Set

attack strength		<i>normal</i>			<i>hard</i>		
performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	
Defence	<i>d</i>	20	0.85 (0.85)	0.10 (0.10)	1	0.74 (0.00)	0.09 (0.00)
	<i>d<sub>m</sub></i>	19	0.84 (0.84)	0.05 (0.05)	0	0.84 ( - )	0.05 ( - )
Reception	<i>r</i>	27	1.00 (1.00)	0.00 (0.00)	7	0.90 (0.86)	0.10 (0.14)
	<i>r<sub>m</sub></i>	61	0.95 (0.95)	0.02 (0.02)	3	0.97 (1.00)	0.01 (0.00)
Set	<i>s</i>	128	0.98 (0.98)	0.00 (0.00)	-	-	-
performance	#	<i>direct point over net but no point fault misses ball</i>					
Block	<i>b</i>	200	0.12	0.13	0.14	0.62	

**Table 14** Input data from all matches except final: Alison Cerutti – Serves and Attack-Hits

target fields		P11-P14		P21-P24			P31-P34		
performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Serve									
<i>S<sub>F</sub></i>	Q01 - Q04	45	0.89 (0.89)	0.00 (0.00)	46	0.96 (0.96)	0.04 (0.04)	-	-
<i>S<sub>J</sub></i>		37	0.70 (0.70)	0.05 (0.05)	17	0.76 (0.76)	0.24 (0.24)	-	-
Attack-Hit									
<i>F<sub>SM</sub></i>	out	0	0.87 ( - )	0.07 ( - )	0	0.87 ( - )	0.07 ( - )	-	-
	Q11-Q14	0	0.87 ( - )	0.07 ( - )	0	0.87 ( - )	0.07 ( - )	-	-
	Q21-Q24	47	0.89 (0.89)	0.04 (0.04)	23	0.83 (0.83)	0.09 (0.09)	-	-
	Q31-Q34	8	0.96 (1.00)	0.02 (0.00)	4	0.73 (0.50)	0.23 (0.50)	-	-
<i>F<sub>E</sub></i>	out	0	0.90 ( - )	0.00 ( - )	0	0.90 ( - )	0.00 ( - )	-	-
	Q11-Q14	1	0.91 (1.00)	0.00 (0.00)	1	0.91 (1.00)	0.00 (0.00)	-	-
	Q21-Q24	4	0.85 (0.75)	0.00 (0.00)	4	0.94 (1.00)	0.00 (0.00)	-	-
	Q31-Q34	0	0.90 ( - )	0.00 ( - )	0	0.90 ( - )	0.00 ( - )	-	-
<i>F<sub>P</sub></i>	out	0	0.94 ( - )	0.00 ( - )	0	0.94 ( - )	0.00 ( - )	0	0.94 ( - )
	Q11-Q14	0	0.94 ( - )	0.00 ( - )	0	0.94 ( - )	0.00 ( - )	0	0.94 ( - )
	Q21-Q24	2	0.86 (0.50)	0.00 (0.00)	10	0.99 (1.00)	0.00 (0.00)	0	0.94 ( - )
	Q31-Q34	1	0.95 (1.00)	0.00 (0.00)	5	0.97 (1.00)	0.00 (0.00)	0	0.94 ( - )

**Table 15** Input data from all matches except final: Alison Cerutti – Defence, Reception, Set, Block

attack strength		<i>normal</i>			<i>hard</i>		
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Defence	<i>d</i>	29	0.83 (0.83)	0.03 (0.03)	8	0.47 (0.38)	0.20 (0.25)
	<i>d<sub>m</sub></i>	17	0.76 (0.76)	0.18 (0.18)	5	0.55 (0.40)	0.42 (0.60)
Reception	<i>r</i>	26	1.00 (1.00)	0.00 (0.00)	5	1.00 (1.00)	0.00 (0.00)
	<i>r<sub>m</sub></i>	36	0.97 (0.97)	0.00 (0.00)	1	0.86 (0.00)	0.12 (1.00)
Set	<i>s</i>	180	0.98 (0.98)	0.00 (0.00)	-	-	-
	performance	#	<i>direct point over net but no point fault misses ball</i>				
Block	<i>b</i>	254	0.13	0.13	0.15	0.59	

**Table 16** Input data from all matches except final: Emanuel Rego – Serves and Attack-Hits

target fields		P11-P14		P21-P24			P31-P34			
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Serve										
$S_F$	Q01 - Q04	42	0.98 (0.98)	0.02 (0.02)	44	0.84 (0.84)	0.14 (0.14)	-	-	-
$S_J$		32	1.00 (1.00)	0.00 (0.00)	15	0.73 (0.73)	0.20 (0.20)	-	-	-
Attack-Hit										
$F_{SM}$	out	0	0.87 ( - )	0.06 ( - )	0	0.87 ( - )	0.06 ( - )	-	-	-
	Q11-Q14	0	0.87 ( - )	0.06 ( - )	0	0.87 ( - )	0.06 ( - )	-	-	-
	Q21-Q24	83	0.90 (0.90)	0.02 (0.02)	57	0.81 (0.81)	0.11 (0.11)	-	-	-
	Q31-Q34	12	1.00 (1.00)	0.00 (0.00)	4	0.73 (0.50)	0.22 (0.50)	-	-	-
$F_E$	out	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )	0.00 ( - )	-	-	-
	Q11-Q14	1	1.00 (1.00)	0.00 (0.00)	0	1.00 ( - )	0.00 ( - )	-	-	-
	Q21-Q24	12	1.00 (1.00)	0.00 (0.00)	5	1.00 (1.00)	0.00 (0.00)	-	-	-
	Q31-Q34	0	1.00 ( - )	0.00 ( - )	1	1.00 (1.00)	0.00 (0.00)	-	-	-
$F_P$	out	0	0.96 ( - )	0.00 ( - )	0	0.96 ( - )	0.00 ( - )	0	0.96 ( - )	0.00 ( - )
	Q11-Q14	0	0.96 ( - )	0.00 ( - )	0	0.96 ( - )	0.00 ( - )	0	0.96 ( - )	0.00 ( - )
	Q21-Q24	4	0.97 (1.00)	0.00 (0.00)	18	0.94 (0.94)	0.00 (0.00)	1	0.96 (1.00)	0.00 (0.00)
	Q31-Q34	0	0.96 ( - )	0.00 ( - )	2	0.97 (1.00)	0.00 (0.00)	0	0.96 ( - )	0.00 ( - )

**Table 17** Input data from all matches except final: Emanuel Rego – Defence, Reception, Set, Block

attack strength		<i>normal</i>			<i>hard</i>		
performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	
Defence	<i>d</i>	22 0.86 (0.86)	0.05 (0.05)	24	0.17 (0.17)	0.50 (0.50)	
	<i>d<sub>m</sub></i>	36 0.81 (0.81)	0.17 (0.17)	22	0.41 (0.41)	0.36 (0.36)	
Reception	<i>r</i>	69 0.94 (0.94)	0.06 (0.06)	19	0.68 (0.68)	0.05 (0.05)	
	<i>r<sub>m</sub></i>	48 0.98 (0.98)	0.00 (0.00)	3	0.99 (1.00)	0.00 (0.00)	
Set	<i>s</i>	95 1.00 (1.00)	0.00 (0.00)	-	-	-	
performance		#	<i>direct point over net but no point fault misses ball</i>				
Block	<i>b</i>	11	0.09	0.36	0.09	0.45	

## 5 Only final match

**Table 18** Input data from final match: Julius Brink – Serves and Attack-Hits (final match only)

target fields		Q11-Q14		Q21-Q24		Q31-Q34				
performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	
Serve										
$S_F$	P01 - P04	11	1.00 (1.00)	0.00 (0.00)	10	1.00 (1.00)	0.00 (0.00)	-	-	-
$S_J$		4	0.81 (0.75)	0.00 (0.00)	1	0.89 (1.00)	0.00 (0.00)	-	-	-
Attack-Hit										
$F_{SM}$	out	0	0.94 ( - )	0.00 ( - )	0	0.94 ( - )	0.00 ( - )	-	-	-
	P11-P14	0	0.94 ( - )	0.00 ( - )	0	0.94 ( - )	0.00 ( - )	-	-	-
	P21-P24	10	0.99 (1.00)	0.00 (0.00)	4	0.87 (0.75)	0.00 (0.00)	-	-	-
	P31-P34	2	0.95 (1.00)	0.00 (0.00)	1	0.95 (1.00)	0.00 (0.00)	-	-	-
$F_E$	out	0	0.75 ( - )	0.11 ( - )	1	0.76 (1.00)	0.11 (0.00)	-	-	-
	P11-P14	0	0.75 ( - )	0.11 ( - )	0	0.75 ( - )	0.11 ( - )	-	-	-
	P21-P24	1	0.67 (0.00)	0.11 (0.00)	2	0.68 (0.50)	0.20 (0.50)	-	-	-
	P31-P34	0	0.75 ( - )	0.11 ( - )	0	0.75 ( - )	0.11 ( - )	-	-	-
$F_P$	out	0	0.95 ( - )	0.02 ( - )	0	0.95 ( - )	0.02 ( - )	0	0.95 ( - )	0.02 ( - )
	P11-P14	0	0.95 ( - )	0.02 ( - )	0	0.95 ( - )	0.02 ( - )	0	0.95 ( - )	0.02 ( - )
	P21-P24	1	0.96 (1.00)	0.01 (0.00)	5	1.00 (1.00)	0.00 (0.00)	0	0.95 ( - )	0.02 ( - )
	P31-P34	0	0.95 ( - )	0.02 ( - )	0	0.95 ( - )	0.02 ( - )	0	0.95 ( - )	0.02 ( - )

**Table 19** Input data from final match: Julius Brink – Defence, Reception, Set, Block (final match only)

attack strength		<i>normal</i>			<i>hard</i>		
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Defence	<i>d</i>	4	0.57 (0.75)	0.38 (0.25)	9	0.36 (0.33)	0.54 (0.56)
	<i>d<sub>m</sub></i>	16	0.25 (0.25)	0.75 (0.75)	16	0.25 (0.25)	0.75 (0.75)
Reception	<i>r</i>	1	0.59 (0.00)	0.23 (1.00)	1	0.59 (0.00)	0.14 (0.00)
	<i>r<sub>m</sub></i>	11	0.91 (0.91)	0.00 (0.00)	3	0.95 (1.00)	0.00 (0.00)
Set	<i>s</i>	40	0.98 (0.98)	0.00 (0.00)	-	-	-
	performance	#	<i>direct point over net but no point fault misses ball</i>				
Block	<i>b</i>	1	0.00	0.00	0.00	1.00	

**Table 20** Input data from final match: Jonas Reckermann – Serves and Attack-Hits (final match only)

target fields		Q11-Q14		Q21-Q24			Q31-Q34				
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	
Serve											
<i>S<sub>F</sub></i>	P01 - P04	14	0.93 (0.93)	0.00 (0.00)	8	0.99 (1.00)	0.00 (0.00)	-	-	-	
<i>S<sub>J</sub></i>		3	1.00 (1.00)	0.00 (0.00)	4	1.00 (1.00)	0.00 (0.00)	-	-	-	
Attack-Hit											
<i>F<sub>SM</sub></i>	out	0	0.93 ( - )	0.00 ( - )	0	0.93 ( - )	0.00 ( - )	-	-	-	
	P11-P14	0	0.93 ( - )	0.00 ( - )	0	0.93 ( - )	0.00 ( - )	-	-	-	
	P21-P24	19	0.89 (0.89)	0.00 (0.00)	8	0.98 (1.00)	0.00 (0.00)	-	-	-	
	P31-P34	0	0.93 ( - )	0.00 ( - )	0	0.93 ( - )	0.00 ( - )	-	-	-	
<i>F<sub>E</sub></i>	out	0	0.92 ( - )	0.00 ( - )	0	0.92 ( - )	0.00 ( - )	-	-	-	
	P11-P14	0	0.92 ( - )	0.00 ( - )	0	0.92 ( - )	0.00 ( - )	-	-	-	
	P21-P24	0	0.92 ( - )	0.00 ( - )	0	0.92 ( - )	0.00 ( - )	-	-	-	
	P31-P34	0	0.92 ( - )	0.00 ( - )	0	0.92 ( - )	0.00 ( - )	-	-	-	
<i>F<sub>P</sub></i>	out	0	0.92 ( - )	0.00 ( - )	0	0.92 ( - )	0.00 ( - )	0	0.92 ( - )	0.00 ( - )	
	P11-P14	0	0.92 ( - )	0.00 ( - )	0	0.92 ( - )	0.00 ( - )	0	0.92 ( - )	0.00 ( - )	
	P21-P24	1	0.83 (0.00)	0.00 (0.00)	10	0.99 (1.00)	0.00 (0.00)	0	0.92 ( - )	0.00 ( - )	
	P31-P34	0	0.92 ( - )	0.00 ( - )	1	0.92 (1.00)	0.00 (0.00)	0	0.92 ( - )	0.00 ( - )	

**Table 21** Input data from final match: Jonas Reckermann – Defence, Reception, Set (final match only)

attack strength		<i>normal</i>			<i>hard</i>		
performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	
Defence	<i>d</i>	8	0.88 (0.88)	0.00 (0.00)	1	0.89 (1.00)	0.01 (0.00)
	<i>d<sub>m</sub></i>	6	0.78 (0.83)	0.22 (0.17)	1	0.68 (0.00)	0.31 (1.00)
Reception	<i>r</i>	7	0.97 (1.00)	0.00 (0.00)	5	0.86 (0.80)	0.00 (0.00)
	<i>r<sub>m</sub></i>	14	0.93 (0.93)	0.07 (0.07)	7	0.77 (0.71)	0.13 (0.14)
Set	<i>s</i>	24	0.88 (0.88)	0.04 (0.04)	-	-	-
performance	#	<i>direct point over net but no point fault misses ball</i>					
Block	<i>b</i>	63	0.08	0.10	0.16	0.67	

**Table 22** Input data from final match: Alison Cerutti – Serves and Attack-Hits (final match only)

target fields		P11-P14		P21-P24		P31-P34			
performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Serve									
<i>S<sub>F</sub></i>	Q01 - Q04	6	0.70 (0.67)	0.00 (0.00)	2	0.80 (1.00)	0.00 (0.00)	-	-
<i>S<sub>J</sub></i>		15	0.80 (0.80)	0.07 (0.07)	2	0.86 (1.00)	0.05 (0.00)	-	-
Attack-Hit									
<i>F<sub>SM</sub></i>	out	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )	0.00 ( - )	-	-
	Q11-Q14	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )	0.00 ( - )	-	-
	Q21-Q24	9	1.00 (1.00)	0.00 (0.00)	1	1.00 (1.00)	0.00 (0.00)	-	-
	Q31-Q34	1	1.00 (1.00)	0.00 (0.00)	2	1.00 (1.00)	0.00 (0.00)	-	-
<i>F<sub>E</sub></i>	out	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )	0.00 ( - )	-	-
	Q11-Q14	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )	0.00 ( - )	-	-
	Q21-Q24	1	1.00 (1.00)	0.00 (0.00)	1	1.00 (1.00)	0.00 (0.00)	-	-
	Q31-Q34	0	1.00 ( - )	0.00 ( - )	1	1.00 (1.00)	0.00 (0.00)	-	-
<i>F<sub>P</sub></i>	out	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )
	Q11-Q14	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )
	Q21-Q24	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )
	Q31-Q34	1	1.00 (1.00)	0.00 (0.00)	0	1.00 ( - )	0.00 ( - )	0	1.00 ( - )

**Table 23** Input data from final match: Alison Cerutti – Defence, Reception, Set, Block (final match only)

attack strength		<i>normal</i>			<i>hard</i>		
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Defence	<i>d</i>	7	0.57 (0.57)	0.29 (0.29)	0	0.56 ( - )	0.30 ( - )
	<i>d<sub>m</sub></i>	6	0.59 (0.67)	0.35 (0.33)	2	0.41 (0.00)	0.39 (0.50)
Reception	<i>r</i>	6	0.69 (0.67)	0.00 (0.00)	1	0.77 (1.00)	0.00 (0.00)
	<i>r<sub>m</sub></i>	5	0.98 (1.00)	0.00 (0.00)	0	0.91 ( - )	0.00 ( - )
Set	<i>s</i>	52	1.00 (1.00)	0.00 (0.00)	-	-	-
performance		#	<i>direct point over net but no point fault misses ball</i>				
Block	<i>b</i>	49	0.06	0.16	0.10	0.67	

**Table 24** Input data from final match: Emanuel Rego – Serves and Attack-Hits (final match only)

target fields		P11-P14		P21-P24			P31-P34			
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Serve										
$S_F$	Q01 - Q04	10	0.90 (0.90)	0.00 (0.00)	9	0.99 (1.00)	0.00 (0.00)	-	-	-
		9	0.89 (0.89)	0.00 (0.00)	2	0.93 (1.00)	0.00 (0.00)	-	-	-
$S_J$	Q01 - Q04	10	0.90 (0.90)	0.00 (0.00)	9	0.99 (1.00)	0.00 (0.00)	-	-	-
		9	0.89 (0.89)	0.00 (0.00)	2	0.93 (1.00)	0.00 (0.00)	-	-	-
Attack-Hit										
$F_{SM}$	out	0	0.86 ( - )	0.05 ( - )	0	0.86 ( - )	0.05 ( - )	-	-	-
	Q11-Q14	0	0.86 ( - )	0.05 ( - )	0	0.86 ( - )	0.05 ( - )	-	-	-
	Q21-Q24	21	0.90 (0.90)	0.05 (0.05)	14	0.79 (0.79)	0.07 (0.07)	-	-	-
	Q31-Q34	5	0.83 (0.80)	0.02 (0.00)	4	0.91 (1.00)	0.03 (0.00)	-	-	-
$F_E$	out	0	0.91 ( - )	0.03 ( - )	0	0.91 ( - )	0.03 ( - )	-	-	-
	Q11-Q14	0	0.91 ( - )	0.03 ( - )	0	0.91 ( - )	0.03 ( - )	-	-	-
	Q21-Q24	2	0.93 (1.00)	0.02 (0.00)	0	0.91 ( - )	0.03 ( - )	-	-	-
	Q31-Q34	0	0.91 ( - )	0.03 ( - )	1	0.92 (1.00)	0.02 (0.00)	-	-	-
$F_P$	out	0	0.90 ( - )	0.00 ( - )	0	0.90 ( - )	0.00 ( - )	0	0.90 ( - )	0.00 ( - )
	Q11-Q14	0	0.90 ( - )	0.00 ( - )	0	0.90 ( - )	0.00 ( - )	0	0.90 ( - )	0.00 ( - )
	Q21-Q24	1	0.91 (1.00)	0.00 (0.00)	7	0.87 (0.86)	0.00 (0.00)	0	0.90 ( - )	0.00 ( - )
	Q31-Q34	0	0.90 ( - )	0.00 ( - )	2	0.92 (1.00)	0.00 (0.00)	0	0.90 ( - )	0.00 ( - )



**Table 25** Input data from final match: Emanuel Rego – Defence, Reception, Set, Block (final match only)

attack strength		<i>normal</i>			<i>hard</i>		
	performance	#	<i>succ</i>	<i>fault</i>	#	<i>succ</i>	<i>fault</i>
Defence	<i>d</i>	7	0.75 (0.86)	0.20 (0.14)	13	0.38 (0.38)	0.38 (0.38)
	<i>d<sub>m</sub></i>	18	0.33 (0.33)	0.61 (0.61)	7	0.19 (0.14)	0.69 (0.71)
Reception	<i>r</i>	13	0.85 (0.85)	0.00 (0.00)	1	0.71 (0.00)	0.00 (0.00)
	<i>r<sub>m</sub></i>	27	0.85 (0.85)	0.04 (0.04)	0	0.85 ( - )	0.04 ( - )
Set	<i>s</i>	12	1.00 (1.00)	0.00 (0.00)	-	-	-
	performance	#	<i>direct point over net but no point fault misses ball</i>				
Block	<i>b</i>	2	0.00	0.50	0.00	0.50	

## References

1. Hoffmeister, S., Rambau, J.: Strategy Optimization in Sports – A Two-Scale Approach via Markov Decision Problems (2017). URL [http://www.wm.uni-bayreuth.de/de/download/xcf2d3wd41kj2/preprint{\\\_}sso{\\\_}bv.pdf](http://www.wm.uni-bayreuth.de/de/download/xcf2d3wd41kj2/preprint{\_}sso{\_}bv.pdf)
2. Mitchell, T.M.: Estimating Probabilities. In: Machine Learning, chap. 2, pp. 1–11 (2017)