The best known (n, r)-arcs in PG(2, 13)

Rumen Daskalov
Department of Mathematics
Technical University of Gabrovo
5300 Gabrovo, Bulgaria
daskalovrn@gmail.com

Alfred Wassermann
Department of Mathematics
University of Bayreuth
D-95440 Bayreuth, Germany
alfred.wassermann@uni-bayreuth.de

After the absurd death of Axel Kohnert in 2013 several hardware errors on the site of the University of Bayreuth have occurred and the database for (n, r)-arcs is no longer available. For that reason we decided to reconstruct the database.

Lower and upper bounds on $m_r(2,13)$ [1]

r=2	r=3	r=4	r = 5	r = 6	r = 7
14	23	38-40	49–53	64–66	79
r=8	r = 9	r = 10	r = 11	r = 12	
92	105	118–119	132-133	145–147	

1. A (23,3)-arc ([2])

The points in the arc are

$$(0,0,1),\ (0,1,0),\ (1,0,0),\ (1,1,7),\ (1,8,6),\ (1,12,7),\ (1,5,6),\ (1,1,12),\ (1,8,1),\ (1,12,12),\ (1,5,1),\ (1,10,4),\ (1,2,9),\ (1,3,4),\ (1,11,9),\ (1,10,3),\ (1,2,10),\ (1,3,3),\ (1,11,10),\ (1,4,11),\ (1,6,2),\ (1,9,11),\ (1,7,2)$$

The secant distribution of the arc is $\tau = (44, 26, 43, 70, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

The arc is found by prescribing the group generated by

$$\left\langle \begin{pmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{pmatrix}, \begin{pmatrix} 12 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 12 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 12 \end{pmatrix} \right\rangle.$$

The order of the group is 8.

2. A (38,4)-arc ([2])

The points in the arc are

$$(0,0,1),\ (0,1,0),\ (1,1,1),\ (1,9,3),\ (1,12,12),\ (1,3,9),\ (1,10,4),\ (1,4,10),\ (1,1,8),\ (1,9,11),\ (1,8,1),\ (1,5,12),\ (1,3,7),\ (1,11,9),\ (1,2,4),\ (1,7,3),\ (1,12,5),\ (1,6,10),\ (1,4,2),\ (1,10,6),\ (1,1,9),\ (1,9,1),\ (1,4,12),\ (1,3,3),\ (1,12,4),\ (1,10,10),\ (1,11,6),\ (1,8,5),\ (1,6,11),\ (1,7,2),\ (1,5,8),\ (1,2,7),\ (1,11,2),\ (1,8,6),\ (1,2,11),\ (1,7,5),\ (1,6,8),\ (1,5,7)$$

The secant distribution of the arc is $\tau = (24, 14, 19, 24, 102, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

The arc is found by prescribing the group generated by

$$\left\langle \begin{pmatrix} 1 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \begin{pmatrix} 12 & 0 & 0 \\ 0 & 0 & 12 \\ 0 & 12 & 0 \end{pmatrix}, \begin{pmatrix} 12 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \right\rangle.$$

The order of the group is 12.

3. A (49,5)-arc ([2])

The points in the arc are

(0,1,5), (0,1,6), (0,1,7), (0,1,9), (0,1,12), (1,0,1), (1,0,8), (1,0,12), (1,0,5), (1,1,0), (1,8,0), (1,12,0), (1,5,0), (1,1,1), (1,8,8), (1,12,12), (1,5,5), (1,1,3), (1,8,11), (1,12,10), (1,5,2), (1,1,4), (1,8,6), (1,12,9), (1,5,7), (1,1,12), (1,8,5), (1,12,1), (1,5,8), (1,10,5), (1,2,1), (1,3,8), (1,11,12), (1,10,7), (1,2,4), (1,3,6), (1,11,9), (1,10,9), (1,2,7), (1,3,4), (1,11,6), (1,10,6), (1,2,9), (1,3,7), (1,11,4), (1,4,7), (1,6,4), (1,9,6), (1,7,9)

The secant distribution of the arc is $\tau = (5, 34, 0, 8, 52, 84, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

The arc is found by prescribing the group generated by

$$\left\langle \begin{pmatrix} 4 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} \right\rangle.$$

The order of the group is 12.

4. A (64,6)-arc ([2])

The points in the arc are

 $\begin{array}{l} (1,0,0),\ (1,0,1),\ (1,8,10),\ (1,2,8),\ (1,12,5),\ (1,1,3),\ (1,11,12),\ (1,5,0),\ (1,0,3),\ (1,11,4),\\ (1,6,11),\ (1,10,2),\ (1,3,9),\ (1,7,10),\ (1,2,0),\ (1,0,10),\ (1,2,9),\ (1,7,2),\ (1,3,11),\ (1,10,4),\\ (1,6,3),\ (1,11,0),\ (1,0,12),\ (1,5,3),\ (1,11,5),\ (1,1,8),\ (1,12,10),\ (1,2,1),\ (1,8,0),\ (1,1,2),\\ (1,3,2),\ (1,3,5),\ (1,1,9),\ (1,7,7),\ (1,4,9),\ (1,7,5),\ (1,1,12),\ (1,5,11),\ (1,10,7),\ (1,4,7),\ (1,4,11),\\ (1,10,12),\ (1,5,5),\ (1,2,10),\ (1,2,12),\ (1,5,6),\ (1,9,9),\ (1,7,6),\ (1,9,12),\ (1,5,10),\ (1,12,1),\\ (1,8,2),\ (1,3,6),\ (1,9,6),\ (1,9,2),\ (1,3,1),\ (1,8,8),\ (1,11,1),\ (1,8,7),\ (1,4,4),\ (1,6,7),\ (1,4,1),\\ (1,8,3),\ (1,11,3) \end{array}$

The secant distribution of the arc is $\tau = (15, 0, 0, 14, 14, 42, 98, 0, 0, 0, 0, 0, 0, 0, 0)$

The arc is found by prescribing the group generated by

$$\left\langle \begin{pmatrix} 9 & 0 & 0 \\ 0 & 0 & 7 \\ 0 & 7 & 12 \end{pmatrix} \right\rangle.$$

The order of the group is 21.

5. A (79,7)-arc (Barlotti's construction, [3])

The points in the arc are

(0,1,0), (1,1,1), (1,1,2), (1,1,6), (1,1,7), (1,1,11), (1,1,12), (1,2,0), (1,2,2), (1,2,3), (1,2,4), (1,2,9), (1,2,10), (1,2,11), (1,3,2), (1,3,4), (1,3,5), (1,3,8), (1,3,9), (1,3,11), (1,4,1), (1,4,2), (1,4,4), (1,4,9), (1,4,11), (1,4,12), (1,5,0), (1,5,1), (1,5,4), (1,5,6), (1,5,7), (1,5,9), (1,5,12), (1,6,0), (1,6,1), (1,6,3), (1,6,5), (1,6,8), (1,6,10), (1,6,12), (1,7,0), (1,7,1), (1,7,2), (1,7,5), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,1), (1,7,2), (1,7,5), (1,7,2),

(1,7,8), (1,7,11), (1,7,12), (1,8,0), (1,8,4), (1,8,5), (1,8,6), (1,8,7), (1,8,8), (1,8,9), (1,9,3), (1,9,5), (1,9,6), (1,9,7), (1,9,8), (1,9,10), (1,10,1), (1,10,3), (1,10,6), (1,10,7), (1,10,10), (1,10,12), (1,11,0), (1,11,2), (1,11,3), (1,11,6), (1,11,7), (1,11,10), (1,11,11), (1,12,3), (1,12,4), (1,12,5), (1,12,8), (1,12,9), (1,12,10)

The secant distribution of the arc is $\tau = (13, 1, 0, 0, 0, 0, 78, 91, 0, 0, 0, 0, 0, 0, 0)$

6. A (92,8)-arc (Barlotti's construction, [3])

The points in the arc are

 $(0,1,0), (1,0,0), (1,1,1), (1,1,2), (1,1,5), (1,1,6), (1,1,7), (1,1,8), (1,1,11), (1,1,12), (1,2,0), \\ (1,2,2), (1,2,3), (1,2,4), (1,2,9), (1,2,10), (1,2,11), (1,3,2), (1,3,4), (1,3,5), (1,3,6), (1,3,7), \\ (1,3,8), (1,3,9), (1,3,11), (1,4,1), (1,4,2), (1,4,3), (1,4,4), (1,4,9), (1,4,10), (1,4,11), (1,4,12), \\ (1,5,0), (1,5,1), (1,5,4), (1,5,6), (1,5,7), (1,5,9), (1,5,12), (1,6,0), (1,6,1), (1,6,3), (1,6,5), \\ (1,6,8), (1,6,10), (1,6,12), (1,7,0), (1,7,1), (1,7,2), (1,7,5), (1,7,8), (1,7,11), (1,7,12), \\ (1,8,0), (1,8,4), (1,8,5), (1,8,6), (1,8,7), (1,8,8), (1,8,9), (1,9,2), (1,9,3), (1,9,5), (1,9,6), \\ (1,9,7), (1,9,8), (1,9,10), (1,9,11), (1,10,1), (1,10,3), (1,10,4), (1,10,6), (1,10,7), (1,10,9), \\ (1,10,10), (1,10,12), (1,11,0), (1,11,2), (1,11,3), (1,11,6), (1,11,7), (1,11,10), (1,11,11), \\ (1,12,1), (1,12,3), (1,12,4), (1,12,5), (1,12,8), (1,12,9), (1,12,10), (1,12,12)$

The secant distribution of the arc is $\tau = (0, 14, 0, 0, 0, 0, 0, 78, 91, 0, 0, 0, 0, 0, 0)$

7. A (105,9)-arc ([2])

The points in the arc are

 $\begin{array}{l} (0,1,2), \ (1,2,0), \ (1,7,0), \ (1,0,7), \ (0,1,7), \ (1,0,2), \ (0,1,3), \ (1,3,0), \ (1,9,0), \ (1,0,9), \ (0,1,9), \\ (1,0,3), \ (0,1,5), \ (1,5,0), \ (1,8,0), \ (1,0,8), \ (0,1,8), \ (1,0,5), \ (0,1,11), \ (1,11,0), \ (1,6,0), \ (1,0,6), \\ (0,1,6), \ (1,0,11), \ (1,1,3), \ (1,3,1), \ (1,9,9), \ (1,1,4), \ (1,4,1), \ (1,10,10), \ (1,1,5), \ (1,5,1), \ (1,8,8), \\ (1,1,6), \ (1,6,1), \ (1,11,11), \ (1,1,7), \ (1,7,1), \ (1,2,2), \ (1,1,8), \ (1,8,1), \ (1,5,5), \ (1,1,9), \ (1,9,1), \\ (1,3,3), \ (1,1,10), \ (1,10,1), \ (1,4,4), \ (1,1,11), \ (1,11,1), \ (1,6,6), \ (1,10,7), \ (1,2,4), \ (1,7,2), \\ (1,2,7), \ (1,7,10), \ (1,4,2), \ (1,11,9), \ (1,2,6), \ (1,7,3), \ (1,3,7), \ (1,9,11), \ (1,6,2), \ (1,10,5), \\ (1,7,4), \ (1,2,8), \ (1,8,2), \ (1,5,10), \ (1,4,7), \ (1,11,7), \ (1,3,6), \ (1,9,2), \ (1,2,9), \ (1,7,11), \ (1,6,3), \\ (1,10,2), \ (1,8,4), \ (1,5,7), \ (1,7,5), \ (1,2,10), \ (1,4,8), \ (1,11,8), \ (1,9,6), \ (1,3,5), \ (1,5,3), \ (1,8,11), \\ (1,6,9), \ (1,10,8), \ (1,6,4), \ (1,11,5), \ (1,5,11), \ (1,8,10), \ (1,4,6), \ (1,10,6), \ (1,11,4), \ (1,6,11), \\ (1,11,6), \ (1,6,10), \ (1,4,11), \ (1,12,5), \ (1,8,12), \ (1,5,8), \ (1,8,5), \ (1,5,12), \ (1,12,8) \\ \end{array}$

The secant distribution of the arc is $\tau = (5, 0, 3, 0, 0, 0, 6, 27, 39, 103, 0, 0, 0, 0, 0)$

The arc is found by prescribing the group generated by

$$\left\langle \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \right\rangle.$$

The order of the group is 6.

8. A (118,10)-arc ([2])

The points in the arc are

 $(0,1,1),\ (1,1,0),\ (1,0,1),\ (1,12,0),\ (1,0,12),\ (0,1,12),\ (0,1,11),\ (1,11,0),\ (1,0,6),\ (1,2,0),\ (1,0,7),\ (0,1,2),\ (0,1,4),\ (1,4,0),\ (1,0,10),\ (1,9,0),\ (1,0,3),\ (0,1,9),\ (0,1,5),\ (1,5,0),\ (1,0,8),\ (1,8,0),\ (1,0,5),\ (0,1,8),\ (0,1,6),\ (1,6,0),\ (1,0,11),\ (1,7,0),\ (1,0,2),\ (0,1,7),\ (1,1,11),\ (1,11,1),\ (1,12,2),\ (1,6,6),\ (1,2,12),\ (1,11,12),\ (1,7,7),\ (1,6,7),\ (1,7,6),\ (1,2,1),\ (1,1,2),\ (1,12,11),\ (1,1,10),\ (1,10,1),\ (1,12,3),\ (1,4,4),\ (1,3,12),\ (1,10,12),\ (1,9,9),\ (1,4,9),\ (1,9,4),\ (1,3,1),\ (1,1,10$

(1,1,3), (1,12,10), (1,1,4), (1,4,1), (1,12,9), (1,10,10), (1,9,12), (1,4,12), (1,3,3), (1,10,3), (1,3,10), (1,9,1), (1,1,9), (1,12,4), (1,10,6), (1,11,4), (1,3,7), (1,11,6), (1,2,9), (1,11,9), (1,2,7), (1,11,7), (1,2,6), (1,2,4), (1,10,7), (1,3,6), (1,11,5), (1,4,6), (1,2,8), (1,8,10), (1,9,7), (1,4,7), (1,5,3), (1,8,3), (1,5,10), (1,9,6), (1,11,8), (1,2,5), (1,10,11), (1,5,4), (1,3,2), (1,6,8), (1,8,9), (1,5,9), (1,7,5), (1,6,5), (1,7,8), (1,8,4), (1,10,2), (1,3,11), (1,10,4), (1,3,4), (1,3,9), (1,10,9), (1,4,11), (1,6,10), (1,9,2), (1,6,11), (1,7,3), (1,6,3), (1,7,2), (1,6,2), (1,7,11), (1,7,10), (1,4,2), (1,9,11)

The secant distribution of the arc is $\tau = (4, 0, 0, 0, 0, 6, 0, 6, 12, 66, 89, 0, 0, 0, 0)$

The arc is found by prescribing the group generated by

$$\left\langle \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 5 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix} \right\rangle.$$

The order of the group is 48.

9. A (132,11)-arc ([2])

The points in the arc are

 $(0,0,1), \ (0,1,0), \ (1,0,0), \ (0,1,1), \ (1,1,0), \ (1,0,1), \ (0,1,2), \ (1,2,0), \ (1,0,2), \ (1,0,7), \ (1,7,0), \ (0,1,7), \ (0,1,3), \ (1,3,0), \ (1,0,3), \ (1,0,9), \ (1,9,0), \ (0,1,9), \ (0,1,10), \ (1,10,0), \ (1,0,10), \ (1,0,10), \ (1,0,4), \ (1,4,0), \ (0,1,4), \ (0,1,5), \ (1,5,0), \ (1,0,5), \ (1,0,8), \ (1,8,0), \ (0,1,8), \ (1,1,1), \ (1,1,1,2), \ (1,2,1), \ (1,7,7), \ (1,1,4), \ (1,4,1), \ (1,10,10), \ (1,1,9), \ (1,9,1), \ (1,3,3), \ (1,1,11), \ (1,11,1), \ (1,11,1), \ (1,6,6), \ (1,1,12), \ (1,12,1), \ (1,12,12), \ (1,2,3), \ (1,8,7), \ (1,7,8), \ (1,9,5), \ (1,5,9), \ (1,3,2), \ (1,10,7), \ (1,2,4), \ (1,4,2), \ (1,2,7), \ (1,7,2), \ (1,7,10), \ (1,2,5), \ (1,9,7), \ (1,7,9), \ (1,8,3), \ (1,3,8), \ (1,5,2), \ (1,11,9), \ (1,2,6), \ (1,6,2), \ (1,3,7), \ (1,7,12), \ (1,12,7), \ (1,2,11), \ (1,11,12), \ (1,7,5), \ (1,5,7), \ (1,2,10), \ (1,11,2), \ (1,12,11), \ (1,10,4), \ (1,3,4), \ (1,4,3), \ (1,10,9), \ (1,9,10), \ (1,4,10), \ (1,11,8), \ (1,9,6), \ (1,6,9), \ (1,5,3), \ (1,3,5), \ (1,8,11), \ (1,3,9), \ (1,3,12), \ (1,12,10), \ (1,10,8), \ (1,6,4), \ (1,4,6), \ (1,5,11), \ (1,11,5), \ (1,8,10), \ (1,10,6), \ (1,11,4), \ (1,4,11), \ (1,11,6), \ (1,6,11), \ (1,6,10), \ (1,12,5), \ (1,8,12), \ (1,12,8), \ (1,8,5), \ (1,5,8), \ (1,5,12)$

The secant distribution of the arc is $\tau = (0, 0, 0, 0, 0, 9, 6, 0, 9, 9, 36, 114, 0, 0, 0)$

The arc is found by prescribing the group generated by

$$\left\langle \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right\rangle.$$

The order of the group is 6.

10. A (145,12)-arc ([2])

The points in the arc are

 $(1,7,12),\ (1,6,12),\ (1,2,2),\ (1,11,2),\ (1,2,11),\ (1,7,1),\ (1,1,7),\ (1,12,6),\ (1,10,6),\ (1,11,4),\ (1,3,7),\ (1,11,6),\ (1,2,9),\ (1,11,9),\ (1,2,7),\ (1,11,7),\ (1,2,6),\ (1,2,4),\ (1,10,7),\ (1,3,6),\ (1,11,5),\ (1,4,6),\ (1,2,8),\ (1,8,10),\ (1,9,7),\ (1,4,7),\ (1,5,3),\ (1,8,3),\ (1,5,10),\ (1,9,6),\ (1,11,8),\ (1,2,5),\ (1,10,11),\ (1,5,4),\ (1,3,2),\ (1,6,8),\ (1,8,9),\ (1,5,9),\ (1,7,5),\ (1,6,5),\ (1,7,8),\ (1,8,4),\ (1,10,2),\ (1,3,11),\ (1,10,5),\ (1,7,4),\ (1,3,8),\ (1,8,2),\ (1,6,9),\ (1,7,9),\ (1,5,11),\ (1,8,11),\ (1,5,2),\ (1,6,4),\ (1,10,8),\ (1,3,5),\ (1,4,11),\ (1,6,10),\ (1,9,2),\ (1,6,11),\ (1,7,3),\ (1,6,3),\ (1,7,2),\ (1,6,2),\ (1,7,11),\ (1,7,10),\ (1,4,2),\ (1,9,11),\ (1,4,10),\ (1,9,10),\ (1,9,3),\ (1,4,3)$

The secant distribution of the arc is $\tau = (0, 0, 0, 0, 4, 0, 0, 0, 18, 22, 36, 103, 0, 0)$

The arc is found by prescribing the group generated by

$$\left\langle \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 5 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix} \right\rangle.$$

The order of the group is 48.

References

- [1] Ball S., Three-dimensional linear codes, Online table, http://mat-web.upc.edu/people/simeon.michael.ball/codebounds.html.
- [2] M. Braun, A. Kohnert, A. Wassermann, Construction of (n, r)-arcs in PG(2, q), *Innov. Incid. Geometry*, 1, 133–141, 2005.
- [3] A. Barlotti, Some Topics in Finite Geometrical Structures, Institute of Statistics Mimeo Series, vol. 439, University of North Carolina, 1965, 439.