Model Predictive Control of Residential Energy Systems Using Energy Storage & Controllable Loads

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Abstract Local energy storage and smart energy scheduling can be used to flatten energy profiles with undesirable peaks. Extending a recently developed model to allow controllable loads, we present Centralized and Decentralized Model Predictive Control algorithms to reduce these peaks. Numerical results show that the additional degree of freedom leads to improved performance.

1 Introduction

Widespread uptake of local electricity generation technologies such as solar photovoltaics and wind turbines are leading to undesirable voltage swings in electricity distribution networks. Large variations in the grid profile, resulting from periods of high local energy generation followed by periods of high power demand require significant network infrastructure and can lead to a degradation of power quality and even outages. In response to these challenges local energy storage is increasingly considered to reduce the peak demand [4], [5]. Additionally, a recent study [1] suggests that up to 60% of the consumption of a household, in the form of appliances such as air conditioners and refrigerators, is elastic or schedulable. Therefore, an alternate,

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but complementary, approach to the use of energy storage devices to reduce the grid variations involves energy consumption scheduling [3], [7].

We consider a small, neighborhood-level, electricity network consisting of several residences. Each residence comprises a Residential Energy System (RES), consisting of a residential load, a local energy storage element, and solar photovoltaic panels. Each RES is connected to the wider electricity network. For the sake of a simplicity, we refer to the storage element as a battery though fuel cells also satisfy our proposed energy storage model and constraints. The important contribution with respect to our previous work [8] is the extension of the model to handle controllable or elastic loads. While the extension of the model is trivial, the resulting constraints are not obvious.

The paper is organized as follows. The extended model is introduced in Section 2 together with two performance metrics. Section 3 introduces two Model Predictive Control algorithms and shows how to incorporate controllable loads in a receding horizon algorithm. The paper concludes with numerical results in Section 4.

2 The Residential Energy System

Let $\mathcal{I} \in \mathbb{N}$ be the number of RESs connected in the local area under consideration. A simple model of the RES of user $i \in \{1, \dots, \mathcal{I}\}$ is:

$$\begin{aligned} x_i(k+1) &= x_i(k) + T u_{i_1}(k), \\ z_i(k) &= w_i(k) + u_{i_1}(k) + u_{i_2}(k) \end{aligned} \tag{1}$$

where x_i is the state of charge of the battery in kWh, u_{i_1} is the battery charge/discharge rate in kW, u_{i_2} is the controllable load in kW, w_i is the static load minus the local generation in kW, and z_i is the power supplied by/to the grid in kW. Here, T represents the length of the sampling interval in hours; e.g., $T = 0.5$ corresponds to 30 minutes. The RES network is then defined by the following discrete-time system

$$x(k+1) = f(x(k), u(k)), \tag{2}$$

$$z(k) = h(u(k), w(k)) \tag{3}$$

where $x, w \in \mathbb{R}^{\mathcal{I}}$, $u \in \mathbb{R}^{2\mathcal{I}}$, and the definitions of f and h are obvious from (1). We assume constraints on the battery capacity and charge/discharge rates are given by $C_i, \overline{u}_i \in \mathbb{R}_{>0}$ and $\underline{u}_i \in \mathbb{R}_{<0}$ so that for each RESs i, $i \in \{1, \dots, \mathcal{I}\}$:

$$0 \le x_i(k) \le C_i \quad \text{and} \quad \underline{u}_{i_1} \le u_{i_1}(k) \le \overline{u}_{i_1} \quad \forall k \in \mathbb{N}_0. \tag{4}$$

Note that this model adequately captures elements of fuel cells as energy storage devices since the conversion of electricity to hydrogen, and vice versa, is rate-limited and fuel cells have a fixed storage capacity.

We assume that the load can be split into two parts: controllable and static load. The static load is included in w. The controllable loads $\{w_c(k)\}_{k\in\mathbb{N}} \subset \mathbb{R}^{\mathcal{I}}$ must be scheduled during a certain time window. More precisely $w_{c_i}(k+\overline{N}-1)$ can be scheduled during the time interval from $\max\{0,k\}$ to $k+\overline{N}-1$ for a given $\overline{N} \in \mathbb{N}$. This leads to the time-dependent constraints

$$\sum_{j=0}^{k} w_{c_i}(j) - \sum_{j=0}^{k-1} u_{i_2}(j) \leq u_{i_2}(k) \leq \sum_{j=0}^{k+\overline{N}-1} w_{c_i}(j) - \sum_{j=0}^{k-1} u_{i_2}(j) \quad \forall k \in \mathbb{N}_0 \tag{5}$$

for each RES $i \in \{1,\ldots,\mathcal{I}\}$. Observe that at time k, $u_{i_2}(j)$ is fixed for all $j < k$, rather than a control variable, since it is a control action that was already applied. We introduce upper and lower bounds on u_{i_2} reflecting the fact that only a certain amount of the controllable load can be scheduled in one time step, i.e., for each RES i, $i \in \{1,\ldots,\mathcal{I}\}$, and given $\underline{w}_{c_i}$, $\overline{w}_{c_i} \in \mathbb{R}$,

$$\underline{w}_{c_i} \leq u_{i_2}(k) \leq \overline{w}_{c_i} \quad \forall k \in \mathbb{N}_0. \tag{6}$$

We assume that the $\underline{w}_{c_i}$, $\overline{w}_{c_i}$ are chosen such that conditions (5) and (6) can be simultaneously satisfied.

Our goal is to flatten the performance output z. We introduce two relevant performance metrics. The average power demand at time k defined as $\Pi(k) := \frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} z_i(k)$ and let $\mathcal{N}$ denote the simulation length in number of samples. The performance metric of peak-to-peak (PTP) variation of the average demand of all RESs is given by

$$\left(\max_{k\in\{0,\ldots,\mathcal{N}\}} \Pi(k)\right) - \left(\min_{k\in\{0,\ldots,\mathcal{N}\}} \Pi(k)\right). \tag{PTP}$$

The second performance metric is the root-mean-square (RMS) deviation from the average; i.e., the average $\Upsilon := \frac{1}{\mathcal{N}\mathcal{I}}\sum_{k=0}^{\mathcal{N}-1}\sum_{i=1}^{\mathcal{I}}(w_i(k)+w_{c_i}(k))$ is calculated and the respective deviations are quadratically penalized:

$$\sqrt{\frac{1}{\mathcal{N}}\sum_{k=0}^{\mathcal{N}-1}\left(\Pi(k)-\Upsilon\right)^2}. \tag{RMS}$$

3 Model Predictive Control Approaches

We present two Model Predictive Control (MPC) algorithms for the control of a network of RESs. The first approach is a Centralized MPC algorithm. This scheme requires full communication of all relevant variables for the entire network as well as a known model of the network. The second approach is a Decentralized MPC approach where each RES implements its own local

MPC controller. This requires no communication or cooperation between RESs. Both schemes use a receding horizon controller.

MPC iteratively minimizes an optimization criterion with respect to predicted trajectories and implements the first part of the resulting optimal control sequence until the next optimization is performed (see, e.g., [6] or [2]). We propose such a predictive controller for (1). In order to do this, we assume that we have predictions of the residential load and generation some time into the future that is coincident with the horizon of the predictive controller. In other words, given a prediction horizon $N \in \mathbb{N}$, we assume knowledge of $w_i(j)$, $w_{c_i}(j)$ for all $j \in \{k, \dots, k+N-1\}$, where $k \in \mathbb{N}_0$ is the current time.

Before defining the cost function for the MPC approaches, we rewrite the constraints (5) in a receding horizon fashion. The constraints on $u_{i_2}(j)$ in the prediction horizon are captured by

$$\lambda_i^q(k) := \sum_{j=0}^{k+q} w_{c_i}(j) - \sum_{j=0}^{k-1} u_{i_2}(j) \leq \sum_{j=k}^{k+q} u_{i_2}(j) \tag{7a}$$

$$\Lambda_i^q(k) := \sum_{j=0}^{k+\min\{q+\overline{N},N\}-1} w_{c_i}(j) - \sum_{j=0}^{k-1} u_{i_2}(j) \geq \sum_{j=k}^{k+q} u_{i_2}(j) \tag{7b}$$

for $q \in \{0, \dots, N-1\}$ and $i \in \{1, \dots, \mathcal{I}\}$. The term $\min\{q+\overline{N}, N\}$ reflects that we predict only N steps ahead and therefore only controllable load with a deadline during the prediction horizon is considered. Observe that the bounds can be easily updated by $\lambda_i^q(k+1) = \lambda_i^q(k) + w_{c_i}(k+q+1) - u_{i_2}(k)$ and $\Lambda_i^q(k+1) = \Lambda_i^q(k) + w_{c_i}(k + \min\{q+\overline{N}, N\}) - u_{i_2}(k)$.

3.1 Centralized Model Predictive Control

Define the predicted average power usage for the i^{th} RES as

$$\zeta_i(k) := \frac{1}{N}\left(\lambda_i^0(k) - w_{c_i}(k) + \sum_{j=k}^{k+N-1} (w_i(j) + w_{c_i}(j))\right). \tag{8}$$

To implement the Centralized MPC algorithm, we compute the overall average on the prediction horizon by $\bar{\zeta}(k) := \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \zeta_i(k)$ and then minimize the joint cost function

$$\min_{\hat{u}(\cdot)} \sum_{j=k}^{k+N-1} \Big(\bar{\zeta}(k) - \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \underbrace{(w_i(j) + \hat{u}_{i_1}(j) + \hat{u}_{i_2}(j))}_{\hat{z}_i(j)}\Big)^2 \tag{9}$$

with respect to the predicted control input $\hat{u}(k), \hat{u}(k+1), \ldots, \hat{u}(k+N-1)$ with $\hat{u}(\cdot) = (\hat{u}_1(\cdot), \hat{u}_2(\cdot), \ldots, \hat{u}_{\mathcal{I}}(\cdot))^T$ subject to the system dynamics (1), the current state $x(k) = (x_1(k), \ldots, x_{\mathcal{I}}(k))^T$, and the constraints (4), (6), and (7) for all $i \in \{1, \ldots, \mathcal{I}\}$.

Here, and in what follows, we denote predicted controls and outputs in the MPC algorithm by hats; i.e., for the i^{th} RES at time j the predicted control is $\hat{u}_i(j)$ and the predicted performance output is $\hat{z}_i(j)$.

3.2 Decentralized Model Predictive Control

The Centralized MPC approach presented above requires a significant amount of communication overhead. A further drawback of the Centralized MPC approach is that the central entity requires full knowledge of the network model, in particular (4), (6), and (7) for each $i \in \{1, \ldots, \mathcal{I}\}$. Therefore, any change in the network such as the addition of new generation or storage resources requires an update of the central model. As a remedy we propose a decentralized control approach that alleviates the communication and computation difficulties.

A straightforward option in order to flatten the energy profile of the i^{th} RES is to penalize deviations from its (anticipated) average usage defined in (8). With a quadratic cost function, this leads to the finite-horizon optimal control problem

$$\min_{\hat{u}_i(\cdot)} \sum_{j=k}^{k+N-1} (\zeta_i(k) - \underbrace{(w_i(j) + \hat{u}_{i_1}(j) + \hat{u}_{i_2}(j))}_{\hat{z}_i(j)})^2$$

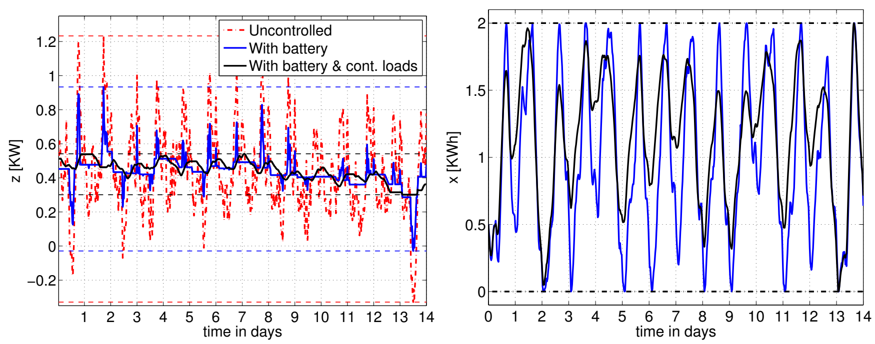
subject to the system dynamics (1), the current state of charge $x_i(k)$ of the energy storage, the constraints (4), (6), and (7) corresponding to the controllable loads. With each RES solving its own optimization problem with no reference to the rest of the network, the aforementioned communication and computation difficulties of Centralized MPC are not present in the Decentralized MPC algorithm.

4 Numerical Results

In this section, we compare the discussed controllers and the impact of controllable load in the model by considering the load and generation profiles for a group of 20 customers drawn from the Australian electricity distribution company Ausgrid. The data from these customers was collected as part of the *Smart Grid, Smart City* project. We use two weeks starting on 1 March

2011. As already mentioned in the introduction, motivated by [1], we split the given load profile into 60% static load and 40% controllable load. Figure 1 visualizes the impact of the energy storage and the controllable loads on the uncontrolled grid profile. Table 1 summarizes the results with respect to the introduced metrics. All simulations use the prediction horizon $N = 48$, $T = 0.5$ and initial battery state $x_i = 0.5$ for all $i \in \{1, \dots, \mathcal{I}\}$. For simplicity we assume that all systems have the same box constraints which for the simulations means $-\underline{u}_i = \overline{u}_i = 0.3$, $C_i = 2$, $\underline{w}_{c_i} = 0$ and $\overline{w}_{c_i} = 1.25$ for all $i \in \{1, \dots, \mathcal{I}\}$.

a) Centralized MPC: aggregated grid (left) and battery (right) profiles



b) Decentralized MPC: aggregated grid (left) and battery (right) profiles

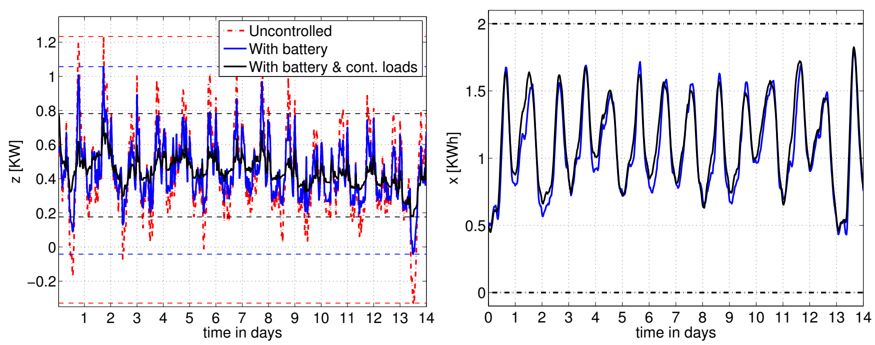


Fig. 1 Australian data for 20 systems. Flattened aggregated grid profile using an energy storage and controllable loads with $\overline{N} = 12$.

The addition of controllable loads yields the expected improvement in the defined performance metrics. Additionally Centralized MPC outperforms Decentralized MPC due to the lack of global coordination in the decentralized setting. $\overline{N}$ seems to play a minor role (assuming that $\overline{N}$ is big enough to call the loads controllable). In the centralized setting we obtain the smallest PTP variation for $\overline{N} = 12$, an observation that requires further investigation.

Table 1 Australian data: Peak-to-peak variation and RMS deviation from the average for 20 RESs and a simulation length of two weeks. Results without Controllable Load (C.L.) and for differing controllable load horizons $\overline{N}$.

	Without C. L.		$\overline{N} = 12$		$\overline{N} = 24$		$\overline{N} = 36$	
	PTP	RMS	PTP	RMS	PTP	RMS	PTP	RMS
No Battery Storage	1.5621	0.2506						
Decentralized MPC	1.0986	0.1663	0.6050	0.0845	0.5555	0.0777	0.5555	0.0777
Centralized MPC	0.9621	0.0952	0.2401	0.0609	0.2915	0.0594	0.2915	0.0594

5 Conclusion

In this paper we have extended our earlier Residential Energy System (RES) model introduced in [8] by adding controllable loads. Numerically we have shown that the additional degree of freedom leads to the expected improvements with respect to the grid profile.

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