On the role of dissipativity in economic model predictive control ⋆

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Abstract: In this paper, we examine and discuss the role of dissipativity in economic model predictive control. We review some recent results relating dissipativity with the concept of optimal steady-state operation, and we show that *strict* dissipativity is necessary and sufficient for a slightly stronger property than optimal steady-state operation. We discuss the importance of this result for giving closed-loop performance guarantees in economic MPC. Furthermore, we present extensions for the case of optimal periodic operation.

Keywords: Economic MPC, Dissipativity, Optimal steady-state operation, Optimal periodic operation

1. INTRODUCTION

Economic model predictive control (MPC) is a variant of MPC where, in contrast to standard tracking MPC, the control objective is not necessarily the stabilization of an a priori given setpoint (or trajectory), but the optimization of some general performance criterion, possibly related to the economics of the considered system. In recent years, different economic MPC schemes have been proposed and studied in the literature, using different assumptions and/or additional terminal constraints or cost terms, see, e.g., (Angeli et al., 2012; Amrit et al., 2011; Heidarinejad et al., 2012; Müller et al., 2013b; Grüne, 2013; Ferramosca et al., 2014) and the recent survey article by Ellis et al. (2014).

Due to the use of a general performance criterion, the optimal operating regime for the considered system might not be stationary, but can be some periodic orbit or even more complex. Hence an interesting question is to classify what the optimal operating regime is for a given system and a given cost function. Furthermore, it is desirable to guarantee that the closed-loop system, resulting from application of an economic MPC scheme, "finds" the optimal operating behavior, i.e., converges to the optimal trajectory. To this end, a certain dissipativity condition has turned out to play a crucial role. Namely, dissipativity with respect to a supply rate involving the employed stage cost function is both necessary and sufficient such that the optimal operating regime is stationary, i.e., at some steady-state (Angeli et al., 2012; Müller et al., 2013a, 2015; Faulwasser et al., 2014). Furthermore, the same dissipativity property (strengthened to strict dissipativity) can be used to conclude that the optimal steady-state is an asymptotically stable equilibrium point for the resulting closed-loop system, see, e.g., Angeli et al. (2012); Amrit et al. (2011); Grüne (2013); Zanon et al. (2014). For the case where periodic operation is optimal, some first generalizations of these results have recently been studied by Grüne and Zanon (2014) and Müller and Grüne (2015a,b).

The contribution of this paper is to provide a comprehensive treatment of the role played by dissipativity in the context of economic MPC. To this end, we first review some of the results mentioned above concerning the relation between dissipativity and optimal steady-state operation. After that, we show that *strict* dissipativity is both necessary and sufficient for a slightly stronger property than optimal steady-state operation (see Section 3). The implications and importance of this result, also for establishing desired convergence properties for the closed-loop system, are then discussed in Section 4. Section 5 provides extensions of the previous results to the case where periodic operation in contrast to steady-state operation is optimal.

2. PRELIMINARIES AND SETUP

Denote by $\mathbb{I}$ the set of integer numbers, by $\mathbb{I}_{[a,b]}$ the set of integers in the interval $[a,b] \subseteq \mathbb{R}$, and by $\mathbb{I}_{\geq a}$ ($\mathbb{I}_{\leq a}$) the set of integers greater (less) than or equal to a. We consider discrete-time nonlinear systems of the form

$$x(t+1) = f(x(t), u(t)), \qquad x(0) = x_0, \tag{1}$$

where $f : \mathbb{X} \times \mathbb{U} \to \mathbb{R}^n$, $x(t) \in \mathbb{X} \subseteq \mathbb{R}^n$ and $u(t) \in \mathbb{U} \subseteq \mathbb{R}^m$ are the system state and the control input, respectively, at time $t \in \mathbb{I}_{\geq 0}$, and $x_0 \in \mathbb{X}$ is the initial condition. The system is subject to pointwise-in-time state and input constraints

$$(x(t), u(t)) \in \mathbb{Z} \subseteq \mathbb{X} \times \mathbb{U} \tag{2}$$

for all $t \in \mathbb{I}_{\geq 0}$. For a given control sequence $u = (u(0), \dots, u(K)) \in \mathbb{U}^{K+1}$ (or $u = (u(0), \dots) \in \mathbb{U}^\infty$), denote by $x_u(t, x_0)$ the corresponding solution of system (1)

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with initial condition $x_u(0, x_0) = x_0$. For a given $x \in \mathbb{X}$, by $\mathbb{U}^N(x)$ we denote the set of all feasible control sequences of length N, i.e., $\mathbb{U}^N(x) := \{u \in \mathbb{U}^N : (x_u(k, x), u(k)) \in \mathbb{Z}\ \forall k \in \mathbb{I}_{[0,N-1]}\}$. Similarly, the set of all feasible control sequences of infinite length is denoted by $\mathbb{U}^\infty(x)$. Define the set $\mathbb{Z}^0$ as the largest "forward invariant" set contained in $\mathbb{Z}$, i.e., the set which contains all elements in $\mathbb{Z}$ which are part of a feasible state/input sequence pair:

$$\mathbb{Z}^0 := \{(x, u) \in \mathbb{Z} : \exists v \in \mathbb{U}^\infty(x) \text{ s.t. } v(0) = u\} \subseteq \mathbb{Z}. \quad (3)$$

Denote by $\mathbb{X}^0$ the projection of $\mathbb{Z}^0$ on $\mathbb{X}$, i.e., $\mathbb{X}^0 := \{x \in \mathbb{X} : \mathbb{U}^\infty(x) \neq \emptyset\}$.

System (1) is equipped with a stage cost function $\ell : \mathbb{X} \times \mathbb{U} \to \mathbb{R}$ specifying the performance criterion to be minimized. In the context of economic MPC, ℓ can be some general function, and need not be positive definite with respect to a setpoint (or more general, set) to be stabilized as in standard tracking MPC. In economic MPC, the control input to system (1) is now computed at each time instant $t \in \mathbb{I}_{\geq 0}$ with current system state $x = x(t)$ by minimizing, with respect to $u \in \mathbb{U}^N(x)$, the following finite-horizon cost function:

$$J_N(x, u) := \sum_{t=0}^{N-1} \ell(x_u(t, x), u(t)) \quad (4)$$

Then, the first element of the optimal input sequence [1] $u^*_{N,x}$ is applied to system (1) and the procedure is repeated again at time $t + 1$. As discussed in the introduction, an additional terminal cost term and/or suitable terminal constraints are added to the above optimization problem in various economic MPC schemes available in the literature.

Let S be defined as the set of all feasible state/input equilibrium pairs of system (1), i.e.,

$$S := \{(x, u) \in \mathbb{Z} : x = f(x, u)\}, \quad (5)$$

which is assumed to be non-empty. In the following, we assume that a (possibly non-unique) optimal state/input equilibrium pair (x^*, u^*) exists, i.e., (x^*, u^*) satisfies

$$\ell(x^*, u^*) = \inf_{(x,u)\in S} \ell(x, u). \quad (6)$$

For a given $M \in \mathbb{I}_{\geq 1}$, denote by $\mathcal{C}_M$ the set of states which can be steered to x^* in M steps in a feasible way, i.e.,

$$\mathcal{X}_M := \{x \in \mathbb{X} : \exists u \in \mathbb{U}^M(x) \text{ s.t. } x_u(M, x) = x^*\}. \quad (7)$$

Next, let $\mathcal{R}_M$ be the set of states which can be reached from x^* in M steps in a feasible way, i.e.,

$$\mathcal{R}_M := \{x \in \mathbb{X} : \exists u \in \mathbb{U}^M(x^*) \text{ s.t. } x_u(M, x^*) = x\}. \quad (8)$$

Note that $\mathcal{C}_M \cap \mathcal{R}_M \neq \emptyset$, as by definition x^* is contained in both $\mathcal{C}_M$ and $\mathcal{R}_M$. Now define the set $\mathcal{Z}_M$ as the set of state/input pairs which are part of a feasible state/input sequence pair staying in $\mathcal{C}_M \cap \mathcal{R}_M$ for all times:

$$\mathcal{Z}_M := \{(x, u) \in \mathbb{Z} : \exists v \in \mathbb{U}^\infty(x) \text{ s.t. } v(0) = u, \\ x_v(t, x) \in \mathcal{C}_M \cap \mathcal{R}_M\ \forall t \in \mathbb{I}_{\geq 0}\} \subseteq \mathbb{Z}^0. \quad (9)$$

As already discussed in the introduction, in this paper we study and discuss the role of dissipativity in economic MPC. The concept of dissipativity dates back to Willems (1972) (see also (Byrnes and Lin, 1994) for a discrete time version) and is as follows.

[1] In the following, we assume that for all $x \in \mathbb{X}^0$, a minimizing control sequence $u^*_{N,x} \in \mathbb{U}^N(x)$ exists, i.e., such that $J_N(x, u^*_{N,x}) = \inf_{u\in\mathbb{U}^N(x)} J_N(x, u)$.

Definition 1. The system (1) is dissipative on a set $\mathbb{W} \subseteq \mathbb{Z}$ with respect to the supply rate $s : \mathbb{W} \to \mathbb{R}$ if there exists a storage function [2] $\lambda : \mathbb{W}_\mathbb{X} \to \mathbb{R}_{\geq 0}$ such that the following inequality is satisfied for all $(x, u) \in \mathbb{W}$:

$$\lambda(f(x, u)) - \lambda(x) \leq s(x, u). \quad (10)$$

If there exists $\rho \in \mathcal{K}_\infty$ such that for all $(x, u) \in \mathbb{W}$

$$\lambda(f(x, u)) - \lambda(x) \leq -\rho(|x - x^*|) + s(x, u), \quad (11)$$

then system (1) is strictly dissipative on $\mathbb{W}$.

An equivalent characterization of dissipativity can be obtained via the so-called *available storage*, defined as

$$S_a(x) := \sup_{T\geq 0, u\in\mathbb{U}^\infty(x)} \sum_{t=0}^{T-1} -s(x_u(t, x), u(t)). \quad (12)$$

Namely, it was shown by [3] Willems (1972) that system (1) is dissipative on $\mathbb{Z}^0$ with respect to the supply rate s if and only if $S_a(x) < \infty$ for all $x \in \mathbb{X}^0$. Furthermore, in an analogous fashion one can show that that system (1) is dissipative on $\mathbb{Z}^0$ with respect to the supply rate s and with a storage function λ which is bounded on $\mathbb{X}^0$ if and only if S_a is bounded on $\mathbb{X}^0$, i.e., $S_a(x) \leq c < \infty$ for all $x \in \mathbb{X}^0$ and some $c \geq 0$.

3. DISSIPATIVITY AND OPTIMAL STEADY-STATE OPERATION

Given the system dynamics (1), the constraint set $\mathbb{Z}$ and the cost function ℓ, an interesting question is to determine what the optimal operating regime looks like, i.e., what system behavior results in an optimal performance. To this end, the following definition of optimal steady-state operation was considered in Angeli et al. (2012).

Definition 2. System (1) is *optimally operated at steady-state*, if for each $x_0 \in \mathbb{X}^0$ and each $u \in \mathbb{U}^\infty(x)$ the following holds for all $t \in \mathbb{I}_{\geq 0}$:

$$\liminf_{T\to\infty} \frac{\sum_{t=0}^{T-1} \ell(x_u(t, x), u(t))}{T} \geq \ell(x^*, u^*). \quad (13)$$

System (1) is *suboptimally operated off steady-state*, if in addition for each $x_0 \in \mathbb{X}^0$ and each $u \in \mathbb{U}^\infty(x)$ at least one of the following two conditions holds:

$$\liminf_{T\to\infty} \frac{\sum_{t=0}^{T-1} \ell(x_u(t, x), u(t))}{T} > \ell(x^*, u^*) \quad (14a)$$

$$\liminf_{t\to\infty} |x_u(t, x) - x^*| = 0 \quad (14b)$$

The definition of optimal steady-state operation means that no feasible solution can have an (asymptotic) average performance which is better than the performance of the best steady-state, while suboptimal operation off steady-state means that each solution has an (asymptotic) average performance which is strictly worse than the performance of the best steady-state, or "passes by" the optimal steady-state infinitely often. The following theorem from Angeli et al. (2012) shows that a certain dissipativity property is sufficient for optimal steady-state operation of system (1).

[2] Here, $\mathbb{W}_\mathbb{X}$ denotes the projection of $\mathbb{W}$ on $\mathbb{X}$.

[3] We note that while this was established by Willems (1972) for continuous-time systems without constraints, the same result can be obtained in an analogous fashion for our setting of discrete-time systems with state and input constraints.

Theorem 3. Suppose that system (1) is dissipative (strictly dissipative) on $\mathbb{Z}^0$ with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$. Then the system (1) is optimally operated at steady-state (suboptimally operated off steady-state). □

In general, the converse statement of Theorem 3 is not true, as was shown in (Müller et al., 2013a) by means of two counterexamples. Nevertheless, dissipativity is in fact necessary for steady-state operation of a system under an additional controllability condition, as shown in the following result by Müller et al. (2015).

Theorem 4. Consider an arbitrary $M \in \mathbb{I}_{\geq 1}$ and suppose that system (1) is optimally operated at steady-state and ℓ is bounded from above on[4] $\mathcal{Z}_M$. Then, system (1) is dissipative on $\mathcal{Z}_M$ with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$. □

The proof of this theorem in (Müller et al., 2015) proceeds by showing that if system (1) is not dissipative with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$, then one can construct a feasible state/input sequence pair $(x(\cdot),u(\cdot))$ which has a better average performance than the performance of the optimal steady-state, contradicting optimal steady-state operation. Note that in case that $\mathcal{Z}_M = \mathbb{Z}^0$ for some $M \in \mathbb{I}_{\geq 1}$ (which means that the system is weakly reversible (Sontag, 1998, Section 4.3)), by combining Theorems 3 and 4 it follows that dissipativity with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$ is a necessary and sufficient condition for optimal steady-state operation.

While both sufficiency and necessity of dissipativity for optimal steady-state operation have been established as discussed above, the question is still open whether the same is true for *strict* dissipativity and suboptimal operation off steady-state. This question is not only interesting and relevant in order to study whether strict dissipativity is only a (possibly conservative) sufficient condition, but also for being able to derive desired statements for the closed-loop system, as discussed in more detail in Section 4. The following simple example shows that strict dissipativity is not necessary for suboptimal operation off steady-state.

Example 5. Consider the system $x(t+1) = u(t)$ with cost function $\ell(x,u) = (x^2+u^2)((x-1)^2+u^2)(x^2+(u-1)^2)$ and constraint set $\mathbb{Z} = [-1,1] \times [-1,1]$. For this system, $\mathcal{Z}_M = \mathbb{Z}^0 = \mathbb{Z}$ for all $M \in \mathbb{I}_{\geq 1}$. The function ℓ has three global minima $\ell(x,u) = 0$ for $(x,u) \in \{(0,0),(0,1),(1,0)\}$. Hence the system is (trivially) optimally operated at the optimal steady-state $(x^*,u^*) = (0,0)$, and it is easy to show that each feasible solution which satisfies (13) with equality must satisfy (14b), which means that the system is suboptimally operated off steady-state. On the other hand, the system cannot be strictly dissipative with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$, since summing up the strict dissipation inequality (11) along the solution with initial condition $x_0 = 0$ and input sequence $u = (1,0) \in \mathbb{U}^2(0)$ yields $0 \leq \sum_{t=0}^{1} -\rho(|x_u(t,0) - x^*|) = -\rho(1)$, which cannot be satisfied for any function $\rho \in \mathcal{K}_\infty$. □

In the following, we show that strict dissipativity with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$ is in fact necessary and sufficient for a property which is slightly stronger than suboptimal operation off steady-state, which we call *uniform* suboptimal operation off steady-state. Here, for each solution which has the same (asymptotic) average performance as the performance of the best steady-state, we do not just require as in (14b) that it "passes by" the optimal steady-state infinitely often, but define how often this has to happen in each time interval.

Definition 6. System (1) is *uniformly suboptimally operated off steady-state* if it is optimally operated at steady-state and in addition there exist $\bar{\delta} > 0$ and $d \in \mathcal{K}_\infty$ such that for each $\delta > 0$ and each $\varepsilon > 0$ there exists $R_{\varepsilon,\delta} \in \mathbb{I}_{\geq 0}$ such that $\delta / R_{\varepsilon,\delta} \geq d(\varepsilon)$ for all $\delta \geq \bar{\delta}$ and such that for each $x \in \mathbb{X}^0$, each $u \in \mathbb{U}^\infty(x)$, and each $T \in \mathbb{I}_{\geq 0}$ at least one of the following two conditions holds:

$$\sum_{t=0}^{T-1} \big(\ell(x_u(t,x),u(t)) - \ell(x^*,u^*)\big) > \delta \tag{15a}$$

$$\#\{t \in \mathbb{I}_{[0,T-1]} : |x_u(t,x) - x^*| > \varepsilon\} \leq R_{\varepsilon,\delta} \tag{15b}$$

In the definition of *uniform* suboptimal operation off steady-state, uniformity is with respect to all initial conditions and feasible solutions. Namely, for each time $T \in \mathbb{I}_{\geq 0}$, each feasible solution has a transient performance (relative to the optimal steady-state) greater than δ or the number of time instants in the interval $[0, T-1]$ for which the state is "far away" from the optimal steady-state x^* is bounded by $R_{\varepsilon,\delta}$. In particular, for each solution which satisfies $\sum_{k=0}^{T-1}(\ell(x_u(t,x),u(t)) - \ell(x^*,u^*)) \leq \delta$ for some $\delta > 0$ and all $T \in \mathbb{I}_{\geq 0}$, condition (15b) implies that it has a turnpike property (Dorfman et al., 1958) with respect to the optimal steady-state; such turnpike properties have recently been studied in the context of economic MPC for both discrete-time (Grüne, 2013; Damm et al., 2014) and continuous-time (Faulwasser et al., 2014; Trélat and Zuazua, 2015) systems. On the other hand, for solutions for which $\sum_{t=0}^{T-1}(\ell(x_u(t,x),u(t)) - \ell(x^*,u^*)) \to \infty$ as $T \to \infty$, (15a)–(15b) give conditions "how often" and "how far" it can be away from the optimal steady-state x^* in each time interval $[0, T-1]$, depending on how large the transient performance is during this time interval.

Remark 7. We note that Definition 6 is slightly stricter than the definition of uniform suboptimal operation off steady-state which was used by Müller et al. (2015). There, it was shown that under local controllability of system (1) at the optimal steady-state x^*, dissipativity is necessary for this slightly weaker notion of uniform suboptimal operation off steady-state. However, this is *not* the case for strict dissipativity. On the other hand, under the assumption of local controllability at the optimal steady-state, strict dissipativity with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$ is necessary and sufficient for uniform suboptimal operation off steady-state as in Definition 6, as shown in the following. □

Theorem 8. Suppose that system (1) is strictly dissipative on $\mathbb{Z}^0$ with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$ and with a storage function λ which is bounded on $\mathbb{X}^0$. Then the system (1) is uniformly suboptimally operated off steady-state.

[4] This is, e.g., the case if ℓ is continuous and $\mathcal{Z}_M$ is compact. The latter is true if, e.g., f is continuous, $\mathbb{U}$ is compact, and $\mathbb{Z}$ is closed.

Proof: Sufficiency of dissipativity for optimal operation at steady-state follows from Theorem 3. Hence it remains to show that (15a) or (15b) is satisfied for each feasible solution of system (1). To this end, it follows from (Grüne, 2013, Theorem 5.3) that for each feasible solution of system (1) for which (15a) does not hold, (15b) is satisfied with $R_{\varepsilon,\delta} = (\delta + c)/\rho(\varepsilon)$ for some $c > 0$. The proof is then concluded by noting that

$$\frac{\delta}{R_{\varepsilon,\delta}} = \frac{\delta\rho(\varepsilon)}{\delta + c} \geq \frac{\bar{\delta}}{\bar{\delta} + c}\rho(\varepsilon) =: d(\varepsilon)$$

for all $\delta \geq \bar{\delta}$ and arbitrary $\bar{\delta} > 0$. □

Theorem 9. Suppose that system (1) is uniformly suboptimally operated off steady-state and locally controllable [5] at x^* in τ steps for some $\tau \in \mathbb{I}_{\geq 0}$, and that ℓ is locally bounded and bounded from below on [6] $\mathbb{Z}^0$. Then, system (1) is strictly dissipative on $\mathbb{Z}^0$ with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$ and with a storage function λ which is bounded on $\mathbb{X}^0$.

Proof: Consider a two sided strictly increasing sequence ε_i, $i \in \mathbb{I}$, with $\varepsilon_i \to \infty$ as $i \to \infty$, $\varepsilon_i \to 0$ as $i \to -\infty$, and $d(\varepsilon_0) = 1$ for d from Definition 6. For each such ε_i, define $\rho(\varepsilon_i) := d(\varepsilon_{i-1})^2/8$ for $i \in \mathbb{I}_{\leq 1}$ and $\rho(\varepsilon_i) := \sqrt{d(\varepsilon_{i-1})}/4$ for $i \in \mathbb{I}_{\geq 2}$. Next, extend ρ such that it is defined for all $\varepsilon \geq 0$ by setting $\rho(0) = 0$ and by linearly interpolating between two values ε_i and ε_{i+1}, i.e., $\rho(\varepsilon) := \rho(\varepsilon_i) + (\rho(\varepsilon_{i+1}) - \rho(\varepsilon_i))(\varepsilon - \varepsilon_i)/(\varepsilon_{i+1} - \varepsilon_i)$ for all $\varepsilon \in (\varepsilon_i, \varepsilon_{i+1})$ and all $i \in \mathbb{I}$. The function ρ as defined above on the interval $[0, \infty)$ is continuous, strictly increasing, unbounded, and $\rho(0) = 0$, i.e., $\rho \in \mathcal{K}_\infty$. In the following, let $\rho_i := \rho(\varepsilon_i)$.

Now consider arbitrary $x \in \mathbb{X}^0$, $u \in \mathbb{U}^\infty(x)$, and $T \in \mathbb{I}_{\geq 0}$, and let $Q_i := \#\{t \in \mathbb{I}_{[0,T-1]} : |x_u(t,x) - x^*| \in (\varepsilon_i, \varepsilon_{i+1}]\}$ for all $i \in \mathbb{I}$. Since at most T of the values Q_i are nonzero, there exists $m \in \mathbb{I}_{\geq 0}$ such that

$$\sum_{t=0}^{T-1} \rho(|x_u(t,x) - x^*|) \leq \sum_{i=-\infty}^{\infty} Q_i \rho_{i+1} = \sum_{i=-m}^{m} Q_i \rho_{i+1}.$$

Now let $\delta := \max\{\sum_{t=0}^{T-1}(\ell(x_u(t,x), u(t)) - \ell(x^*,u^*)), \bar{\delta}\}$ with $\bar{\delta}$ from Definition 6. By uniform suboptimal operation off steady-state, it follows that (15b) is satisfied for this choice of δ, and hence $\kappa_j := \sum_{i=j}^{\infty} Q_i \leq R_{\varepsilon_j,\delta}$. Since $Q_i = \kappa_i - \kappa_{i+1}$, we obtain

$$\begin{aligned}
\sum_{i=-m}^{m} Q_i \rho_{i+1} &= \sum_{i=-m}^{m} (\kappa_i - \kappa_{i+1})\rho_{i+1} \\
&= \kappa_{-m}\rho_{-m+1} + \sum_{i=-m+1}^{m} \kappa_i(\rho_{i+1} - \rho_i) - \kappa_{m+1}\rho_{m+1} \\
&\leq R_{\varepsilon_{-m},\delta}\rho_{-m+1} + \sum_{i=-m+1}^{m} R_{\varepsilon_i,\delta}(\rho_{i+1} - \rho_i),
\end{aligned}$$

where in the last step we took into account that the choice of m implies $\kappa_{m+1} = 0$. Using the fact that $R_{\varepsilon_i,\delta} \leq \delta/d(\varepsilon_i)$ (since $\delta \geq \bar{\delta}$) and the definition of ρ_{-m+1}, we obtain

$$R_{\varepsilon_{-m},\delta}\rho_{-m+1} \leq \delta d(\varepsilon_{-m})^2/(8d(\varepsilon_{-m})) \leq \delta/8, \tag{16}$$

where the last inequality follows from the fact that $d(\varepsilon_{-m}) \leq 1$ by definition of the sequence ε_i. Using again the fact that $R_{\varepsilon_i,\delta} \leq \delta/d(\varepsilon_i)$ and that the definition of ρ_i implies $d(\varepsilon_{i-1}) = 2\sqrt{2\rho_i}$ for $i \in \mathbb{I}_{\leq 1}$ and $d(\varepsilon_{i-1}) = 16\rho_i^2$ for $i \in \mathbb{I}_{\geq 2}$, we furthermore obtain

$$\begin{aligned}
&\sum_{i=-m+1}^{m} R_{\varepsilon_i,\delta}(\rho_{i+1} - \rho_i) \leq \delta \sum_{i=-m+2}^{m+1} \frac{\rho_i - \rho_{i-1}}{d(\varepsilon_{i-1})} \\
&= \delta \sum_{i=-m+2}^{1} \frac{\rho_i - \rho_{i-1}}{2\sqrt{2\rho_i}} + \delta \sum_{i=2}^{m+1} \frac{\rho_i - \rho_{i-1}}{16\rho_i^2} \\
&\leq \delta \int_0^{1/8} \frac{1}{2\sqrt{2x}} dx + \delta \int_{1/8}^{\infty} \frac{1}{16x^2} dx = \delta(\frac{1}{4} + \frac{1}{2}) = \frac{3\delta}{4},
\end{aligned}$$

where in the third step we used that the respective sums are lower Riemann sums for the respective integrals since the integrands are strictly decreasing, and $\rho_1 = 1/8$. Summarizing all the above, we have shown that

$$\sum_{t=0}^{T-1} \rho(|x_u(t,x) - x^*|) \leq \delta/8 + 3\delta/4 \leq \delta.$$

For the case that $\delta = \sum_{t=0}^{T-1}(\ell(x_u(t,x), u(t)) - \ell(x^*,u^*))$, the above implies that

$$\begin{aligned}
&\sum_{t=0}^{T-1} \big(\ell(x_u(t,x), u(t)) - \ell(x^*,u^*) - \rho(|x_u(t,x) - x^*|)\big) \\
&\geq \delta - \delta = 0.
\end{aligned}$$

In case that $\delta = \bar{\delta}$, consider the following. From Theorem 4.12 in Müller (2014) (compare also Theorem 4 in Müller et al. (2015)) it follows that under the given assumptions, system (1) is dissipative on $\mathbb{Z}^0$ with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$ and with a storage function λ which is bounded on $\mathbb{X}^0$. As discussed below Equation (12), this is equivalent to the fact that the available storage S_a is bounded on $\mathbb{X}^0$, i.e., there exists a constant $c < \infty$ such that $\sum_{t=0}^{T-1} -(\ell(x_u(t,x), u(t)) - \ell(x^*,u^*)) \leq c$ for all $x \in \mathbb{X}^0$, all $u \in \mathbb{U}^\infty(x)$, and all $T \in \mathbb{I}_{\geq 0}$. Hence we obtain

$$\begin{aligned}
&\sum_{t=0}^{T-1} \big(\ell(x_u(t,x), u(t)) - \ell(x^*,u^*) - \rho(|x_u(t,x) - x^*|)\big) \\
&\geq -c - \bar{\delta}.
\end{aligned} \tag{17}$$

Combining the above, it follows that (17) is satisfied for all $x \in \mathbb{X}^0$, all $u \in \mathbb{U}^\infty(x)$, and all $T \in \mathbb{I}_{\geq 0}$. But this means that the available storage S_a as defined in (12) with supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*) - \rho(|x - x^*|)$ is bounded on $\mathbb{X}^0$. Hence system (1) is dissipative on $\mathbb{Z}^0$ with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*) - \rho(|x - x^*|)$ and with a storage function λ which is bounded on $\mathbb{X}^0$, which implies that system (1) is strictly dissipative on $\mathbb{Z}^0$ with respect to the supply rate $s(x,u) = \ell(x,u) - \ell(x^*,u^*)$ and with a storage function λ which is bounded on $\mathbb{X}^0$. □

Remark 10. Theorems 8 and 9 can be extended to the cases where (i) a finite number of multiple optimal state/input equilibrium pairs exists and (ii) strictness in the dissipation inequality (11) is not only required with

[5] System (1) is *locally controllable* (Sontag, 1998, Section 3.7) at x^* in τ steps if for every $\varepsilon > 0$, there exists $\delta > 0$ such that for each pair of states $y', y'' \in \mathbb{X}^0$ with $|y' - x^*| \leq \delta$ and $|y'' - x^*| \leq \delta$, there exists $u' \in \mathbb{U}^\tau(y')$ such that $x_{u'}(\tau, y') = y''$ and $|(x_{u'}(t,y'), u'(t)) - (x^*,u^*)| \leq \varepsilon$ for all $t \in \mathbb{I}_{[0,\tau-1]}$.

[6] Note that local boundedness of ℓ implies boundedness from below if $\mathbb{Z}^0$ is compact, which is, e.g., the case if f is continuous and $\mathbb{Z}$ is compact. Furthermore, a sufficient condition for local boundedness of ℓ is continuity.

respect to x but with respect to x and u. In the former case, Theorems 8 and 9 are still correct if the term $\rho(|x-x^*|)$ in (11) is replaced by $|x|_{S_{\mathbb{X}}^*}$ and the term $|x_u(t,x)-x^*|$ in (15b) is replaced by $|x_u(t,x)|_{S_{\mathbb{X}}^*}$, where $S_{\mathbb{X}}^* := \{x^* : \exists u^* \text{ s.t. } (x^*,u^*) \in S \text{ and (6) holds}\}$, and system (1) is locally controllable at x^* for all $x^* \in S_{\mathbb{X}}^*$. In the latter case, Theorems 8 and 9 are still correct if the term $\rho(|x-x^*|)$ in (11) is replaced by $\rho(|(x,u)-(x^*,u^*)|)$ and the term $|x_u(t,x)-x^*|$ in (15b) is replaced by $|(x_u(t,x),u(t))-(x^*,u^*)|$. Note that both extensions can also be combined.

4. DISCUSSION

We now discuss some of the implications of the preceding results and their significance for giving closed-loop guarantees in economic MPC. Namely, in case that the optimal operating regime for a system is steady-state operation, it is desirable that also the closed-loop system resulting from application of an economic MPC scheme "finds" this optimal behavior, i.e., converges to the optimal steady-state. Convergence (or even (practical) asymptotic stability) of the closed-loop system to the optimal steady-state has previously been established using the same strict dissipativity condition as above, both for suitably defined economic MPC schemes with and without additional terminal constraints, see, e.g., (Angeli et al., 2012; Amrit et al., 2011; Grüne, 2013). However, for general nonlinear systems and nonconvex cost functions, computing a storage function λ in order to verify strict dissipativity is a very hard task, and no general systematic procedure is available to this end. On the other hand, the results of the previous section allow to conclude convergence of the closed-loop system *without* having to verify the strict dissipation inequality (11). Namely, Theorem 9 guarantees that if the optimal operating regime for a system is steady-state operation (in its strict form as in Definition 2), then it is strictly dissipative, which in turn can be used to conclude that the closed-loop system converges to the optimal steady-state. Loosely speaking, this means that the closed-loop system "does the right thing", i.e., it "finds" the optimal operating regime. Note that this is true in both economic MPC settings with and without additional terminal constraints. An interesting question is whether the same is true if not steady-state operation is optimal, but periodic operation. This will be treated in Section 5.

To summarize the above a little more pointedly, or results show that for guaranteeing desired closed-loop behavior, the explicit computation of a storage function λ in order to verify strict dissipativity is *not* necessary, since its existence follows from controllability and uniform suboptimal operation off steady-state.

5. OPTIMAL PERIODIC OPERATION

We now turn our attention to the case where the optimal operating regime for system (1) is not stationary, but some periodic orbit. In this case, the results of Section 3 can be extended to show that a modified dissipativity condition is necessary and sufficient for optimal periodic operation. To this end, we first formally define the notion of a periodic orbit and optimal periodic operation, analogous to the steady-state case in Section 3.

Definition 11. A *feasible P-periodic orbit* of system (1) with $P \in \mathbb{I}_{\geq 1}$ is a set of state/input pairs $\Pi = \{(x_0^p,u_0^p),\dots,(x_{P-1}^p,u_{P-1}^p)\}$ such that $(x_k^p,u_k^p) \in \mathbb{Z}$ for all $k \in \mathbb{I}_{[0,P-1]}$, $x_{k+1}^p = f(x_k^p,u_k^p)$ for all $k \in \mathbb{I}_{[0,P-2]}$, and $x_0^p = f(x_{P-1}^p,u_{P-1}^p)$.

In the following, denote by $\Pi_{\mathbb{X}}$ the projection of Π on $\mathbb{X}$ and let $\widetilde{\Pi}_{\mathbb{X}} := \{(x_k^p,\dots,x_{P-1}^p,x_0^p,\dots,x_{k-1}^p) : k \in \mathbb{I}_{[0,P-1]}\}$ denote the set of all state sequences starting at some point $x_k^p \in \Pi_{\mathbb{X}}$ and then following once the periodic orbit Π.

Definition 12. System (1) is *optimally operated at a periodic orbit* Π if for each $x \in \mathbb{X}^0$ and each $u \in \mathbb{U}^\infty(x)$ the following inequality holds:

$$\liminf_{T\to\infty} \frac{\sum_{t=0}^{T-1} \ell(x_u(t,x),u(t))}{T} \geq \frac{1}{P}\sum_{k=0}^{P-1} \ell(x_k^p,u_k^p) \quad (18)$$

System (1) is *uniformly suboptimally operated off the periodic orbit* Π if in addition there exist $\bar{\delta} > 0$ and $d \in \mathcal{K}_\infty$ such that for each $\delta > 0$ and each $\varepsilon > 0$ there exists $R_{\varepsilon,\delta} \in \mathbb{I}_{\geq 0}$ such that $\delta / R_{\varepsilon,\delta} \geq d(\varepsilon)$ for all $\delta \geq \bar{\delta}$ and such that for each $x \in \mathbb{X}^0$, each $u \in \mathbb{U}^\infty(x)$, and each $T \in \mathbb{I}_{\geq 0}$ at least one of the following two conditions holds:

$$\sum_{t=0}^{TP-1} \ell(x_u(t,x),u(t)) > T\sum_{k=0}^{P-1} \ell(x_k^p,u_k^p) + \delta, \quad (19a)$$

$$\#\{t \in \mathbb{I}_{[0,T-1]} : \sum_{j=0}^{P-1} |x_u(tP+j,x)|_{\Pi_{\mathbb{X}}} > \varepsilon\} \leq R_{\varepsilon,\delta} \quad (19b)$$

Note that for $P=1$, the definitions of optimal steady-state operation and uniform suboptimal operation off steady-state are recovered. Furthermore, we note that analogous to the steady-state case, if system (1) is optimally operated at some periodic orbit $\Pi^* = \{(x_0^{p*},u_0^{p*}),\dots,(x_{P-1}^{p*},u_{P-1}^{p*})\}$, then Π^* is necessarily an optimal periodic orbit for system (1), i.e. we have

$$\frac{1}{P}\sum_{k=0}^{P-1} \ell(x_k^{p*},u_k^{p*}) = \inf_{P\in\mathbb{I}_{\geq 1},\Pi\in S_\Pi^P} \frac{1}{P}\sum_{k=0}^{P-1} \ell(x_k^p,u_k^p), \quad (20)$$

where S_Π^P denotes the set of all feasible P-periodic orbits.

In order to generalize the results of Section 3, we define the P-step system with state $\tilde{x} = (\tilde{x}_0,\dots,\tilde{x}_{P-1}) \in \mathbb{X}^P$, input $\tilde{u} = (\tilde{u}_0,\dots,\tilde{u}_{P-1}) \in \mathbb{U}^P$, dynamics $\tilde{x}(t+1) = f^P(\tilde{x}(t),\tilde{u}(t))$ and initial condition [7] $\tilde{x}_{P-1}(0) = x$, where

$$f^P(\tilde{x},\tilde{u}) := \begin{bmatrix} x_{\tilde{u}}(1,\tilde{x}_{P-1}) \\ \dots \\ x_{\tilde{u}}(P,\tilde{x}_{P-1}) \end{bmatrix} = \begin{bmatrix} f(\tilde{x}_{P-1},\tilde{u}_0) \\ f(f(\tilde{x}_{P-1},\tilde{u}_0),\tilde{u}_1) \\ \dots \end{bmatrix}. \quad (21)$$

The pointwise-in-time state and input constraints $(x,u) \in \mathbb{Z}$ for system (1) translate into pointwise-in-time state and input constraints $(\tilde{x},\tilde{u}) \in \widetilde{\mathbb{Z}}$ for the P-step system, with $\widetilde{\mathbb{Z}} := \{(\tilde{x},\tilde{u}) : (x_{\tilde{u}}(j,\tilde{x}_{P-1}),\tilde{u}_j) \in \mathbb{Z} \ \forall\ j \in \mathbb{I}_{[0,P-1]}\}$. Furthermore, the sets $\widetilde{\mathbb{Z}}^0$ and $\widetilde{\mathcal{Z}}_M$ for the P-step system can then be calculated analogously to (3) and (9), respectively. For a given control sequence $\tilde{u} \in \mathbb{U}^{KP}$ with $K \in \mathbb{I}_{\geq 1}$, the corresponding solution of system (21) is denoted by $\tilde{x}_{\tilde{u}}(t,x)$ for $t \in \mathbb{I}_{[1,K]}$. This means that for a given control sequence $u \in \mathbb{U}^{KP}$ with $K \in \mathbb{I}_{\geq 1}$, partitioned

[7] Initial conditions for the first $P-1$ components of $\tilde{x}$, i.e., $\tilde{x}_0(0),\dots,\tilde{x}_{P-2}(0)$, can be arbitrary.

into $\tilde{u}(t) = (u(tP), \dots, u((t+1)P-1))$ for all $t \in \mathbb{I}_{[0,K-1]}$, we have that $\tilde{x}_{\tilde{u}}(t,x) = (x_u((t-1)P+1,x), \dots, x_u(tP,x))$ for all $t \in \mathbb{I}_{[1,K]}$. Next, for $(\tilde{x},\tilde{u}) \in \widetilde{\mathbb{Z}}$ and a P-periodic orbit Π, define $|(\tilde{x},\tilde{u})|_\Pi := \sum_{j=0}^{P-1} |(x_{\tilde{u}}(j,x_{P-1}),u_j)|_\Pi$ and $|\tilde{x}|_{\Pi_\mathbb{X}} := \sum_{j=0}^{P-1} |x_{\tilde{u}}(j,x_{P-1})|_{\Pi_\mathbb{X}}$. Furthermore, define the cost function associated to the P-step system (21) as $\tilde{\ell}(\tilde{x},\tilde{u}) := \sum_{j=0}^{P-1} \ell(x_{\tilde{u}}(j,\tilde{x}_{P-1}),\tilde{u}_j)$. Then, for an optimal periodic orbit Π^* of system (1), for each $k \in \mathbb{I}_{[0,P-1]}$ the point $\tilde{x}^* = (x_k^{p*}, \dots, x_{P-1}^{p*}, x_0^{p*}, \dots, x_{k-1}^{p*}) \in \widetilde{\Pi}_\mathbb{X}^*$ with corresponding input [8] $\tilde{u}^* = (u_{k-1}^{p*}, \dots, u_{P-1}^{p*}, u_0^{p*}, \dots, u_{k-2}^{p*})$ is an optimal state/input equilibrium pair for the P-step system (21) with corresponding cost $\tilde{\ell}(\tilde{x}^*,\tilde{u}^*) = \sum_{k=0}^{P-1} \ell(x_k^{p*},u_k^{p*})$. We can now state the following result.

Lemma 13. Suppose that ℓ is bounded from below on $\mathbb{Z}^0$. Then system (1) is optimally operated at a P-periodic orbit Π (uniformly suboptimally operated off the P-periodic orbit Π) if and only if the corresponding P-step system (21) is optimally operated at steady-state (uniformly suboptimally operated off steady-state [9]).

Proof: Consider arbitrary $x \in \mathbb{X}^0$ and $u \in \mathbb{U}^\infty(x)$, and define the sequence $\tilde{u}$ as $\tilde{u}(t) = (u(tP), \dots, u((t+1)P-1))$ for all $t \in \mathbb{I}_{\geq 0}$. If the P-step system is optimally operated at steady-state, we obtain

$$\begin{aligned}
&\liminf_{T\to\infty} \frac{\sum_{t=0}^{PT-1} \ell(x_u(t,x),u(t))}{PT} \\
&= \liminf_{T\to\infty} \frac{\sum_{t=0}^{T-1} \tilde{\ell}(\tilde{x}_{\tilde{u}}(t,x),\tilde{u}(t))}{PT} \\
&\overset{(13)}{\geq} \frac{1}{P}\tilde{\ell}(\tilde{x}^*,\tilde{u}^*) = \frac{1}{P}\sum_{k=0}^{P-1} \ell(x_k^{p*},u_k^{p*}),
\end{aligned} \tag{22}$$

where the first equality follows from the definition of $\tilde{\ell}$. Furthermore, we have

$$\begin{aligned}
&\liminf_{T\to\infty} \frac{\sum_{t=0}^{PT-1} \ell(x_u(t,x),u(t))}{PT} \\
&\geq \liminf_{T\to\infty} \frac{\sum_{t=0}^{T-1} \ell(x_u(t,x),u(t))}{T},
\end{aligned} \tag{23}$$

since the sequence on the left hand side is a subsequence of the one on the right hand side, and hence (18) implies (22). On the other hand, since ℓ is assumed to be bounded from below on $\mathbb{Z}^0$, it is straightforward to show that (23) in fact holds with equality, and hence also (22) implies (18). This means that system (1) is optimally operated at a P-periodic orbit Π if and only if the corresponding P-step system is optimally operated at steady-state. Finally, the equivalence between uniform suboptimal operation off the periodic orbit Π for system (1) and uniform suboptimal operation off steady-state for the P-step system (21) follows from the fact that satisfaction of the condition specified by (19) for system (1) implies satisfaction of the condition specified by (15) (with the slight modification of (15b) as described in the theorem) for the P-step system (21), and vice versa. □

[8] For $k = 0$, $\tilde{u}^* = (u_{P-1}^{p*}, u_0^{p*}, \dots, u_{P-2}^{p*})$.

[9] Here, we need the slightly modified definition of uniform suboptimal operation off steady-state as discussed in Remark 10, i.e., where the term $|\tilde{x}_{\tilde{u}}(t,x) - \tilde{x}^*|$ in (15b) is replaced by $|\tilde{x}_{\tilde{u}}(t,x)|_{\Pi_\mathbb{X}}$.

With the help of Lemma 13 (and Remark 10), we immediately arrive at the following corollary of Theorems 3, 4, 8, and 9.

Corollary 14. Suppose that ℓ is bounded from below on $\mathbb{Z}^0$. Then the following statements hold.

(i) If the P-step system (21) is dissipative on $\widetilde{\mathbb{Z}}^0$ with respect to the supply rate $s(\tilde{x},\tilde{u}) = \tilde{\ell}(\tilde{x},\tilde{u}) - \tilde{\ell}(\tilde{x}^*,\tilde{u}^*)$, then system (1) is optimally operated at the periodic orbit Π^*.

(ii) Consider an arbitrary $M \in \mathbb{I}_{\geq 1}$ and suppose that system (1) is optimally operated at a periodic orbit Π^* and $\tilde{\ell}$ is bounded from above on $\widetilde{\mathcal{Z}}_M$. Then the P-step system (21) is dissipative on $\widetilde{\mathcal{Z}}_M$ with respect to the supply rate $s(\tilde{x},\tilde{u}) = \tilde{\ell}(\tilde{x},\tilde{u}) - \tilde{\ell}(\tilde{x}^*,\tilde{u}^*)$.

(iii) Suppose that the P-step system (21) is strictly dissipative [10] on $\widetilde{\mathbb{Z}}^0$ with respect to the supply rate $s(\tilde{x},\tilde{u}) = \tilde{\ell}(\tilde{x},\tilde{u}) - \tilde{\ell}(\tilde{x}^*,\tilde{u}^*)$ and with a storage function $\tilde{\lambda}$ which is bounded on $\widetilde{\mathbb{X}}^0$. Then system (1) is uniformly suboptimally operated off the periodic orbit Π^*.

(iv) Suppose that system (1) is uniformly suboptimally operated off the periodic orbit Π^*, that the P-step system (21) is locally controllable at each $\tilde{x}^* \in \widetilde{\Pi}_\mathbb{X}^*$ in τ steps for some $\tau \in \mathbb{I}_{\geq 0}$, and that $\tilde{\ell}$ is locally bounded on $\widetilde{\mathbb{Z}}^0$. Then the P-step system (21) is strictly dissipative on $\widetilde{\mathbb{Z}}^0$ with respect to the supply rate $s(\tilde{x},\tilde{u}) = \tilde{\ell}(\tilde{x},\tilde{u}) - \tilde{\ell}(\tilde{x}^*,\tilde{u}^*)$ and with a storage function $\tilde{\lambda}$ which is bounded on $\widetilde{\mathbb{X}}^0$.

Given the above, similar statements as in Section 4 can now be made for the case of optimal periodic operation. Namely, for economic MPC schemes with (periodic) terminal constraints, Grüne and Zanon (2014) discuss that strict dissipativity of the P-step system results in convergence of system (1) to the optimal periodic orbit, which is currently under further investigation (Zanon et al., 2015). For economic MPC without terminal constraints, our recent work (Müller and Grüne, 2015b) established optimal closed-loop performance of a P-step MPC scheme under a periodic dissipativity condition for system (1). The same results as well as convergence to the optimal periodic orbit can be established using instead the above strict dissipativity condition for the P-step system [11] (see Müller and Grüne (2015a)). Thus, similar to the case where steady-state operation is optimal, it follows that the closed-loop system resulting from a suitably defined economic MPC scheme (with or without terminal constraints) will "do the right thing", i.e., converge to the optimal periodic orbit if periodic operation (in its strict form) is optimal, and this can again be concluded *without* having to verify the corresponding strict dissipativity condition for the P-step system.

[10] Here, we need the slightly modified definition of strict dissipativity as discussed in Remark 10, i.e., where the term $|\tilde{x} - \tilde{x}^*|$ in (10) is replaced by $|\tilde{x}|_{\Pi_\mathbb{X}}$. The same holds in item (iv) of this corollary.

[11] Here, strict dissipativity with respect to x and u is needed, and hence the considerations of Remark 10 have to be taken into account, i.e., the term $|\tilde{x} - \tilde{x}^*|$ in (10) has to be replaced by $|(\tilde{x},\tilde{u})|_\Pi$.

6. CONCLUSIONS

In this paper, we analyzed and discussed the role of dissipativity in the context of economic MPC. In particular, we established that strict dissipativity conditions are necessary and sufficient for classifying the optimal operating regime for a system, both in case of optimal steady-state operation and optimal periodic operation. This allows us to conclude that the closed-loop system resulting from application of an economic MPC scheme will "do the right thing" without having to verify the dissipativity property.

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