GEOMETRY

with your finger

sketchometry

WORKSHOP

CMLDT

2 Geometry with your Finger

First Steps with sketchometry

Dynamic mathematics software for visualising geometric relationships has been used successfully in mathematics lessons for many years.
sketchometry is a gesture-based dynamic mathematics software that allows you to quickly and easily create and explore interactive constructions with your finger on tablets, smartphones or electronic whiteboards. The innovative gesture control of sketchometry turns mobile devices into an electronic sketching tool that can be used directly in the classroom. sketchometry is available free of charge as an app or directly as a live version (QR code) in the browser:
https://start.sketchometry.org

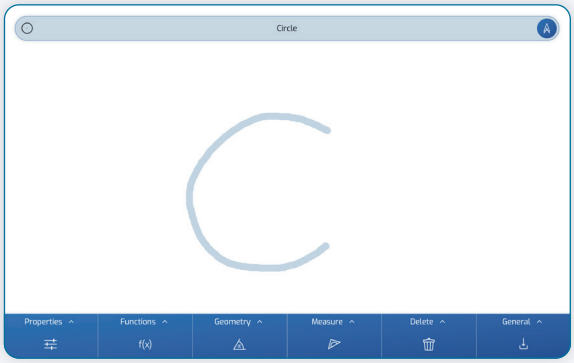
Workshop – Geometry

This workshop provides a step-by-step introduction to the concept and examples of how sketchometry can be used in the geometry classroom. It takes about 90 minutes to complete.
Uses a new board for each of the individual examples, which you can open in the sketchometry gallery via ⊞ *New board*.

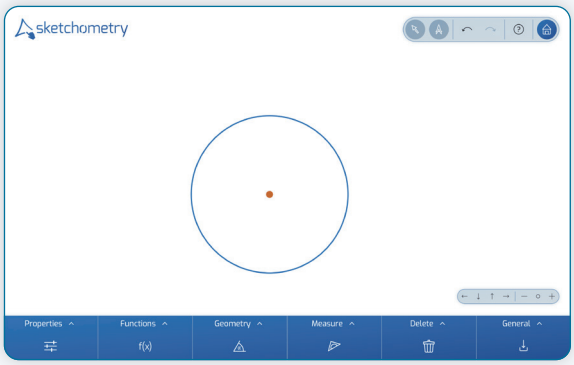
You can access the sketchometry gallery via the icon ⌂ in the menu bar.

Sketching Tool

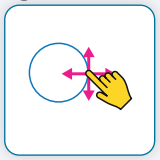
Sketch a circle with your finger on the board. The recognized object *Circle* is displayed in the info bar.



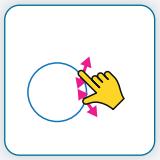
As soon as you lift your finger, the circle and its center point appear.



The special feature of sketchometry: You don‘t have to select your own „circle“ tool in advance. sketchometry automatically recognises the finger sketch (= gesture).
You can change the position of the circle with your finger by dragging either the circle line or the center point.



You can change the size of a circle with two fingers: Touch the circle line at two points and move your fingers towards or away from each other, just as you would do when zooming photos on your smartphone.



Drag and Construct

sketchometry has two basic tools, *Drag* and *Construct*. These can be used independently. The currently active tool is highlighted in the menu bar and displayed in the background of the board.

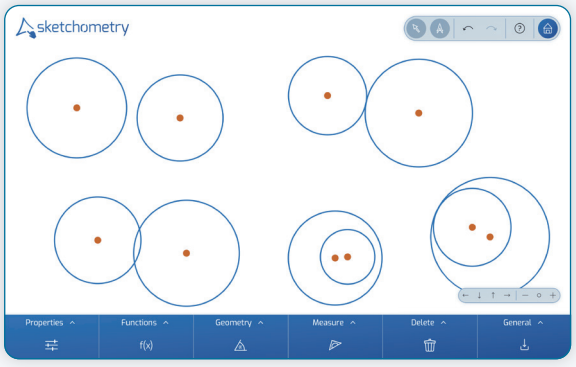
Drag *Construct* *Drag & Construct*

To begin with, it is advisable to use *Drag* and *Construct* separately: If you want to draw a new object, only activate *Construct*. This will prevent you from accidentally moving the existing objects. If you want to change the position of an object, switch *Drag* on and *Construct* off. With a little practice, you will later be able to use both tools at the same time.

Initial experience can be gained with the following task:

> *Draw two circles and examine the positional relationships between the two circles. To do this, change the position and size of the circles.*

By moving, enlarging and reducing the circles, the students observe and describe different positional relationships.

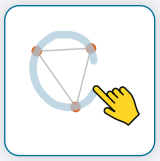


If you want to undo the last construction step – because a gesture was incorrectly recognized, for example – you can do this via *Undo* in the menu bar.

Circumcircle of a Triangle

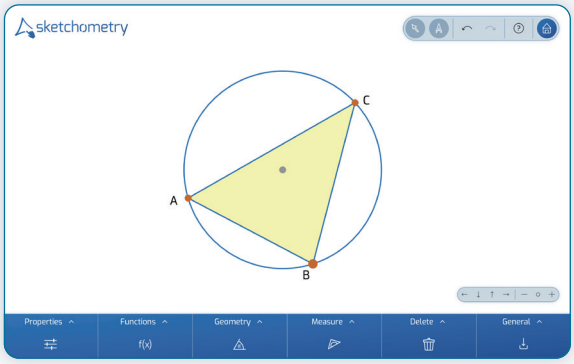
However, sketchometry offers even more. You can create the circumcircle of a triangle by simply sketching it.

Start with a triangle on a new board and then draw a circle through the vertices of the triangle.



Make sure that you touch the three vertices with your finger. They are highlighted by a gray ring.

sketchometry creates the circumcircle (including the center point) of the triangle.



Close to the Object

The motto of sketchometry is „close to the object“. Circles, points, intersections, straight lines, segments and even triangles are sketched. The software recognizes these gestures and uses them to create exact objects that can also be modified.

This means that sketchometry does not have a complex menu structure, which is particularly favorable on small screens.

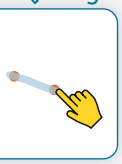
Figures and constructions can be quickly created and analysed on a smartphone, tablet or electronic whiteboard.

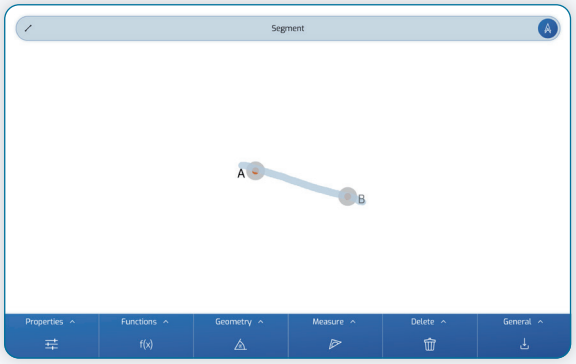
It is not necessary to select a „circumcircle“ tool here either. This direct way of creating a circumcircle is useful if the construction of the circumcircle and its properties have already been covered in class, for example, the following task can be completed using this construction:

Which special triangle shapes are created when the center of the circumcircle is inside, outside or on one side of the triangle?

Perpendicular Line and Circumcircle

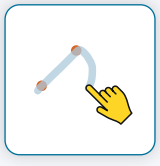
Open a new board and switch off *Drag*. Place two points A and B on the board and connect them with a sketched segment. Again, make sure that you touch both points with your finger so that they are marked:





First construct the perpendicular line “in the classic way“ as a set of all points that are equidistant from the two given points: To do this, create two circles. The first circle with center point A runs through point B and vice versa, the second circle goes around point B through point A.

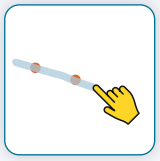
Here you use another gesture. Start with the first circle with center point A. The *Circle (Radius)* gesture consists of two parts that must be sketched without interruption:

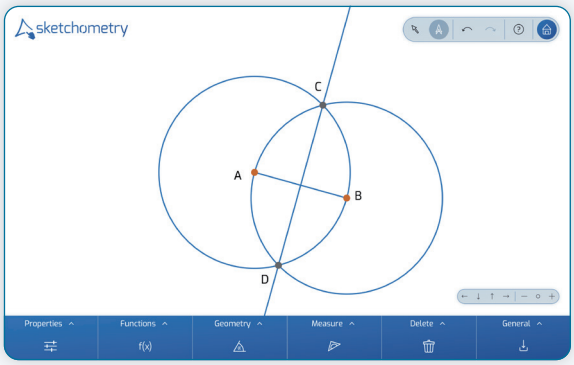


1. Start the gesture just before point A and move your finger past point B. Both points must be marked. This indicates the radius of the circle (see segment gesture).
2. Now indicate a quarter-circle arc without stopping.

sketchometry recognizes this as the *Circle (Radius)* gesture and creates the circle. The circle around B through A is created in the same way.

In the next step, tap the two intersections of the circles one after the other and the intersection points appear. Finally, sketch the center line as a straight line through the two intersection points. In contrast to the *Segment* gesture, you start clearly before the first (intersection) point and end the indicated straight line clearly after the second (intersection) point.

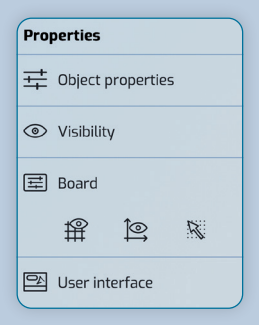




Properties

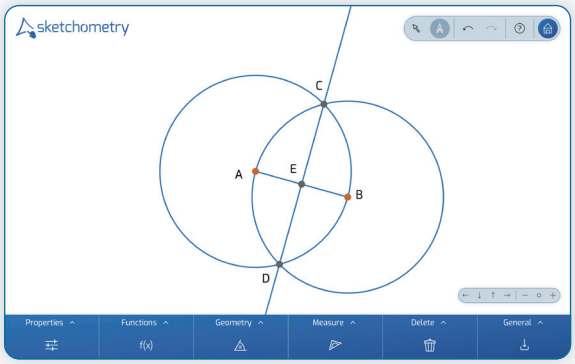
Use the *Object properties* tool to adjust the appearance and behavior of objects.

Use the *Visibility* tool to show or hide objects.

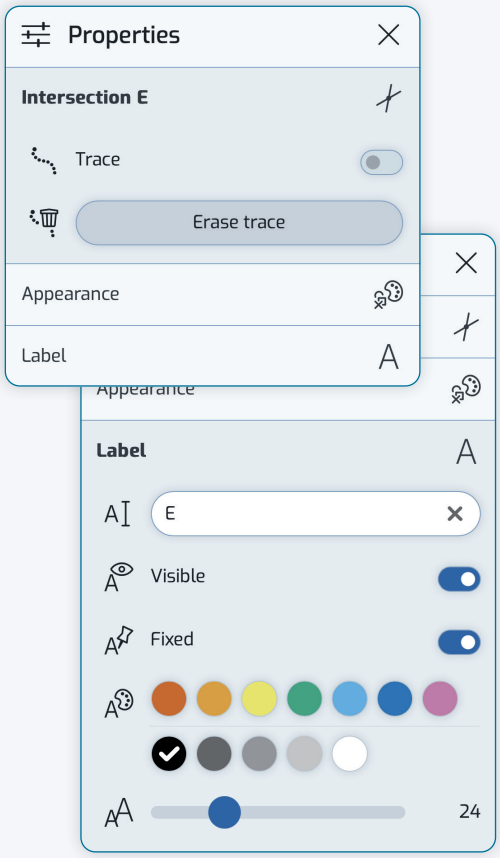


You can use the *Board* tool to show the coordinate axes or the grid, for example, or to change the background color of a board. You can change the position of the menu bar and the toolbar or the language with *User Interface*.

Finally, mark the intersection point of the segment with the perpendicular line, which bisects the segment.
sketchometry names actively created points alphabetically starting with A. Therefore this intersection point is labeled by E.



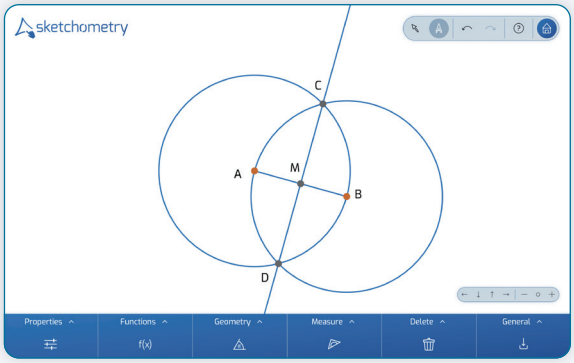
You can change the name, font size, color settings and other properties at any time using the *Object properties* tool in the toolbar (via *Properties*).
Change the name of point E to M. To do this, activate the tool *Object properties* and tap on point E.
A selection list appears. Select point E. sketchometry now displays the properties of the intersection point E. These are divided into individual categories. Select category *Label* and change the name E to M.



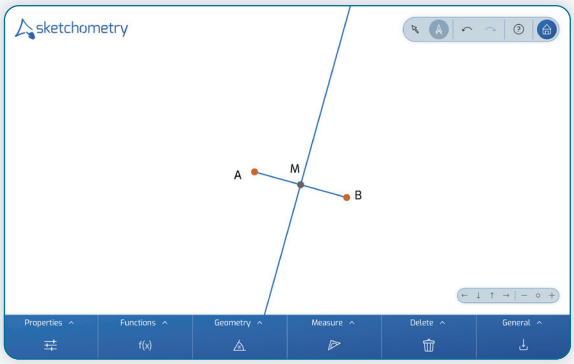
Visibility – Hide Objects

Activate the tool *Visibility* via *Properties* in the toolbar. Tap the objects you want to hide one after the other. As soon as you select another tool, the hidden objects disappear completely (but remain internally available). If there are several objects at the selected position, a selection list appears.
If you want to show an object again, activate *Visibility*. The hidden objects reappear pale. Tap on the object you want to display. It will become visible again.

Close *Properties* with ×. The intersection point is now called M.



Finally, hide the “auxiliary objects“ (here the two circles and their intersection points) using the tool *Visibility*. The construction of the perpendicular line is now complete.



To find the center of the circumcircle of a triangle, you would have to perform this construction on at least two sides of the triangle. This is quite time-consuming and can quickly become a little bit messy.
Therefore, use the two sketchometry gestures *Midpoint* and *Perpendicular Line*:

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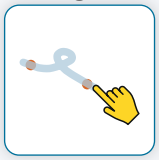
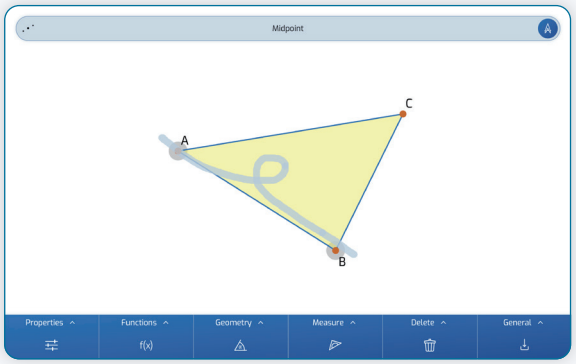
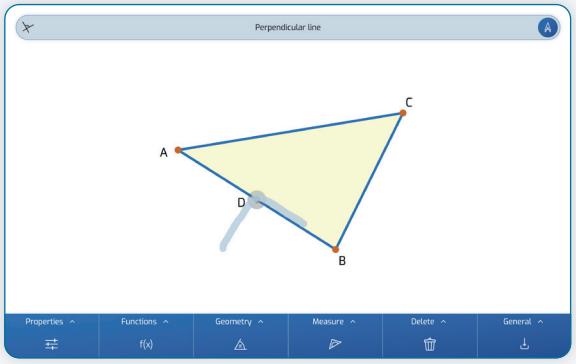
Create a triangle in a new board and then the midpoint of a side of the triangle.The *Midpoint* gesture looks like a loop. When sketching the gesture, mark the two end points of the chosen side of the triangle:

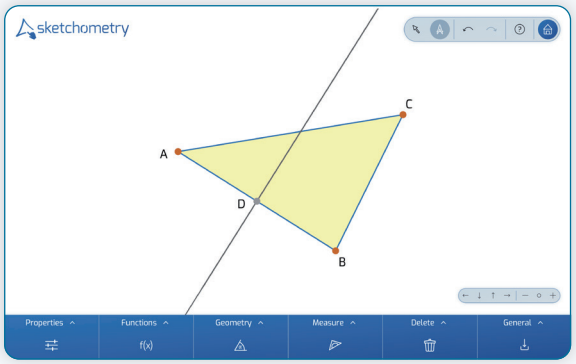




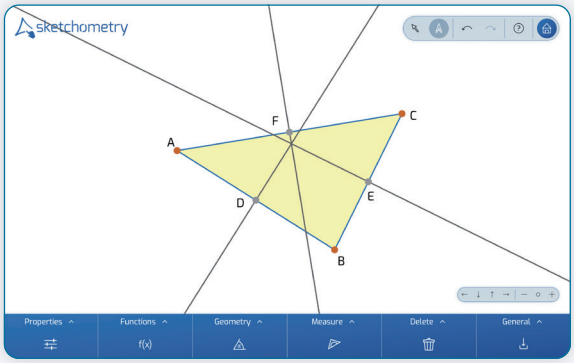
Now draw a perpendicular line to the side of the triangle through its midpoint. Again with a gesture, of course:



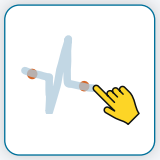
To do this, draw a right angle. When drawing the gesture, make sure that you mark the center point at the gesture bend (gray ring).



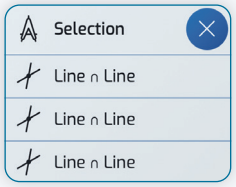
Repeat these two steps for the remaining two sides of the triangle.



This procedure is much quicker and clearer than the step-by-step construction. But there is an even simpler way: Instead of the *Midpoint* and *Perpendicular Line* gestures, use the *Perpendicular Bisector* gesture as shown.

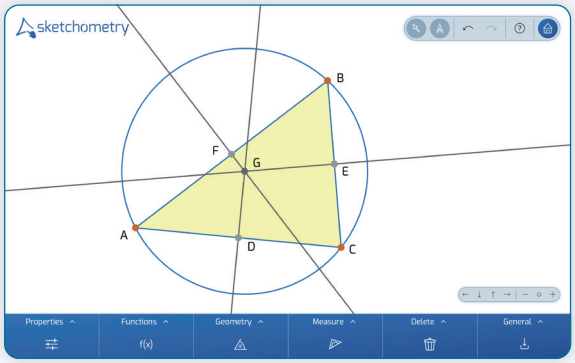


The three perpendicular lines intersect at one point. Activate *Drag* and change the position of the vertices of the triangle.
An explanation of the intersection property should be discussed in class.
Finally, the center point and the circumcircle must be created. Tap on the intersection of the perpendicular lines. sketchometry may display a selection list with the intersection options. To finish draw the circumcircle using the *Circle (Radius)* gesture.



Intersection of More Than Two Objects

An intersection point is clearly defined based on the position of *two* objects (lines, circles). However, if there are three or more objects at the point in question, sketchometry will display a list of possible intersections of two objects. One of the intersection options can be selected by tapping the corresponding list entry. If the finger remains on the list entry, the corresponding objects are highlighted on the board. If the finger is moved to another list entry, the new selected objects are highlighted. The intersection point is only created when the finger is lifted.



Now the construction is complete. Depending on the students' prior knowledge a suitable construction option can be selected: either the step-by-step approach or the "ready-made" gesture tool.

Inscribed circle of a triangle

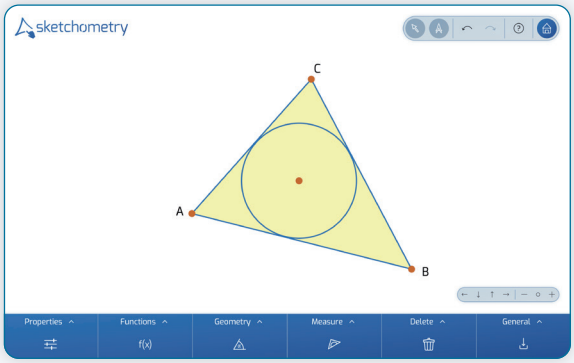
Change of perspective:
The next example is about the inscribed circle. The following question can be asked in class:

> *Is there a circle that touches all three sides of a triangle from the inside?*

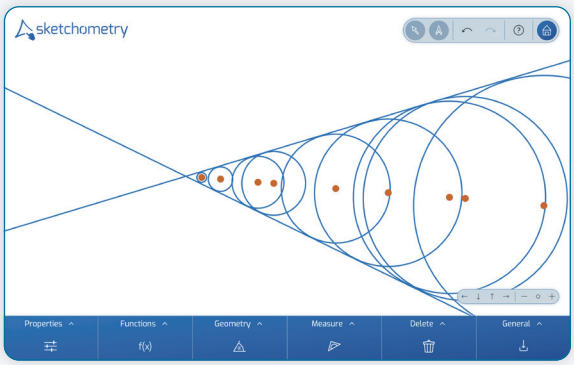
Students can explore this question using the following procedure: Draw a triangle and a circle. Use two fingers to position the circle so that it touches the three sides of the triangle.

Learning with Your Finger

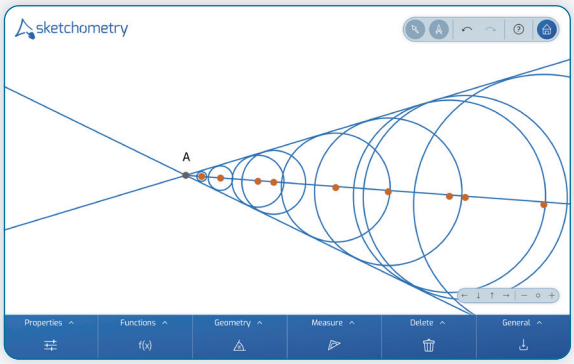
In sketchometry, the gestures are based on the shape of the objects to be drawn. To draw a circle sketch the shape of the circle with your finger; to draw a line sketch a line with your finger, for example.
All the sketchometry gestures are designed in that way. Compared to conventional geometry software, in which the center point and a point on the circle line have to be selected separately to create a circle, for example, sketchometry allows this direct approach.
Recent studies have highlighted the importance of pointing and tracing when learning. These finger movements increase attention and activate additional areas of the brain, leading to a better memorising abilty. sketchometry makes consistent use of these findings.



Of course, this is not a construction. Here sketchometry is again used as a sketching tool: change the shape of the triangle and reposition the circle. Assumption: There will always be a circle that has the desired property. The construction of the inscribed circle is now developed step by step.
In a new board, first consider two intersecting straight lines and mark the point of intersection. Draw circles with different radii and position them so that they touch both straight lines:



The centers of the circles are equidistant from the two straight lines (radius!). Draw a straight line that runs through the intersection of the two lines and one of the circle centers.



New board

Mobile UI

sketchometry

Selected Tool

Toobar

Properties | Functions | Geometry

Properties

- Object properties
- Visibility
- Board
- Shortcuts
- User interface

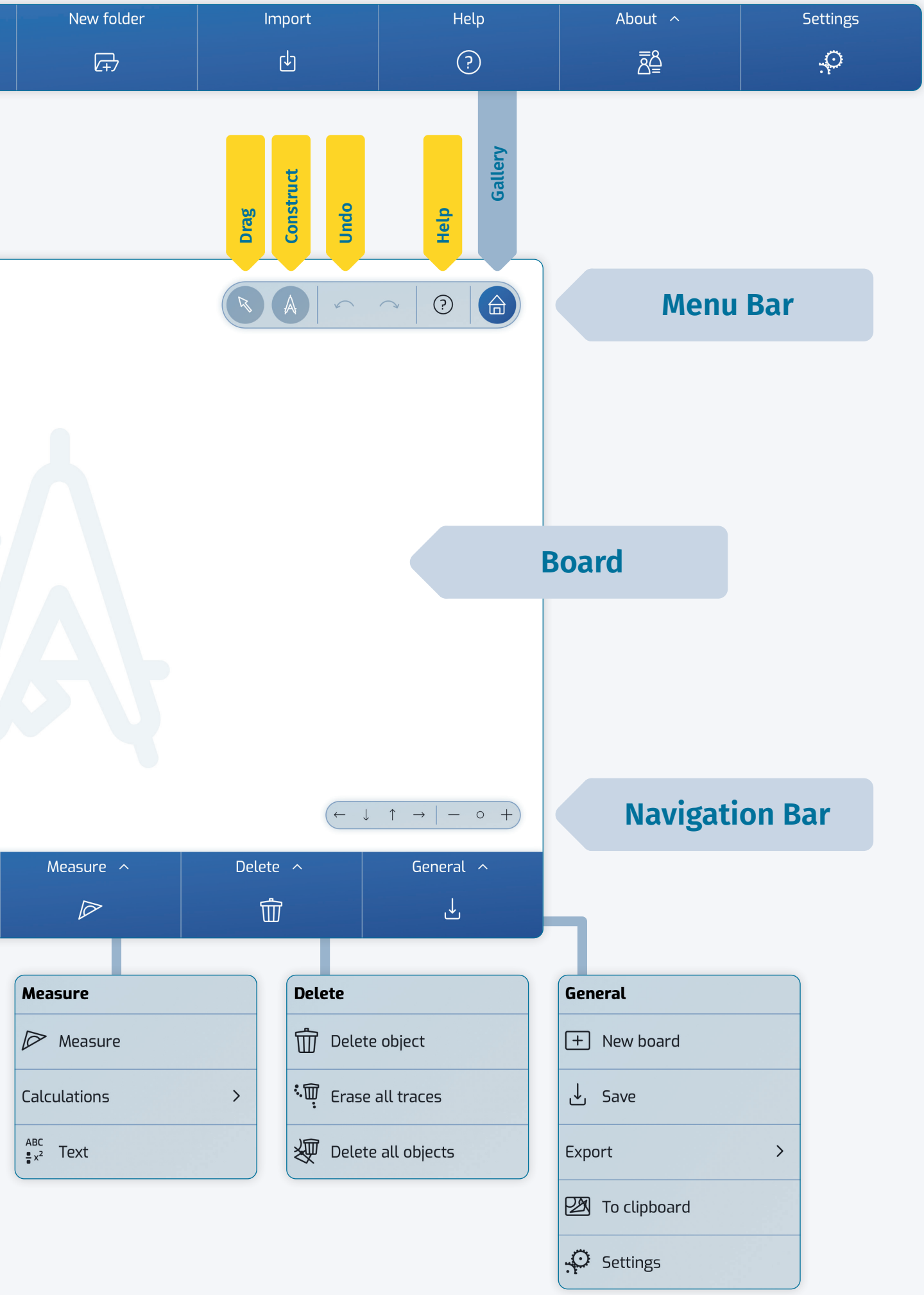
Functions

- Function graph
- Slider
- Sketched curve

Geometry

- Parallelograms
- Regular polygons
- Segment (fixed length)
- Angles & Sectors

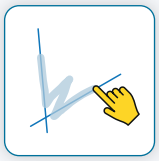
More Tools


New folder
Import
Help
About
Settings
Drag
Construct
Undo
Help
Gallery
Menu Bar
Board
Navigation Bar
Measure
Delete
General
Measure
Measure
Calculations
Text
Delete
Delete object
Erase all traces
Delete all objects
General
New board
Save
Export
To clipboard
Settings

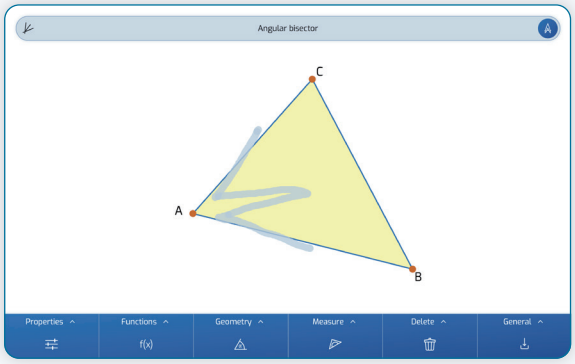
Possible work orders:

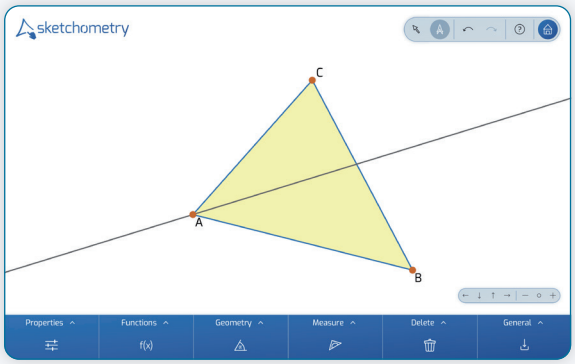
- *What do you find?*
- *What property does this straight line have with respect to the two initial straight lines?*
- *Give reasons for your assumption!*

With these considerations, the significance of the angle bisector for the construction of the inscribed circle becomes clear.
Open a new board and deactivate *Drag*. To make it easier to construct the inscribed circle of a triangle, there is a simple gesture to create the angle bisector. This looks like the letter W.



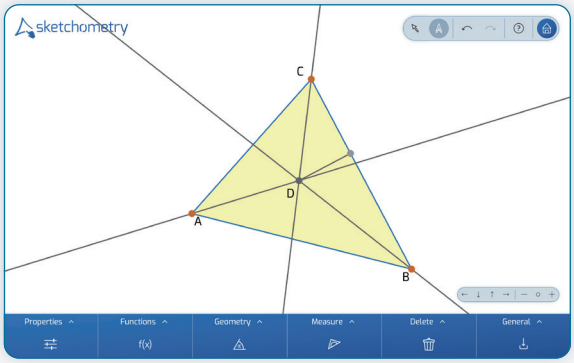
According to the motto "close to the object", when drawing the gesture, first mark the first leg of the angle, then the sector in which the angle bisector lies and finally the second leg:



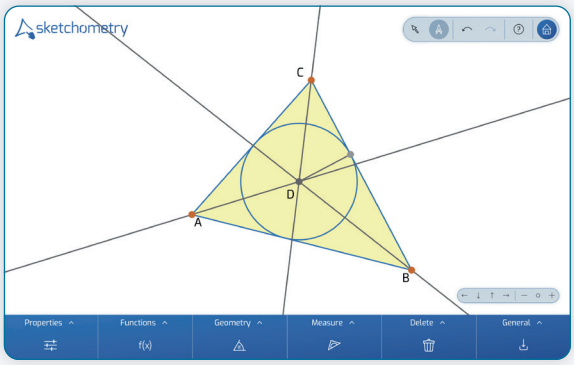


Add the angle bisectors of the other two angles. As in the case of the perpendicular bisectors, you can show that the angle bisectors intersect at a point. Mark this point of intersection. It has the same distance from all sides of the triangle.
Now the radius is still required to draw the inscribed circle. To do this, use the familiar perpendicular gesture with a minor variation:

start the gesture directly at the intersection point and not before it. In addition to the perpendicular segment, sketchometry also creates the corresponding foot of the perpendicular.



Use the *Circle (Radius)* gesture to create the inscribed circle and the construction is finished.



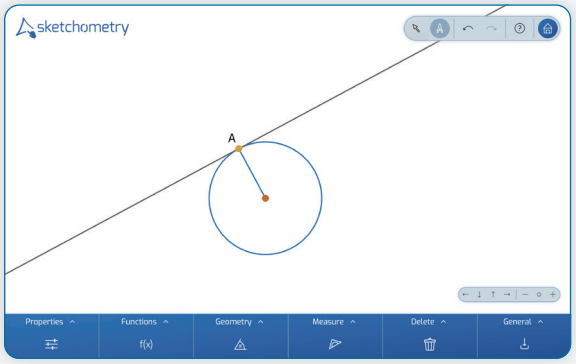
Another Point of View – Tangents

The inscribed circle touches all three sides of a given triangle. Now a circle is given and you are looking for a triangle whose sides touch this circle?
Draw a circle in a new board. Since the sides of the triangle are to touch the circle, they must be tangents to the circle. Two options are now considered:
For the first option, you already know all the necessary gestures:

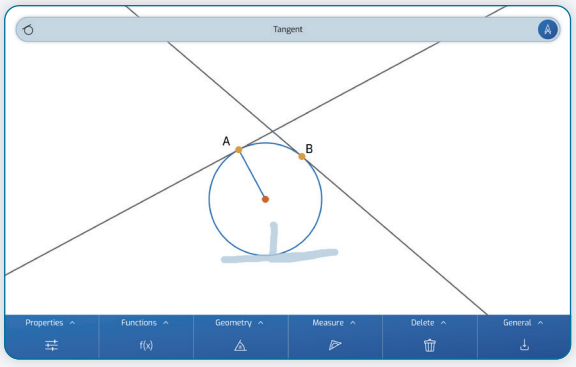
- Place a point on the circle line. It appears as a *glider* that can only move along the circle line.
- Draw the radius as the distance from the center point to this glider.
- Create the tangent using the familiar gesture for the perpendicular line. Make sure that you mark the glider at the bend of the gesture.

Gliders and Dependent Points

If you tap with your finger on an empty spot on the board, a free movable point is created. sketchometry shows this point in red. If there is already an object (line, segment, curve, ...) at this spot, sketchometry creates a so-called *glider* that you can move along the object.
If you tap on a spot where two objects intersect, an intersection point is created that cannot be moved. It is shown in gray. For example, the foot of a perpendicular is shown in gray because it cannot be moved.



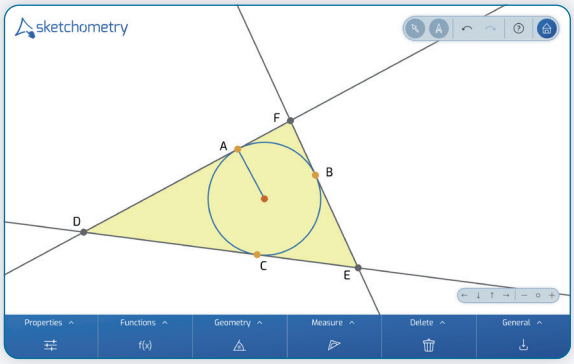
The second option with a gesture is much more elegant:
Now sketch a capital T on the circle line. The shape of the letter T indicates the tangent direction and also the direction of the radius.



Further information

On sketchometry.org gesture overviews and printable worksheets as well as other teaching materials are available for free download.

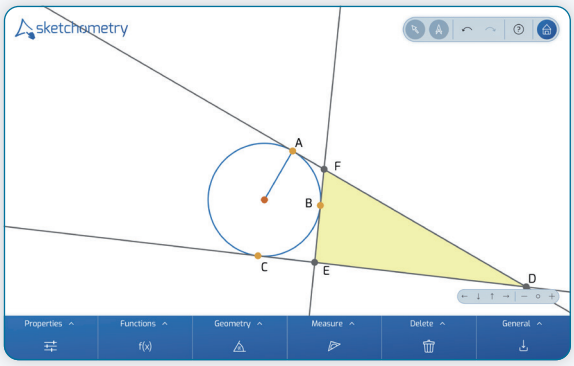
The gesture is easy to remember “T for tangent”. It also works for function graphs and curves. Together with the tangent, sketchometry also creates the point of tangency as a glider. After you have added the three tangents to the circle, mark the intersection points of the tangents. Finally, draw a triangle through the three intersection points to highlight the triangle area.



The original circle is now the inscribed circle of the triangle. Change the position of the points and observe the construction.

Variations

Now position the three gliders (points of tangency) so that they are only in one “half” of the circle:



What happens? And, above all, why? A so-called excircle is created, which leads to further questions:

- *How can an excircle be constructed?*
- *How many excircles does a triangle have?*
- *What is the relationship between the inscribed circle and the excircles of a triangle?*

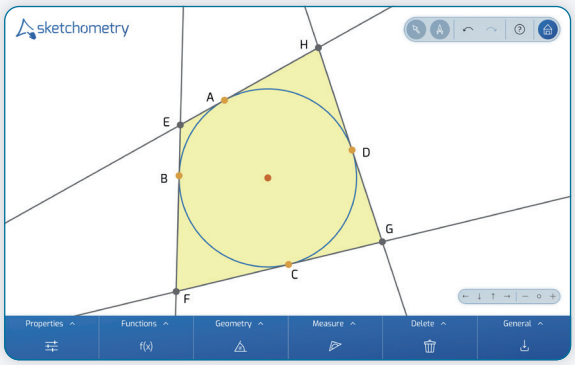
These questions are the basis for differentiation in the classroom, for example. Substantially the approach to excircles is

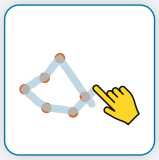
oriented towards that of the incircle, but is somewhat more demanding.

Tangent Quadrilaterals

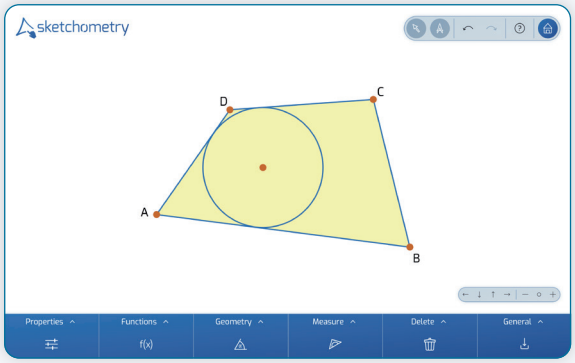
Now you draw four tangents to the circle instead of three. Use the points of intersection of the tangents to create a quadrilateral, a so-called tangent quadrilateral.

- *Modify the quadrilateral so that you get a square or a rhombus.*
- *Are there any other special quadrilateral shapes that can be created in this way?*
- *Explain why you cannot create a rectangle?*
- *Are there any quadrilaterals that have both an incircle and a circumcircle?*





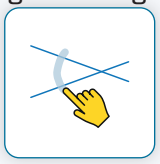
Change the point of view here too. Sketch a quadrilateral in a new board, draw an additional circle and try to position it so that it touches all four sides of the quadrilateral. Compared to the triangle, there is not always a solution. Therefore, it makes sense to examine different quadrilateral shapes, too.

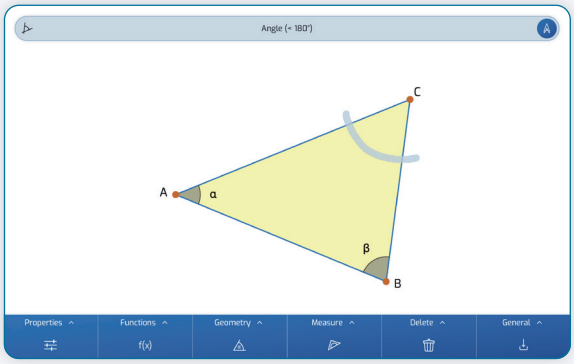


The circle has to be repositioned again and again. The students discuss their observations. Here, they use sketchometry as a sketching tool.

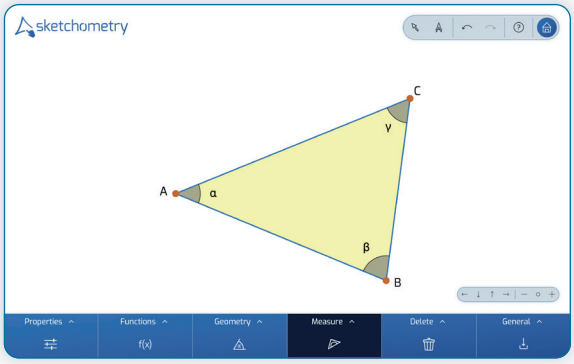
Measurement

In addition to numerous gestures, sketchometry also offers various measuring tools: draw a triangle and mark its interior angles. How?
In the same way as on the blackboard: sketch an arc from the first leg of the angle to the second leg. Note that sketchometry always treats angles counterclockwise. Also add the other two interior angles:

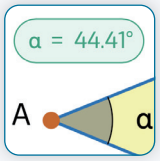


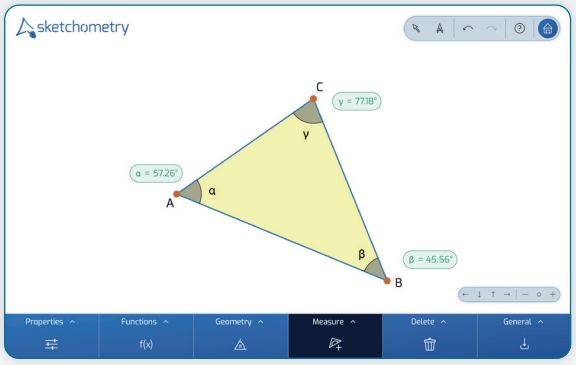


As soon as you have created an angle object, you can measure its size. To do this, activate the tool *Measure* in the toolbar.



Tap on the angle and the measurement appears. As long as *Measure* is activated, you can add further measurement objects and also drag them (without having to switch to *Drag*).





As soon as a measurement object is modified, the result is automatically adjusted. sketchometry opens a list box if there are several objects that can be measured at the spot you tap. Lengths and areas are measured in the same way:

- Activate *Measure*.
- Tap on the object to be measured (and select it from the list box if necessary).

Calculations

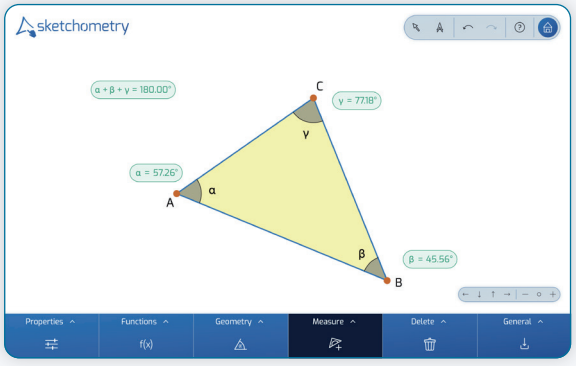
In addition to *Measure*, sketchometry also allows calculations:

- *Sum*
- *Difference*
- *Product*
- *Ratio*

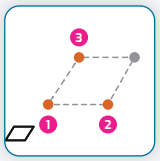
In the example above, determine the sum of the angles of the triangle. To do this, activate *Sum* in the toolbar and tap on the three measurements one after the other. A preview appears in the info bar:

α + β + γ = 180.00°

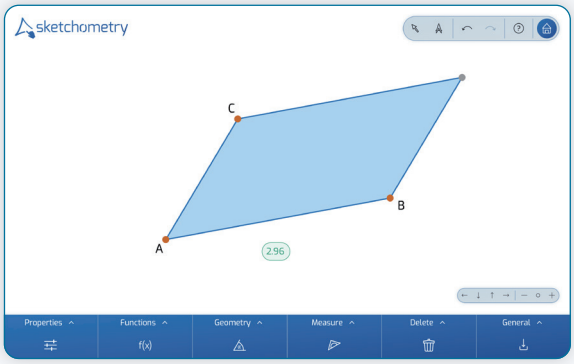
Then tap on a free spot on the board. The sum of the measurements appears.



However, the tools *Sum*, *Difference*, *Product* and *Ratio* are even more flexible. With the *Ratio* tool, for example, you can also click directly on the objects and do not have to create individual measurements first, as the following example shows:



Place three points on a new board, select the *Parallelogram* tool and tap the points one after the other. A parallelogram appears. Now select the tool *Ratio* and tap first on the parallelogram area (numerator) and then on the (base) side of the parallelogram (divisor). Place the result (=ratio) on the board. It indicates the length of the height of the parallelogram.



Finally, calculations can also be part of other calculations.

Outlook

This workshop provides a first insight into working with the sketching tool sketchometry. In addition to numerous geometry gestures, tools for calculus are also available. For example, a slope triangle can be easily sketched on an interactive function graph. The following gesture overview is useful in the classroom.

Author

Carsten Miller

CMDT
Center for Mobile Learning with Digital Technology

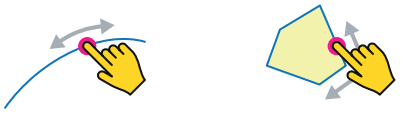
University of Bayreuth
95440 Bayreuth
Germany

sketchometry.org

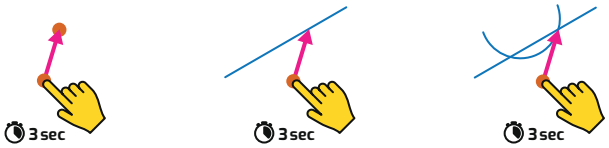
sketchometry

Point

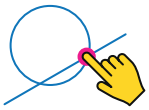
Glider



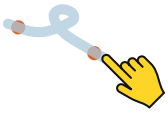
Merge point



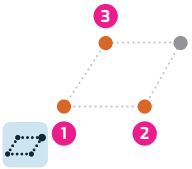
Intersection



Midpoint



Parallelogram point



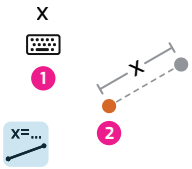
Split point

Line

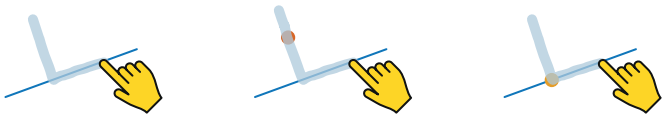
Ray

Segment

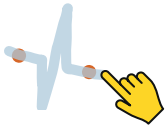
Segment (fixed length)



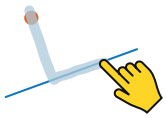
Perpendicular line



Perpendicular bisector



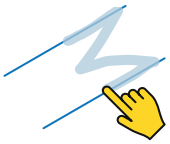
Perpendicular segment



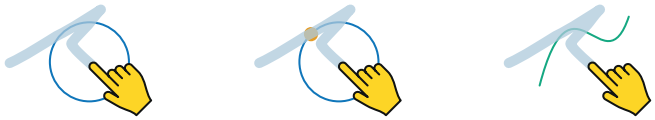
Rotate

Parallel line

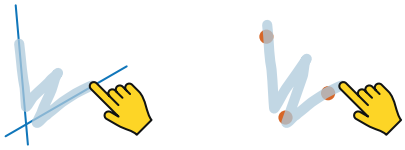
Centerline



Tangent



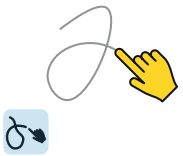
Angular bisector



Vector

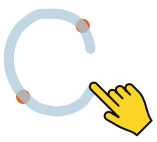
Copy vector

Sketched curve

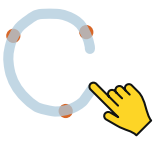


Circle

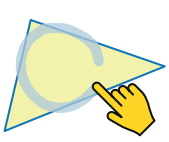
Circle (diameter)



Circumcircle



Inscribed circle

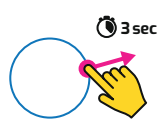


Resize radius

Circle (radius)

Copy length

Copy circle

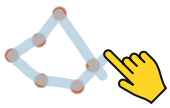


sketchometry

Triangle

Quadrilateral

Polygon

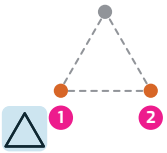


Drag

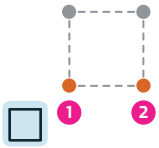
Rotate

Copy polygon

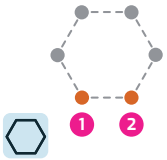
Equilateral triangle



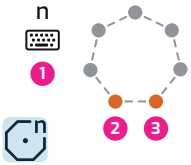
Square



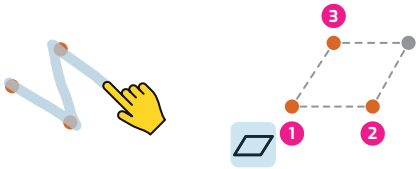
Regular hexagon



Regular polygon

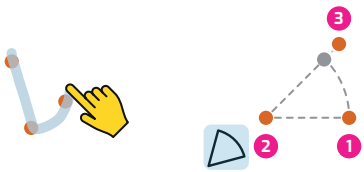


Parallelogram

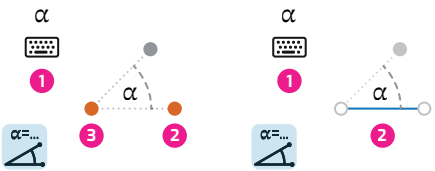


Angle

Sector



Angle (fixed size)

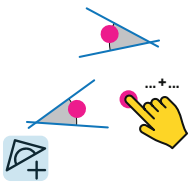


Interior angle

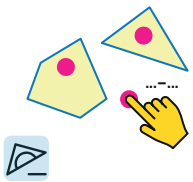
Measurement

Reflection through an axis

Sum

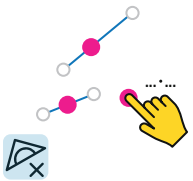


Difference

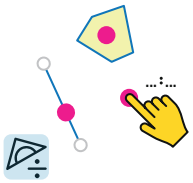


Reflection through a point

Product



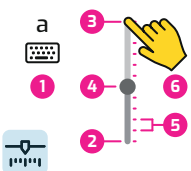
Ratio



Function graph

f(x)

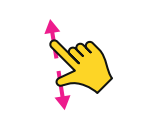
Slider



Slope triangle

Zoom out

Zoom in



Drag

Construct

Visibility

Delete

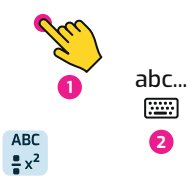
Settings

Share

Gallery

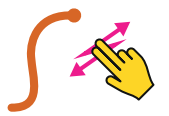
Object properties

Text



Move board

Erase trace



sketchometry

WORKSHOP

CMLDT