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Do zombies rise when interest rates fall: A relationship banking model

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ABSTRACT

A relationship bank or market investors finance an entrepreneur's risky project. Unlike investors, the bank can identify and liquidate bad projects at an interim stage. If the entrepreneur can provide only limited capital, the optimal loan contract induces an inefficient continuation decision, i.e., the bank engages in zombie lending. In the short run – for a given contract – the bank's incentive to roll over bad loans is enhanced if the base interest rate drops. In the long run, however, the bank adjusts the contract to a drop in the interest rate, and the effect on zombification is reversed.

1. Introduction

Zombie firms are the walking dead of an economy: unable to cover their debt obligations with current profits, yet kept alive by banks through loan extensions or favorable refinancing. The term *zombie lending* was coined by Caballero et al. (2008) in their analysis of Japan's 'lost decade' of the 1990s. Following the Global Financial Crisis (GFC), zombie lending regained attention as evidence mounted that many developed economies harbored an alarmingly high share of zombie firms (Adalet McGowan et al., 2018; Banerjee and Hofmann, 2018). For 14 advanced economies, Banerjee and Hofmann (2018) estimate that the zombie share rose from 2% in the late 1980s to 12% in 2016, attributing this trend to reduced financial pressure stemming from expansionary monetary policy and persistently low interest rates. Their findings are supported by further empirical studies (De Martiis and Peter, 2021; Banerjee and Hofmann, 2022; Ciżkowicz et al., 2023).

The connection between low interest rates and zombie lending has also entered public debate (Banerjee and Hofmann, 2022).[1] For example, the *Washington Post* (2020) argued[2]:

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[1] Examples are the following publications: Financial Times on February 5, 2020: "How to avoid a corporate zombie apocalypse" https://www.ft.com/content/1d87c9ec-4762-11ea-aeb3-955839e06441; New York Times on June 15, 2019: "When Dead Companies Don't Die" https://www.nytimes.com/2019/06/15/opinion/sunday/economy-recession.html; The Economist on September 26, 2020: "Why covid-19 will make killing zombie firms off harder" https://www.economist.com/finance-and-economics/2020/09/26/why-covid-19-will-make-killing-zombie-firms-off-harder.

[2] "Here's one more economic problem the government's response to the virus has unleashed: Zombie firms". Washington Post, June 23, 2020, https://www.washingtonpost.com/business/2020/06/23/economy-debt-coronavirus-zombie-firms/.

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> "Years of ultralow interest rates intended to stimulate the economy after each of three 21st-century recessions created the conditions for zombies to proliferate [...] Weak growth prompts the central bank to cut interest rates, which allows zombies to multiply".

Similarly, *The Guardian* (2020) reported that[3]:

> "As many as one in seven UK firms are potentially "under sustained financial strain" and had been able to "stagger on" partly thanks to low interest rates [...]".

While there is substantial evidence on the connection between the prevalence of zombie firms and the level of interest rates, the precise mechanism through which low interest rates foster zombie activity remains unclear. Intuitively, a decline in interest rates should first and foremost reduce firms' debt-servicing costs and thereby lower the share of zombies (Banerjee and Hofmann, 2018). To resolve this apparent puzzle, we develop a theoretical framework that examines how interest rates shape banks' incentives to roll over non-performing loans. Our model focuses on one specific zombification channel inspired by Hu and Varas (2021). In their framework, continued bank financing enhances an entrepreneur's reputation; sufficiently reputable entrepreneurs can then obtain cheap market financing. This dynamic incentivizes privately informed banks to help zombie firms build a false reputation of creditworthiness, enabling the banks to transfer credit to uninformed investors and recover full repayment. Empirical support for this risk-shifting channel is provided by Won (2023).

Building on this idea, we construct a contract-theoretic model of relationship banking that links banks' rollover incentives directly to the central bank's policy rate. In our setup, an entrepreneur can finance a risky project of ex-ante unknown quality through either the bank or the market. The bank faces higher capital costs but has an informational advantage: it learns the project's quality at an interim stage. At this point, the bank decides whether to liquidate the project or roll over the loan. Importantly, the rollover decision itself sends a positive signal to market investors, who may then provide ex post financing.[4] The loan contract between the relationship bank and the entrepreneur specifies (i) the bank's initial outlay and (ii) the ex post repayment. If the entrepreneur has sufficient internal funds, the repayment terms induce efficient continuation, maximizing joint surplus. However, if the entrepreneur is cash-constrained ex ante, the second-best contract features an inefficiently high repayment. In this case, the bank continues projects that should be liquidated from a welfare perspective, thereby engaging in zombie lending.

Our contribution is threefold. First, we develop a simple and tractable contract-theoretic model in which it is in the bank's best interest to extend zombie loans. The mechanism arises from the bank's trade-off between rent extraction and efficiency, which it resolves by rolling over zombie loans. Second, we analyze how changes in the interest rate affect this zombie-lending mechanism in the short run, i.e., for a given loan contract. Put differently, we consider a situation in which the bank holds legacy debt when the central bank unexpectedly changes the interest rate. Third, we examine the long-run implications by studying how interest rate changes influence the granting of new loans and the zombification mechanism, for example in the case of a persistently low-interest-rate environment.

We begin our comparative static analysis by examining an unanticipated change in the interest rate under a given second-best contract. Since cheaper financing implies that more projects should be continued from a welfare perspective, one might expect zombie lending to decrease. However, we show the opposite: the bank becomes more patient as the interest rate falls, making continuation and extraction of inefficiently high repayments more attractive. This effect, however, is mitigated or even reversed when loan contracts feature variable rates tied to a benchmark rate, e.g., the central bank rate.

In the long run, the bank adjusts its loan contract to the prevailing interest rate. In this case, we find that a decline in the interest rate reduces the probability of zombie lending. The mechanism is that lower interest rates increase market investors' willingness to fund risky projects ex ante, which forces the bank to offer the entrepreneur a more favorable contract. This contract specifies a lower ex-post repayment, reducing the bank's incentive to roll over zombie loans. The effect diminishes if falling interest rates make alternative investment opportunities – such as investments in capital-intensive industries – more attractive to market investors.

Finally, we extend our analysis by linking the model to additional empirical patterns. In our baseline setup, relationship banks engage in zombie lending irrespective of their capital structure. We show that more leveraged banks (with lower equity shares) have stronger incentives to roll over bad loans. Moreover, the probability of zombie lending rises during economic downturns. Both results are consistent with empirical evidence, e.g., Giannetti and Simonov (2013), Schivardi et al. (2021), and De Martiis and Peter (2021).

The paper is organized as follows. We begin with a discussion of the related literature. Section 2 introduces the model, outlines the first-best outcome, and defines zombie lending. In Section 3, we solve the bank's contracting problem and derive conditions under which zombie lending arises in optimum. Section 4 presents comparative statics with respect to interest rate changes. Section 4.2 examines the impact of an interest rate change on the bank's continuation decision under a given and fixed loan contract, while Section 4.3 incorporates contract adjustments. Sections 5 and 6 extend the model and show its robustness. Policy implications are discussed in Section 7, and Section 8 concludes. Proofs are collected in Appendix A, with additional material available in the online Appendix B.

[3] "Zombie firms' a major drag on UK economy, analysis shows". The Guardian, May 6, 2019, https://www.https://www.theguardian.com/business/2019/may/06/zombie-firms-a-major-drag-on-uk-economy-analysis-shows.

[4] Evidence that recent bank loans are perceived as positive signals by public investors is documented by Ma et al. (2019), who show that borrowers with fresh private loans receive more favorable terms for subsequent bond issues. Bittner et al. (2021) report a similar effect in trade credit, where suppliers interpret loan rollovers as a signal of creditworthiness. Classic contributions also highlight this signaling role of bank credit: James (1987) documents that announcements of new bank loans trigger positive stock price reactions, while Diamond (1991) provides a theoretical argument based on moral hazard that early bank loans can build borrower reputation and facilitate future access to direct debt markets.

Related literature

The literature on zombie lending starts with Caballero et al. (2008) and Peek and Rosengren (2005), who analyze the impact of the Japanese asset price bubble in the 1990s on the banking industry.[5] Zombie lending gained renewed attention in the aftermath of the GFC and the European debt crisis. Adalet McGowan et al. (2018) and Banerjee and Hofmann (2018) document a high share of zombie firms in various developed economies in recent years. Several articles investigate the role of fiscal stimulus and central bank policies on the prevalence of zombification.[6] For instance, Acharya et al. (2021a) find that under-capitalized banks that relied heavily on support from the European Central Bank (ECB) increased their zombie lending. Relatedly, investigating the ECB's Outright Monetary Policy (OMT), Acharya et al. (2019) document zombie lending for banks that remained undercapitalized post OMT.[7] Closer related to our paper are the empirical contributions investigating the connection between the base interest rate and zombie lending (Borio, 2018; De Martiis and Peter, 2021; Banerjee and Hofmann, 2022; Blažková and Chmelíková, 2022). For instance, the estimates by Banerjee and Hofmann (2022, p.32) suggest that "the roughly 10 percentage point decline in nominal interest rates across advanced economies since the mid-1980s can account for around 17 percent of the rise in the zombie share [...]".[8]

An important branch of the theoretical literature on zombie lending models weakly capitalized banks with limited liability. These banks have incentives to 'gamble for resurrection' by keeping their insolvent borrowers alive (Bruche and Llobet, 2014; Acharya et al., 2021b). In Bruche and Llobet (2014), banks privately learn the number of bad loans they possess at an interim stage. At that stage, the return of bad loans is uncertain, and thus banks that possess many bad loans have the incentive to hide losses and gamble for resurrection.[9] Bruche and Llobet (2014) propose a regulatory regime that induces banks to disclose their bad loans. Relying on a related explanation for zombie lending, Acharya et al. (2021b) build a model with heterogeneous firms and heterogeneous banks. Firms differ in productivity and risk, whereas banks differ in equity share. The model gives rise to 'diabolic sorting': poorly capitalized banks lend to firms with low productivity.[10] Acharya et al. (2021b) also analyze the impact of conventional (interest rate) and unconventional (forbearance) monetary policy on zombification. They point out that, in a dynamic setting, myopic policies result in low interest rates and high forbearance that keep zombies alive and productivity low. In contrast to our findings, low interest rates without forbearance do not promote zombie lending.

Furthermore, and more closely related to our study, is the extant literature that relies on models of relationship banking to explain zombie lending (Hu and Varas, 2021; Aragon, 2022; Faria-e Castro et al., 2024).[11] Faria-e Castro et al. (2024) develop a model in which relationship banks evergreen loans by offering better credit terms to less productive and more indebted firms. Unlike market investors, the relationship bank owns a firm's legacy debt, and thus has the incentive to increase the continuation value of its firm. As a result, financially distressed firms receive 'discounted' credit terms from relationship banks to reduce their probability of default. Aragon (2022) analyzes competition for firm financing between an incumbent bank, which holds the firm's legacy debt, and a competing lender. He shows that debt overhang grants the incumbent monopoly power, enabling it to extract higher rents. This incentive to extract rents may discourage the firm from investing in a new, generally profitable technology, thereby contributing to zombification.

Regarding the modeled zombification mechanism, the article closest related to our study is by Hu and Varas (2021). They consider a dynamic continuous-time model where an entrepreneur initially chooses between bank and market finance. The bank has higher capital costs but receives private information regarding the quality of the entrepreneur's project over time. The quality of the project is either good or bad. Once the bank (and the entrepreneur) learns that the project is bad, continued financing is costly. However, if the bank finances the project for sufficiently many periods, market investors believe that its quality is high, and are thus willing to pay a high price for it.[12] This incentivizes the bank to continue projects that turn out to be of bad quality at interim points in time. These projects are sold later to market investors, who are 'deceived' by the roll-over decision. Similar to our short-run finding, Hu and Varas (2021) find that a reduction in the interest rate applied by the bank and the entrepreneur – but not by market investors – prolongs the duration of the zombie-lending period. Importantly, while in Hu and Varas (2021) the good project should always obtain financing and the bad one should always be liquidated, the welfare optimal quality threshold is endogenous in our model. Specifically, in our model, continuing more projects when interest rates are low is efficient, in line with the immediate effect of a falling interest rate discussed in Banerjee and Hofmann (2018).

Finally, Rocheteau (2024) also develops a model linking low interest rates to the rise of zombie firms. He considers a dynamic general equilibrium framework that departs only minimally from a frictionless Arrow–Debreu economy and shows that unsubsidized zombie firms emerge when interest rates are low. Importantly, his analysis is set in a disintermediated economy without a role

[5] Related articles that investigate the Japanese banking sector are Hoshi (2000), Giannetti and Simonov (2013), and Kwon et al. (2015).

[6] The interaction of regulatory forbearance and zombie lending is investigated by Chari et al. (2021). Blattner et al. (2023) document that capital requirements affect zombie lending, especially by low-capitalized banks.

[7] Zombie lending in the aftermath of the European debt crisis is also documented by Acharya et al. (2024). They report that zombie lending led to excess production capacity, which in turn led to significantly higher pressure on prices, and thus lower inflation. Further empirical studies on zombie lending include Gouveia and Osterhold (2018), Andrews and Petroulakis (2019), and Jordà et al. (2022).

[8] In a VOXeu column, Laeven et al. (2000) question whether there is a clear link between low interest rates and zombification.

[9] A related model where banks have the incentive to roll over loans to hide the loan quality from the market is analyzed by Rajan (1994).

[10] Tracey (2025) proposes a further model where zombie lending helps low-productivity firms to survive.

[11] According to most models, zombie lending has negative implications for the economy. An exception is Jaskowski (2015), who builds a model in which zombie lending improves ex-ante lending and can prevent ex-post fire sales, thereby enhancing overall efficiency.

[12] Somewhat related, Puri (1999) builds a model where the bank's decision at an intermediate stage affects investor evaluations of securities the bank underwrites. In her model, investors may effectively repay the firm's bank loan.

for banks. By contrast, in our explanation of zombification, banks play a central role, making our approach orthogonal to that of Rocheteau (2024).

An important distinction from the existing literature is that we investigate both the short-run and long-run implications of low interest rate environments. Specifically, we analyze how a bank's incentives to engage in zombification are affected by low interest rates when it holds legacy debt, as well as how persistently low rates influence the granting of new loans and the potential evergreening of these. This distinction enables us to show that different policy responses are required to mitigate zombification in the short run versus the long run.

2. The model

We consider an economy over three dates $t = 0, 1, 2$. There are three types of risk-neutral agents: an entrepreneur (she), a relationship bank, and investors. We denote all variables in terms of their respective date $t = 2$ future values. In Section 4, we explicitly present how the variables depend on the interest rate.

At $t = 0$, the entrepreneur owns a risky business project of ex-ante unknown quality θ. The project requires an initial investment of $I > 0$ at $t = 0$. If the project is initiated at $t = 0$, then it generates a payoff of $\gamma\theta$, with $\gamma > 0$, at the end of date $t = 1$, and a payoff of θ at date $t = 2$. The project quality is distributed according to c.d.f. $F(\theta)$ and density $f(\theta) > 0$ on $[\underline{\theta}, \bar{\theta}]$. The expected quality

$$\mu := \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta)\, \mathrm{d}\theta > 0 \tag{1}$$

is assumed to be strictly positive. The entrepreneur's initial wealth is $w \geq 0$. We assume that $w < I$ so that the entrepreneur requires external finance to implement her business project. The entrepreneur can sign a loan contract with the bank or borrow from (sell the project to) investors. She can also decide not to implement the business project.

At $t = 0$ the bank can make a take-it-or-leave-it loan contract (d, R) offer to the entrepreneur. The bank finances $I - d$ of the project, and the entrepreneur invests equity capital d. The contract also specifies the gross repayment R from the entrepreneur to the bank at $t = 2$. For ease of exposition, we assume that the contract transfers the date $t = 1$ cash flow and control rights to the bank. In Section 6.1, we examine the case where the loan takes the form of a standard two-period debt contract. The contract specifies repayments in both $t = 1$ and $t = 2$, while the entrepreneur retains the project's cash flow and control rights as long as she meets these repayment obligations. We show that optimal contracts involving a transfer of cash flow and control rights are outcome-equivalent to the optimal standard debt contracts. At $t = 1$, the bank has the cost of $c > 0$ for engaging in this relationship lending, which can be interpreted as monitoring costs. Due to this monitoring, the bank learns the quality of the project θ at the beginning of date $t = 1$. The bank then decides whether to continue the project or liquidate it. In case of liquidation, the project pays a liquidation value $L > 0$ at the end of date $t = 1$. This liquidation value L is independent of the project's quality θ. A continued project generates a return of $\gamma\theta$ at the end of date $t = 1$ and of θ at date $t = 2$. Finally, the parties commit at $t = 0$ to terminate the relationship at the beginning of $t = 2$ and to sell the project to investors. In other words, the project sell-off to the investors, and thus R, is made before the return θ is realized. Let $L < (1 + \gamma)\mu$.

A large group of investors act in a perfectly competitive financial market. Investors can either purchase (finance) the project at a price P_0 at date $t = 0$ or at a price P_2 at the beginning of date $t = 2$.[13] If investors purchase the project at date $t = 0$, they learn its quality only indirectly at the end of date $t = 1$, where it pays out $\gamma\theta$. At this point, it is no longer possible to liquidate the project in $t = 1$ (and there is no liquidation opportunity in $t = 2$). Thus, the disadvantage of market finance compared to bank finance is that projects with low returns cannot be terminated at the intermediate date $t = 1$. The advantage of market finance is that the market does not incur any costs. If investors purchase the project at the beginning of date $t = 2$, they pay a price P_2 to the entrepreneur and receive the return θ at the end of date $t = 2$. Importantly, if the project is initially financed via the bank, there is asymmetric information at date $t = 2$ between the bank/entrepreneur and investors. The investors are uncertain about the quality of the project. However, they observe the signed loan contract and correctly understand the bank's incentives to continue projects at date $t = 1$, thus updating their belief regarding the offered project's quality accordingly.

The timeline of our model, particularly the project's investment and returns at the three dates, is depicted in Fig. 1.

Throughout, we assume that the bank can make a profitable loan offer to the entrepreneur. Roughly speaking, this is the case if the bank's cost c and the required investment I is not too high.[14]

Next, we outline the first-best outcome, that is, the maximization of the joint surplus of entrepreneur and bank in the absence of contractual frictions (welfare maximization). This serves as a clear benchmark for defining zombie lending. Bank finance is efficient relative to market finance if the expected gains from liquidating low-quality projects early exceed the bank's operating costs. Under market finance, investors only learn about the project's quality at the end of period $t = 1$. In contrast, with bank finance, the project's quality becomes observable at the beginning of $t = 1$. This enables the bank to liquidate low-quality projects immediately at $t = 1$.

[13] With all parties being risk-neutral, the assumption that investors purchase the whole project at $t = 0$ is without loss in generality. To see this, suppose the entrepreneur sells shares α of her project to investors to finance $I - w$. The lowest share that investors are willing to accept is $\hat{\alpha} = (I - w)/[(1 + \gamma)\mu]$. The expected profit of the entrepreneur from selling share $\hat{\alpha}$ of the project is $\mathbb{E}[-w + (1 - \hat{\alpha})\gamma\theta + (1 - \hat{\alpha})\theta] = (1 + \gamma)\mu - I$. Moreover, note that risk-neutral investors could also finance the project at the beginning of date $t = 1$. This, however, will never happen in equilibrium because the monitoring cost is sunk at the beginning of $t = 1$, but the liquidation decision (usage of the information) is not yet made.

[14] The precise conditions under which each form of financing arises in equilibrium, in the first-best outcome, or when the project is not financed at all are analyzed in the online Appendix B.

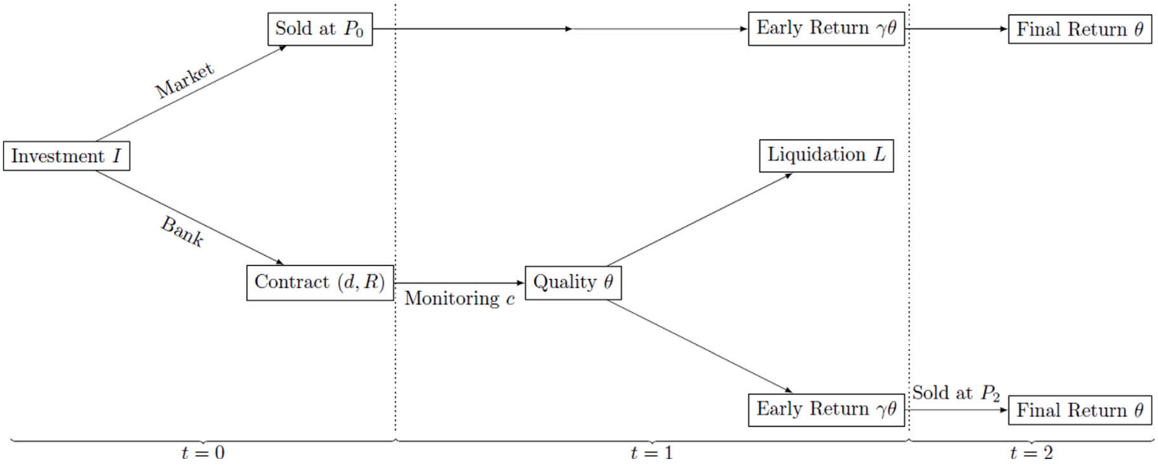

Fig. 1. Timeline of the project's investment, liquidation, and returns.

At that point, continuation is efficient if the project's total return exceeds its liquidation value, i.e., if $\gamma\theta + \theta \geq L$. This inequality can be rewritten as

$$\theta \geq \frac{L}{1+\gamma} =: \theta^*. \tag{2}$$

We refer to θ^* as the efficient quality threshold. It increases with the liquidation value L and decreases with the $t = 1$ share of the project's return γ.

Having established the first-best continuation rule for the bank, we can now formally define zombie lending:

Definition 1 (*Zombie Lending*)**.** At date $t = 1$, if the bank continues financing (rolls over credit for) a project whose quality is below the efficient threshold, $\theta < \theta^*$, this constitutes zombie lending.

In other words, zombie lending arises when a project is not liquidated even though liquidation would maximize total surplus.

Assumptions and Limitations: A few remarks regarding the imposed assumptions and the implied limitations of the model are in order.

The loan contracts we are analyzing are optimal under two conditions. First, the parties – bank and entrepreneur – can commit at $t = 0$ to sell the project to investors at the beginning of $t = 2$. This is efficient as the bank has higher operating costs ($c > 0$) than the market, and once θ is learned, there is no benefit from bank monitoring. Furthermore, note that ex post, at $t = 2$, the entrepreneur prefers to sell only 'bad' projects and to keep 'good' ones since the return $\theta - R$ accrues to the entrepreneur. The bank, however, does not benefit from not selling 'good' projects to the market as it obtains at most the repayment R. The bank suffers if only bad projects are sold to market investors who anticipate this adverse selection, and thus $R > P_2$. Note that the entrepreneur has an incentive to sell only those projects with quality $\theta \leq R$, which implies $P_2 = \mathbb{E}[\theta | R \geq \theta \geq \hat{\theta}] < R$, where $\hat{\theta}$ denotes the bank's roll-over threshold. Consequently, it is in the bank's best interest to maintain a reputation for adhering to the original contract – selling continued projects to the market at $t = 2$ – rather than engaging in renegotiation with the entrepreneur. This behavior is consistent with the empirical evidence reported by Won (2023, p.2), who documents that banks "actively evade losses by persuading uninformed market participants to provide credit, enabling full repayment to the bank". Moreover, such commitment is in the joint interest of both the bank and the entrepreneur ex ante. In the absence of commitment, an adverse selection problem emerges, which reduces the total surplus from bank lending available for distribution between the two parties. Importantly, neither party benefits from limiting the scope of market finance ex post, as market investors – due to competition – always earn zero profits. Finally, "selling the project" need not be interpreted literally. Rather, the bank exits the project before the final return is realized, requiring the entrepreneur to seek financing from an alternative source (e.g., the financial market). This commitment is effectively enforced by the maturity structure of the loan contract.

Second, the parties cannot commit ex ante to a specific roll-over decision. In particular, it is not feasible to specify in the contract a precise quality threshold $\hat{\theta}$ that determines whether the project should be continued. This assumption – that a court cannot verify project quality at date $t = 1$ – is consistent with our further assumption that market investors likewise do not observe project quality at $t = 1$ when financing occurs through a bank loan. Consequently, the roll-over decision can only be incentivized indirectly through the repayment structure to the bank.

Moreover, our analysis focuses on a specific mechanism behind increases in the zombie-firm share: banks' incentives to roll over non-performing loans in order to offload them to uninformed non-bank investors, and the role of the interest-rate environment in amplifying this channel. However, the zombie share is shaped by several other forces. Banks facing default risk may roll over non-performing loans as a 'gambling-for-resurrection' strategy. In addition, financing sources beyond bank lending – such as equity funding (Tuuli, 2024) and trade credit (Shiraishi and Yano, 2021) – can sustain zombie firms. Beyond the provision of capital, institutional factors matter: inadequate insolvency-resolution frameworks can impede exit (Andrews and Petroulakis, 2019), and since the CoViD-19 crisis, government support programs have received particular attention as potential contributors to zombification (Favara et al., 2021; Haynes et al., 2021). For a comprehensive overview of these mechanisms, see Yamada et al. (2025).

3. Financing analysis

3.1. Bank's optimization problem

Before stating the bank's contracting problem, we first investigate its continuation decision at date $t = 1$. The bank rolls over the loan at $t = 1$ if and only if

$$\gamma\theta + \min\{R, P_2 + w - d\} \geq L. \tag{3}$$

In the case of roll-over, the bank receives the cash flow $\gamma\theta$ at the end of date $t = 1$ and the repayment R at date $t = 2$. If, however, the entrepreneur cannot repay R, she goes bankrupt and the bank instead obtains her remaining capital, $P_2 + w - d$. Note that it cannot be optimal to set a repayment so high that the entrepreneur will never be able to repay it. Thus, without loss of generality, we may focus on $\min\{R, P_2 + w - d\} = R$, so that the bank continues all projects with qualities $\theta \geq \hat{\theta}(R)$, where

$$\hat{\theta}(R) = \frac{1}{\gamma}(L - R). \tag{RD}$$

Hence, the bank must take into account how the offered loan contract (d, R) affects its roll-over decision (RD).

The loan contract (d, R) offered by the bank maximizes its expected profit

$$\pi_B(d, R) = F(\hat{\theta}(R))L + \gamma \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta + [1 - F(\hat{\theta}(R))]R - c - I + d \tag{4}$$

subject to

$$\pi_E(d, R) \geq \max\{P_0, 0\}, \tag{PC}$$

$$d \leq w, \tag{LL}$$

where the entrepreneur's net expected profit is given by

$$\pi_E(d, R) = -d + [1 - F(\hat{\theta}(R))][P_2(\hat{\theta}(R)) - R]. \tag{5}$$

The bank's expected profit, $\pi_B(d, R)$, is composed of four elements. If project quality θ falls below the threshold $\hat{\theta}$, the bank receives the liquidation value L. Conversely, when quality exceeds the threshold, the bank obtains the interim cash flow $\gamma\theta$ at $t = 1$ and the repayment R at $t = 2$. The bank finances $I - d$ and bears operating costs c.

The entrepreneur accepts the loan offer only if the participation constraint (PC) is satisfied; that is, if her expected profit from accepting the bank loan weakly exceeds the profit obtainable from her next-best alternative. Recall that a large number of risk-neutral market investors compete. At date $t = 0$, these investors are willing to pay

$$P_0 := (1 + \gamma)\mu - I \tag{6}$$

for a project of unknown quality. At date $t = 2$, investors update their quality expectations conditional on the bank's roll-over decision, and are thus willing to pay

$$P_2(\hat{\theta}(R)) = \mathbb{E}[\theta \mid \theta \geq \hat{\theta}(R)] = \frac{1}{1 - F(\hat{\theta}(R))} \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta. \tag{7}$$

The entrepreneur's net expected profit, $\pi_E(d, R)$, consists of two components. First, she invests the amount d out of own funds initially. Second, if the project is of sufficiently high quality and thus continued at $t = 1$, i.e., if $\theta \geq \hat{\theta}(R)$, the entrepreneur sells the project to market investors at price $P_2(\hat{\theta}(R))$ at the beginning of $t = 2$ and makes repayment R to the bank.

Finally, the entrepreneur's initial outlay cannot exceed her wealth, which implies that the limited liability constraint (LL) must hold.

3.2. Optimal loan contract

Note that the amount initially invested by the entrepreneur herself, d, is an ex-ante one-to-one transfer between the bank and the entrepreneur. A higher d increases the bank's expected profit, does not affect the bank's roll-over decision, and the bank's and entrepreneur's joint surplus is independent of d. Thus, if the limited-liability constraint (LL) is slack, the bank optimally offers a loan contract that maximizes the rents generated by bank financing while extracting as much of these rents as possible through d, the entrepreneur's required initial investment. Overall expected rents are maximized if and only if the roll-over decision is efficient. The bank makes an efficient roll-over decision if and only if $\hat{\theta}(R) = \theta^*$. This is achieved for the repayment

$$R^* = \frac{L}{1 + \gamma} = \theta^*. \tag{8}$$

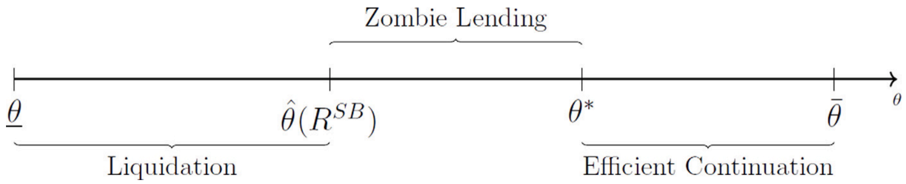

Fig. 2. The bank's decision at date $t = 1$ under a second-best contract.

Let d^* be the entrepreneur's initial outlay that satisfies the participation constraint with equality for $R = R^*$, implicitly given by $\pi_E(d^*, R^*) = \max\{P_0, 0\}$. Given that the entrepreneur's initial outlay cannot exceed her wealth, $d \leq w$, the first-best loan contract (d^*, R^*) is feasible, and thus offered if $d^* \leq w$.

Proposition 1 (*First-best Contract*)**.** *The loan contract* (d, R) *offered by the bank induces the efficient roll-over decision at* $t = 1$ *if*

$$w \geq \int_{\theta^*}^{\bar{\theta}} [\theta - \theta^*] f(\theta)\, d\theta - \max\{P_0, 0\} =: d^*. \tag{9}$$

The loan contract specifies

$$d = d^* \text{ and } R = R^* = \theta^*. \tag{10}$$

If the entrepreneur lacks sufficient own funds, $w < d^*$, the bank cannot extract the full surplus generated by efficient bank lending. In this case, the bank sets the entrepreneur's initial outlay at the maximum feasible level, i.e., $d = w$. Observe that for $d = w$ and $R = R^* = \theta^*$, the participation constraint (PC) is non-binding. At this point, the bank faces a trade-off between rent extraction and allocative efficiency.

As the bank's expected profit increases and the entrepreneur's expected profit decreases in the final repayment R, the bank optimally chooses the highest repayment that the entrepreneur is willing to accept. In other words, the optimal repayment renders the entrepreneur indifferent between accepting the bank loan and her next-best alternative. With this contract, the bank continues a project at $t = 1$ if its quality θ exceeds the threshold $\hat{\theta}(R) = \gamma^{-1}(L - R)$. Since $R > R^*$ and $\hat{\theta}(R^*) = \theta^*$, the bank applies a threshold $\hat{\theta} < \theta^*$, which is too lenient from a welfare perspective: the bank engages in zombie lending.

Before formally stating the second-best optimal contract, it is useful to note that there exists an upper bound on the repayment R. The higher the repayment, the more projects are continued at $t = 1$, and thus the lower is the expected quality that market investors infer at $t = 2$. This implies the existence of a maximum feasible repayment $\bar{R}$, implicitly defined by

$$\mathbb{E}\big[\theta \mid \theta \geq \hat{\theta}(\bar{R})\big] = \bar{R}, \tag{11}$$

with $\bar{R} > R^*$. Substituting $d = w$ and $R = \bar{R}$ into the entrepreneur's expected profit (5) yields $\pi_E = -w$. Hence, all repayments $R > \bar{R}$ violate the participation constraint (PC).

We are now in a position to formally state the solution to the bank's optimization problem for the case in which both (LL) and (PC) are binding constraints.

Proposition 2 (*Second-best Contract*)**.** *Suppose* $w < d^*$. *Then, the bank offers the second-best optimal loan contract* (d^{SB}, R^{SB})*, with* $d^{SB} = w$ *and* $R^{SB} \in (\theta^*, \bar{R}]$ *implicitly defined by* $\pi_E(d^{SB}, R^{SB}) = \max\{P_0, 0\}$.

If the entrepreneur is effectively cash-constrained, the loan contract entails an excessively high repayment, $R^{SB} > R^*$, from an efficiency perspective. As a result, the bank continues projects whose quality falls below the efficient threshold θ^*. In other words, the bank engages in zombie lending, as illustrated in Fig. 2.

Corollary 1. *Under the second-best loan contract* (d^{SB}, R^{SB}) *zombie lending takes place for projects of quality* $\theta \in [\hat{\theta}(R^{SB}), \theta^*)$.

This is a very important observation: In case the entrepreneur is effectively cash-constrained, $w < d^*$, there is scope for (inefficient) zombie lending. The ex ante probability of zombie lending – inefficient roll-over decision at $t = 1$ – is given by

$$Z = Prob(\theta \in [\hat{\theta}, \theta^*)). \tag{12}$$

4. Interest rates and zombification

4.1. Research question and notation

We are particularly interested in how a change in the interest rate affects zombie lending. We assume that all agents – the entrepreneur, the bank, and the investors – discount future payments based on an identical interest rate $r \geq 0$. This interest rate can be interpreted as being determined, albeit only indirectly, by a central bank's policy.[15]

As explained in Section 2, all variables can be interpreted as the respective variable's date $t = 2$ future value. We denote the actual numerical value of each variable with a tilde. Thus, we can introduce the following variable transformation:

$$\gamma = (1+r)\tilde{\gamma}, \quad c = (1+r)\tilde{c},$$
$$L = (1+r)\tilde{L}, \quad I = (1+r)^2\tilde{I},$$
$$w = (1+r)^2\tilde{w}, \quad d = (1+r)^2\tilde{d}.$$

Note that variables occurring at date $t = 2$ need no transformation, e.g., the repayment still denotes R.

We are interested in how a change in the interest rate affects a bank's decision to roll over zombie credit. Therefore, we focus on the financing scenario where the entrepreneur and the bank sign a second-best loan contract (d^{SB}, R^{SB}).

The efficient roll-over quality threshold is

$$\theta^*(r) = \frac{(1+r)\tilde{L}}{1+(1+r)\tilde{\gamma}}. \tag{13}$$

A change in the interest rate affects the efficient quality threshold as follows:

$$\frac{d\theta^*}{dr} = \frac{\tilde{L}}{[1+(1+r)\tilde{\gamma}]^2} > 0. \tag{14}$$

Thus, a decline in the interest rate renders it welfare-optimal to roll over a larger set of loans. Intuitively, a lower interest rate increases the relative importance of the date $t = 2$ project return θ compared to the date $t = 1$ liquidation value $\tilde{L}$. Put differently, continuation becomes less costly when the interest rate falls. Consequently, the primary effect of an interest rate reduction in our model is a decline in zombie lending, as it becomes efficient to continue a greater number of projects.

Under a second-best loan contract, the bank rolls over all loans of quality θ weakly larger than

$$\hat{\theta}(r, R^{SB}) = \frac{(1+r)\tilde{L} - R^{SB}}{(1+r)\tilde{\gamma}}. \tag{15}$$

The following considers two scenarios. First, we investigate the effects of changes in the interest rate on a given loan contract (short-run analysis). Then, we take the impact of a change in the interest rate on the offered contract into account.

4.2. Short-run effects of interest rate changes

4.2.1. Baseline model

As a first step, we investigate the effect of an adjustment in the interest rate r on the probability of zombie lending $Z(r) = Prob(\theta \in [\hat{\theta}, \theta^*))$, for a given second-best loan contract (d^{SB}, R^{SB}). This effect can be interpreted as the effect of an unanticipated change in the interest rate. Namely, the entrepreneur and the bank signed a second-best loan contract at date $t = 0$. At the beginning of date $t = 1$, the interest rate changes, and this change was not expected by the bank or the entrepreneur. Thus, at date $t = 1$, the contract is given, but the bank can adjust its roll-over decision. If the interest rate increases, the bank applies a stricter roll-over rule, i.e.,

$$\frac{\partial\hat{\theta}}{\partial r} = \frac{R^{SB}}{\tilde{\gamma}(1+r)^2} > 0. \tag{16}$$

The intuition is analogous to the efficient threshold argument. To obtain a clear-cut finding in this section, we assume the following:

Assumption 1. For all $\theta \in [\underline{\theta}, \bar{\theta}]$ it holds that $f'(\theta) \leq 0$.

According to Assumption 1, projects of higher quality are less likely, i.e., 'unicorns' are rare. An alternative sufficient condition to Assumption 1, which permits symmetric and single-peaked distributions such as the (truncated) normal distribution, is as follows. Suppose the density function $f(\theta)$ is single-peaked with mode θ^m. Then, for infinitesimal changes in r, the condition $\theta^m > \hat{\theta}$ is sufficient. We are now able to make the following proposition.

[15] Investigating the optimal central bank policy is outside the scope of this paper. The central bank may set an interest rate that appears inefficient from our model's perspective, as it takes into account factors not included in our analysis.

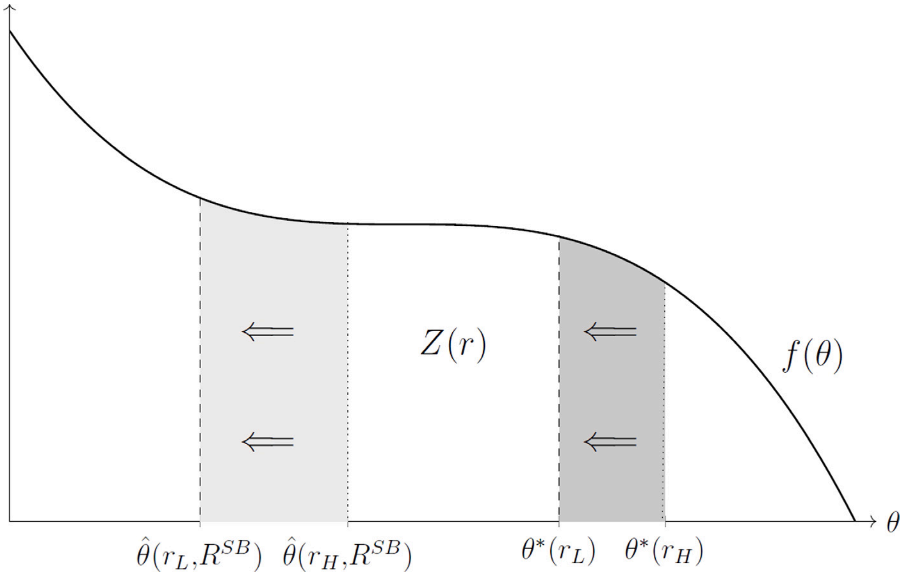

Fig. 3. The bank's adjusted roll-over decision for an unexpected drop in interest rates from r_H to $r_L < r_H$.

Proposition 3. *Suppose that Assumption 1 holds and that the entrepreneur and the bank signed a second-best loan contract. Then, an unanticipated reduction in the interest rate increases the probability of zombie lending, i.e.,*

$$Z(r) = \int_{\hat{\theta}(r,R^{SB})}^{\theta^*(r)} f(\theta)\,\mathrm{d}\theta \tag{17}$$

is strictly decreasing in r.

Proposition 3 states that if the entrepreneur and the bank engage in a long-term lending relationship and the interest rate drops unexpectedly during this relationship, then the bank rolls over even more loans compared to the efficient continuation decision.

As highlighted in Fig. 3, the probability of zombie lending $Z(r)$ increases with decreasing interest rates r for any density function $f(\theta)$, with $f'(\theta) \leq 0$. Note that any drop (rise) in the interest rate r increases (decreases) the zombie lending interval, $\theta \in [\hat{\theta}(r), \theta^*(r))$. Specifically, the mass of qualities θ in the interval of $\hat{\theta}(r_L, R^{SB})$ and $\hat{\theta}(r_H, R^{SB})$ is strictly larger than the corresponding mass in the interval of $\theta^*(r_L)$ and $\theta^*(r_H)$, for $r_H > r_L$. Conveying the result to the real world, this scenario may very well resemble many lending relationships between commercial banks and companies following the financial crisis in the EU, i.e., in the early 2010s. Thus, according to our theory, the – to some degree – unexpectedly continued loose monetary policy of the ECB after the financial crisis may have augmented the problem of zombie lending in the euro area.

Proposition 3 also has implications regarding the probability of zombie lending under a formerly first-best contract. Under the first-best contract, the repayment is $R^*(r) = \theta^*(r)$ so that the bank applies the efficient quality threshold $\hat{\theta}(r, R^*(r)) = \theta^*(r)$. Now, suppose the interest rate drops from r_H to $r_L < r_H$. This decreases the first-best threshold from $\theta^*(r_H)$ to $\theta^*(r_L)$. Given that the interest rate drop was unexpected, the repayment stays at $R^*(r_H)$ while the bank applies the quality threshold $\hat{\theta}(r_L, R^*(r_H))$. It can readily be shown that $\hat{\theta}(r_L, R^*(r_H)) < \theta^*(r_L)$, and thus zombie lending occurs for qualities $\theta \in [\hat{\theta}, \theta^*)$. In other words, an unanticipated drop in the interest rate also increases the scope for zombie lending under the formerly first-best loan contract (d^*, R^*).

4.2.2. Variable interest rate

Up to this point, we have assumed that the entrepreneur finances her project with a fixed-rate loan — meaning the repayment amount is predetermined and unaffected by interest rate shocks at $t = 1$. In practice, however, many commercial loans carry a variable interest rate that adjusts with a benchmark such as EURIBOR in the EU or LIBOR in the US. If repayments adjust fully to an interest rate shock, Proposition 3 no longer applies, and therefore an unexpected decline in the interest rate does not expand the scope for zombie lending.

Although variable-rate lending is common in corporate loans to non-financial firms, it is not universal. In particular, small and medium-sized enterprises (SMEs) often borrow at fixed rates (Athavale et al., 2003), and such firms are more likely to be non-growing zombie companies. For example, Cowling and Wong (2025) report that nearly 50% of UK Enterprise Finance Guarantee loans – government-backed loans to small firms – had fixed rates in 2020. Similarly, Fischer and Kampl (2019) show that in Austria in 2018, roughly one in five loans to non-financial institutions carried a fixed rate.

These patterns do not necessarily imply that zombie firms systematically choose fixed-rate loans, as required by Proposition 3. Still, there is evidence pointing in this direction: Göbel and Tavares (2022) find that zombie firms rely more heavily on fixed-rate bank loans than on revolving credit facilities, which usually have variable rates. Moreover, Vickery (2008) shows that credit-constrained firms – a condition frequently observed among zombie firms – are generally more likely to opt for fixed-rate borrowing.

We now formally analyze how loan contracting changes when the repayment is linked to a variable rate. At date $t = 0$, when the loan contract is signed, the interest rate is $\bar{r}$, which also represents the expected rate for date $t = 1$. The realized rate at $t = 1$ is r_1. Define R^{ind} as the "repayment net of interest", such that $(1+\bar{r})R^{ind} = R^{SB}$. The actual repayment at date $t = 2$ is then

$$[1+\bar{r}+\alpha(r_1-\bar{r})]R^{ind}, \tag{18}$$

which adjusts with the realized rate r_1. The parameter $\alpha \in [0,1]$ captures the extent of indexation: $\alpha = 0$ corresponds to a fixed-rate contract (our benchmark); $\alpha = 1$ corresponds to full adjustment, i.e., repayment moves one-to-one with r_1. In practice, floating rates typically adjust fully to benchmark rates. Alternatively, α may be interpreted as the share of the entrepreneur's fixed-rate borrowing, or as the degree of monetary policy pass-through to the benchmark rate. Under such a floating-rate contract, the bank rolls over a loan of quality θ at $t = 1$ if and only if

$$\tilde{\gamma}\theta + \frac{1+\bar{r}+\alpha(r_1-\bar{r})}{1+r_1}R^{ind} \geq \tilde{L}. \tag{19}$$

Proposition 4. *Suppose that Assumption 1 holds and that the entrepreneur and the bank signed a second-best loan contract with a variable interest rate. Then, an unanticipated reduction in the interest rate increases the probability of zombie lending if and only if the degree of indexation is sufficiently low, i.e.,*

$$\alpha < \frac{1+\tilde{\gamma}r_1}{1+\tilde{\gamma}(1+r_1)}\,\frac{\theta^*}{R^{SB}} \in (0,1). \tag{20}$$

For full adjustment ($\alpha = 1$), the result in Proposition 3 is reversed. In this case, the bank's roll-over threshold remains unchanged, while the first-best threshold decreases when the interest rate falls. Consequently, a lower interest rate reduces the probability of zombie lending. By contrast, as shown in Proposition 4, if indexation is incomplete (or if only a fraction of contracts carry a variable rate), the result of Proposition 3 may still apply.

4.3. Long-run effects of interest rate changes

4.3.1. Baseline model

This section assumes that the interest rate changes before the parties sign a loan contract. We remain in the scenario where the entrepreneur and the bank sign a second-best loan contract. We investigate how this loan contract adapts to a change in the interest rate. In particular, we are interested in how the repayment $R^{SB} = R^{SB}(r)$ adjusts and how this affects the bank's roll-over decision at $t = 1$. Under the second-best contract, the amount financed by the entrepreneur $\tilde{d}$ equals her initial wealth $\tilde{w}$, thus not depending on the interest rate r.

The efficient quality threshold θ^* depends on the interest rate r only directly, and thus, the long-run effect is equal to the short-run effect. The quality threshold applied by the bank, $\hat{\theta}(r, R^{SB}(r))$, on the other hand, is not only directly a function of the interest rate r but also indirectly via the repayment $R^{SB}(r)$. The total change of this threshold is

$$\frac{d\hat{\theta}}{dr} = \frac{\partial\hat{\theta}}{\partial r} + \frac{\partial\hat{\theta}}{\partial R^{SB}}\,\frac{dR^{SB}}{dr}. \tag{21}$$

We know that $\partial\hat{\theta}/\partial r > 0$ and that $\partial\hat{\theta}/\partial R^{SB} < 0$. Thus, if the repayment R^{SB} is increasing in the interest rate, the long-run effect of an interest rate change on the likelihood of zombie lending is weaker than the short-run effect. An interest rate change affects the considerations of all three agents: the entrepreneur, the bank, and the investors. An increase in the interest rate makes the entrepreneur less patient, and thus selling the project at $t = 0$ to investors becomes more attractive. Therefore, the repayment needs to be lower to make the entrepreneur accept the bank loan. On the other hand, an increase in the interest rate decreases the expected net present value of the project, and thus reduces investors' willingness to pay at $t = 0$. This allows the bank to demand a higher repayment. Finally, the bank is incentivized to liquidate more projects at $t = 1$ for a higher interest rate. The higher interest rate not only decreases the probability of the entrepreneur profitably selling the project at $t = 2$ but also, in case of a sale, leads to a higher project price P_2. A sufficient (but not necessary) condition for $dR^{SB}/dr > 0$ is that a rise in the interest rate r increases – ceteris paribus – the advantage of bank finance over market finance.[16] In other words, the possibility of early liquidation is particularly valuable if interest rates are high. To obtain an unambiguous result, we, therefore, impose the following simple sufficient condition:

Assumption 2. The quality of a project is non-negative, i.e., $\underline{\theta} \geq 0$.

[16] The expected advantage of bank finance over market finance in terms of $t = 1$ values is

$$\psi(r,\hat{\theta}) = F(\hat{\theta})\tilde{L} + \left(\tilde{\gamma} + \frac{1}{1+r}\right)\left[\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - \mu\right].$$

Note that $\partial\psi/\partial r > 0$ if and only if $\int_{\underline{\theta}}^{\hat{\theta}} \theta f(\theta)\,\mathrm{d}\theta > 0$.

According to Assumption 2, no project in itself makes negative returns. However, note that $\underline{\theta} \geq 0$ does not exclude projects with a negative net present value at $t = 0$ nor liquidation being the efficient decision at $t = 1$. We can then make the following proposition.

Proposition 5. *Suppose that Assumption 2 holds and that $P_0 = [1 + (1 + r)\tilde{\gamma}]\mu - (1 + r)^2 \tilde{I} > 0$. Then,*

(i) the repayment of the second-best contract R^{SB} is strictly increasing in the interest rate r;
(ii) under the second-best loan contract, the probability of zombie lending is strictly increasing in the interest rate; i.e.,

$$Z(r) = \int_{\hat{\theta}(r,R^{SB}(r))}^{\theta^*(r)} f(\theta)\,\mathrm{d}\theta \tag{22}$$

is strictly increasing in r.

According to Proposition 5, an anticipated drop (rise) in the interest rate decreases (increases) the probability of zombie lending. The proof reveals that the bank's quality threshold $\hat{\theta}$ decreases in the interest rate. Thus, apparent from (21), the indirect effect of contract adaptation on the bank's quality threshold must outweigh the direct effect. While this result may be surprising at first, the rough intuition of the finding can be argued as follows: An increase in the interest rate makes risk-neutral investors less willing to pay for the entrepreneur's project at date $t = 0$, and thus P_0 becomes smaller. In return, the bank adapts the loan contract by demanding a higher repayment R^{SB} from the entrepreneur (participation constraint) ex ante. This higher repayment ultimately leads to a higher incentive for the bank to continue projects at date $t = 1$, thus increasing zombie lending.[17]

In summary, we find that a mere reduction in interest rates does not generate persistent zombification; rather, it has a diminishing effect over time. Translating this result to real-world settings, low interest rate environments may foster zombie lending within relationship banking in the short run, but not in the long run. Put differently, if interest rates remain low in a monetary area for an extended period, the economy is not necessarily at risk of being dominated by zombie firms. This result stands in contrast to the findings of several empirical studies that document a sustained rise in the share of zombies even during a decade of low interest rates, e.g., Banerjee and Hofmann (2018) and Albuquerque and Iyer (2024). At the same time, there is also empirical evidence consistent with our result. For instance, Albuquerque and Mao (2025) report a decline in the share of zombie firms between 2016 and 2019 across advanced and emerging economies, covering both listed and private firms. Beer et al. (2021) document a pronounced decline in the Austrian zombie share from 2015 to 2017, while De Jonghe et al. (2025) show that in Belgium, the share peaked in 2011 and gradually decreased thereafter. In line with this, Banerjee and Hofmann (2022) find weakly declining zombie shares after 2010 in Japan, Denmark, and Germany.[18] Recall that our relationship banking framework is particularly suited to bank-oriented economies, such as Germany and Japan, rather than market-oriented economies, like the US or the UK.

4.3.2. Alternative investment opportunities by investors

One main driver behind Proposition 5 is that a reduction in the interest rate makes it more attractive for investors to finance the project at the initial date $t = 0$. This effect can be described as a competition effect: the lower the interest rate, the stronger the competition between investors and the bank to get selected as the financial backer for the entrepreneur's project. Due to this effect, a lower interest rate decreases the repayment under the second-best contract and increases the bank's quality threshold. In the long-run, this makes zombie lending less likely for low interest rates.

A reduction in the interest rate may, however, positively affect the return on alternative investments that are available to the investors. For instance, the reduction in interest rates may cause an increase in the demand for corporate stocks, leading to higher expected returns from investing in stocks.[19] Moreover, capital-intensive industries benefit from low interest rates and can thus generate higher profits. In the following, we augment our baseline model by incorporating the latter channel.

A central bank determines the basis interest rate r^*. For simplicity, we assume that the relationship bank uses this basis interest rate, i.e., $r_B = r^*$. The interest rate applied by the entrepreneur, r_E, reflects her idiosyncratic time preference and is independent of r^*. The interest rate investors use r_M is the net return they can achieve from alternative investments.

There is a large number of homogeneous firms that operate each with a fixed amount of equity k_E.[20] Each firm chooses an amount of outside capital k_O. A firm invests in $t = 0$ (and in $t = 1$) and generates a gross return of $B(k_E + k_O)$ in $t+1$, with $B'(\cdot) > 0$ and $B''(\cdot) < 0$. A firm's profit (net present value) is $\pi(r^*) = B(k_E + k^*_O) - (1 + r^*)k^*_O$, where $k^*_O(r^*)$ is the profit-maximizing amount of outside capital.[21] Thus, the net return on equity is

$$r_M(r^*) = \frac{\pi(r^*)}{k_E} - 1. \tag{23}$$

[17] We investigate the channels behind this finding in more detail in the online Appendix B, where we allow for different interest rates for the three types of agents.

[18] Comparable evidence for Germany is also reported by Blažková and Chmelíková (2022, p. 8), who note: "for Germany, the share of zombies has increased during the crisis, but after 2009 it has been gradually declining". For the period 2010–2014, Storz et al. (2017) reports mixed evidence across several EU countries.

[19] Daniel et al. (2021) report that low interest rates drive up demand and prices for high-dividend stocks and high-yield bonds. Somewhat related, Domian et al. (1996) find that drops in interest rates are followed by excessive stock returns. A theoretical mechanism of how lower nominal interest rates that make liquidity cheaper translate into higher asset prices and investments is proposed by Drechsler et al. (2018).

[20] Assuming a fixed amount of equity has the advantage that profit-maximization is equivalent to maximizing the rate of return on equity.

[21] We assume that k^*_O is determined by the first-order condition of profit maximization. Imposing the Inada conditions $\lim_{k\to 0} B'(k_E + k) = \infty$ and $\lim_{k\to\infty} B'(k_E + k) = 0$ is sufficient.

Each investor can decide to finance such a firm instead of the entrepreneur's project. An investor prefers to finance the entrepreneur's project if it has an expected net return that is weakly larger than $r_M(r^*)$.

We focus on situations where market finance is the entrepreneur's best alternative to bank finance, i.e., we assume that

$$P_0 := \frac{\mu}{(1+r_M)^2} + \frac{\tilde{\gamma}\mu}{1+r_M} - \tilde{I} > 0. \tag{24}$$

We can now state the following result.

Proposition 6. *Suppose that $P_0 > 0$. An increase in the basis interest rate r^**

(i) decreases the net return investors demand from the entrepreneur, $dr_M/dr^* = -k_O^*/k_E < 0$;
(ii) increases the quality threshold $\hat{\theta}(R^{SB})$ that the bank applies under the second-best contract, $d\hat{\theta}/dr^* > 0$.

Moreover, the bank's quality threshold $\hat{\theta}(R^{SB})$ reacts stronger to a change in the basis interest rate r^*, the stronger the net return r_M reacts, i.e., the larger $|dr_M/dr^*|$ is.

If the central bank interest rate r^* increases, firms' productivity declines, reducing the return on equity, part (i) of Proposition 6. An increase in the interest rate r^* has two effects on the bank's quality threshold $\hat{\theta}$. First, there is the direct positive effect on $\hat{\theta}$: If the interest rate is higher, the bank has the incentive to liquidate more often. Second, a change in the basis interest rate changes the second-best repayment R^{SB}. Regarding the repayment, there are two opposing effects. On the one hand, the bank liquidates more often, which increases the second-period price P_2. This allows the bank to demand a higher repayment. On the other hand, if the interest rate r^* increases, financing the entrepreneur rather than one of the homogeneous firms becomes more attractive for investors. This forces the bank to reduce the repayment. The former effect dominates if $|dr_M/dr^*| \approx 0$, while the latter dominates if $|dr_M/dr^*|$ is large. In any case, the overall effect on the quality thresholds is unambiguous: a higher interest rate r^* increases the bank's quality threshold.

Proposition 6 alludes to the concern that a low basis interest rate may lead to more zombie lending not only in the short-run but also in the long-run. This concern can be mitigated by strict financial regulations, e.g. capital requirements. A higher required share of equity to outside capital reduces the leverage of the publicly traded companies, and thus their return on equity. To see this mathematically, note that $|dr_M/dr^*| = k_O^*/k_E$ is strictly decreasing in k_E.

5. Extensions and further implications

5.1. Bank's capital structure

Empirical evidence suggests that zombie lending is a more pronounced problem if the lender (the bank) is itself in a weak financial position (Peek and Rosengren, 2005; Acharya et al., 2022; Blattner et al., 2023). In other words, a bank with a lower equity-to-outside-capital ratio has a stronger incentive to roll over loans of poor quality. In the following, we consider a simple extension of the baseline model.

To address the issue of bank capital structure, we now assume that the bank finances the investment partially with equity and partially with outside finance. More precisely, share $\alpha \in (0, 1]$ of the investment $\tilde{I} - \tilde{d}$ is financed by bank equity and share $1 - \alpha$ by deposits. The bank pays an interest $r_D < r$ on deposits. To rule out trivial cases, we assume that the bank can repay the deposits also in case of project liquidation. Moreover, we focus on the second-best loan contract with $\tilde{d}^{SB} = \tilde{w}$. Under the second-best contract, the repayment $R = R^{SB}$ is determined by the entrepreneur's participation constraint, and thus is independent of the bank's capital structure. The bank keeps the deposits on the balance sheet for two periods if the entrepreneur's loan is continued at $t = 1$ but only for one period if the loan is terminated at $t = 1$.

The bank prefers to roll over the entrepreneur's loan at $t = 1$ if and only if

$$\tilde{\gamma}\theta + \frac{R^{SB}}{1+r} - (1-\alpha)\frac{(1+r_D)^2(\tilde{I}-\tilde{w})}{1+r} \geq L - (1-\alpha)(1+r_D)(\tilde{I}-\tilde{w}). \tag{25}$$

The difference between (25) and the respective condition in the baseline model is that the bank needs to repay the deposits $(\tilde{I} - \tilde{w})$ plus interest payments. The next result is readily obtained from (25).

Proposition 7. *Suppose the bank's equity share is α and it pays an interest $r_D < r$ on deposits. Then, the bank's quality threshold is higher, the higher the equity share: $\partial\hat{\theta}/\partial\alpha > 0$.*

The lower a bank's quality threshold $\hat{\theta}$, the higher is the scope for zombie lending — i.e., roll-over of loans from projects with inefficiently low returns. Thus, according to Proposition 7, weakly capitalized or even under-capitalized banks are particularly likely to engage in zombification.

5.2. Booms and busts

Zombification seems to be particularly pronounced during economic downturns. Banerjee and Hofmann (2022) and De Martiis and Peter (2021) report that the share of zombie firms rises during recessions. For instance, De Martiis and Peter (2021) analyze the share of zombie firms in eight European countries from 1990 until 2018. For this period, they investigate how three recession events, the Dot-com Bubble, the GFC, and the European Debt Crisis, affected the likelihood of zombie lending. They point out that recession events are likely to be a primary cause for firms to become over-indebted. The recession alone, however, can hardly explain why these non-viable firms stay alive as they do according to the data of De Martiis and Peter (2021).

In the following, we investigate how an (unexpected) change in the economic conditions at the beginning of $t = 1$ – i.e., for given contracts – affects the probability of zombie lending. If there is an economic downturn at the beginning of $t = 1$, this affects the prospects regarding the project's returns in $t = 1$ and likely also in $t = 2$. Moreover, in an economic downturn, prices may drop, affecting the value of the entrepreneur's assets, e.g. the collateral and the value of the company's physical capital. In other words, the liquidation value of the project is reduced in an economic downturn. We model this by assuming that the project's quality is $\alpha\theta$ and the liquidation value is $\alpha\tilde{L}$, with $\alpha > 0$. For $\alpha < 1$ the economy is in a recession and for $\alpha > 1$ in a boom. We focus on a given second-best contract (d^{SB}, R^{SB}), where R^{SB} is optimal for the neutral economic condition $\alpha = 1$. We restrict the attention to drops in values that are not too severe, i.e., we assume that α is sufficiently large so that $P_2 = \mathbb{E}[\alpha\theta|\theta \geq \hat{\theta}(\alpha)] > R^{SB}$. The entrepreneur's price at $t = 2$ is larger than the repayment, and thus, the bank always obtains R^{SB} in $t = 2$.

First, note that the efficient quality threshold θ^* is independent of α because all relevant payments from $t = 1$ onward – both the project revenues and the liquidation value – are scaled by α. The bank, however, prefers to roll over the loan if and only if

$$\tilde{\gamma}\alpha\theta + \frac{R^{SB}}{1+r} \geq \alpha\tilde{L}. \tag{26}$$

The roll-over decision of the bank hinges on the economic state α because the repayment is fixed ex-ante and does not depend on the economic situation.

Proposition 8. *The probability of zombie lending $Z(\alpha) = \int_{\hat{\theta}(\alpha)}^{\bar{\theta}} f(\theta)\,\mathrm{d}\theta$ increases (decreases) in a recession (boom), i.e., $dZ/d\alpha < 0$.*

According to Proposition 8 and in line with empirical evidence, zombie lending increases if the economy turns into a recession. With the repayment being fixed ex-ante, the bank has the incentive to continue the project for more quality levels if the liquidation value and the project's returns decrease. Intuitively, the relationship bank prefers to 'speculate' on obtaining the (ex-ante) contracted repayment in the future rather than realizing the busted liquidation value.

6. Robustness and discussion

Throughout the robustness section, we adopt the notation from Sections 2 and 3, denoting payments as $t = 2$ future values without explicitly indicating dependence on the interest rate r.

6.1. Standard debt contract

Consider a standard debt contract offered by the bank, which provides a loan of size $I - d$ and has a maturity of two periods. The borrower – i.e., the entrepreneur – is obliged to make repayments R_t at the end of each period $t = 1, 2$. Formally, we represent the contract as $C = (d, R_1, R_2)$.

Suppose the entrepreneur accepts the standard debt contract at $t = 1$. If the generated cash flow together with the remaining capital at $t = 1$ is sufficient to service the contract (i.e., to repay R_1), then the entrepreneur pays R_1 to the bank and retains $\gamma\theta - R_1$ for herself. This is the case if and only if $w - d + \gamma\theta \geq R_1$, which is equivalent to

$$\theta \geq \frac{1}{\gamma}(R_1 + d - w) =: \check{\theta}. \tag{27}$$

If the generated cash flow is insufficient, the entrepreneur cannot make the contracted repayment and thus defaults. In this case, the bank either liquidates the company and receives L, or renegotiates the contract. Under renegotiation, the bank reduces the $t = 1$ repayment (the interest payment on the debt) to $w - d + \gamma\theta$, preventing insolvency and allowing the firm to continue into $t = 2$. The bank prefers renegotiation, i.e., rolling over the loan, if and only if $\gamma\theta + R_2 \geq L$, which is equivalent to

$$\theta \geq \frac{1}{\gamma}(L - R_2) =: \hat{\theta}. \tag{28}$$

Note that $R_1 = w - d + \gamma\check{\theta}$ and $R_2 = L - \gamma\hat{\theta}$, which allows us to frame the bank's problem as a problem of choosing a contract $\tilde{C} = (d, \check{\theta}, \hat{\theta})$. The bank's expected profit at $t = 0$ amounts to

$$\pi_B(d, \check{\theta}, \hat{\theta}) = F\left(\min\{\check{\theta}, \hat{\theta}\}\right) L + \int_{\min\{\check{\theta},\hat{\theta}\}}^{\check{\theta}} \gamma\theta f(\theta)\,\mathrm{d}\theta + \int_{\check{\theta}}^{\bar{\theta}} R_1 f(\theta)\,\mathrm{d}\theta + \left[1 - F\left(\min\{\check{\theta}, \hat{\theta}\}\right)\right] R_2 - c - (I - d) \tag{29}$$

The firm is liquidated if $\theta < \min\{\check{\theta}, \hat{\theta}\}$ in which case the bank receives L. If $\min\{\check{\theta}, \hat{\theta}\} \leq \theta < \check{\theta}$ the contract is renegotiated: the bank receives $\gamma\theta < R_1$ in period $t = 1$ and R_2 in period $t = 2$. Finally, if $\theta \geq \check{\theta}$, the entrepreneur services the contract, paying R_1 in $t = 1$ and R_2 in $t = 2$.[22]

The entrepreneur's net expected profit from contract $\tilde{C} = (d, \check{\theta}, \hat{\theta})$ is

$$\pi_E(d, \check{\theta}, \hat{\theta}) = -d + \int_{\check{\theta}}^{\bar{\theta}} (\gamma\theta - R_1) f(\theta)\,\mathrm{d}\theta + \left[1 - F\left(\min\{\check{\theta}, \hat{\theta}\}\right)\right](P_2 - R_2), \tag{30}$$

where $P_2 = \mathbb{E}[\theta | \theta \geq \min\{\check{\theta}, \hat{\theta}\}]$.

For the case of standard debt contracts with a two-period maturity, the bank solves the following program:

$$\max_{d, \check{\theta}, \hat{\theta}} \quad \pi_B(d, \check{\theta}, \hat{\theta}) \tag{31}$$

subject to

$$\pi_E(d, \check{\theta}, \hat{\theta}) \geq \max\{P_0, 0\} \tag{PC}$$

$$d \leq w \tag{LL}$$

If the entrepreneur has sufficient equity capital, then the limited liability constraint (LL) is not binding. In this case, the bank offers a contract that induces the first-best outcome. If, on the other hand, constraint (LL) is binding, the bank offers a second-best contract that induces a distorted continuation decision.

Proposition 9.

(i) If $w \geq d^*$, the bank offers a contract that induces the first-best outcome; e.g. $(d^{FB}, R_1^{FB}, R_2^{FB}) = (d^*, \gamma\bar{\theta} + w - d^*, \theta^*)$.

(ii) If $w < d^*$, the bank offers the second-best contract $(d^{SB}, R_1^{SB}, R_2^{SB}) = (w, \gamma\bar{\theta}, R^{SB})$, where $R^{SB} \in (\theta^*, \bar{R}]$ is implicitly defined by $\pi_E = \max\{P_0, 0\}$.

Crucially, the two cases and their corresponding outcomes align exactly with those in Propositions 1 and 2. Hence, restricting attention to 'simple' debt contracts – defined by a single repayment at $t = 2$ and the assignment of the intermediate payoff to the bank – entails no loss of generality relative to standard two-period debt contracts. In particular, the results established in Sections 4 and 5 carry over directly to standard two-period debt contracts.

The intuition behind this result can be understood by considering the contract $C = (d, R_1, R_2)$. If the entrepreneur has sufficient equity capital, the bank offers a contract that maximizes the joint surplus of bank and entrepreneur, i.e., one that induces efficient continuation. The bank uses the initial outlay – allowing for a one-to-one transfer of rents between bank and entrepreneur – to extract as many rents as possible. If the entrepreneur is instead cash constrained, constraint (LL) binds, while constraint (PC) becomes slack under any contract that ensures efficient continuation. The bank can then extract additional rents by increasing either R_1 or R_2. Raising $R_1 \geq L$ (implying that $\check{\theta} \geq \hat{\theta}$) does not distort the roll-over decision but increases the bank's expected repayment. Once $R_1 = \gamma\bar{\theta}$, however, further increases no longer raise expected profits. At that point, the bank has an incentive to demand a repayment $R_2 > \theta^*$. The resulting contract, $C = (d = w, R_1 = \gamma\bar{\theta}, R_2)$, is equivalent to the second-best contract analyzed in the previous sections.

6.2. Bank competition

Throughout the paper, we assumed that a monopolistic bank learns the quality of the project at an intermediate date and makes a take-it-or-leave-it contract offer. The bank's offer is constrained by the risk-neutral investors' offer to the entrepreneur at date $t = 0$. In the baseline model, however, no other bank can monitor the project and is willing to finance it. The terms of the second-best contract, under which zombie lending occurs, are determined by the entrepreneur's participation constraint. In the following, we show that zombie lending can also occur under bank competition where the entrepreneur's participation constraint does not determine the equilibrium repayment.

Suppose that there are several – at least two – banks that can create a relationship with the entrepreneur. These banks, who are all identical, compete at date $t = 0$ à la Bertrand by making a loan contract offer (d, R). To simplify the exposition, we assume a cashless entrepreneur, i.e., $w = 0$, and that there is no financial market at $t = 0$. Note that if bank finance occurs in equilibrium, the next best alternative for the entrepreneur is to take up a loan from another bank. Thus, we can abstract from market finance without loss in generality.

Furthermore, we assume that a bank that fully finances the project ($d = 0$) and charges the highest feasible repayment $\bar{R}$ makes a strictly positive expected profit.

Assumption 3. $\pi_B(0, \bar{R}) > 0$.

[22] Strictly speaking, the relevant threshold is $\min\{\check{\theta}, \bar{\theta}\}$ rather than $\check{\theta}$. For clarity of exposition, we adopt the simpler notation.

The assumption implies that the expected surplus generated by efficient bank finance ($R = \theta^*$) is strictly positive. To be able to state a concise result, we define the following threshold

$$\bar{d} := -F(\theta^*)L - \gamma \int_{\theta^*}^{\bar{\theta}} \theta f(\theta)\, \mathrm{d}\theta - [1 - F(\theta^*)]\theta^* + c + I.$$

Finally, we focus on symmetric equilibria of the bank competition game.

Proposition 10. *Suppose that Assumption 3 holds. If $\bar{d} > 0$, the equilibrium loan contract (d^C, R^C) under bank competition specifies $d^C = 0$ and $R^C \in (R^*, \bar{R})$ so that $\pi_B(0, R^C) = 0$.*

According to Proposition 10, if the expected bank profit from the contract that induces efficient continuation ($R = \theta^*$) is negative, the equilibrium contract specifies an inefficiently high repayment $R^C > R^* = \theta^*$. Thus, under the competitive loan contract (d^C, R^C), zombie lending takes place for projects of quality $\theta \in [\hat{\theta}(R^C), \theta^*)$. Moreover, as a bank's continuation decision for a given contract is independent of the degree of bank competition, Proposition 3 still applies. In other words, if the entrepreneur signed an equilibrium loan contract under bank competition (d^C, R^C), an unanticipated drop in the interest rate increases the probability of zombie lending.

7. Policy implications

This section summarizes the policy implications that follow directly from the model. The analysis proceeds by taking the model at face value, with the caveat that it captures only one specific zombification mechanism and relies on several strong assumptions.

The most immediate implication is that an unanticipated reduction in the central bank's policy rate may unintentionally increase the prevalence of zombie firms. Banks holding legacy debt of low-productivity firms face strong incentives to evergreen such loans, even though these firms – despite lower interest rates – are economically non-viable and should exit the market from an efficiency perspective. This problem is particularly pronounced when low-productivity firms predominantly rely on fixed-rate debt contracts. By contrast, if most potentially zombifiable firms are financed through variable-rate contracts that adjust with the policy rate, an unexpected rate cut does not raise the zombie share in the economy.

For newly originated contracts, the model suggests that low policy rates reduce the scope for zombie lending, independent of whether interest rates are fixed or floating. The underlying mechanism is competitive pressure: in a low-rate environment, profitable investment opportunities are scarce. However, if reduced rates simultaneously fuel higher returns in alternative investments, then the same environment may become conducive to zombification. Consequently, a central bank that maintains low interest rates for an extended period – and that is concerned about an increase in the zombie share – should carefully monitor whether its policy stance contributes to excessive stock market valuations. One mitigating instrument is the imposition of strict capital requirements on publicly listed firms, which reduce leverage and thereby dampen excess returns generated by cheap financing.

In summary, policymakers should recognize that temporary or unexpected cuts in interest rates heighten the risk of zombification, whereas stable and predictable low-rate environments are less problematic. Sudden rate changes should therefore be avoided where possible. In addition, policymakers should monitor how monetary policy affects not only loan markets but also the allocation of capital across asset classes.

Regulators should also be particularly vigilant with respect to weakly capitalized banks, which have stronger incentives to engage in zombie lending. Supervisory scrutiny should therefore focus on these institutions' credit allocation practices, particularly their restructuring and forbearance policies toward borrowers experiencing financial distress. The risk of zombie lending by weakly capitalized banks can be mitigated through the enforcement of strict capital requirements, which help ensure that institutions remain adequately capitalized. Enhanced monitoring of lending practices is especially important during recessions, when incentives to extend credit to zombie firms are strongest. In such periods, countercyclical measures – such as targeted restructuring programs and support for orderly firm exit – may be required to prevent an increase in zombie firms.

Finally, the mechanism described here presumes that banks and entrepreneurs are able to renegotiate loan terms when default risk arises. This presupposes that banks are legally permitted to restructure debt and apply forbearance policies to non-performing loans (NPL). Transparent information about loan performance can constrain banks' ability to offload weak borrowers. Consequently, strict regulation of NPL management may effectively close this zombification channel – provided such rules cannot be circumvented, for example by offering new loans to ensure that technically no arrears emerge.[23]

8. Conclusion

In this paper, we have analyzed a simple mechanism of zombie lending in the context of relationship banking. We considered the case of an entrepreneur with ex-ante uncertain project quality and showed that under a second-best contract – arising when the entrepreneur is cash-constrained – the relationship bank may continue projects of inefficiently low quality. This form of zombie lending originates from the binding upper bound on the entrepreneur's initial outlay, which translates into an inefficiently high ex-post repayment and distorts the bank's continuation decision. This mechanism provides a coherent explanation for how contractual frictions – not just bank capital structure or regulatory forbearance – can give rise to zombie lending in otherwise standard credit relationships.

[23] For a discussion of EU legal rules on NPL, see Montanaro (2019) and the guidance by the European Central Bank (2017).

We then analyze how changes in the interest rate affect this mechanism over time. In the short run, when interest rate movements are unexpected and contractual terms are fixed, a decline in the policy rate strengthens the bank's incentive to roll over low-quality loans. The reason is straightforward: lower rates increase the value of delayed repayment relative to early liquidation under an unchanged contract. This short-run amplification arises in particular when a substantial share of lending is conducted under fixed-rate contracts, whereas full indexation of repayments reverses this effect.

In the long run, when banks can adjust contractual terms before origination, the effects differ fundamentally. As shown in our contracting analysis, lower interest rates intensify competition from market investors, forcing the bank to offer a contract with a lower repayment. This reduces the bank's incentive to evergreen loans and makes zombie lending less prevalent. However, when declining rates simultaneously raise returns in investors' alternative investment opportunities, this competitive pressure weakens, and the long-run decline in zombie lending becomes smaller or might even reverse. Together with our extensions on bank capitalization and business-cycle conditions, these results illustrate how broader economic forces shape the continuation incentives of relationship banks.

Overall, our findings highlight the interplay between contractual frictions, interest rate dynamics, and banks' roll-over decisions. They suggest that policies targeting the prevalence of zombie lending need to account not only for the structure of financial contracts but also for the broader interest rate environment – including its short-run versus long-run implications – and its impact on outside investment opportunities.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Mathematical appendix: Proofs

Proof of Proposition 1. For $R = R^*$, we have $\hat{\theta}(R) = \theta^*$ and $P_2 = \mathbb{E}[\theta \mid \theta \geq \theta^*]$. This implies that for repayment R^* the entrepreneur is indifferent between accepting the bank loan (d, R^*) and her next best alternative if and only if

$$d = [1 - F(\theta^*)]\left\{\mathbb{E}[\theta \mid \theta \geq \theta^*] - \theta^*\right\} - \max\{(1+\gamma)\mu - I, 0\} \tag{A.1}$$

$$= \int_{\theta^*}^{\bar{\theta}} [\theta - \theta^*] f(\theta)\, \mathrm{d}\theta - \max\{(1+\gamma)\mu - I, 0\}. \tag{A.2}$$

Note that $P_0 = (1+\gamma)\mu - I$. If bank finance is efficient and all the additional surplus from bank finance is extracted by the bank – i.e., participation is binding – then offering a loan contract that implements efficient continuation maximizes the bank's profits. □

Proof of Proposition 2. The bank maximizes its profit subject to the entrepreneur's participation constraint, $\pi_E(d, R) \geq \max\{P_0, 0\}$, and the limited liability constraint, $d \leq w$. The first-best contract (d^*, R^*) satisfies the participation but violates the limited liability constraint, $w < d^*$. With d being an ex ante one-to-one transfer between the entrepreneur and the bank, the second-best optimal amount financed by the entrepreneur is $d^{SB} = w$.

The expected profit of the bank is

$$\pi_B(d^{SB}, R) = F(\hat{\theta}(R))[L - c - I + w] + \left[1 - F(\hat{\theta}(R))\right]\left\{\gamma \mathbb{E}[\theta | \theta \geq \hat{\theta}(R)] + R - c - I + w\right\}. \tag{A.3}$$

Simplifying the above expression yields

$$\pi_B(d^{SB}, R) = F(\hat{\theta}(R))L + \gamma \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\, \mathrm{d}\theta + [1 - F(\hat{\theta}(R))]R - (c + I - w). \tag{A.4}$$

Taking the derivative of π_B with respect to the repayment R yields

$$\begin{aligned}\frac{\partial \pi_B}{\partial R} &= f(\hat{\theta})\frac{d\hat{\theta}}{dR}L - \gamma\hat{\theta} f(\hat{\theta})\frac{d\hat{\theta}}{dR} + [1 - F(\hat{\theta})] - f(\hat{\theta})\frac{d\hat{\theta}}{dR}R \\ &= -f(\hat{\theta})\frac{1}{\gamma}[\underbrace{L - \gamma\hat{\theta} - R}_{=0}] + 1 - F(\hat{\theta}) > 0\end{aligned} \tag{A.5}$$

The term in square brackets equals zero by the definition of $\hat{\theta}$. Thus, the bank strictly prefers a higher repayment R.

The expected profit of the entrepreneur is

$$\pi_E(d^{SB}, R) = F(\hat{\theta}(R))(-w) + [1 - F(\hat{\theta}(R))]\{\mathbb{E}[\theta|\theta \geq \hat{\theta}(R)] - R - w\}$$
$$= \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1 - F(\hat{\theta}(R))]R - w. \tag{A.6}$$

Note that $\pi_E(d^{SB}, R^*) > \max\{P_0, 0\}$ because $\pi_E(d^*, R^*) = \max\{P_0, 0\}$ and $d^* > w = d^{SB}$. Moreover, $\pi_E(d^{SB}, \bar{R}) = -w$, which implies that for $R > \bar{R}$ the participation constraint is violated. Recall that $\bar{R}$ is implicitly defined by $\mathbb{E}[\theta|\theta \geq \hat{\theta}(\bar{R})] = \bar{R}$. Hence, $R^{SB} \in (R^*, \bar{R}]$.

Taking the partial derivative of the entrepreneur's expected profit with respect to R yields

$$\frac{\partial \pi_E}{\partial R} = -\hat{\theta} f(\hat{\theta})\frac{d\hat{\theta}}{dR} - [1 - F(\hat{\theta})] + Rf(\hat{\theta})\frac{d\hat{\theta}}{dR}$$
$$= -[R - \hat{\theta}]f(\hat{\theta})\frac{1}{\gamma} - [1 - F(\hat{\theta})]. \tag{A.7}$$

For $R > R^*$ we have $\hat{\theta}(R) < \theta^*$ and, thus, $\partial \pi_E/\partial R < 0$.

The bank's expected profit is strictly increasing in R and the entrepreneur's expected profit is strictly decreasing in R. Thus, the second-best optimal repayment R^{SB} solves $\pi_E(d^{SB}, R) = \max\{P_0, 0\}$. □

Proof of Corollary 1. The finding follows directly from the observation that $R^{SB} > R^*$ for $w < d^*$. □

Proof of Proposition 3. Taking the derivative of $Z(r)$ with respect to r – for a constant repayment R^{SB} – yields

$$Z'(r) = f(\theta^*)\frac{d\theta^*}{dr} - f(\hat{\theta})\frac{\partial \hat{\theta}}{\partial r}. \tag{A.8}$$

To sign the above expression, we first need to determine $d\theta^*/dr$ and $\partial\hat{\theta}/dr$. Taking the partial derivative of (15) with respect to r yields

$$\frac{\partial \hat{\theta}}{\partial r} = \frac{\tilde{L}(1+r)\tilde{\gamma} - \tilde{\gamma}[(1+r)\tilde{L} - R^{SB}]}{\tilde{\gamma}(1+r)^2}$$
$$= \frac{R^{SB}}{\tilde{\gamma}(1+r)^2} > 0. \tag{A.9}$$

Taking the partial derivative of (13) with respect to r yields

$$\frac{d\theta^*}{dr} = \frac{\tilde{L}[1 + (1+r)\tilde{\gamma}] - \tilde{\gamma}(1+r)\tilde{L}}{[1 + (1+r)\tilde{\gamma}]^2}$$
$$= \frac{\tilde{L}}{[1 + (1+r)\tilde{\gamma}]^2} > 0. \tag{A.10}$$

Using the definition of θ^* allows us to write the above derivative as

$$\frac{d\theta^*}{dr} = \frac{\theta^*}{(1+r)[1 + (1+r)\tilde{\gamma}]}. \tag{A.11}$$

By Assumption 1 it holds that $f(\theta^*) \leq f(\hat{\theta})$. Thus, $Z'(r) \leq f(\hat{\theta})[d\theta^*/dr - \partial\hat{\theta}/\partial r]$, which implies that $Z'(r) < 0$ for $\partial\hat{\theta}/\partial r > d\theta^*/dr$. Note that $\partial\hat{\theta}/\partial r > d\theta^*/dr$ is equivalent to

$$R^{SB}(1+r)[1 + \tilde{\gamma}(1+r)] > \theta^*\tilde{\gamma}(1+r)^2 \tag{A.12}$$
$$\iff R^{SB}(1+r) + \tilde{\gamma}(1+r)^2[R^{SB} - \theta^*] > 0. \tag{A.13}$$

The above claim is true because $R^{SB} > \theta^*$ by the assumption that the parties signed the second-best contract. □

Proof of Proposition 4. From (19), the bank's quality threshold is given by

$$\hat{\theta}(r_1, R^{ind}) = \frac{1}{\tilde{\gamma}}\left[\tilde{L} - \frac{1 + \bar{r} + \alpha(r_1 - \bar{r})}{1 + r_1}R^{ind}\right]. \tag{A.14}$$

Following the logic of Proposition 3, we compare the sensitivity of the bank's threshold with that of the first-best threshold. Specifically, we show that

$$\frac{\partial \hat{\theta}}{\partial r_1} > \frac{d\theta^*}{dr_1}. \tag{A.15}$$

First, compute the derivative of the bank's threshold:

$$\frac{\partial \hat{\theta}}{\partial r_1} = \frac{1}{\tilde{\gamma}} \cdot \frac{(1-\alpha)(1+\bar{r})}{(1+r_1)^2}R^{ind} \tag{A.16}$$
$$= \frac{1}{\tilde{\gamma}} \cdot \frac{1-\alpha}{(1+r_1)^2}R^{SB}. \tag{A.17}$$

Hence, the bank's threshold reacts more strongly than the first-best threshold if and only if

$$\frac{1}{\tilde{\gamma}} \cdot \frac{1-\alpha}{(1+r_1)^2} R^{SB} > \frac{\tilde{L}}{[1+\tilde{\gamma}(1+r_1)]^2}. \tag{A.18}$$

Using the definition of θ^*, we can write

$$\tilde{L} = \frac{1+\tilde{\gamma}(1+r_1)}{1+r_1}\theta^*. \tag{A.19}$$

Substituting this expression into the inequality yields the threshold condition for α stated in the proposition. □

Proof of Proposition 5. Under the second-best loan contract, the repayment $R^{SB} \in (R^*, \bar{R})$ solves

$$\frac{1}{(1+r)^2}\left(\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1-F(\hat{\theta})]R^{SB}\right) - \tilde{w} = \frac{\mu}{1+r}\left(\tilde{\gamma} + \frac{1}{1+r}\right) - \tilde{I}, \tag{A.20}$$

where

$$\hat{\theta}(r, R^{SB}(r)) = \frac{(1+r)\tilde{L} - R^{SB}}{(1+r)\tilde{\gamma}}. \tag{A.21}$$

In the above condition determining $R^{SB}(r)$ we use the fact that the entrepreneur's best alternative to bank finance is market finance, i.e., that $P_0 > 0$. The implicit differentiation of (A.20) with respect to r yields

$$\begin{aligned} &\frac{-2}{(1+r)^3}\left\{\int_{\hat{\theta}}^{\bar{\theta}} -[1-F(\hat{\theta})]R^{SB}\right\} + \frac{1}{(1+r)^2}\left\{-\hat{\theta} f(\hat{\theta})\frac{d\hat{\theta}}{dr} + f(\hat{\theta})R^{SB}\frac{d\hat{\theta}}{dr} - [1-F(\hat{\theta})]\frac{dR^{SB}}{dr}\right\} \\ &= \frac{-2\mu}{(1+r)^3} - \frac{\gamma\mu}{(1+r)^2}. \end{aligned} \tag{A.22}$$

Note that

$$\frac{d\hat{\theta}}{dr} = \frac{1}{(1+r)\tilde{\gamma}}\left[\frac{R^{SB}}{1+r} - \frac{dR^{SB}}{dr}\right]. \tag{A.23}$$

Inserting (A.23) in (A.22) and rearranging yields

$$\frac{dR^{SB}}{dr} = \frac{2\tilde{\gamma}[\mu - \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta] + \tilde{\gamma}^2(1+r)\mu}{(1+r)\tilde{\gamma}[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\hat{\theta})} + \frac{2(1+r)\tilde{\gamma}[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\hat{\theta})}{(1+r)\tilde{\gamma}[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\hat{\theta})}\,\frac{R^{SB}}{1+r}. \tag{A.24}$$

By Assumption 2 it holds that $\mu - \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta > 0$ and thus $dR^{SB}/dr > 0$.

We proceed by inserting (A.24) into (A.23) and obtain

$$\frac{d\hat{\theta}}{dr} = \frac{-1}{(1+r)\tilde{\gamma}}\left\{\frac{(1+r)\tilde{\gamma}^2\mu + 2\tilde{\gamma}[\mu - \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta]}{\tilde{\gamma}(1+r)[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\theta)} + \frac{R^{SB}}{1+r}\,\frac{\tilde{\gamma}(1+r)[1-F(\hat{\theta})]}{\tilde{\gamma}(1+r)[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\theta)}\right\} < 0. \tag{A.25}$$

Finally, recall that $d\theta^*/dr > 0$ and thus $Z(r) = \int_{\hat{\theta}}^{\theta^*} f(\theta)\,\mathrm{d}\theta$ is strictly increasing in r. □

Proof of Proposition 6. First, we prove part (i): Note that

$$r_M(r^*) = \frac{B(k_E + k_O^*(r^*)) - (1+r^*)k_O^*(r^*)}{k_E} - 1. \tag{A.26}$$

Taking the derivative with respect to r^* yields

$$\begin{aligned} \frac{dr_M}{dr^*} &= \frac{1}{k_E}\left[B'(k_E + k_O^*)\frac{dk_O^*}{dr^*} - (1+r^*)\frac{dk_O^*}{dr^*} - k_O^*\right] \\ &= -\frac{k_O^*}{k_E} < 0. \end{aligned} \tag{A.27}$$

Next, we prove part (ii). The second-best repayment $R^{SB} = R^{SB}(r^*)$ makes the entrepreneur indifferent between bank finance and market finance:

$$\frac{1}{(1+r_M(r^*))^2}\left[\int_{\hat{\theta}(r^*)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1-F(\hat{\theta}(r^*))]R^{SB}(r^*)\right] - \tilde{w} = \frac{\mu}{(1+r_M(r^*))^2} + \frac{\tilde{\gamma}\mu}{1+r_M(r^*)} - \tilde{I}. \tag{A.28}$$

Recall that

$$\frac{d\hat{\theta}}{dr^*} = \frac{1}{(1+r^*)\tilde{\gamma}}\left[\frac{R^{SB}}{1+r^*} - \frac{dR^{SB}}{dr^*}\right]. \tag{A.29}$$

Multiplying both sides of (A.28) with $(1+r_E)^2$ and then implicitly differentiating with respect to r^* yields

$$-\hat{\theta} f(\hat{\theta})\frac{d\hat{\theta}}{dr^*} + f(\hat{\theta})\frac{d\hat{\theta}}{dr^*}R^{SB} - [1-F(\hat{\theta})]\frac{dR^{SB}}{dr^*} = (1+r_E)^2\left[\frac{-2\mu}{(1+r_M)^3}\frac{dr_M}{dr^*} - \frac{\tilde{\gamma}\mu}{(1+r_M)^2}\frac{dr_M}{dr^*}\right]. \tag{A.30}$$

We insert (A.29) into (A.30) and solve for

$$\frac{dR^{SB}}{dr^*} = \frac{(R^{SB} - \hat{\theta})f(\hat{\theta})}{(R^{SB} - \hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1 - F(\hat{\theta})]} \frac{R^{SB}}{1+r^*} + \frac{(1+r^*)\tilde{\gamma}(1+r_E)^2\mu[2+\tilde{\gamma}(1+r_M)]}{(1+r_M)^3\{(R^{SB} - \hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1 - F(\hat{\theta})]\}} \frac{dr_M}{dr^*}. \tag{A.31}$$

Inserting (A.31) into (A.29) yields

$$\frac{d\hat{\theta}}{dr^*} = \frac{1}{(1+r^*)\tilde{\gamma}} \left[\frac{(1+r^*)\tilde{\gamma}[1 - F(\hat{\theta})]}{(R^{SB} - \hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1 - F(\hat{\theta})]} \frac{R^{SB}}{1+r^*} - \frac{(1+r^*)\tilde{\gamma}(1+r_E)^2\mu[2+\tilde{\gamma}(1+r_M)]}{(1+r_M)^3\{(R^{SB} - \hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1 - F(\hat{\theta})]\}} \frac{dr_M}{dr^*} \right]. \tag{A.32}$$

The above equation allows us to conclude that $d\hat{\theta}/dr^* > 0$ because $R^{SB} > \hat{\theta}(R^{SB})$ and $dr_M/dr^* < 0$ by (A.27). □

Proof of Proposition 7. Solving (25) for θ yields

$$\theta \geq \frac{\tilde{L}(1+r) - R^{SB}}{1+r} - (1-\alpha)(\tilde{I} - \tilde{w})\frac{1+r_D}{1+r}(r - r_D) =: \hat{\theta}. \tag{A.33}$$

We differentiate (A.33) with respect to α and obtain

$$\frac{\partial\hat{\theta}}{\partial\alpha} = (\tilde{I} - \tilde{w})\frac{1+r_D}{1+r}(r - r_D) > 0, \tag{A.34}$$

which concludes the proof. □

Proof of Proposition 8. From Eq. (26) it follows directly that the quality threshold applied by the bank is given by

$$\hat{\theta}(\alpha) = \frac{\tilde{L}}{\tilde{\gamma}} - \frac{R^{SB}}{\tilde{\gamma}(1+r)\alpha}. \tag{A.35}$$

The change in the threshold due to a change in α is

$$\frac{d\hat{\theta}}{d\alpha} = \frac{R^{SB}}{\tilde{\gamma}(1+r)\alpha^2} > 0. \tag{A.36}$$

Finally, note that

$$\begin{aligned} \frac{dZ}{d\alpha} &= -f(\hat{\theta})\frac{d\hat{\theta}}{d\alpha} \\ &= -f(\hat{\theta})\frac{R^{SB}}{\tilde{\gamma}(1+r)\alpha^2} < 0. \quad \square \end{aligned} \tag{A.37}$$

Proof of Proposition 9. First, suppose that (LL) is slack. If this is the case, (PC) binds in optimum. To see this, note that π_B is increasing and π_E is decreasing in d (and strictly so if $\check{\theta} < \bar{\theta}$). Moreover, note that $\partial R_1/\partial d = -1$. Thus, d is implicitly defined by the binding participation constraint (PC):

$$d = \int_{\check{\theta}}^{\bar{\theta}} (\gamma\theta - R_1)f(\theta)\,\mathrm{d}\theta + [1 - F(\min\{\check{\theta},\hat{\theta}\})](P_2 - R_2) - \max\{P_0, 0\}. \tag{A.38}$$

Inserting this initial outlay into the bank's profit function yields

$$\pi_B = F(\min\{\check{\theta},\hat{\theta}\})L + \int_{\min\{\check{\theta},\hat{\theta}\}}^{\bar{\theta}} \gamma\theta f(\theta)\,\mathrm{d}\theta + [1 - F(\min\{\check{\theta},\hat{\theta}\})]P_2(\min\{\check{\theta},\hat{\theta}\}) - c - I - \max\{P_0, 0\}, \tag{A.39}$$

solely as a function of $\min\{\check{\theta},\hat{\theta}\}$. Moreover, the bank's optimization problem reduces to the unconstrained maximization of the joint surplus of bank and entrepreneur. Hence, the optimal contract specifies

$$\min\{\check{\theta},\hat{\theta}\} = \theta^*, \tag{A.40}$$

thereby implementing the first-best outcome.

There are several contracts that achieve this allocation; one example is given by

$$R_1^{FB} = \gamma\bar{\theta} + w - d^{FB}, \quad R_2^{FB} = \theta^*, \tag{A.41}$$

together with

$$d^{FB} = [1 - F(\theta^*)](P_2 - \theta^*) - \max\{P_0, 0\} = d^*. \tag{A.42}$$

Here, d^* corresponds to the term defined in Proposition 1.

The constraint (LL) is indeed slack if and only if

$$w \geq \int_{\check{\theta}}^{\bar{\theta}} (\gamma\theta - R_1)f(\theta)\,\mathrm{d}\theta + [1 - F(\min\{\check{\theta},\hat{\theta}\})](P_2 - R_2) - \max\{P_0, 0\}. \tag{LL'}$$

Implementation of the first-best outcome requires that $\min\{\check{\theta},\hat{\theta}\} = \theta^*$. Increasing $\check{\theta}$ and thus R_1, while maintaining $\min\{\check{\theta},\hat{\theta}\} = \theta^*$, relaxes (LL'). Similarly, decreasing $\hat{\theta}$ and thus increasing R_2, while keeping $\min\{\check{\theta},\hat{\theta}\} = \theta^*$, also relaxes (LL'). Hence, (LL') is least likely to bind – in the sense of set inclusion – if $\check{\theta}$ is as large as possible (i.e. $\check{\theta} = \bar{\theta}$) and $\hat{\theta}$ is as low as possible ($\hat{\theta} = \theta^*$). This is achieved for the contract $(d^{FB}, R_1^{FB}, R_2^{FB})$. For this contract, the limited liability constraint holds if and only if $w \geq d^*$.

If $w < d^*$, then constraint (LL) binds and the bank optimally specifies $d = w$. The first-period repayment now is $R_1 = \gamma\check{\theta}$.

The bank maximizes its profit

$$\pi_B = F(\min\{\check{\theta},\hat{\theta}\})L + \int_{\min\{\check{\theta},\hat{\theta}\}}^{\check{\theta}} \gamma\theta f(\theta)\,\mathrm{d}\theta + [1 - F(\check{\theta})][\gamma(\check{\theta} - \hat{\theta}) + L] + [F(\check{\theta}) - F(\min\{\check{\theta},\hat{\theta}\})](L - \gamma\hat{\theta}) + w - c - I, \tag{A.43}$$

subject to the participation constraint

$$PC \geq \max\{P_0, 0\} + w, \tag{A.44}$$

where

$$PC = \int_{\check{\theta}}^{\bar{\theta}} (\gamma + 1)\theta f(\theta)\,\mathrm{d}\theta + \int_{\min\{\check{\theta},\hat{\theta}\}}^{\check{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1 - F(\check{\theta})][\gamma(\check{\theta} - \hat{\theta}) + L] - [F(\check{\theta}) - F(\min\{\check{\theta},\hat{\theta}\})](L - \gamma\hat{\theta}) \tag{A.45}$$

It is worth noting that the first-best contract $(d^{FB}, R_1^{FB}, R_2^{FB}) = (d^*, \gamma\bar{\theta} + w - d^*, \theta^*)$ satisfies (PC) with equality but violates (LL). By contrast, under the contract $(d, R_1, R_2) = (w, \gamma\bar{\theta}, \theta^*)$, the limited liability constraint (LL) is satisfied, while the participation constraint (PC) is over-satisfied, implying that the entrepreneur receives a positive rent. Hence, the participation constraint (PC) can bind only if $R_2 > \theta^*$ (in particular, when $R_1 < \gamma\bar{\theta}$). In this case, we obtain $\min\{\check{\theta},\hat{\theta}\} < \theta^*$.

First, suppose that in the optimum $\check{\theta} \leq \hat{\theta}$. Keeping $\min\{\check{\theta},\hat{\theta}\} = \check{\theta}$, an infinitesimal change in the thresholds $\check{\theta}$ and $\hat{\theta}$ has the following effects on the bank's profit and the participation constraint. First, we investigate a change in $\hat{\theta}$:

$$\frac{\partial \pi_B}{\partial \hat{\theta}} = -\gamma[1 - F(\check{\theta})], \tag{A.46}$$

$$\frac{\partial PC}{\partial \hat{\theta}} = \gamma[1 - F(\check{\theta})]. \tag{A.47}$$

A marginal decrease in $\hat{\theta}$ that increases the bank's profit by one unit translates into a one-to-one tightening of the participation constraint.

A change in $\check{\theta}$ has the following effects:

$$\frac{\partial \pi_B}{\partial \check{\theta}} = \gamma[1 - F(\check{\theta})] + \gamma(\hat{\theta} - \check{\theta})f(\check{\theta}), \tag{A.48}$$

$$\frac{\partial PC}{\partial \check{\theta}} = -\gamma[1 - F(\check{\theta})] - \gamma(\hat{\theta} - \check{\theta})f(\check{\theta}) + f(\check{\theta})[L - (1 + \gamma)\check{\theta}]. \tag{A.49}$$

Note that by arguments made above $L > (1 + \gamma)\check{\theta}$ because $\min\{\check{\theta},\hat{\theta}\} < \theta^*$. Thus, a marginal increase in $\check{\theta}$ that increases the bank's profit by one unit tightens the participation constraint by less than one unit. Hence, it is optimal to increase $\check{\theta}$ and if (PC) binds to simultaneously increase $\hat{\theta}$. Note that in the extreme, for $\check{\theta} = \hat{\theta} = \bar{\theta}$ we have $R_1 = \gamma\bar{\theta} = R_1^{FB}$ and $R_2(\bar{\theta}) = L - \gamma\hat{\theta} < \theta^* = R_2^{FB}$ implying that (PC) is slack. Thus, $\check{\theta} \leq \hat{\theta}$ cannot be optimal.

Now, suppose that in the optimum $\hat{\theta} < \check{\theta}$. Keeping $\min\{\check{\theta},\hat{\theta}\} = \hat{\theta}$, an infinitesimal change in the thresholds $\hat{\theta}$ and $\check{\theta}$ has the following effects on the bank's profit and the participation constraint. First, we investigate a change in $\hat{\theta}$:

$$\frac{\partial \pi_B}{\partial \hat{\theta}} = -\gamma[1 - F(\hat{\theta})], \tag{A.50}$$

$$\frac{\partial PC}{\partial \hat{\theta}} = \gamma[1 - F(\hat{\theta})] + f(\hat{\theta})[L - (1 + \gamma)\hat{\theta}]. \tag{A.51}$$

Note that $L > (1 + \gamma)\hat{\theta}$ because $\hat{\theta} < \theta^*$. A change in $\check{\theta}$ has the following effects:

$$\frac{\partial \pi_B}{\partial \check{\theta}} = \gamma[1 - F(\check{\theta})], \tag{A.52}$$

$$\frac{\partial PC}{\partial \check{\theta}} = -\gamma[1 - F(\check{\theta})]. \tag{A.53}$$

An infinitesimal change in $\check{\theta}$ has a one-to-one effect on the bank's profit and the participation constraint. By contrast, a decrease in $\hat{\theta}$ has a stronger effect on the participation constraint than on the bank's profit. Hence, it is optimal to increase $\check{\theta}$ as much as possible. Note that for $\check{\theta} = \bar{\theta}$ and $\hat{\theta} = \theta^*$, the participation constraint is slack implying that $\check{\theta} = \bar{\theta}$ is optimal. Note further that it is optimal to decrease $\hat{\theta}$ – and thereby to increase R_2 – until (PC) binds. The resulting repayment R_2^{SB} coincides with the repayment R^{SB} defined in Proposition 2. □

Proof of Proposition 10. The equilibrium loan contract under perfect bank competition solves

$$\max_{d,R} \quad \pi_E(d, R)$$

subject to

$$\pi_B(d, R) \geq 0, \tag{PC}$$

$$d \leq w = 0, \tag{LL}$$

and taking into account that $\hat{\theta}(R) = (L - R)/\gamma$. In equilibrium, banks make a zero profit. Solving (PC) as an equality for d and inserting this into the target function yields

$$F(\hat{\theta}(R))L + (\gamma + 1) \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\, \mathrm{d}\theta - c - I. \tag{A.54}$$

Thus, if (LL) is slack, the equilibrium contract maximizes the joint surplus of the entrepreneur and the bank. This is achieved for

$$R = \frac{L}{1+\gamma} = \theta^*.$$

The corresponding part financed by the entrepreneur is

$$\bar{d} = -F(\theta^*)L - \gamma \int_{\theta^*}^{\bar{\theta}} \theta f(\theta)\, \mathrm{d}\theta - [1 - F(\theta^*)]\theta^* + c + I. \tag{A.55}$$

Thus, for $\bar{d} \leq 0$, the equilibrium loan contract is $(\bar{d}, R^*)$ and the first-best allocation is implemented.

For $\bar{d} > 0$, the contract $(\bar{d}, R^*)$ is not feasible. The highest feasible d is $d = 0$. Now, the repayment needs to be increased in order to satisfy (PC). Thus, the equilibrium repayment R^C solves $\pi_B(0, R^C) = 0$. Note that $\pi_B(0, R)$ is strictly increasing in $R \leq \bar{R}$. Moreover, by Assumption 3, $\pi_B(0, \bar{R}) > 0$. As a result, there exists a unique $R^C \in (R^*, \bar{R})$ that solves $\pi_B(0, R^C) = 0$. Finally, as $\pi_E(0, R)$ is strictly decreasing in $R \in [R^*, \bar{R}]$ and $\pi_E(0, \bar{R}) = 0$, we know that $\pi_E(0, R^C) > 0$, implying that the entrepreneur accepts a loan contract $(0, R^C)$. □

Appendix B. Online supplement: additional theoretical results and supporting derivations.

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.euroecorev.2025.105218.

Data availability

No data was used for the research described in the article.

References

Acharya, V.V., Borchert, L., Jager, M., Steffen, S., 2021a. Kicking the can down the road: Government interventions in the European banking sector. Rev. Financ. Stud. 34 (9), 4090–4131.
Acharya, V.V., Crosignani, M., Eisert, T., Eufinger, C., 2024. Zombie credit and (dis-) inflation: Evidence from Europe. J. Financ. 79 (3), 1883–1929.
Acharya, V.V., Crosignani, M., Eisert, T., Steffen, S., 2022. Zombie lending: Theoretical, international, and historical perspectives. Annu. Rev. Financ. Econ. 14, 21–38.
Acharya, V.V., Eisert, T., Eufinger, C., Hirsch, C., 2019. Whatever it takes: The real effects of unconventional monetary policy. Rev. Financ. Stud. 32 (9), 3366–3411.
Acharya, V.V., Lenzu, S., Wang, O., 2021b. Zombie Lending and Policy Traps. Working Paper 29606, National Bureau of Economic Research, http://dx.doi.org/10.3386/w29606, URL: http://www.nber.org/papers/w29606.
Adalet McGowan, M., Andrews, D., Millot, V., 2018. The walking dead? Zombie firms and productivity performance in OECD countries. Econ. Policy 33 (96), 685–736.
Albuquerque, B., Iyer, R., 2024. The rise of the walking dead: Zombie firms around the world. J. Int. Econ. 152, 104019. http://dx.doi.org/10.1016/j.jinteco.2024.104019, URL: https://www.sciencedirect.com/science/article/pii/S0022199624001466.
Albuquerque, B., Mao, C., 2025. Unveiling the zombie lending channel of monetary policy. SUERF Policy Brief 1093, 20.
Andrews, D., Petroulakis, F., 2019. Breaking the Shackles: Zombie Firms, Weak Banks and Depressed Restructuring in Europe. European Central Bank: ECB Working Paper, Available at SSRN: https://ssrn.com/abstract=3957694.
Aragon, N., 2022. Debt Overhang, Risk Shifting and Zombie Lending. Working Paper 1/2022, National Bank of Ukraine.
Athavale, M., Edmister, R.O., et al., 2003. Fixed or variable rate choice in the commercial bank business loan market. J. Appl. Bus. Res. (JABR) 19 (4).
Banerjee, R., Hofmann, B., 2018. The rise of zombie firms: Causes and consequences. BIS Q. Rev..
Banerjee, R., Hofmann, B., 2022. Corporate zombies: Anatomy and life cycle. Econ. Policy 37 (112), 757–803. http://dx.doi.org/10.1093/epolic/eiac027, arXiv:https://academic.oup.com/economicpolicy/article-pdf/37/112/757/48354155/eiac027.pdf.
Beer, C., Ernst, N., Waschiczek, W., 2021. The share of zombie firms among Austrian nonfinancial companies. Monet. Policy & Econ. Q2/21, 35–58, URL: https://ideas.repec.org/a/onb/oenbmp/y2021iq2-21b2.html.
Bittner, C., Fecht, F., Georg, C.-P., 2021. Contagious Zombies. Discussion Paper, Deutsche Bundesbank.
Blattner, L., Farinha, L., Rebelo, F., 2023. When losses turn into loans: The cost of weak banks. Am. Econ. Rev. 113 (6), 1600–1641. http://dx.doi.org/10.1257/aer.20190149, URL: https://www.aeaweb.org/articles?id=10.1257/aer.20190149.
Blažková, I., Chmelíková, G., 2022. Zombie firms during and after crisis. J. Risk Financ. Manag. 15 (7), URL: https://www.mdpi.com/1911-8074/15/7/301.
Borio, C., 2018. A blind spot in today's macroeconomics. Bank Int. Settlements: BIS Manag. Speeches URL: https://www.bis.org/speeches/sp180110.htm.
Bruche, M., Llobet, G., 2014. Preventing zombie lending. Rev. Financ. Stud. 27 (3), 923–956, URL: http://www.jstor.org/stable/24465697.
Caballero, R.J., Hoshi, T., Kashyap, A.K., 2008. Zombie lending and depressed restructuring in Japan. Am. Econ. Rev. 98 (5), 1943–1977.
Faria-e Castro, M., Paul, P., Sánchez, J.M., 2024. Evergreening. J. Financ. Econ. 153, 103778.
Chari, A., Jain, L., Kulkarni, N., 2021. The Unholy Trinity: Regulatory Forbearance, Stressed Banks and Zombie Firms. Working Paper 28435, National Bureau of Economic Research, http://dx.doi.org/10.3386/w28435, URL: http://www.nber.org/papers/w28435.
Ciżkowicz, P., Parosa, G., Rzonca, A., 2023. Ultra-low interest rates, optimal risk, and the rise of zombies. Available At SSRN 4561003.
Cowling, M., Wong, K.P., 2025. The fixed or variable rate loan decision: Which small businesses prefer the certainty of fixed rate loans and which banks offer them? Manch. Sch. 93 (3), 267–280.

Daniel, K., Garlappi, L., Xiao, K., 2021. Monetary policy and reaching for income. J. Financ. 76 (3), 1145–1193. http://dx.doi.org/10.1111/jofi.13004, URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.13004. arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1111/jofi.13004.

De Jonghe, O., Mulier, K., Samarin, I., 2025. Bank specialization and zombie lending. Manag. Sci. 71 (2), 1260–1286.

De Martiis, A., Peter, F.J., 2021. When Companies Don't Die: Analyzing Zombie and Distressed Firms in a Low Interest Rate Environment. University of Bern, Available at SSRN 3890788.

Diamond, D.W., 1991. Monitoring and reputation: The choice between bank loans and directly placed debt. J. Political Econ. 99 (4), 689–721.

Domian, D.L., Gilster, J.E., Louton, D.A., 1996. Expected inflation, interest rates, and stock returns. Financ. Rev. 31 (4), 809–830. http://dx.doi.org/10.1111/j.1540-6288.1996.tb00898.x, URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6288.1996.tb00898.x. arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1540-6288.1996.tb00898.x.

Drechsler, I., Savov, A., Schnabl, P., 2018. A model of monetary policy and risk premia. J. Financ. 73 (1), 317–373. http://dx.doi.org/10.1111/jofi.12539, URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.12539. arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1111/jofi.12539.

European Central Bank, 2017. Guidance to Banks on Non-Performing Loans. European Central Bank: Banking Supervision, ECB, Frankfurt am Main. Germany.

Favara, G., Minoiu, C., Perez-Orive, A., 2021. US Zombie Firms: How Many and How Consequential?. vol. July 30, Board of Governors of the Federal Reserve System, Washington, URL: https://doi.org/10.17016/2380-7172.2954.

Fischer, E.O., Kampl, L.-M., 2019. How to choose between fixed-and variable-rate loans. J. Bank. Financ. Res. (BankArchiv) 67 (2), 125–135.

Giannetti, M., Simonov, A., 2013. On the real effects of bank bailouts: Micro evidence from Japan. Am. Econ. J.: Macroecon. 5 (1), 135–167.

Göbel, M., Tavares, N., 2022. Zombie-Lending in the United States: Prevalence versus Relevance. Working Papers REM 2022/0231, ISEG - Lisbon School of Economics and Management, REM, Universidade de Lisboa, URL: https://ideas.repec.org/p/ise/remwps/wp02312022.html.

Gouveia, A.F., Osterhold, C., 2018. Fear the Walking Dead: Zombie Firms, Spillovers and Exit Barriers. OECD Productivity Working Papers 13, URL: https://www.oecd-ilibrary.org/content/paper/e6c6e51d-en.

Haynes, J., Hope, P., Talbot, H., 2021. Non-performing loans–new risks and policies? NPL resolution after COVID-19: Main differences to previous crises. Study Prepared by the Economic Governance Support Unit (EGOV) at the Request of the Committee on Economic and Monetary Affairs (ECON).

Hoshi, T., 2000. Naze nihon wa ryudosei no wana kara nogarerarenainoka? (why is the Japanese economy unable to get out of a liquidity trap?). In: Fukuo, M., Yoshikawa, H. (Eds.), Zero Kinri To Nihon Keizai (Zero Interest Rate and the Japanese Economy). Nihon Keizai Shimbunsha, Tokyo, pp. 233–266.

Hu, Y., Varas, F., 2021. A theory of zombie lending. J. Financ. 76 (4), 1813–1867.

James, C., 1987. Some evidence on the uniqueness of bank loans. J. Financ. Econ. 19 (2), 217–235.

Jaskowski, M., 2015. Should zombie lending always be prevented? Int. Rev. Econ. Financ. 40, 191–203. http://dx.doi.org/10.1016/j.iref.2015.02.023, URL: https://www.sciencedirect.com/science/article/pii/S1059056015000453.

Jordà, Ò., Kornejew, M., Schularick, M., Taylor, A.M., 2022. Zombies at large? Corporate debt overhang and the macroeconomy. Rev. Financ. Stud. 35 (10), 4561–4586, arXiv:https://academic.oup.com/rfs/article-pdf/35/10/4561/45892728/hhac018.pdf.

Kwon, H.U., Narita, F., Narita, M., 2015. Resource reallocation and zombie lending in Japan in the 1990s. Rev. Econ. Dyn. 18 (4), 709–732.

Laeven, L., Scheppens, G., Schnabel, I., 2000. Zombification in europe in times of pandemic. VOXeu https://voxeu.org/article/zombification-europe-times-pandemic.

Ma, Z., Stice, D., Williams, C., 2019. The effect of bank monitoring on public bond terms. J. Financ. Econ. 133 (2), 379–396.

Montanaro, E., 2019. Non-performing loans and the European union legal framework. In: Chiti, M.P., Santoro, V. (Eds.), The Palgrave Handbook of European Banking Union Law. Springer International Publishing, Cham, Switzerland, pp. 213–246.

Peek, J., Rosengren, E.S., 2005. Unnatural selection: Perverse incentives and the misallocation of credit in Japan. Am. Econ. Rev. 95 (4), 1144–1166.

Puri, M., 1999. Commercial banks as underwriters: Implications for the going public process. J. Financ. Econ. 54 (2), 133–163.

Rajan, R.G., 1994. Why bank credit policies fluctuate: A theory and some evidence. Q. J. Econ. 109 (2), 399–441.

Rocheteau, G., 2024. A model of zombie firms and the perils of negative real interest rates. J. Political Econ. Macroecon. 2 (2), 272–335. http://dx.doi.org/10.1086/729502.

Schivardi, F., Sette, E., Tabellini, G., 2021. Credit misallocation during the European financial crisis. Econ. J. 132 (641), 391–423. http://dx.doi.org/10.1093/ej/ueab039, arXiv:https://academic.oup.com/ej/article-pdf/132/641/391/41954551/ueab039.pdf.

Shiraishi, M., Yano, G., 2021. Do 'zombie firms' emerge among private firms in China? A survival analysis approach that pays attention to the reception of trade credit. J. Chin. Econ. Bus. Stud. 19 (1), 1–34.

Storz, M., Koetter, M., Setzer, R., Westphal, A., 2017. Do We Want These Two to Tango? On Zombie Firms and Stressed Banks in Europe. ECB Working Paper.

Tracey, B., 2025. The real effects of zombie lending in europe. Oxf. Bull. Econ. Stat. 87 (1), 122–154.

Tuuli, S., 2024. Who funds zombie firms: Banks or non-banks? Int. Rev. Financ. Anal. 95, 103420. http://dx.doi.org/10.1016/j.irfa.2024.103420, URL: https://www.sciencedirect.com/science/article/pii/S1057521924003521.

Vickery, J., 2008. How and why do small firms manage interest rate risk? J. Financ. Econ. 87 (2), 446–470.

Won, J., 2023. Zombies in the Syndicated Bank Loan Market: Credit Lines versus Term Loans. SSRN Working Paper, University of Virginia - Darden School of Business, 4152089, available at https://ssrn.com/abstract=4152089.

Yamada, K., Minoura, Y., Nakajima, J., Yagi, T., 2025. Zombie firms and resource allocation: research trends and developments during the spread of COVID-19. J. Appl. Econ. 28 (1), 2410625.