Three Papers on Strategic Decision-Making in Economics and Finance

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Maximilian Philipp Kähny
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Dekan: Prof. Dr. Claas Christian Germelmann (Universität Bayreuth)
Vorsitzender der Prüfungskommission: Prof. Dr. Martin Leschke (Universität Bayreuth)
Erstberichterstatter: Prof. Dr. Fabian Herweg (Universität Bayreuth)
Zweitberichterstatter: Prof. Dr. Stefan Napel (Universität Bayreuth)
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Chapter 1

Introduction

> "It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard for their own interest." — *Adam Smith, 1776*

1.1 The Role of Microeconomic Theory

ECONOMICS, at its core, seeks to understand the process by which scarce resources are allocated to their most efficient use and thus (simply put) to formalize the exchange of goods and services. While market settings play an integral role, analyzing any economic system is to understand the fundamental mechanisms that drive economic actors within. In this context, microeconomic theory focuses on individual decision-making and its collective impact on a society's scarce resource allocation.

Historically, microeconomic theory was limited to formally analyzing the most straightforward exchanges of goods and services between economic actors until the 1950s. However, with the introduction of state-contingent securities by Arrow (1964) and Debreu (1954) alongside the theory of choice under uncertainty developed by von Neumann and Morgenstern (1944), the field experienced a paradigm shift. These new concepts allowed economists to formalize the exchange and allocation of risk as well as individual risk preferences, thereby enriching the discourse on economic decision-making. In the 1960s, microeconomic theory reached another milestone by investigating the delegation of tasks from a principal to an agent with private information, particularly in the context of contracts. Central to these analyses were the formal representation of decentralized information and conflicting objectives, the two cornerstones of what is now commonly known as the theory of incentives. The introduction of decentralized information and conflicting objectives gave rise to the frictions of moral hazard and adverse selection, to which incentive theory provided foundational insights into incentive-compatibility and incentives for truth-telling.

The evolution of microeconomic theory has laid the groundwork for theoretical models related to firms, corporate finance, and, more broadly, economic institutions. Until now, its concepts have been an integral part of modern research on investments under risk and portfolio choice, as well as on information economics. In fact, microeconomic theory has spurred diverse research areas where the modeling of strategic decision-making has become highly relevant. Fields such as labor economics, organization theory, behavioral economics, and industrial organization have all benefited from these insights. While many of the concepts in microeconomic theory operate at a high level of abstraction, their applications are both practical and widespread. From firms vying for market share to consumers navigating choices in the marketplace and even politicians competing for electoral votes, the principles of strategic decision-making permeate our everyday lives.[1]

This dissertation, titled "Three Papers on Strategic Decision-Making in Economics and Finance", aims to explore the applications further. Each paper delves into distinct aspects of strategic decision-making, applying core concepts of microeconomic theory to different settings in economics and finance. The following section will outline each paper's key contributions and findings, providing an overview of the insights gained from exploring the strategic behavior of individuals and firms and its effect on society.

1.2 This Dissertation

Chapter 2, coauthored with Fabian Herweg, examines through a contract-theoretic model the relationship between the interest rate and the rise of "zombie firms" – non-viable companies that continue operating due to continued financial support from creditors.[2] Initially observed in Japan following the Japanese asset price bubble, zombie lending has re-emerged as a pressing concern in the aftermath of the Global Financial Crisis. The cause of this concern is studies revealing a substantial increase in the share of zombie firms in Western economies, with researchers and the media attributing this trend to central banks' low interest rate policies (Adalet McGowan *et al.*, 2018; Banerjee and Hofmann, 2018). Nevertheless, it is not obvious how low interest rates favor zombie lending as the direct effect of a drop in the interest rate is a reduction of interest expenses and, thus, would suggest a reduction of zombie firms.

[1]The introduction to microeconomic theory is inspired by the contents of Laffont and Martimort (2002), Bolton and Dewatripont (2005) and Bernheim and Whinston (2007).

[2]The working paper version is Herweg and Kähny (2022).

First, we contribute to the existing literature by establishing a simple and tractable model where it is optimal for banks to continue projects that should be liquidated from a welfare perspective, i.e., where banks engage in zombie lending. Our second contribution is to elaborate on the precise mechanism of how interest rate changes affect banks' incentives for zombie credits. Specifically, we show that an unexpected drop in interest rates encourages banks to engage in zombie lending. On this account, our model may help explain "questionable" lending decisions of commercial banks after the financial crisis and attest to the opinion that the – to some degree – unexpected loose monetary policy of the European Central Bank may have propelled the problem of zombie lending in the eurozone. However, our analysis also shows that once banks adjust loan contracts to the new lower rates, the incentive for zombie lending diminishes. As such, we argue that while short-term interest rate reductions can fuel zombie lending, they do not cause long-term zombification. With our results best fitting to bank-oriented economies and considering Europe's (mainly) bank-centric financial system, our findings suggest that the European Central Bank's prolonged loose monetary policies have not necessarily warranted countermeasures against zombification.

Chapter 3 presents a venture capital model that examines the role of active corporate governance by venture capitalists, explicitly focusing on the provision of human capital and its impact on portfolio composition.[3] Human capital has been widely recognized as a critical driver of the exceptional performance of venture capitalists, not only by increasing the likelihood of start-up success but also through intra-portfolio dynamics. The existing theoretical literature suggests that venture capitalists strategically invest in firms with overlapping target markets to credibly threaten human capital reallocation across ventures, thereby leveraging the competition among entrepreneurs to extract larger rents (Fulghieri and Sevilir, 2009). However, recent trends in the venture capital industry – in particular, the decline in active corporate governance and the rise of the "spray and pray" portfolio tactic – challenge this intuition and motivate an exploration of alternative channels that might influence human capital provision and its impact on venture capital portfolios (see Ewens *et al.*, 2018; Lerner and Nanda, 2020).

The contribution of this paper is to introduce a venture capital model that considers not only entrepreneurs' intra-portfolio competition at the financing but also at the market stage. The central idea is that venture capitalists' strategy of investing in rival firms can incentivize individual entrepreneurs to create negative externalities on competitors through anti-competitive actions, e.g., price dumping or market segmentation. By reducing the profitability of competing portfolio firms through these actions,

[3] The working paper version is Kähny (2024).

which I refer to as *preemptive differentiation*, the model shows that entrepreneurs may discourage venture capitalists from reallocating human capital and, in extreme cases, even abandon active governance despite it being welfare-optimal. Thus, accounting for market-based externalities arising from the threat of human capital reallocation may offer a new perspective on venture capitalists' recent trend of reduced active governance. Furthermore, the analysis demonstrates that venture debt as an alternative funding type does not mitigate this friction between venture capitalists and entrepreneurs but, in fact, enhances the scope for preemptive differentiation. This finding is consistent with the observation that traditional lenders, such as banks, are not known for providing entrepreneurs with human capital or strategic guidance but only equity investors (see Barry and Mihov, 2015).

Chapter 4, joint work with Tim Baule and Jonathan Bothner, adopts a theoretical model on network goods to rationalize individuals' protest participation and uses survey data from the American National Election Studies to test its validity.[4] Whether peaceful or violent, protests are a crucial tool for citizens to instigate social and political change. One prominent example is the mass mobilization during the Arab Spring and its profound consequences on the political and economic landscape, illustrating the importance of understanding the mechanisms behind protest formation. While the literature identifies critical factors in the individual participation decision – i.e., personal attitudes toward protests, network effects, and coordination issues – the existing theoretical models struggle to explain the emergence of protests without imposing a quasi-public goods property when integrating these insights (Acemoglu and Robinson, 2005; Ellis and Fender, 2011).

The first contribution is to combine the abovementioned factors into a unified theoretical framework that explains protest formation regardless of protests' goods property. By arguing that an individual's participation is non-pivotal to the success of the protest objective, we show the rationale for participation and derive equilibria of the aggregated number of protesters. With personal attitudes (standalone benefits) and network benefits determining the resulting equilibrium, the analysis also highlights the crucial role of coordination when network benefits drive protest participation. The second contribution is the empirical approach of employing a spatial autoregressive model, for which we construct synthetic networks, to identify network benefits across a representative sample of US citizens. Our findings support the model's implication that standalone and network benefits significantly shape individuals' protest participation, providing tentative insights into the varying sizes of US protest movements and how individual motivations and social networks shape protest outcomes.

[4] The working paper version is Baule *et al.* (2024).

Chapter 2

Do Zombies Rise when Interest Rates Fall? A Relationship Banking Model

Abstract: A relationship bank or market investors finance an entrepreneur's risky project. Unlike investors, the bank can identify and liquidate bad projects at an interim stage. If the entrepreneur can provide only limited capital, the optimal loan contract induces an inefficient continuation decision, i.e., the bank engages in zombie lending. In the short run – for a given contract – the bank's incentive to roll over bad loans is enhanced if the base interest rate drops. In the long run, however, the bank adjusts the contract to a drop in the interest rate, and the effect on zombification is reversed.

Keywords: Evergreening; Interest rates; Loan rollover; Relationship banking; Zombie firms.

JEL classification: D82; D86; G21; G33.

2.1 Introduction

Zombie firms are the walking dead of an economy: unable to cover their debt obligations with current profits but still staggering on. Banks often keep zombie firms alive by extending or granting loans at favorable terms. The term *zombie lending* was coined by Caballero *et al.* (2008), who analyze the so-called lost decade in Japan in the 1990s. Early contributions – but also recent ones – investigating zombie lending point out that weak banks may have incentives to roll over (evergreen) loans of non-viable firms instead of realizing the losses (Peek and Rosengren, 2005; Caballero *et al.*, 2008; Storz *et al.*, 2017; Schivardi *et al.*, 2021).

In the aftermath of the Global Financial Crisis (GFC), zombie lending received renewed interest as studies showed that several developed countries had an alarmingly high proportion of zombie firms (Adalet McGowan *et al.*, 2018; Banerjee and Hofmann, 2018). According to the estimates of Banerjee and Hofmann (2018) for 14 advanced economies, the zombie share increased from 2% in the late 1980s to 12% in 2016. Banerjee and Hofmann (2018) attribute this development to reduced financial pressure rooted in worldwide expansionary monetary policies and low interest rates. The claim that low interest rates constitute favorable conditions for zombie firms is further supported by empirical studies such as De Martiis and Peter (2021), Banerjee and Hofmann (2022) and Ciżkowicz *et al.* (2023).

Zombie lending and the channel of low interest rates have also attracted public attention (Banerjee and Hofmann, 2022).[1]

> "Years of ultralow interest rates intended to stimulate the economy after each of three 21st-century recessions created the conditions for zombies to proliferate [...] Weak growth prompts the central bank to cut interest rates, which allows zombies to multiply." — *Washington Post, 2020*[2]

> "As many as one in seven UK firms are potentially "under sustained financial strain" and had been able to "stagger on" partly thanks to low interest rates

[1] Examples are the following publications: Financial Times, February 5, 2020: "How to avoid a corporate zombie apocalypse" `https://www.ft.com/content/1d87c9ec-4762-11ea-aeb3-955839e06441`; New York Times, June 15, 2019: "When Dead Companies Don't Die" `https://www.nytimes.com/2019/06/15/opinion/sunday/economy-recession.html`; The Economist, September 26, 2020: "Why covid-19 will make killing zombie firms off harder" `https://www.economist.com/finance-and-economics/2020/09/26/why-covid-19-will-make-killing-zombie-firms-off-harder`; last accessed on May 5, 2024.

[2] "Here's one more economic problem the government's response to the virus has unleashed: Zombie firms." Washington Post, June 23, 2020, `https://www.washingtonpost.com/business/2020/06/23/economy-debt-coronavirus-zombie-firms/`; last accessed on May 5, 2024.

[...].” — *The Guardian, 2020*[3]

While there is evidence regarding the connection between zombie shares in an economy and the interest rate, the precise mechanism of how low interest rates create a favorable environment for zombie firms is not fully understood. On the contrary, the first effect of a drop in the interest rate should be the reduction of interest expenses, and thus the share of zombie firms (Banerjee and Hofmann, 2018). Therefore, we investigate in a theoretical framework how the interest rate affects banks' incentives to roll over loans of non-viable firms. We model one particular zombification channel inspired by the theoretical explanation of Hu and Varas (2021). In Hu and Varas (2021), continued bank financing enhances an entrepreneur's reputation, and sufficiently reputable entrepreneurs obtain cheap market financing in the future. This creates an incentive for privately informed banks to enable zombie firms to build a false reputation of creditworthiness that allows the bank to transfer the credit to uniformed investors and obtain the full repayment itself. Evidence for the risk-shifting hypotheses of Hu and Varas (2021) is provided by Won (2023), who analyzes a rich data set of publicly traded US firms from 1993 to 2021. He shows that privately informed relationship banks allow zombie firms to build a facade of creditworthiness through generous credit conditions. After the reputation building, banks of zombie firms shift their credit exposure to non-bank investors.

We build a contract-theoretic relationship banking model to address the link between banks' incentives to roll over loans of zombie firms and the base (central bank) interest rate. An entrepreneur can choose between bank or market finance for a risky investment project of an ex ante unknown quality. The bank has higher capital costs but can identify the project's quality earlier than the market – at an interim stage. At this stage, the bank can decide whether to liquidate the project or roll over the loan. Rolling over the loan is a positive signal about the project's quality to market investors who may finance the project at the ex post stage.[4] The loan contract between the relationship bank and the entrepreneur specifies (i) the bank's initial outlay and (ii)

[3]“Zombie firms' a major drag on UK economy, analysis shows.” The Guardian, May 6, 2019, `https://www.https://www.theguardian.com/business/2019/may/06/zombie-firms-a-major-drag-on-uk-economy-analysis-shows`; last accessed on May 5, 2024.

[4]Evidence that a recent bank loan is considered a positive signal by public investors is shown by Ma *et al.* (2019). They document that a borrower who recently obtained a private loan receives more favorable terms for its public bond issuance. Similarly, Bittner *et al.* (2021) find that suppliers (falsely) interpret the bank's roll-over decision as a positive signal about the firm's creditworthiness and are willing to extend trade credits. Already James (1987) points out that bank loans are distinct from other forms of financing. He reports that the announcement of a new bank credit triggered a positive reaction in the borrower's stock prices. A classic theoretical argument based on moral hazard that an early bank loan can enhance borrower reputation, allowing them to switch to direct debt issuance, is provided by Diamond (1991).

the ex post repayment. If the entrepreneur has deep pockets, the contracted repayment induces efficient continuation, i.e., the contract maximizes the joint surplus of the bank and the entrepreneur. If, however, the entrepreneur is effectively cash constrained ex ante, a second-best loan contract with an inefficiently high repayment is signed. With the repayment being too high, some projects that should be liquidated from a welfare perspective are then continued by the bank at the interim stage: The bank engages in zombie lending.

Our contribution is twofold. First, we propose a simple and tractable contract-theoretic model in which it is in the bank's best interest to make zombie loans. The bank faces a trade-off between rent extraction and efficiency, which it resolves by making zombie loans. Second, we investigate how a change in the interest rate affects the zombie lending mechanism. Note that a decrease in the interest rate leads to cheaper financing; hence, more project qualities should be continued from a welfare point of view. First, we analyze an unanticipated change in the interest rate, i.e., analyzing the effects of an interest rate drop for a given second-best contract. In this case, the bank is incentivized to roll over even more loans, and the probability of zombie lending increases. The rough intuition is that the bank becomes more patient if the interest rate drops, and thus continuing the project and receiving the inefficiently high ex post repayment becomes more attractive. In the long run, the bank adjusts the offered loan contract to interest rate changes. In this scenario, we can show that the probability of zombie lending decreases with a drop in the interest rate. The reason lies in the market investors' increasing willingness to pay for the risky project ex ante if interest rates decrease. This, in turn, forces the bank to make a more favorable loan contract offer to the entrepreneur. As a result, the adapted loan contract specifies a lower ex post repayment which ultimately reduces the bank's incentive to roll over loans of zombie projects.

To gain a better intuition for our main findings, we extend our baseline model by allowing the three agents – the entrepreneur, the bank, and the market investors – to discount future profits at different rates. The more patient the entrepreneur and the bank are and the less patient the investors are, the more projects are continued at the interim stage. Moreover, to link our results to additional empirical findings, we incorporate the bank's capital structure and overall economic conditions in further extensions. While the relationship bank engages in zombie lending irrespective of its capital structure in our baseline model, we show that banks with lower equity share, and thus higher leverage have higher incentives to roll over bad loans. In addition, we show that the probability of zombie lending increases in the wake of an economic downturn. These findings align with empirical observations, e.g., Giannetti and Simonov (2013) and De Martiis and Peter (2021).

The paper is structured as follows. After discussing the related literature in the following paragraphs, we introduce the model in Section 2.2. In Subsection 2.2.2 we derive the first-best outcome and provide a clear definition of zombie lending. Next, we investigate the equilibrium outcome in Section 2.3, providing conditions for zombie lending to occur in equilibrium. Thereafter, in Section 2.4, we derive comparative static results concerning changes in the interest rate. In Subsection 2.4.2 we analyze the effects of an interest rate change on the bank's continuation decision for a given and fixed loan contract. In Subsection 2.4.3 we take contract adjustments into account. We discuss the extensions and robustness of our model in Sections 2.5 and 2.6, respectively. Finally, we conclude in Section 2.7. All proofs are deferred to Appendix 2.A.

Related Literature

The literature on zombie lending starts with Caballero *et al.* (2008) and Peek and Rosengren (2005), who analyze the impact of the Japanese asset price bubble in the 1990s on the banking industry. They highlight that the housing crisis and the international capitalization requirements (Basel capital standards) pressured banks into not writing off loans. The perverse bank incentives to continue lending relationships with otherwise insolvent firms resulted in a prolonged economic stagnation in Japan, featuring depressed market prices and a general misallocation of resources.[5]

Zombie lending gained renewed attention in the aftermath of the GFC and the European debt crisis. Adalet McGowan *et al.* (2018) and Banerjee and Hofmann (2018) document a high share of zombie firms in various developed economies in recent years. Several articles investigate the role of fiscal stimulus and central bank policies on the prevalence of zombification.[6] For instance, Acharya *et al.* (2021a) find that undercapitalized banks which relied heavier on support from the European Central Bank (ECB) increased their zombie lending. Relatedly, investigating the ECB's Outright Monetary Policy (OMT), Acharya *et al.* (2019) document zombie lending for banks that remained undercapitalized post OMT.[7] Closer related to our paper are the empirical contributions investigating the connection between the base interest rate and

[5] Related articles that investigate the Japanese banking sector are Hoshi (2000), Giannetti and Simonov (2013) and Kwon *et al.* (2015).

[6] The interaction of regulatory forbearance and zombie lending is investigated by Chari *et al.* (2021). Blattner *et al.* (2023) document that capital requirements affect zombie lending, especially by low-capitalized banks.

[7] Zombie lending in the aftermath of the European debt crisis is also documented by Acharya *et al.* (2024). They report that zombie lending led to excess production capacity, which in turn led to significantly higher pressure on prices, and thus lower inflation. Further empirical studies on zombie lending include Gouveia and Osterhold (2018), Andrews and Petroulakis (2019), and Jordà *et al.* (2022).

zombie lending (Borio, 2018; De Martiis and Peter, 2021; Banerjee and Hofmann, 2022; Blažková and Chmelíková, 2022). For instance, the estimates by Banerjee and Hofmann (2022, p.32) suggest that "the roughly 10 percentage point decline in nominal interest rates across advanced economies since the mid-1980s can account for around 17 percent of the rise in the zombie share [...]". Similarly, De Martiis and Peter (2021) report evidence suggesting that low short-term interest rates are favorable for zombie firms.[8]

The theoretical literature on zombie lending can be decomposed into two strands. First, the branch that models weakly capitalized banks with limited liability, which have incentives to 'gamble for resurrection' by keeping their insolvent borrowers alive (Bruche and Llobet, 2014; Acharya *et al.*, 2021b). In Bruche and Llobet (2014), banks privately learn the number of bad loans they possess at an interim stage. At that stage, the return of bad loans is uncertain, and thus banks that possess many bad loans have the incentive to hide losses and gamble for resurrection.[9] Bruche and Llobet (2014) propose a regulatory regime that induces banks to disclose their bad loans. Relying on a related explanation for zombie lending, Acharya *et al.* (2021b) build a model with heterogeneous firms and heterogeneous banks. Firms differ in productivity and risk, whereas banks differ in equity share. The model gives rise to 'diabolic sorting': poorly capitalized banks lend to firms with low productivity.[10] Acharya *et al.* (2021b) also analyze the impact of conventional (interest rate) and unconventional (forbearance) monetary policy on zombification. They point out that, in a dynamic setting, myopic policies result in low interest rates and high forbearance that keep zombies alive and productivity low. In contrast to our findings, low interest rates alone without forbearance do not promote zombie lending.

Second, and more closely related to our study, is the extant literature that relies on models of relationship banking to explain zombie lending (Hu and Varas, 2021; Aragon, 2022; Faria-e Castro *et al.*, 2024).[11] Faria-e Castro *et al.* (2024) develop a model in which relationship banks evergreen loans by offering better credit terms to less productive and more indebted firms. Different from market investors, the relationship bank owns a firm's legacy debt, and thus has the incentive to increase the continuation value of its firm. As a result, financially distressed firms receive 'discounted' credit terms from relationship banks to reduce their probability of default. It follows that

[8]In a VOXeu column, Laeven *et al.* (2000) question whether there is a clear link between low interest rates and zombification.

[9]A related model where banks have the incentive to roll over loans to hide the loan quality from the market is analyzed by Rajan (1994).

[10]Tracey (2021) proposes a further model where zombie lending helps low productivity firms to survive.

[11]According to most models, zombie lending has negative implications for the economy. An exception is Jaskowski (2015) who builds a model where zombie lending improves ex ante lending and can prevent ex post fire sales, thereby improving overall efficiency.

relationship banking leads to dispersion in firms' marginal product of capital, and thus an inefficient capital allocation. Moreover, banks' evergreening of loans leads to higher levels of debt and lower aggregate productivity. Aragon (2022) models competition for firm financing between an incumbent bank owning the firm's legacy debt and a competitor. He shows that the firm's debt overhang creates monopoly power for the incumbent bank, which may be used to extract larger rents. This rent extraction incentive by the bank may prevent the borrowing firm from investing in a new and on average profitable technology. According to Aragon (2022), zombification describes the situation where a firm that cannot fully repay its outstanding debt is kept alive but is unable to invest, and thus unable to improve its productivity.

Regarding the modeled zombification mechanism, the article closest related to our study is by Hu and Varas (2021). They consider a dynamic continuous time model where an entrepreneur initially chooses between bank finance and market finance. The bank has higher costs of capital but receives private information regarding the quality of the entrepreneur's project over time. The quality of the project is either good or bad. Once the bank (and the entrepreneur) learns that the project is bad, continued financing is costly. However, if the project is financed for sufficiently many periods by the bank, market investors believe that its quality is high, and are thus willing to pay a high price for it.[12] This creates an incentive for the bank to continue projects that turn out to be of bad quality at interim points in time. These projects are sold later to market investors, who are 'deceived' by the roll-over decision. While in Hu and Varas (2021) good projects should always obtain financing and bad ones should always be liquidated, the welfare optimal quality threshold is endogenous in our model. In other words, it is optimal to liquidate fewer projects if interest rates are low. Moreover, the implications of interest rate changes on a bank's incentive to engage in zombie lending are not at the heart of Hu and Varas (2021).

2.2 The Model

2.2.1 Players & Timing

We consider an economy over three dates $t = 0, 1, 2$. There are three types of risk-neutral agents: an entrepreneur (she), a relationship bank, and investors. We denote all variables in terms of their respective date $t = 2$ future values.[13]

[12]Somewhat related, Puri (1999) builds a model where the bank's decision at an intermediate stage affects investor evaluations of securities the bank underwrites. In her model, investors may effectively repay the firm's bank loan.

[13]From Section 2.4 onward, we explicitly express the variables' dependence on the interest rate. As an example, suppose the interest rate is $r \geq 0$ and the project requires at $t = 0$ an initial investment

At $t = 0$, the entrepreneur owns a risky business project of ex ante unknown quality θ. The project requires an initial investment of $I > 0$ at $t = 0$. If the project is initiated at $t = 0$, then it generates a payoff of $\gamma\theta$, with $\gamma > 0$, at the end of date $t = 1$, and a payoff of θ at date $t = 2$. The project quality is distributed according to c.d.f. $F(\theta)$ and density $f(\theta) > 0$ on $[\underline{\theta}, \bar{\theta}]$. The expected quality

$$\mu := \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta) \, \mathrm{d}\theta > 0 \tag{2.1}$$

is assumed to be strictly positive. The entrepreneur's initial wealth is $w \geq 0$. We assume that $w < I$ so that the entrepreneur requires external finance to implement her business project. The entrepreneur can sign a loan contract with the bank or borrow from (sell the project to) investors. She can also decide not to implement the business project.

At $t = 0$ the bank can make a take-it-or-leave-it loan contract (d, R) offer to the entrepreneur. The bank finances $I - d$ of the project, and the entrepreneur invests equity capital d. The contract also specifies the gross repayment R from the entrepreneur to the bank at $t = 2$. For ease of exposition, we assume that the contract transfers the date $t = 1$ cash flow and control rights to the bank (instead of specifying next to the final repayment also an interim repayment).[14] At $t = 1$, the bank has the cost of $c > 0$ for engaging in this relationship lending, which can be interpreted as monitoring costs. Due to this monitoring, the bank learns the quality of the project θ at the beginning of date $t = 1$. The bank then decides whether to continue the project or liquidate it. In case of liquidation, the project pays a liquidation value $L > 0$ at the end of date $t = 1$. This liquidation value L is independent of the project's quality θ. A continued project generates a return of $\gamma\theta$ at the end of date $t = 1$ and of θ at date $t = 2$. Finally, the parties commit at $t = 0$ to terminate the relationship at the beginning of $t = 2$ and to sell the project to investors. In other words, the project sell-off to the investors, and thus R, is made before the return θ is realized. Let $L < (1 + \gamma)\mu$.

There is a large group of investors that act in a perfectly competitive financial market. Investors can either purchase (finance) the project at a price P_0 at date $t = 0$ or at a price P_2 at the beginning of date $t = 2$.[15] If investors purchase the project at

of $\tilde{I}$. The date $t = 2$ future value of this investment is $I = (1 + r)^2 \tilde{I}$.

[14] The assumption that the bank obtains the project's full return and control rights at $t = 1$ is not crucial for our results. In Section 2.6.1, we consider the case where the loan contract specifies repayments in $t = 1$ and $t = 2$, and the entrepreneur keeps the cash flow and control rights (as long as she can make the repayment). The results are qualitatively identical.

[15] With all parties being risk-neutral, the assumption that investors purchase the whole project at $t = 0$ is without loss in generality. To see this, suppose the entrepreneur sells shares α of her project to investors to finance $I - w$. The lowest share that investors are willing to accept is $\hat{\alpha} =$

date $t = 0$, they learn the project's quality only indirectly at the end of date $t = 1$ where it pays out $\gamma\theta$. At this point, it is no longer possible to liquidate the project in $t = 1$ (and there is no liquidation opportunity in $t = 2$). Thus, the disadvantage of market finance compared to bank finance is that projects with low returns can not be terminated at the intermediate date $t = 1$. The advantage of market finance is that the market does not have any costs. If investors purchase the project at the beginning of date $t = 2$, they pay a price P_2 to the entrepreneur and receive the return θ at the end of date $t = 2$. Importantly, if the project is initially financed via the bank, there is asymmetric information at date $t = 2$ between the bank/entrepreneur and investors. The investors do not know the quality of the project. However, they do observe the signed loan contract and correctly understand the bank's incentives to continue projects at date $t = 1$, and thus update their belief regarding the offered project's quality accordingly.

The timeline of our model, in particular the project's investment and returns at the three dates, are depicted in Figure 2.1.

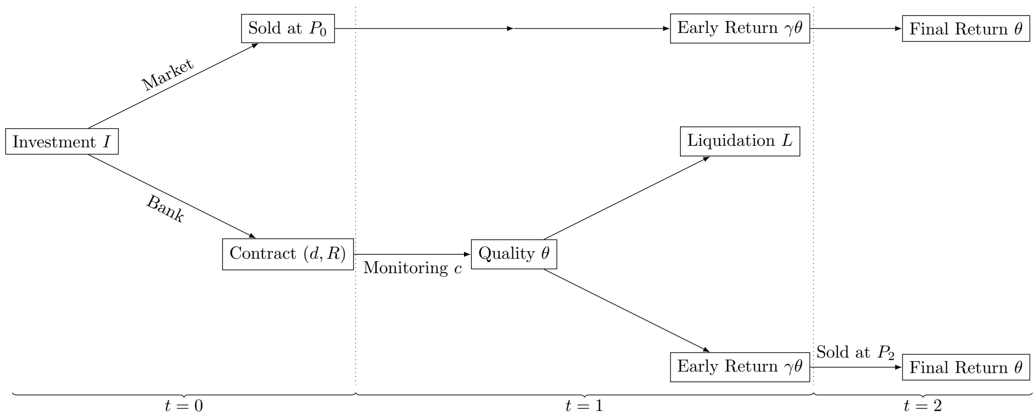

Figure 2.1: Timeline of the project's investment, liquidation, and returns.

A few remarks regarding the optimality of the allowed contracts are in order. The loan contracts we are analyzing are optimal under two conditions. First, the parties – bank and entrepreneur – can commit at $t = 0$ to sell the project to investors at the beginning of $t = 2$. This is efficient as the bank has higher operating costs ($c > 0$) than the market, and once θ is learned, there is no benefit from bank monitoring. Ex

$(I - w)/[(1 + \gamma)\mu]$. The expected profit of the entrepreneur from selling share $\hat{\alpha}$ of the project is $\mathbb{E}[-w + (1 - \hat{\alpha})\gamma\theta + (1 - \hat{\alpha})\theta] = (1 + \gamma)\mu - I$. Moreover, note that risk-neutral investors could also finance the project at the beginning of date $t = 1$. This, however, will never happen in equilibrium because the monitoring cost is sunk at the beginning of $t = 1$ but the liquidation decision (usage of the information) is not yet made.

post, at $t = 2$, the entrepreneur prefers to sell only 'bad' projects and to keep 'good' ones since the return $\theta - R$ accrues to the entrepreneur. The bank, however, does not benefit from not selling 'good' projects to the market. It obtains at most the repayment R. The bank suffers if only bad projects are sold to market investors who anticipate this adverse selection, and thus $R > P_2$. Hence, it is in the bank's interest to have a reputation for sticking to the original contract – selling continued projects to the market at $t = 2$ – instead of engaging in renegotiation with the entrepreneur.

Second, the parties cannot commit to a certain roll-over decision, i.e., it is not feasible to specify a critical quality threshold $\hat{\theta}$ directly in the contract. The idea is that at the beginning of $t = 1$, the quality is privately observed by the bank (and the entrepreneur). An outsider, say a court, cannot observe and verify θ. Thus, the roll-over decision can only be incentivized indirectly via payments to the bank at dates $t = 1$ and $t = 2$.

Finally, note that there is no scope for signaling the project quality by financing parts with own funds in the spirit of Leland and Pyle (1977). Ex ante, at $t = 0$, there is symmetric information between all three types of agents. Ex post, at $t = 2$, the project is always sold to the market, and thus there is no inefficiency arising from adverse selection that can be mitigated by signaling (via selling only shares of the project to the market).

2.2.2 First-best Benchmark and Definition of Zombie Lending

In the case of market finance via investors, information is only revealed at the end of date $t = 1$, implying that early liquidation is not optimal. Thus, the expected surplus generated by market finance at $t = 0$ is

$$(1 + \gamma)\mu - I. \tag{2.2}$$

In the case of bank finance, the project's quality is observed at the beginning of date $t = 1$. This allows liquidating low-quality projects at date $t = 1$. The continuation of a project is efficient at $t = 1$ if the project's total return is higher than the liquidation value, i.e., if $\gamma\theta + \theta \geq L$. This inequality is equivalent to

$$\theta \geq \frac{L}{1 + \gamma} =: \theta^*. \tag{2.3}$$

We call θ^* the efficient quality threshold. The efficient quality threshold θ^* is increasing in the liquidation value L and decreasing in the $t = 1$ share of the project's return γ.

The expected surplus generated by efficient bank financing is

$$\int_{\underline{\theta}}^{\bar{\theta}} \max\{(1+\gamma)\theta, L\} f(\theta)\,\mathrm{d}\theta - c - I. \tag{2.4}$$

The following result summarizes the first-best outcome.

Observation 1 (First-best Finance)**.** *In the first-best situation, the project is*

(i) financed by the bank if $c \leq \bar{c}^{FB}$*;*

(ii) financed by investors (financial market) if $c > \bar{c}^{FB}$ *and* $I \leq (1+\gamma)\mu$*;*

(iii) not financed in all remaining cases.

The threshold value for the monitoring cost is

$$\bar{c}^{FB} := \begin{cases} \int_{\underline{\theta}}^{\bar{\theta}} \max\{(1+\gamma)\theta, L\} f(\theta)\,\mathrm{d}\theta - (1+\gamma)\mu & \text{for } I \leq (1+\gamma)\mu, \\ \int_{\underline{\theta}}^{\bar{\theta}} \max\{(1+\gamma)\theta, L\} f(\theta)\,\mathrm{d}\theta - I & \text{for } I > (1+\gamma)\mu. \end{cases} \tag{2.5}$$

Note that $\bar{c}^{FB} > 0$ for $I < (1+\gamma)\mu$. Having characterized the first-best outcome, and in particular the first-best continuation decision of the bank, we are now in the position to define zombie lending.

Definition 1 (Zombie Lending)**.** *If at date* $t = 1$ *the bank continues a project (rolls over the credit) of quality less than the efficient threshold,* $\theta < \theta^*$*, we define this as zombie lending.*

According to our definition, zombie lending occurs if a project is not liquidated even though liquidation maximizes the generated surplus.

2.3 Financing Analysis

2.3.1 Bank's Optimization Problem

Suppose the bank and the entrepreneur can sign a contract that is profitable for both parties. The loan contract (d, R) offered by the bank maximizes its expected profit

$$\pi_B(d, R) = F(\hat{\theta}(R))L + \gamma \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta + [1 - F(\hat{\theta}(R))]R - c - I + d \tag{2.6}$$

subject to

$$\pi_E(d,R) \geq \max\{P_0, 0\}, \tag{PC}$$

$$\hat{\theta}(R) = \frac{1}{\gamma}(L-R), \tag{RD}$$

$$d \leq w \tag{LL}$$

where

$$\pi_E(d,R) = -d + [1 - F(\hat{\theta}(R))][P_2(\hat{\theta}(R)) - R]. \tag{2.7}$$

denotes the entrepreneur's net expected profit.

The offer made by the bank is accepted by the entrepreneur only if the participation constraint (PC) is satisfied. Recall that there is a large number of risk-neutral investors. At date $t = 0$, these investors are willing to pay

$$P_0 := (1+\gamma)\mu - I \tag{2.8}$$

for a project of unknown quality. At $t = 2$ investors update their quality expectation taking the bank's roll-over decision into account, and are thus willing to pay

$$P_2(\hat{\theta}(R)) = \mathbb{E}[\theta \mid \theta \geq \hat{\theta}(R)] = \frac{1}{1 - F(\hat{\theta}(R))} \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta. \tag{2.9}$$

Moreover, the bank considers how the signed loan contract (d, R) affects its own roll-over decision, constraint (RD). The bank rolls over the loan at $t = 1$ if and only if

$$\gamma\theta + \min\{R, P_2 + w - d\} \geq L. \tag{2.10}$$

In case of roll-over, the bank obtains $\gamma\theta$ at the end of date $t = 1$ and the repayment R at date $t = 2$. If, however, the entrepreneur cannot repay R, then the entrepreneur is bankrupt and the bank obtains her remaining capital, $P_2 + w - d$. Note that it can not be optimal to set a repayment so high that the entrepreneur will never be able to pay it. Thus, without loss of generality, we can focus on $\min\{R, P_2 + w - d\} = R$, and the bank continues all projects with qualities $\theta \geq \hat{\theta}(R)$.

Finally, the entrepreneur's initial outlay cannot exceed her wealth, i.e., the limited liability constraint (LL) must hold.

2.3.2 Optimal Loan Contract

Note that the amount initially invested by the entrepreneur herself, d, is an ex ante one-to-one transfer between the bank and the entrepreneur. A higher d increases the bank's expected profit, does not affect the bank's roll-over decision, and the joint surplus of the bank and entrepreneur is independent of d. Thus, if constraint (LL) is slack, the bank has the incentive to offer a loan contract that maximizes the rents generated by bank finance. These rents are maximized if and only if the roll-over decision is efficient. The bank makes an efficient roll-over decision if and only if $\hat{\theta}(R) = \theta^*$. This is achieved for the repayment

$$R^* = \frac{L}{1+\gamma} = \theta^*. \tag{2.11}$$

Let d^* be the entrepreneur's initial outlay that satisfies the participation constraint with equality for $R = R^*$, implicitly given by $\pi_E(d^*, R^*) = \max\{(1+\gamma)\mu - I, 0\}$. Given that the entrepreneur's initial outlay can not exceed her wealth, $d \leq w$, the first-best loan contract (d^*, R^*) is feasible, and thus offered if $d^* \leq w$.

Proposition 1 (First-best Contract)**.** *Suppose bank lending is efficient. Then, the loan contract (d, R) offered by the bank induces the efficient roll-over decision at $t = 1$ if*

$$w \geq \int_{\theta^*}^{\bar{\theta}} [\theta - \theta^*] f(\theta)\, \mathrm{d}\theta - \max\{P_0, 0\} =: d^*. \tag{2.12}$$

The loan contract specifies

$$d = d^* \text{ and } R = R^* = \theta^*. \tag{2.13}$$

If the entrepreneur does not have sufficiently deep pockets, $w < d^*$, the bank cannot extract the full additional surplus that is generated by efficient bank lending. The bank will specify the highest feasible initial outlay by the entrepreneur, i.e., $d = w$. In this case, the bank faces a trade-off between rent extraction and efficiency. The bank can increase its expected profit by increasing the repayment R above the efficient level $R^* = \theta^*$. This, however, distorts the continuation decision at date $t = 1$. The bank continues a project if the quality θ is above $\hat{\theta}(R) = \gamma^{-1}(L - R)$, with $d\hat{\theta}/dR = -\gamma^{-1} < 0$. Note that for R^* it holds that $\hat{\theta}(R^*) = \theta^*$. Thus, for $R > R^*$ it holds that $\hat{\theta} < \theta^*$. The financial market anticipates the bank's lenient roll-over decision, and thus reduces its willingness to pay for the project at date $t = 2$. There is a maximum feasible repayment $\bar{R}$, implicitly defined by

$$\mathbb{E}[\theta \mid \theta \geq \hat{\theta}(\bar{R})] = \bar{R}. \tag{2.14}$$

Note that $\bar{R} > R^*$. Inserting $d = w$ and $R = \bar{R}$ into the entrepreneur's expected profit

(2.7) yields $\pi_E = -w$. All repayments $R > \bar{R}$ violate the participation constraint (PC). The expected profit of the bank $\pi_B(d = w, R)$ is strictly increasing in the repayment $R \leq \bar{R}$ with

$$\frac{\partial \pi_B}{\partial R} = 1 - F(\hat{\theta}) > 0. \tag{2.15}$$

This implies that the bank specifies the highest repayment that the entrepreneur is just willing to accept, i.e., the repayment that makes the entrepreneur indifferent between the offered bank loan and her best alternative option.

Proposition 2 (Second-best Contract)**.** *Suppose $w < d^*$ and that the bank can make a profitable offer that is accepted by the entrepreneur. Then, the bank offers the second-best optimal loan contract (d^{SB}, R^{SB}), with $d^{SB} = w$ and $R^{SB} \in (\theta^*, \bar{R}]$ implicitly defined by $\pi_E(d^{SB}, R^{SB}) = \max\{P_0, 0\}$.*

If the entrepreneur is effectively cash constrained but bank finance nevertheless occurs in equilibrium, then a loan contract is signed with a too high repayment $R^{SB} > R^*$ from an efficiency point of view. Thus, the bank rolls over projects with quality below the efficient quality threshold θ^*. In other words, the bank engages in zombie lending, depicted in Figure 2.2.

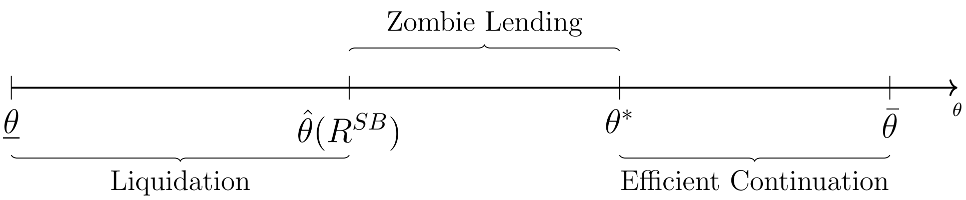

Figure 2.2: The bank's decision at date $t = 1$ under a second-best contract.

Corollary 1. *Under the second-best loan contract (d^{SB}, R^{SB}) zombie lending takes place for projects of quality $\theta \in [\hat{\theta}(R^{SB}), \theta^*)$.*

This is a very important observation: In case the entrepreneur is effectively cash-constrained, $w < d^*$, there is scope for (inefficient) zombie lending. The parameters for which zombification occurs in equilibrium are analyzed in the next section.

2.3.3 Equilibrium Finance

Now, we analyze which form of financing occurs in equilibrium. In particular, we investigate the conditions in which the entrepreneur and the bank sign the second-best loan contract in equilibrium. We depict the findings in Figure 2.3: the horizontal axis scales the investment I and the vertical axis the monitoring cost c.

On the one hand, market finance is only feasible if the initial investment is not too high,

$$I \leq (1+\gamma)\mu. \tag{2.16}$$

On the other hand, first-best bank financing leads to a higher expected surplus than market financing if the monitoring cost is rather low, $c \leq \bar{c}^{FB}$ (see Observation 1). The bank offers the first-best contract (d^*, R^*) only if the entrepreneur possesses sufficient initial wealth, i.e., if $w \geq d^*$. For $I \leq (1+\gamma)\mu$, and thus $P_0 \geq 0$, the condition $w \geq d^*$ is equivalent to

$$I \leq (1+\gamma)\mu + w - \int_{\theta^*}^{\bar{\theta}} [\theta - \theta^*] f(\theta)\,\mathrm{d}\theta =: \bar{I}^{FB}. \tag{2.17}$$

For projects with low initial financing volume, $I \leq \bar{I}^{FB}$ (and $c < \bar{c}^{FB}$), the bank offers the first-best contract. In case of higher required initial investments, the bank either offers the second-best contract or no contract.

A priori, it is not clear whether the critical threshold $\bar{I}^{FB}$ is smaller or larger than $(1+\gamma)\mu$. In the following, we focus on the former case, which applies if the entrepreneur's initial wealth is not too large. In this regard, we impose

Assumption 1. *The entrepreneur's initial wealth is lower than the expected surplus generated by efficient continuation:*

$$w < \int_{\theta^*}^{\bar{\theta}} [\theta - \theta^*] f(\theta)\,\mathrm{d}\theta. \tag{2.18}$$

The bank offers the second-best contract, where $d^{SB} = w$ and R^{SB} is determined by the participation constraint, only if its profit $\pi_B(w, R^{SB})$ from the contract is non-negative. The second-best repayment is determined by $\pi_E(w, R^{SB}) = \max\{(1+\gamma)\mu - I, 0\}$, and thus is a function of the initial investment I but is independent of the monitoring cost c. Formally, $R^{SB} = R^{SB}(I)$. The expected profit of the bank from offering contract $(d^{SB} = w, R^{SB}(I))$ is non-negative if and only if $c \leq \bar{c}^{SB}(I)$, where

$$\bar{c}^{SB}(I) \equiv F(\hat{\theta}(R^{SB}(I)))L + \gamma \int_{\hat{\theta}(R^{SB}(I))}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta + [1 - F(\hat{\theta}(R^{SB}(I)))]R^{SB}(I) - I + w. \tag{2.19}$$

Equation (2.19) defines the cost threshold as a function of the initial investment I.

Importantly, for $I \searrow \bar{I}^{FB}$ it holds that $\bar{c}^{SB}(I) \to \bar{c}^{FB}$.[16] The critical threshold of the monitoring cost $\bar{c}^{SB}(I)$ is a strictly decreasing function in I. For large initial investments I, the threshold $\bar{c}^{SB}$ is negative which implies that second-best bank finance is not profitable.

The equilibrium contracts are summarized in the following proposition.

Proposition 3 (Equilibrium Finance)**.** *Suppose that Assumption 1 holds. Then, the date $t = 0$ equilibrium decision of the entrepreneur is*

(i) market finance if and only if

$$c \geq \begin{cases} \bar{c}^{FB} & \text{for } I \leq \bar{I}^{FB}, \\ \bar{c}^{SB}(I) & \text{for } I \in (I^{FB}, (1+\gamma)\mu]; \end{cases} \tag{2.20}$$

(ii) bank finance if and only if

$$c < \begin{cases} \bar{c}^{FB} & \text{for } I \leq \bar{I}^{FB}, \\ \bar{c}^{SB}(I) & \text{for } I > I^{FB}; \end{cases} \tag{2.21}$$

(iii) no finance in all other cases.

As shown in Figure 2.3, the project is not financed at all if the initial investment is too large. A project with a low or moderately high initial investment is financed in equilibrium. Such a project is financed by the financial market (sold to investors at date $t = 0$) if the bank's monitoring cost is high, otherwise, it is initially financed with a bank loan. The bank offers the first-best contract if the initial investment is low, $I \leq \bar{I}^{FB}$. In this case, bank finance is efficient. For moderately high initial investments, $I \in (\bar{I}^{FB}, (1+\gamma)\mu]$, and low monitoring cost, $c \leq \bar{c}^{SB}$, the bank offers the second-best contract. In this case, first-best bank lending is efficient but the equilibrium outcome is second-best bank lending with a distorted continuation decision. Moreover, for $I \in (\bar{I}^{FB}, (1+\gamma)\mu]$ and $c \in (\bar{c}^{SB}, \bar{c}^{FB})$ first-best bank lending is efficient but in equilibrium, the project is financed by the financial market. Finally, for some projects with $I > (1+\gamma)\mu$ the efficient outcome is bank finance. In equilibrium, however, these projects are either not financed at all or with a second-best loan contract offered by the bank.

In summary, three distortions may arise in equilibrium: First, a project with a strictly positive expected net return from efficient bank lending is not financed in

[16] To see this formally, note that for $I = \bar{I}^{FB}$ we have $R^{SB} = R^* = \theta^*$ and $\hat{\theta} = \theta^*$. Solving $\pi_E(d = w, R^{SB}) = \max\{P_0, 0\}$ for w and inserting this into (2.19) yields the result.

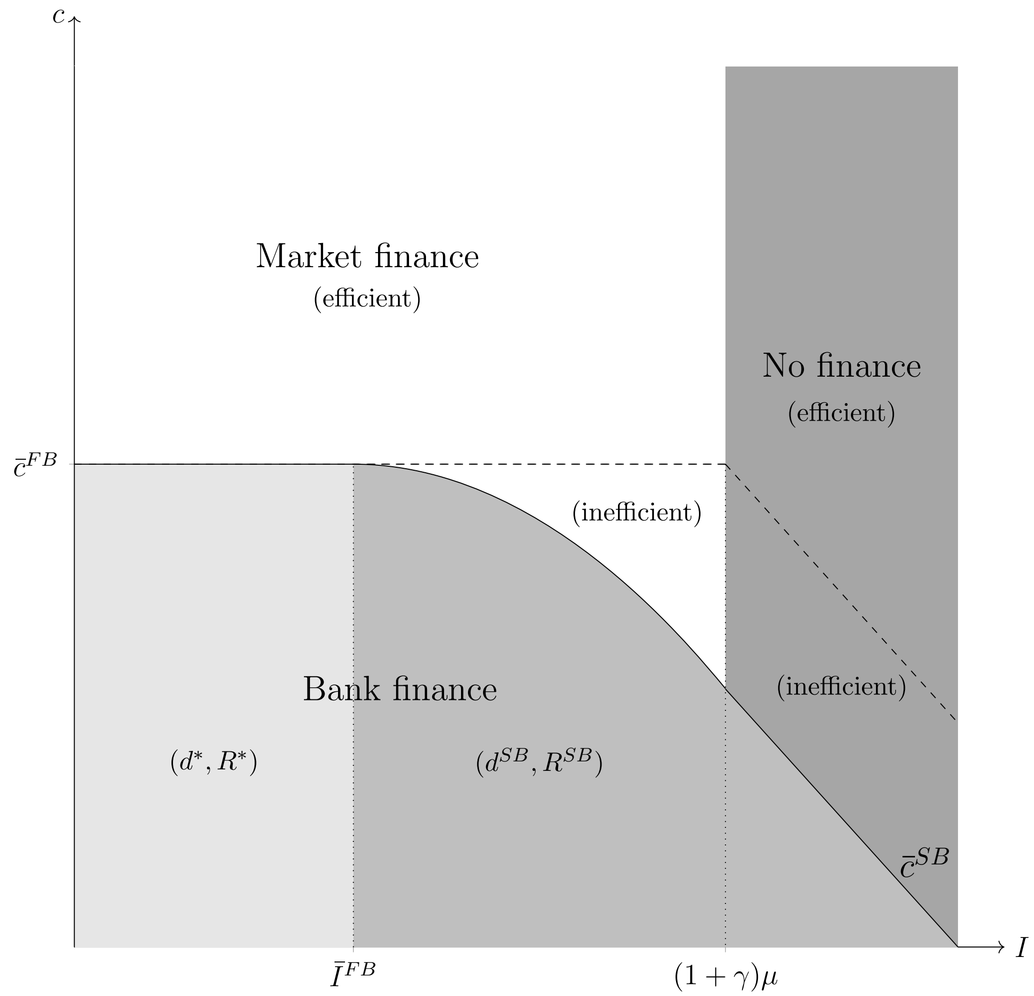

Figure 2.3: Equilibrium finance and efficiency.

equilibrium (credit crunch). Second, a project that – from a welfare perspective – should be financed by the bank, is financed by investors in equilibrium (inefficient financing form). Third, a project is financed via the second-best loan contract rather than efficient bank lending, which creates incentives for zombie lending.

The focus of our paper is on the third inefficiency, zombie lending. The following result summarizes the conditions under which zombie lending – inefficient roll-over decisions – occurs in equilibrium.

Corollary 2 (Zombie Lending)**.** *Suppose that Assumption 1 holds. In equilibrium, the bank and the entrepreneur sign a second-best loan contract* (d^{SB}, R^{SB}) *if and only if* $I > \bar{I}^{FB}$ *and* $c < \bar{c}^{SB}(I)$*. In this case, the bank engages in the zombification of projects of quality* $\theta \in [\hat{\theta}(R^{SB}), \theta^*)$*.*

2.4 Interest Rates and Zombification

2.4.1 Research Question and Notation

Besides explaining the occurrence of zombie lending, we are particularly interested in how a change in the interest rate affects zombie lending. We assume that all agents – the entrepreneur, the bank, and the investors – discount future payments based on an identical interest rate $r \geq 0$. This interest rate can be interpreted as being determined, albeit only indirectly, by the policy of a central bank.[17]

As explained in Section 2.2, all variables can be interpreted as the date $t = 2$ future value of the respective variable. We denote the actual numerical value of each variable with a tilde. Thus, we can introduce the following variable transformation:

$$\begin{aligned} \gamma &= (1+r)\tilde{\gamma}, & c &= (1+r)\tilde{c}, \\ L &= (1+r)\tilde{L}, & I &= (1+r)^2\tilde{I}, \\ w &= (1+r)^2\tilde{w}, & d &= (1+r)^2\tilde{d}. \end{aligned}$$

Note that variables occurring at date $t = 2$ need no transformation, e.g., the repayment still denotes R.

We are interested in how a change in the interest rate affects a bank's decision to roll over credit. Therefore, we focus on the financing scenario where the entrepreneur and the bank sign a second-best loan contract (d^{SB}, R^{SB}).

The efficient roll-over quality threshold is

$$\theta^*(r) = \frac{(1+r)\tilde{L}}{1+(1+r)\tilde{\gamma}}. \tag{2.22}$$

A change in the interest rate affects the efficient quality threshold as follows:

$$\frac{d\theta^*}{dr} = \frac{\tilde{L}}{[1+(1+r)\tilde{\gamma}]^2} > 0. \tag{2.23}$$

Thus, if the interest rate decreases, it is welfare optimal to roll over more loans. This is intuitive because a lower interest rate makes the date $t = 2$ project return θ relatively more important than the date $t = 1$ project liquidation value $\tilde{L}$. In other words, the continuation decision is cheaper if the interest rate decreases. Therefore, the first effect of a drop in the interest rate in our model is the reduction of zombie lending as it

[17] Investigating the optimal central bank policy is outside the scope of this paper. The central bank may induce an interest rate that seems inefficient from our model's point of view because it takes other reasons that are not modeled here into account.

becomes efficient to continue more projects.

Under a second-best loan contract, the bank rolls over all loans of quality θ weakly larger than

$$\hat{\theta}(r, R^{SB}) = \frac{(1+r)\tilde{L} - R^{SB}}{(1+r)\tilde{\gamma}}. \tag{2.24}$$

In the following, we consider two scenarios. First, we investigate the effects of changes in the interest rate for a given loan contract (short-run analysis). Thereafter, we take the impact of a change in the interest rate on the offered contract into account.

2.4.2 Short-run Effects of Interest Rate Changes

As a first step, we investigate the effect of an adjustment in the interest rate r on the probability of zombie lending

$$Z(r) = Prob(\theta \in [\hat{\theta}, \theta^*)), \tag{2.25}$$

for a given second-best loan contract (d^{SB}, R^{SB}). This effect can be interpreted as the effect of an unanticipated change in the interest rate. Namely, the entrepreneur and the bank signed a second-best loan contract at date $t = 0$. At the beginning of date $t = 1$, the interest rate changes, and this change was not expected by the bank or the entrepreneur. Thus, at date $t = 1$, the contract is given, but the bank can adjust its roll-over decision. If the interest rate increases, the bank applies a stricter roll-over rule, i.e.,

$$\frac{\partial \hat{\theta}}{\partial r} = \frac{R^{SB}}{\tilde{\gamma}(1+r)^2} > 0. \tag{2.26}$$

The intuition is analog to the efficient threshold argument. To obtain a clear-cut finding in this section, we assume the following:

Assumption 2. *For all $\theta \in [\underline{\theta}, \bar{\theta}]$ it holds that $f'(\theta) \leq 0$.*

According to Assumption 2, projects of higher quality are less likely, i.e., 'unicorns' are rare. We are then able to make the following proposition.

Proposition 4. *Suppose that Assumption 2 holds and that the entrepreneur and the bank signed a second-best loan contract. Then, an unanticipated reduction in the interest rate increases the probability of zombie lending, i.e.,*

$$Z(r) = \int_{\hat{\theta}(r, R^{SB})}^{\theta^*(r)} f(\theta)\, \mathrm{d}\theta \tag{2.27}$$

is strictly decreasing in r.

Proposition 4 states that if the entrepreneur and the bank engage in a long-term lending relationship and during this relationship the interest rate drops unexpectedly, then the bank rolls over even more loans compared to the efficient continuation decision.

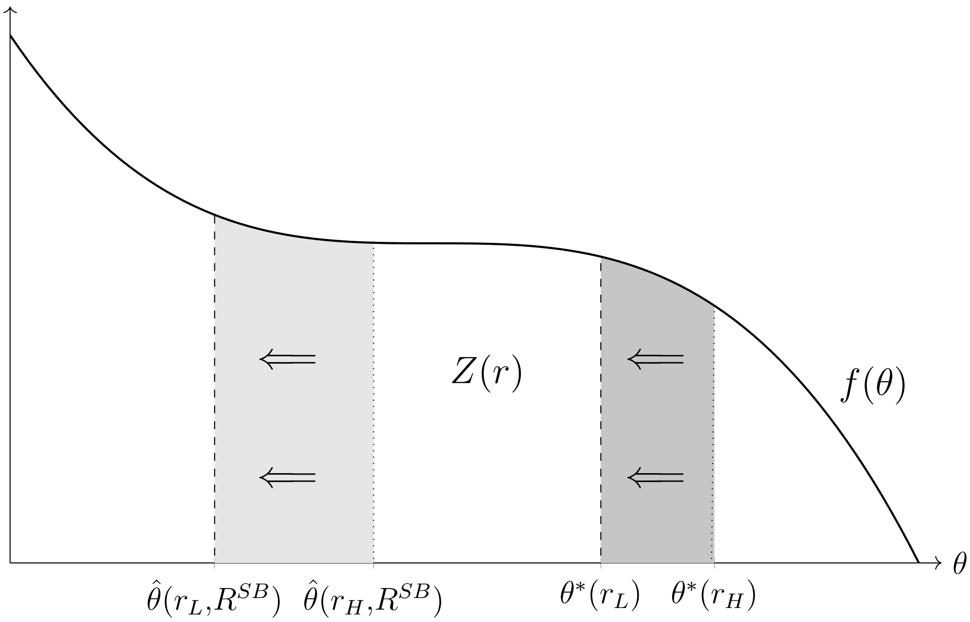

Figure 2.4: The bank's adjusted roll-over decision for an unexpected drop in interest rates from r_H to $r_L < r_H$.

As highlighted in Figure 2.4, the probability of zombie lending $Z(r)$ increases with decreasing interest rates r for any density function $f(\theta)$, with $f'(\theta) \leq 0$. Note that any drop (rise) in the interest rate r increases (decreases) the zombie lending interval, $\theta \in [\hat{\theta}(r), \theta^*(r))$. Specifically, the mass of qualities θ in the interval of $\hat{\theta}(r_L, R^{SB})$ and $\hat{\theta}(r_H, R^{SB})$ is strictly larger than the corresponding mass in the interval of $\theta^*(r_L)$ and $\theta^*(r_H)$, for $r_H > r_L$. Conveying the result to the real world, this scenario may very well resemble many lending relationships between commercial banks and companies following the financial crisis in the EU, i.e., in the early 2010s. Thus, according to our theory, the – to some degree – unexpectedly continued loose monetary policy of the ECB after the financial crisis may have augmented the problem of zombie lending in the euro area.

Proposition 4 also has implications regarding the probability of zombie lending under a formerly first-best contract. Under the first-best contract, the repayment is $R^*(r) = \theta^*(r)$ so that the bank applies the efficient quality threshold $\hat{\theta}(r, R^*(r)) = \theta^*(r)$. Now, suppose the interest rate drops from r_H to $r_L < r_H$. This decreases the first-best threshold from $\theta^*(r_H)$ to $\theta^*(r_L)$. Given that the interest rate drop was unexpected, the repayment stays at $R^*(r_H)$ while the bank applies the quality threshold $\hat{\theta}(r_L, R^*(r_H))$. It can readily be shown that $\hat{\theta}(r_L, R^*(r_H)) < \theta^*(r_L)$, and thus zombie lending occurs for qualities $\theta \in [\hat{\theta}, \theta^*)$. In other words, an unanticipated drop in the interest rate also increases the scope for zombie lending under the formerly first-best loan contract (d^*, R^*).[18]

2.4.3 Long-run Effects of Interest Rate Changes

In this section, we assume that the interest rate changes before the parties sign a loan contract. We remain in the scenario where the entrepreneur and the bank sign a second-best loan contract. We investigate how this loan contract adapts to a change in the interest rate. In particular, we are interested in how the repayment $R^{SB} = R^{SB}(r)$ adjusts and how this affects the bank's roll-over decision at $t = 1$. Under the second-best contract, the amount financed by the entrepreneur $\tilde{d}$ equals her initial wealth $\tilde{w}$, thus not depending on the interest rate r.

The efficient quality threshold θ^* depends on the interest rate r only directly, and thus the long-run effect is equal to the short-run effect. The quality threshold applied by the bank, $\hat{\theta}(r, R^{SB}(r))$, on the other hand, is not only directly a function of the interest rate r but also indirectly via the repayment $R^{SB}(r)$. The total change of this threshold is

$$\frac{d\hat{\theta}}{dr} = \frac{\partial\hat{\theta}}{\partial r} + \frac{\partial\hat{\theta}}{\partial R^{SB}} \frac{dR^{SB}}{dr}. \tag{2.28}$$

We know that $\partial\hat{\theta}/\partial r > 0$ and that $\partial\hat{\theta}/\partial R^{SB} < 0$. Thus, if the repayment R^{SB} is increasing in the interest rate, the long-run effect of an interest rate change on the likelihood of zombie lending is weaker than the short-run effect. An interest rate change affects the considerations of all three agents, the entrepreneur, the bank, and the investors. An increase in the interest rate makes the entrepreneur less patient, and thus selling the project at $t = 0$ to investors becomes more attractive. Therefore, to make the entrepreneur accept the bank loan, the repayment needs to be lower. On the other hand, an increase in the interest rate decreases the expected net present value of the project, and thus reduces investors' willingness to pay at $t = 0$. This allows the bank to demand a higher repayment. Finally, for a higher interest rate, the bank has the incentive to liquidate more projects at $t = 1$. The higher interest rate not only decreases the probability of the entrepreneur profitably selling the project at $t = 2$ but also, in case of a sale, leads to a higher project price P_2. A sufficient (but not

[18]Our results apply to empirically documented zombie firms that borrow from banks at a fixed interest rate (fixed repayment R). Göbel and Tavares (2022) find that zombie firms – compared to their non-zombie counterparts – rely more heavily on bank loans with fixed interest rates than on revolving loan facilities that typically have interest rates of a variable line.

necessary) condition for $dR^{SB}/dr > 0$ is that a rise in the interest rate r increases – ceteris paribus – the advantage of bank finance over market finance.[19] In other words, the possibility of early liquidation is particularly valuable if interest rates are high. To obtain an unambiguous result, we, therefore, impose the following simple sufficient condition:

Assumption 3. *The quality of a project is non-negative, i.e.,* $\underline{\theta} \geq 0$.

According to Assumption 3, no project in itself makes negative returns. Note, however, that $\underline{\theta} \geq 0$ does not exclude projects having a negative net present value at $t = 0$ nor liquidation being the efficient decision at $t = 1$. We can then make the following proposition.

Proposition 5. *Suppose that Assumption 3 holds and that* $P_0 = [1 + (1 + r)\tilde{\gamma}]\mu - (1 + r)^2\tilde{I} > 0$. *Then,*

(i) the repayment of the second-best contract R^{SB} *is strictly increasing in the interest rate* r*;*

(ii) under the second-best loan contract, the probability of zombie lending is strictly increasing in the interest rate; i.e.,

$$Z(r) = \int_{\hat{\theta}(r, R^{SB}(r))}^{\theta^*(r)} f(\theta)\,\mathrm{d}\theta \qquad (2.29)$$

is strictly increasing in r.

According to Proposition 5, an anticipated drop (rise) in the interest rate decreases (increases) the probability of zombie lending. As the proof reveals, the bank's quality threshold $\hat{\theta}$ is decreasing in the interest rate. Thus, apparent from (2.28), the indirect effect of contract adaption on the bank's quality threshold must outweigh the direct effect. While this result may be surprising at first, the rough intuition of the finding can be argued as follows: An increase in the interest rate makes risk-neutral investors less willing to pay for the entrepreneur's project at date $t = 0$, and thus P_0 becomes smaller. In return, the bank adapts the loan contract by demanding a higher repayment R^{SB} from the entrepreneur (participation constraint) ex ante. This higher repayment

[19] The expected advantage of bank finance over market finance in terms of $t = 1$ values is

$$\psi(r, \hat{\theta}) = F(\hat{\theta})\tilde{L} + \left(\tilde{\gamma} + \frac{1}{1+r}\right)\left[\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - \mu\right].$$

Note that $\partial\psi/\partial r > 0$ if and only if $\int_{\underline{\theta}}^{\hat{\theta}} \theta f(\theta)\,\mathrm{d}\theta > 0$.

ultimately leads to a higher incentive for the bank to continue projects at date $t = 1$, and thus zombie lending increases. We investigate the channels behind this finding in more detail in Section 2.5.1, where we allow for different interest rates for the three types of agents.

In summary, we find that a mere drop in interest rate does not cause long-run zombification, but in fact has a diminishing effect. Translating our result to the real world, low interest rate environments may lead to increased zombie lending within relationship banking in the short-run but not in the long-run. In other words, if interest rates are low in a monetary area for a prolonged period, the economy is not at risk of being crowded by zombie firms. Evidence in line with our result is, for instance, Beer *et al.* (2021) who analyze zombie shares in Austria. They report an especially pronounced decline in the zombie share in the years 2015 until 2017. On a similar note, Banerjee and Hofmann (2022) report weakly decreasing zombie shares post the year 2010 for Japan, Denmark and Germany.[20] Recall that our theory of relationship banking fits best to bank-oriented economies such as Germany and Japan rather than market-oriented economies such as the US or UK.

2.5 Extensions and Further Implications

2.5.1 Diverging Time Preferences

In this section, to gain a better understanding of the main drivers behind Proposition 5, we allow for different interest rates across the three types of agents. These diverging interest rates may reflect different time preferences, different opportunity costs, or different alternative investment opportunities. The interest rate of agent $i \in \{B, E, M\}$ is r_i, where subscript B denotes the bank, subscript E the entrepreneur, and subscript M the agents acting in the financial market (the investors).[21] We investigate how a change in the interest rate r_i applied by agent i affects the second-best repayment $R^{SB} = R^{SB}(r_B, r_M, r_E)$ and the quality threshold

$$\hat{\theta}(r_B, R^{SB}) = \frac{(1 + r_B)\tilde{L} - R^{SB}}{(1 + r_B)\tilde{\gamma}} \tag{2.30}$$

[20]The findings regarding Germany are also observed in the data by Blažková and Chmelíková (2022, p. 8), who report “for Germany, the share of zombies has increased during the crisis, but after 2009 it has been gradually declining.”

[21]If the entrepreneur chooses market finance at $t = 0$, selling the whole project is only optimal if $r_E \geq r_M$, i.e., if the entrepreneur is less patient, and thus discounts future profits stronger than investors. To keep the analysis as close as possible to the previous analysis, we assume that this is the case.

applied by the bank.[22]

The second-best repayment R^{SB} makes the entrepreneur indifferent between bank finance and her best alternative (market finance or outside option). Hence, it solves

$$\frac{1}{(1+r_E)^2}\left\{\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1-F(\hat{\theta})]R^{SB}\right\} - \tilde{w} = \max\left\{\frac{\mu}{(1+r_M)^2} + \frac{\tilde{\gamma}\mu}{1+r_M} - \tilde{I},\ 0\right\} \tag{2.31}$$

where $\hat{\theta}(r_B, R^{SB})$ is given by (2.30). The interest rate of investors, r_M, influences the repayment, and thus the threshold $\hat{\theta}$ only if market finance is better than the outside option, i.e., if $P_0 > 0$. Therefore, in the following, we focus on the case $P_0 > 0$.

Proposition 6. *Suppose that* $P_0 = [1 + (1 + r_M)\tilde{\gamma}]\mu - (1 + r_M)^2\tilde{I} > 0$. *Then,*

(i) the repayment R^{SB} *is strictly increasing and the bank's quality threshold* $\hat{\theta}$ *is strictly decreasing in the interest rate of investors (market participants):* $\partial R^{SB}/\partial r_M > 0$ *and* $\partial\hat{\theta}/\partial r_M < 0$*;*

(ii) the repayment R^{SB} *is strictly decreasing and the bank's quality threshold* $\hat{\theta}$ *is strictly increasing in the entrepreneur's interest rate:* $\partial R^{SB}/\partial r_E < 0$ *and* $\partial\hat{\theta}/\partial r_E > 0$*;*

(iii) the repayment R^{SB} *and the bank's quality threshold* $\hat{\theta}$ *are both strictly increasing in the bank's interest rate:* $\partial R^{SB}/\partial r_B > 0$ *and* $d\hat{\theta}/dr_B > 0$.

If the interest rate of investors r_M increases, then purchasing the project at $t = 0$ becomes less attractive to investors. The entrepreneur's best alternative – market finance – becomes less attractive, and thus the bank can demand a higher repayment. The higher repayment directly translates into a lower quality threshold $\hat{\theta}$.

If, on the other hand, the interest rate of the entrepreneur r_E increases, she discounts future profits more heavily, and thus selling the project to investors at $t = 0$ instead of at $t = 2$ (after intermediate run bank finance) becomes more attractive. This implies that the bank is forced to reduce the repayment, which increases its quality threshold.

Finally, the effect of an increase in the bank's interest rate r_B has a more nuanced effect. If the bank discounts future profits stronger, it has the incentive to terminate more projects. Thus, the direct effect of an increase in r_B on the quality threshold $\hat{\theta}$ is positive. A change in the bank's interest rate also affects the second-best repayment.

[22] In this section, we do not investigate how the probability of zombie lending is affected by changes in interest rates. The reason is that for $r_E \neq r_B$ it is not clear how to define the efficient threshold θ^*, and thus zombie lending.

First, the higher quality threshold implies that – ex ante – the project is less likely to be sold at $t = 2$. Second, a project sold at $t = 2$ obtains a higher price P_2 because an increase in $\hat{\theta}$ increases the average quality of continued projects. In the second-best contract, the repayment is too high from a welfare perspective ($R > \hat{\theta}$), implying that the price effect dominates the probability effect. This allows the bank to demand a higher repayment R^{SB}. The effect of an increase of the bank's interest rate on the quality threshold via the repayment is only of second order such that the threshold is strictly increasing in r_B.

According to Proposition 5 – all agents use an identical interest rate $r = r_B = r_E = r_M$ – an increase in the interest rate decreases the bank's quality threshold. Proposition 6 illustrates that the aforementioned comparative static is driven by two effects. First, an increase in the identical interest rate r increases investors' discounting, which leads to an increase in the repayment, and thus a decrease in the quality threshold. Moreover, an increase in the bank's interest rate increases the repayment R^{SB}, which – ceteris paribus – leads to a decrease in the quality threshold. For identical interest rates, these two effects dominate.

2.5.2 Alternative Investment Opportunities by Investors

In the previous section, we learned that one main driver behind Proposition 5 is that a reduction in the interest rate makes it more attractive for investors to finance the project at the initial date $t = 0$. This effect can be described as a competition effect: the lower the interest rate, the stronger the competition between investors and the bank to get selected as the financial backer for the entrepreneur's project. Due to this effect, a lower interest rate decreases the repayment under the second-best contract and increases the bank's quality threshold. In the long-run, this makes zombie lending less likely for low interest rates.

A reduction in the interest rate may, however, positively affect the return on alternative investments that are available to the investors. For instance, the reduction in interest rates may cause an increase in the demand for corporate stocks, leading to higher expected returns from investing in stocks.[23] Moreover, capital intensive industries benefit from low interest rates, and thus can generate higher revenues. In the following, we augment our baseline model by incorporating the latter channel.

A central bank determines the basis interest rate r^*. For simplicity, we assume

[23] Daniel *et al.* (2021) report that low interest rates drive up demand and prices for high-dividend stocks and high-yield bonds. Somewhat related, Domian *et al.* (1996) find that drops in interest rates are followed by excessive stock returns. A theoretical mechanism of how lower nominal interest rates that make liquidity cheaper translate into higher asset prices and investments is proposed by Drechsler *et al.* (2018).

that the relationship bank uses this basis interest rate, i.e., $r_B = r^*$. The interest rate applied by the entrepreneur, r_E, reflects her idiosyncratic time preference and is independent of r^*. The interest rate used by investors r_M is the net return they can achieve from alternative investments.

There is a large number of homogeneous firms that operate each with a fixed amount of equity k_E.[24] Each firm chooses an amount of outside capital k_O. A firm invests in $t = 0$ (and in $t = 1$) and generates a gross return of $B(k_E + k_O)$ in $t+1$, with $B'(\cdot) > 0$ and $B''(\cdot) < 0$. A firm's profit (net present value) is $\pi(r^*) = B(k_E + k_O^*) - (1 + r^*)k_O^*$, where $k_O^*(r^*)$ is the profit-maximizing amount of outside capital.[25] Thus, the net return on equity is

$$r_M(r^*) = \frac{\pi(r^*)}{k_E} - 1. \tag{2.32}$$

Each investor can decide to finance such a firm instead of the entrepreneur's project. An investor prefers to finance the entrepreneur's project if it has an expected net return that is weakly larger than $r_M(r^*)$.

We focus on situations where market finance is the entrepreneur's best alternative to bank finance, i.e., we assume that

$$P_0 := \frac{\mu}{(1 + r_M)^2} + \frac{\tilde{\gamma}\mu}{1 + r_M} - \tilde{I} > 0. \tag{2.33}$$

We can now state the following result.

Proposition 7. *Suppose that $P_0 > 0$. An increase in the basis interest rate r^**

(i) decreases the net return investors demand from the entrepreneur, $dr_M/dr^* = -k_O^*/k_E < 0$;

(ii) increases the quality threshold $\hat{\theta}(R^{SB})$ that the bank applies under the second-best contract, $d\hat{\theta}/dr^* > 0$.

Moreover, the bank's quality threshold $\hat{\theta}(R^{SB})$ reacts stronger to a change in the basis interest rate r^*, the stronger the net return r_M reacts, i.e., the larger $|dr_M/dr^*|$ is.

If the central bank interest rate r^* increases, the productivity of firms declines, which in turn reduces the return on equity, part (i) of Proposition 7. An increase in the interest rate r^* has two effects on the bank's quality threshold $\hat{\theta}$. First, there is the direct positive effect on $\hat{\theta}$: If the interest rate is higher, the bank has the incentive to

[24] Assuming a fixed amount of equity has the advantage that profit-maximization is equivalent to maximizing the rate of return on equity.

[25] We assume that k_O^* is determined by the first-order condition of profit maximization. Imposing the Inada conditions $\lim_{k\to 0} B'(k_E + k) = \infty$ and $\lim_{k\to\infty} B'(k_E + k) = 0$ is sufficient.

liquidate more often. Second, a change in the basis interest rate changes the second-best repayment R^{SB}. Regarding the repayment, there are two opposing effects. On the one hand, the bank liquidates more often, which increases the second-period price P_2. This allows the bank to demand a higher repayment. On the other hand, if the interest rate r^* increases, financing the entrepreneur rather than one of the homogeneous firms becomes more attractive for investors. This forces the bank to reduce the repayment. The former effect dominates if $|dr_M/dr^*| \approx 0$, while the latter dominates if $|dr_M/dr^*|$ is large. In any case, the overall effect on the quality thresholds is unambiguous: a higher interest rate r^* increases the bank's quality threshold.

Proposition 7 alludes to the concern that a low basis interest rate may lead to more zombie lending not only in the short-run but also in the long-run. This concern can be mitigated by strict financial regulations, e.g., capital requirements. A higher required share of equity to outside capital reduces the leverage of the publicly traded companies, and thus their return on equity. To see this mathematically, note that $|dr_M/dr^*| = k_O^*/k_E$ is strictly decreasing in k_E.

2.5.3 Bank's Capital Structure

Empirical evidence suggests that zombie lending is a more pronounced problem if the lender (the bank) is itself in a weak financial position (Peek and Rosengren, 2005; Acharya *et al.*, 2022; Blattner *et al.*, 2023). In other words, a bank with lower equity to outside capital ratio has a stronger incentive to roll over loans of poor quality. In the following, we consider a simple extension of the baseline model.

To address the issue of bank capital structure, we now assume that the bank finances the investment partially with equity and partially with outside finance. More precisely, share $\alpha \in (0, 1]$ of the investment $\tilde{I} - \tilde{d}$ is financed by bank equity and share $1 - \alpha$ by deposits. The bank pays an interest $r_D < r$ on deposits. To rule out trivial cases, we assume that the bank can repay the deposits also in case of project liquidation. Moreover, we focus on the second-best loan contract with $\tilde{d}^{SB} = \tilde{w}$. Under the second-best contract, the repayment $R^{SB} = R$ is determined by the entrepreneur's participation constraint, and thus is independent of the bank's capital structure. The bank keeps the deposits on the balance sheet for two periods if the entrepreneur's loan is continued at $t = 1$ but only for one period if the loan is terminated at $t = 1$.

The bank prefers to roll over the entrepreneur's loan at $t = 1$ if and only if

$$\tilde{\gamma}\theta + \frac{R^{SB}}{1+r} - (1-\alpha)\frac{(1+r_D)^2(\tilde{I} - \tilde{w})}{1+r} \geq L - (1-\alpha)(1+r_D)(\tilde{I} - \tilde{w}). \tag{2.34}$$

The difference between (2.34) and the respective condition in the baseline model is that the bank needs to repay the deposits $(\tilde{I} - \tilde{w})$ plus interest payments. The next result is readily obtained from (2.34).

Proposition 8. *Suppose the bank's equity share is α and it pays an interest $r_D < r$ on deposits. Then, the bank's quality threshold is higher, the higher the equity share: $\partial\hat{\theta}/\partial\alpha > 0$.*

The lower a bank's quality threshold $\hat{\theta}$, the higher is the scope for zombie lending – i.e., roll-over of loans from projects with inefficiently low returns. Thus, according to Proposition 8, weakly capitalized or even under-capitalized banks are particularly likely to engage in zombification.

2.5.4 Booms and Busts

Zombification seems to be particularly pronounced during economic downturns. Banerjee and Hofmann (2022) and De Martiis and Peter (2021) report that the share of zombie firms rises during recessions. For instance, De Martiis and Peter (2021) analyze the share of zombie firms in eight European countries from 1990 until 2018. For this period, they investigate how three recession events, the Dot-com Bubble, the GFC, and the European Debt Crisis, affected the likelihood of zombie lending. They point out that recession events are likely to be a primary cause for firms to become over-indebted. The recession alone, however, can hardly explain why these non-viable firms stay alive as they do according to the data of De Martiis and Peter (2021).

In the following, we investigate how an (unexpected) change in the economic conditions at the beginning of $t = 1$ – i.e., for given contracts – affects the probability of zombie lending. If there is an economic downturn at the beginning of $t = 1$, this affects the prospects regarding the project's returns in $t = 1$ and likely also in $t = 2$. Moreover, in an economic downturn, prices may drop, affecting the value of the entrepreneur's assets, e.g., the collateral and the value of the company's physical capital. In other words, the liquidation value of the project is reduced in an economic downturn. We model this by assuming that the project's quality is $\alpha\theta$ and the liquidation value is $\alpha\tilde{L}$, with $\alpha > 0$. For $\alpha < 1$ the economy is in a recession and for $\alpha > 1$ in a boom. We focus on a given second-best contract (d^{SB}, R^{SB}), where R^{SB} is optimal for the neutral economic condition $\alpha = 1$. We restrict the attention to drops in values that are not too severe, i.e., we assume that α is sufficiently large so that $P_2 = \mathbb{E}[\alpha\theta|\theta \geq \hat{\theta}(\alpha)] > R^{SB}$. The price that the entrepreneur obtains at $t = 2$ is larger than the repayment, and thus the bank always obtains R^{SB} in $t = 2$.

First, note that the efficient quality threshold θ^* is independent of α because all relevant payments from $t = 1$ onward – both the project revenues and the liquidation value – are scaled by α. The bank, however, prefers to roll over the loan if and only if

$$\tilde{\gamma}\alpha\theta + \frac{R^{SB}}{1+r} \geq \alpha\tilde{L}. \tag{2.35}$$

The roll-over decision of the bank hinges on the economic state α because the repayment is fixed ex ante and does not depend on the economic situation.

Proposition 9. *The probability of zombie lending $Z(\alpha) = \int_{\hat{\theta}(\alpha)}^{\bar{\theta}} f(\theta)\,\mathrm{d}\theta$ increases (decreases) in a recession (boom), i.e., $dZ/d\alpha < 0$.*

According to Proposition 9 and in line with empirical evidence, zombie lending increases if the economy turns into a recession. With the repayment being fixed ex ante, the bank has the incentive to continue the project for more quality levels if the liquidation value and the project's returns decrease. Intuitively, the relationship bank prefers to 'speculate' on obtaining the (ex ante) contracted repayment in the future rather than realizing the busted liquidation value.

2.6 Robustness and Discussion

2.6.1 Alternative Contracts: Repayments in $t = 1$ and $t = 2$

Suppose that at $t = 0$, the bank offers a contract $\mathcal{C} = (\tilde{d}, \tilde{R}_1, R_2)$ that specifies (i) the own contribution of the entrepreneur to the investment $\tilde{d} \leq \tilde{w}$, (ii) a repayment $\tilde{R}_1$ to be made at the end of $t = 1$, and (iii) a repayment R_2 to be made at $t = 2$. The entrepreneur keeps the control and cash-flow rights at $t = 1$. If, however, the bank learns at the beginning of $t = 1$ that the entrepreneur will be unable to make the repayment $\tilde{R}_1$, it can force the illiquid entrepreneur to liquidate her business. The bank can also decide to roll over the loan even though the entrepreneur is not able to pay the full obligation $\tilde{R}_1$.

To simplify the exposition, we focus on the case $\tilde{d} = \tilde{w}$. Moreover, by the argument outlined for the baseline model, we know that $R_2 \leq P_2 = \mathbb{E}[\theta|\theta \geq \hat{\theta}(R_2)]$. If $\tilde{\gamma}\theta < \tilde{R}_1$, and thus the entrepreneur is insolvent, the bank prefers the continuation if and only if

$$\tilde{\gamma}\theta + \frac{R_2}{1+r} \geq \min\{\tilde{L}, \tilde{R}_1\} \iff \theta \geq \frac{(1+r)\min\{\tilde{L}, \tilde{R}_1\} - R_2}{\tilde{\gamma}(1+r)} =: \hat{\theta}. \tag{2.36}$$

For $\tilde{R}_1 \geq \tilde{L}$ and $R_2 = \theta^*$ we have $\hat{\theta} = \theta^*$, i.e., the first-best quality threshold is implemented.

If the bank can extract larger rents from the entrepreneur, it can increase its profit by either increasing $\tilde{R}_1$ or R_2. Increasing $\tilde{R}_1 \geq \tilde{L}$ does not distort the roll-over decision but increases the bank's expected total repayment. Once $\tilde{R}_1 = \tilde{\gamma}\bar{\theta}$, a further increase of $\tilde{R}_1$ does not increase the bank's expected profit. If this is the case, the bank has the incentive to demand a repayment $R_2 > \theta^*$. Now, it offers the contract $\mathcal{C} = (\tilde{d} = \tilde{w}, \tilde{R}_1 = \tilde{\gamma}\bar{\theta}, R_2)$ that is equivalent to the second-best contract analyzed in the baseline model. In other words, if the entrepreneur is wealth constrained, $\tilde{w} < \tilde{d}^*$, the loan contract $\mathcal{C} = (\tilde{d}, \tilde{R}_1, R_2)$ specifies a first-period repayment $\tilde{R}_1$ so that the effective continuation-liquidation decision at $t = 1$ is made by the bank.

In practice there can be several reasons why the signed contract leaves a rent to the entrepreneur at $t = 1$, i.e., $\tilde{R}_1 < \tilde{\gamma}\bar{\theta}$ is optimal. One reason could be a non-contractible effort by the entrepreneur that is important for project success. Our simple model abstracts from any moral hazard issues. Note, however, the tighter the constraint on $\tilde{R}_1$ from above (e.g., due to moral hazard issues), the higher the initial investment, $\tilde{I} - \tilde{d}$, or the repayment R_2 that the bank demands. On this account, a further constraint on $\tilde{R}_1$ implies even more scope for zombie lending.

2.6.2 Bank Competition

Throughout the paper, we assumed that a monopolistic bank learns the quality of the project at an intermediate date and makes a take-it-or-leave-it contract offer. The bank's offer is constrained by the risk-neutral investors' offer to the entrepreneur at date $t = 0$. In the baseline model, however, no other bank can monitor the project and is willing to finance it. The terms of the second-best contract, under which zombie lending occurs, are determined by the entrepreneur's participation constraint. In the following, we show that zombie lending can also occur under bank competition where the entrepreneur's participation constraint does not determine the equilibrium repayment.

Suppose that there are several – at least two – banks that can create a relationship with the entrepreneur. These banks, who are all identical, compete at date $t = 0$ à la Bertrand by making a loan contract offer (d, R). To simplify the exposition we assume a cashless entrepreneur, i.e., $w = 0$, and that there is no financial market at $t = 0$. Note that if bank finance occurs in equilibrium, the next best alternative for the entrepreneur is to take up a loan from another bank. Thus, we can abstract from market finance without loss in generality.

Furthermore, we assume that a bank that fully finances the project $(d = 0)$ and charges the highest feasible repayment $\bar{R}$ makes a strictly positive expected profit.

Assumption 4. $\pi_B(0, \bar{R}) > 0$.

The assumption implies that the expected surplus generated by efficient bank finance $(R = \theta^*)$ is strictly positive. To be able to state a concise result, we define the following threshold

$$\bar{d} := -F(\theta^*)L - \gamma \int_{\theta^*}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1 - F(\theta^*)]\theta^* + c + I.$$

Finally, we focus on symmetric equilibria of the bank competition game.

Proposition 10. *Suppose that Assumption 4 holds. If $\bar{d} > 0$, the equilibrium loan contract (d^C, R^C) under bank competition specifies $d^C = 0$ and $R^C \in (R^*, \bar{R})$ so that $\pi_B(0, R^C) = 0$.*

According to Proposition 10, if the expected bank profit from the contract that induces efficient continuation ($R = \theta^*$) is negative, the equilibrium contract specifies an inefficiently high repayment $R^C > R^* = \theta^*$. Thus, under the competitive loan contract (d^C, R^C), zombie lending takes place for projects of quality $\theta \in [\hat{\theta}(R^C), \theta^*)$. Moreover, as a bank's continuation decision for a given contract is independent of the degree of bank competition, Proposition 4 still applies. In other words, if the entrepreneur signed an equilibrium loan contract under bank competition (d^C, R^C), an unanticipated drop in the interest rate increases the probability of zombie lending.

What is the intuition behind Proposition 10? The equilibrium loan contract maximizes the expected profit of the entrepreneur subject to a bank's break-even constraint, the entrepreneur's limited funds (LL), and taking the roll-over decision (RD) into account. If (LL) is slack, the optimal contract maximizes the joint surplus, and thus specifies $R = R^*$ so that the roll-over decision is efficient. If (LL) is binding, and thus $\bar{d} > 0$, the contract specifies $d^C = 0$. In this case, the bank can break even only for repayments higher than $R^* = \theta^*$. As a result, the equilibrium contract specifies a repayment $R^C > R^*$ and zombie lending occurs for qualities $\theta \in [\hat{\theta}(R^C), \theta^*)$. Recall that $\hat{\theta}(R^C) < \hat{\theta}(R^*) = \theta^*$.

2.7 Conclusion

In this paper, we analyze a simple zombie lending mechanism. A relationship bank may grant a loan to an entrepreneur who possesses a project of ex ante unknown quality. We show that within a second-best contract – that arises in equilibrium if the entrepreneur is cash constrained – the relationship bank continues projects of inefficiently low qualities: zombie lending occurs. The reason is that the binding upper bound on the entrepreneur's initial outlay directly translates into an inefficiently high ex post repayment demanded by the relationship bank. The latter fact, in turn, leads to a distorted continuation decision.

Investigating the bank's motive for inefficient roll-over decisions further, we introduce interest rate shocks. In case the interest rate drops unexpectedly, i.e., the bank faces a 'new' continuation decision for a predetermined second-best contract, the probability of zombie lending increases. Intuitively, the bank becomes more patient when the interest rate drops, and hence continuing the project and receiving the inefficiently high ex post repayment becomes more attractive. Interestingly, we find that the relationship between the bank's zombie lending behavior and the interest rate is inverted in the long run, i.e., where contracts are adapted. In other words, the probability of zombie lending decreases with lower interest rates. Since lower interest rates increase the market investors' willingness to pay for the entrepreneur's project, the relationship bank reacts by offering a contract with a lower ex post repayment. As a consequence, the bank's roll-over decision becomes more efficient, i.e., the bank continues fewer zombie projects. In an extension, we show that this effect mitigates if a low interest rate, say a low basic interest rate of the central bank, increases the attractiveness of alternative investment opportunities that market investors have.

2.A Mathematical Appendix

Proof of Observation 1. The result follows readily from comparing the expected surplus of market finance (2.2), the expected surplus from efficient bank finance (2.4), and the surplus from no finance, which is zero. □

Proof of Proposition 1. For $R = R^*$, we have $\hat{\theta}(R) = \theta^*$ and $P_2 = \mathbb{E}[\theta \mid \theta \geq \theta^*]$. This implies that for repayment R^* the entrepreneur is indifferent between accepting the bank loan (d, R^*) and her next best alternative if and only if

$$d = [1 - F(\theta^*)]\big\{\mathbb{E}[\theta \mid \theta \geq \theta^*]\big\} - \max\{(1+\gamma)\mu - I, 0\} \tag{2.37}$$

$$= \int_{\theta^*}^{\bar{\theta}} [\theta - \theta^*] f(\theta)\, \mathrm{d}\theta - \max\{(1+\gamma)\mu - I, 0\}. \tag{2.38}$$

Note that $P_0 = (1+\gamma)\mu - I$. If bank finance is efficient and all the additional surplus from bank finance is extracted by the bank – i.e., participation is binding – then offering a loan contract that implements efficient continuation maximizes the bank's profits. □

Proof of Proposition 2. The bank maximizes its profit subject to the entrepreneur's participation constraint, $\pi_E(d, R) \geq \max\{P_0, 0\}$, and the limited liability constraint, $d \leq w$. The first-best contract (d^*, R^*) satisfies the participation but violates the limited liability constraint, $w < d^*$. With d being an ex ante one-to-one transfer between the entrepreneur and the bank, the second-best optimal amount financed by the entrepreneur is $d^{SB} = w$.

The expected profit of the bank is

$$\begin{aligned}\pi_B(d^{SB}, R) = {} & F(\hat{\theta}(R))[L - c - I + w] \\ & + [1 - F(\hat{\theta}(R))]\big\{\gamma \mathbb{E}[\theta | \theta \geq \hat{\theta}(R)] + R - c - I + w\big\}.\end{aligned} \tag{2.39}$$

Simplifying the above expression yields

$$\pi_B(d^{SB}, R) = F(\hat{\theta}(R))L + \gamma \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\, \mathrm{d}\theta + [1 - F(\hat{\theta}(R))]R - (c + I - w). \tag{2.40}$$

Taking the derivative of π_B with respect to the repayment R yields

$$\begin{aligned}\frac{\partial \pi_B}{\partial R} &= f(\hat{\theta})\frac{d\hat{\theta}}{dR}L - \gamma\hat{\theta} f(\hat{\theta})\frac{d\hat{\theta}}{dR} + [1 - F(\hat{\theta})] - f(\hat{\theta})\frac{d\hat{\theta}}{dR}R \\ &= -f(\hat{\theta})\frac{1}{\gamma}[\underbrace{L - \gamma\hat{\theta} - R}_{=0}] + 1 - F(\hat{\theta}) > 0\end{aligned} \tag{2.41}$$

The term in square brackets equals zero by the definition of $\hat{\theta}$. Thus, the bank strictly prefers a higher repayment R.

The expected profit of the entrepreneur is

$$\begin{aligned}\pi_E(d^{SB}, R) &= F(\hat{\theta}(R))(-w) + [1 - F(\hat{\theta}(R))]\{\mathbb{E}[\theta|\theta \geq \hat{\theta}(R)] - R - w\} \\ &= \int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1 - F(\hat{\theta}(R))]R - w. \end{aligned} \tag{2.42}$$

Note that $\pi_E(d^{SB}, R^*) > \max\{P_0, 0\}$ because $\pi_E(d^*, R^*) = \max\{P_0, 0\}$ and $d^* > w = d^{SB}$. Moreover, $\pi_E(d^{SB}, \bar{R}) = -w$, which implies that for $R > \bar{R}$ the participation constraint is violated. Recall that $\bar{R}$ is implicitly defined by $\mathbb{E}[\theta|\theta \geq \hat{\theta}(\bar{R})] = \bar{R}$. Hence, $R^{SB} \in (R^*, \bar{R}]$.

Taking the partial derivative of the entrepreneur's expected profit with respect to R yields

$$\begin{aligned}\frac{\partial \pi_E}{\partial R} &= -\hat{\theta} f(\hat{\theta})\frac{d\hat{\theta}}{dR} - [1 - F(\hat{\theta})] + Rf(\hat{\theta})\frac{d\hat{\theta}}{dR} \\ &= -[R - \hat{\theta}]f(\hat{\theta})\frac{1}{\gamma} - [1 - F(\hat{\theta})]. \end{aligned} \tag{2.43}$$

For $R > R^*$ we have $\hat{\theta}(R) < \theta^*$ and, thus, $\partial\pi_E/\partial R < 0$.

The bank's expected profit is strictly increasing in R and the entrepreneur's expected profit is strictly decreasing in R. Thus, the second-best optimal repayment R^{SB} solves $\pi_E(d^{SB}, R) = \max\{P_0, 0\}$. □

Proof of Corollary 1. The finding follows directly from the observation that $R^{SB} > R^*$ for $w < d^*$. □

Proof of Proposition 3. The first-best outcome is described in Observation 1. If a project is not financed in the first-best, it is also not financed in equilibrium. Moreover, if market finance is efficient, it also occurs in equilibrium because the full surplus of this channel accrues to the entrepreneur. Similarly, if bank finance is efficient and the first-best loan contract is offered by the bank ($w \leq d^*$), then bank finance occurs in equilibrium. The remaining question is, when is the second-best loan contract (d^{SB}, R^{SB}) offered in equilibrium. The bank's offer just compensates the entrepreneur for her best alternative option. Thus, the second-best loan contract is offered as long as the resulting expected bank profits are non-negative. This is the case if and only if $c \leq \bar{c}^{SB}$, which is characterized by (2.19).

Differentiation of (2.19) with respect to I yields

$$\frac{d\bar{c}^{SB}}{dI} = \frac{dR^{SB}}{dI}\left\{1 - F(\hat{\theta}) - \frac{d\hat{\theta}}{dR}f(\hat{\theta})\underbrace{[\gamma\hat{\theta} + R^{SB} - L]}_{=0}\right\} - 1 \tag{2.44}$$

$$= \frac{dR^{SB}}{dI}[1 - F(\hat{\theta})] - 1. \tag{2.45}$$

The second-best repayment $R^{SB}(I)$ is implicitly defined by

$$\int_{\hat{\theta}(R^{SB})}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1 - F(\hat{\theta}(R^{SB}))] - w = \max\{(1+\gamma)\mu - I, 0\}. \tag{2.46}$$

First, suppose that $(1+\gamma)\mu - I \geq 0$. The implicit differentiation of (2.46) with respect to I yields

$$-\hat{\theta}f(\hat{\theta})\frac{d\hat{\theta}}{dR}\frac{dR^{SB}}{dI} + R^{SB}f(\hat{\theta})\frac{d\hat{\theta}}{dR}\frac{dR^{SB}}{dI} - [1 - F(\hat{\theta})]\frac{dR^{SB}}{dI} = -1. \tag{2.47}$$

Rearranging the above expression and using the fact that $d\hat{\theta}/dR = -\gamma^{-1}$ yields

$$\frac{dR^{SB}}{dI} = \frac{1}{1 - F(\hat{\theta}) + \frac{1}{\gamma}(R^{SB} - \hat{\theta})f(\hat{\theta})} > 0. \tag{2.48}$$

□

Inserting (2.48) in (2.45) yields

$$\frac{d\bar{c}^{SB}}{dI} = -\frac{\frac{1}{\gamma}(R^{SB} - \hat{\theta})f(\hat{\theta})}{1 - F(\hat{\theta}) + \frac{1}{\gamma}(R^{SB} - \hat{\theta})f(\hat{\theta})} \in (-1, 0). \tag{2.49}$$

Recall that $R^{SB} > R^* = \theta^* > \hat{\theta}(R^{SB})$.

Second, suppose that $I > (1+\gamma)\mu$. In this case, R^{SB} is independent of I, which is apparent from (2.46). Thus, $dR^{SB}/dI = 0$. Now, using (2.45), we immediately obtain that

$$\frac{d\bar{c}^{SB}}{dI} = -1. \tag{2.50}$$

This concludes the proof.

Proof of Corollary 2. The finding follows directly from the proof of Proposition 3 in combination with Corollary 1. □

Proof of Proposition 4. Taking the derivative of $Z(r)$ with respect to r – for a constant repayment R^{SB} – yields

$$Z'(r) = f(\theta^*)\frac{d\theta^*}{dr} - f(\hat{\theta})\frac{\partial\hat{\theta}}{\partial r}. \tag{2.51}$$

To sign the above expression, we first need to determine $d\theta^*/dr$ and $\partial\hat{\theta}/dr$. Taking the partial derivative of (2.24) with respect to r yields

$$\begin{aligned}\frac{\partial\hat{\theta}}{\partial r} &= \frac{\tilde{L}(1+r)\tilde{\gamma} - \tilde{\gamma}[(1+r)\tilde{L} - R^{SB}]}{\tilde{\gamma}(1+r)^2} \\ &= \frac{R^{SB}}{\tilde{\gamma}(1+r)^2} > 0.\end{aligned} \tag{2.52}$$

Taking the partial derivative of (2.22) with respect to r yields

$$\begin{aligned}\frac{d\theta^*}{dr} &= \frac{\tilde{L}[1+(1+r)\tilde{\gamma}] - \tilde{\gamma}(1+r)\tilde{L}}{[1+(1+r)\tilde{\gamma}]^2} \\ &= \frac{\tilde{L}}{[1+(1+r)\tilde{\gamma}]^2} > 0.\end{aligned} \tag{2.53}$$

Using the definition of θ^* allows us to write the above derivative as

$$\frac{d\theta^*}{dr} = \frac{\theta^*}{(1+r)[1+(1+r)\tilde{\gamma}]}. \tag{2.54}$$

By Assumption 2 it holds that $f(\theta^*) \leq f(\hat{\theta})$. Thus, $Z'(r) \leq f(\hat{\theta})[d\theta^*/dr - \partial\hat{\theta}/\partial r]$, which implies that $Z'(r) < 0$ for $\partial\hat{\theta}/\partial r > d\theta^*/dr$. Note that $\partial\hat{\theta}/\partial r > d\theta^*/dr$ is equivalent to

$$R^{SB}(1+r)[1+\tilde{\gamma}(1+r)] > \theta^*\tilde{\gamma}(1+r)^2 \tag{2.55}$$

$$\iff R^{SB}(1+r) + \tilde{\gamma}(1+r)^2[R^{SB} - \theta^*] > 0. \tag{2.56}$$

The above claim is true because $R^{SB} > \theta^*$ by the assumption that the parties signed the second-best contract. □

Proof of Proposition 5. Under the second-best optimal loan contract, the repayment $R^{SB} \in (R^*, \bar{R})$ solves

$$\frac{1}{(1+r)^2}\left(\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1-F(\hat{\theta})]R^{SB}\right) - \tilde{w} = \frac{\mu}{1+r}\left(\tilde{\gamma} + \frac{1}{1+r}\right) - \tilde{I}, \tag{2.57}$$

where

$$\hat{\theta}(r, R^{SB}(r)) = \frac{(1+r)\tilde{L} - R^{SB}}{(1+r)\tilde{\gamma}}. \tag{2.58}$$

In the above condition determining $R^{SB}(r)$ we use the fact that the entrepreneur's best alternative to bank finance is market finance, i.e., that $P_0 > 0$. The implicit differentiation of (2.57) with respect to r yields

$$\frac{-2}{(1+r)^3}\left\{\int_{\hat{\theta}}^{\bar{\theta}} -[1-F(\hat{\theta})]R^{SB}\right\} + \frac{1}{(1+r)^2}\left\{-\hat{\theta}f(\hat{\theta})\frac{d\hat{\theta}}{dr} + f(\hat{\theta})R^{SB}\frac{d\hat{\theta}}{dr} - [1-F(\hat{\theta})]\frac{dR^{SB}}{dr}\right\} = \frac{-2\mu}{(1+r)^3} - \frac{\gamma\mu}{(1+r)^2}. \tag{2.59}$$

Note that

$$\frac{d\hat{\theta}}{dr} = \frac{1}{(1+r)\tilde{\gamma}}\left[\frac{R^{SB}}{1+r} - \frac{dR^{SB}}{dr}\right]. \tag{2.60}$$

Inserting (2.60) in (2.59) and rearranging yields

$$\frac{dR^{SB}}{dr} = \frac{2\tilde{\gamma}[\mu - \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta] + \tilde{\gamma}^2(1+r)\mu}{(1+r)\tilde{\gamma}[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\hat{\theta})} + \frac{2(1+r)\tilde{\gamma}[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\hat{\theta})}{(1+r)\tilde{\gamma}[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\hat{\theta})}\,\frac{R^{SB}}{1+r}. \tag{2.61}$$

By Assumption 3 it holds that $\mu - \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta > 0$, and thus $dR^{SB}/dr > 0$.

We proceed by inserting (2.61) into (2.60) and obtain

$$\frac{d\hat{\theta}}{dr} = \frac{-1}{(1+r)\tilde{\gamma}}\left\{\frac{(1+r)\tilde{\gamma}^2\mu + 2\tilde{\gamma}[\mu - \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta]}{\tilde{\gamma}(1+r)[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\theta)} + \frac{R^{SB}}{1+r}\,\frac{\tilde{\gamma}(1+r)[1-F(\hat{\theta})]}{\tilde{\gamma}(1+r)[1-F(\hat{\theta})] + (R^{SB}-\hat{\theta})f(\theta)}\right\} < 0. \tag{2.62}$$

Finally, recall that $d\theta^*/dr > 0$, and thus $Z(r) = \int_{\hat{\theta}}^{\theta^*} f(\theta)\,\mathrm{d}\theta$ is strictly increasing in r. □

Proof of Proposition 6. The second-best repayment $R^{SB} = R^{SB}(r_B, r_E, r_M)$ solves

$$\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1 - F(\hat{\theta})]R^{SB} - (1 + r_E)^2\tilde{w} = \left\{[1 + (1 + r_M)\tilde{\gamma}]\mu - (1 + r_M)^2\tilde{I}\right\}\frac{(1 + r_E)^2}{(1 + r_M)^2}, \quad (2.63)$$

where

$$\hat{\theta}(r_B, R^{SB}) = \frac{\tilde{L}(1 + r_B) - R^{SB}}{\tilde{\gamma}(1 + r_B)}. \quad (2.64)$$

Note that

$$\frac{\partial\hat{\theta}}{\partial r_i} = \frac{-1}{\tilde{\gamma}(1 + r_B)}\,\frac{\partial R^{SB}}{\partial r_i} \quad \text{for } i = E, M. \quad (2.65)$$

First, we investigate the comparative static with respect to r_M. The differentiation of (2.63) with respect to r_M yields

$$-\hat{\theta}f(\hat{\theta})\frac{\partial\hat{\theta}}{\partial r_H} + f(\hat{\theta})\frac{\partial\hat{\theta}}{\partial r_H}R^{SB} - [1 - F(\hat{\theta})]\frac{\partial R^{SB}}{\partial r_H} = (1 + r_E)^2\left[\frac{-2\mu}{(1 + r_M)^3} + \frac{-\tilde{\gamma}\mu}{(1 + r_M)^2}\right]. \quad (2.66)$$

We rearrange the above expression and obtain

$$\frac{\partial R^{SB}}{\partial r_M} = \frac{\tilde{\gamma}(1 + r_B)(1 + r_E)^2[2 + \tilde{\gamma}(1 + r_M)]\mu}{(R^{SB} - \hat{\theta})f(\hat{\theta}) + \tilde{\gamma}(1 + r_B)[1 - F(\hat{\theta})]} > 0. \quad (2.67)$$

From (2.67) together with (2.65) it follows immediately that $\partial\hat{\theta}/\partial r_M < 0$.

Next, we implicitly differentiate (2.63) with respect to r_E and obtain

$$-\hat{\theta}f(\hat{\theta})\frac{\partial\hat{\theta}}{\partial r_E} + f(\hat{\theta})\frac{\partial\hat{\theta}}{\partial r_E}R^{SB} - [1 - F(\hat{\theta})]\frac{\partial R^{SB}}{\partial r_E} - 2(1 + r_E)\tilde{w} = 2(1 + r_E)\tilde{P}_0, \quad (2.68)$$

where

$$\tilde{P}_0 = -\tilde{I} + \frac{\tilde{\gamma}\mu}{1 + r_M} + \frac{\mu}{(1 + r_M)^2} > 0 \quad (2.69)$$

by assumption. We rearrange the above expression and obtain

$$\frac{\partial R^{SB}}{\partial r_E} = -\frac{2\tilde{\gamma}(1 + r_B)(1 + r_E)\tilde{P}_0}{(R^{SB} - \hat{\theta})f(\hat{\theta}) + \tilde{\gamma}(1 + r_B)[1 - F(\hat{\theta})]} < 0. \quad (2.70)$$

From (2.70) together with (2.65) it follows that $\partial\hat{\theta}/\partial r_E > 0$.

Finally, we investigate the comparative static with respect to r_B. First, note that

$$\frac{d\hat{\theta}}{dr_B} = \frac{R^{SB}}{\tilde{\gamma}(1+r_B)^2} - \frac{1}{\tilde{\gamma}(1+r_B)} \frac{\partial R^{SB}}{\partial r_B}. \tag{2.71}$$

The implicit differentiation of (2.63) with respect to r_B yields

$$-\hat{\theta} f(\hat{\theta}) \frac{d\hat{\theta}}{dr_B} + f(\hat{\theta}) \frac{d\hat{\theta}}{dr_B} R^{SB} - [1 - F(\hat{\theta})] \frac{\partial R^{SB}}{\partial r_B} = 0. \tag{2.72}$$

Inserting (2.71) into (2.71) and rearranging yields

$$\frac{\partial R^{SB}}{\partial r_B} = \frac{R^{SB}}{1+r_B} \frac{(R^{SB} - \hat{\theta}) f(\hat{\theta})}{(R^{SB} - \hat{\theta}) f(\hat{\theta}) + \tilde{\gamma}(1+r_B)[1 - F(\hat{\theta})]} > 0. \tag{2.73}$$

To conclude the proof note that

$$\frac{d\hat{\theta}}{dr_B} = \frac{1}{\tilde{\gamma}(1+r_B)} \left[\frac{R^{SB}}{1+r_B} - \frac{\partial R^{SB}}{\partial r_B} \right]. \tag{2.74}$$

Inserting (2.74) into (2.73) reveals that $d\hat{\theta}/dr_B > 0$. □

Proof of Proposition 7. First, we prove part (i): Note that

$$r_M(r^*) = \frac{B(k_E + k_O^*(r^*)) - (1+r^*) k_O^*(r^*)}{k_E} - 1. \tag{2.75}$$

Taking the derivative with respect to r^* yields

$$\begin{aligned} \frac{dr_M}{dr^*} &= \frac{1}{k_E} \left[B'(k_E + k_O^*) \frac{dk_O^*}{dr^*} - (1+r^*) \frac{dk_O^*}{dr^*} - k_O^* \right] \\ &= -\frac{k_O^*}{k_E} < 0. \end{aligned} \tag{2.76}$$

Next, we prove part (ii). The second-best repayment $R^{SB} = R^{SB}(r^*)$ makes the entrepreneur indifferent between bank finance and market finance:

$$\begin{aligned} \frac{1}{(1+r_M(r^*))^2} \left[\int_{\hat{\theta}(r^*)}^{\bar{\theta}} \theta f(\theta) \, \mathrm{d}\theta - [1 - F(\hat{\theta}(r^*))] R^{SB}(r^*) \right] - \tilde{w} \\ = \frac{\mu}{(1+r_M(r^*))^2} + \frac{\tilde{\gamma}\mu}{1+r_M(r^*)} - \tilde{I}. \end{aligned} \tag{2.77}$$

Recall that

$$\frac{d\hat{\theta}}{dr^*} = \frac{1}{(1+r^*)\tilde{\gamma}}\left[\frac{R^{SB}}{1+r^*} - \frac{dR^{SB}}{dr^*}\right]. \tag{2.78}$$

Multiplying both sides of (2.77) with $(1+r_E)^2$ and then implicitly differentiating with respect to r^* yields

$$\begin{aligned} &-\hat{\theta}f(\hat{\theta})\frac{d\hat{\theta}}{dr^*} + f(\hat{\theta})\frac{d\hat{\theta}}{dr^*}R^{SB} - [1-F(\hat{\theta})]\frac{dR^{SB}}{dr^*} \\ &\qquad = (1+r_E)^2\left[\frac{-2\mu}{(1+r_M)^3}\frac{dr_M}{dr^*} - \frac{\tilde{\gamma}\mu}{(1+r_M)^2}\frac{dr_M}{dr^*}\right]. \end{aligned} \tag{2.79}$$

We insert (2.78) into (2.79) and solve for

$$\begin{aligned} \frac{dR^{SB}}{dr^*} = &\frac{(R^{SB}-\hat{\theta})f(\hat{\theta})}{(R^{SB}-\hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1-F(\hat{\theta})]}\frac{R^{SB}}{1+r^*} \\ &+ \frac{(1+r^*)\tilde{\gamma}(1+r_E)^2\mu[2+\tilde{\gamma}(1+r_M)]}{(1+r_M)^3\{(R^{SB}-\hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1-F(\hat{\theta})]\}}\frac{dr_M}{dr^*}. \end{aligned} \tag{2.80}$$

Inserting (2.80) into (2.78) yields

$$\begin{aligned} \frac{d\hat{\theta}}{dr^*} = \frac{1}{(1+r^*)\tilde{\gamma}}\Bigg[&\frac{(1+r^*)\tilde{\gamma}[1-F(\hat{\theta})]}{(R^{SB}-\hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1-F(\hat{\theta})]}\frac{R^{SB}}{1+r^*} \\ &+ \frac{(1+r^*)\tilde{\gamma}(1+r_E)^2\mu[2+\tilde{\gamma}(1+r_M)]}{(1+r_M)^3\{(R^{SB}-\hat{\theta})f(\hat{\theta}) + (1+r^*)\tilde{\gamma}[1-F(\hat{\theta})]\}}\frac{dr_M}{dr^*}\Bigg]. \end{aligned} \tag{2.81}$$

The above equation allows us to conclude that $d\hat{\theta}/dr^* > 0$ because $R^{SB} > \hat{\theta}(R^{SB})$ and $dr_M/dr^* < 0$ by (2.76).

□

Proof of Proposition 8. Solving (2.34) for θ yields

$$\theta \geq \frac{\tilde{L}(1+r) - R^{SB}}{1+r} - (1-\alpha)(\tilde{I}-\tilde{w})\frac{1+r_D}{1+r}(r-r_D) =: \hat{\theta}. \tag{2.82}$$

We differentiate (2.82) with respect to α and obtain

$$\frac{\partial\hat{\theta}}{\partial\alpha} = (\tilde{I}-\tilde{w})\frac{1+r_D}{1+r}(r-r_D) > 0, \tag{2.83}$$

which concludes the proof. □

Proof of Proposition 9. From equation (2.35) it follows directly that the quality threshold applied by the bank is given by

$$\hat{\theta}(\alpha) = \frac{\tilde{L}}{\tilde{\gamma}} - \frac{R^{SB}}{\tilde{\gamma}(1+r)\alpha}. \tag{2.84}$$

The change in the threshold due to a change in α is

$$\frac{d\hat{\theta}}{d\alpha} = \frac{R^{SB}}{\tilde{\gamma}(1+r)\alpha^2} > 0. \tag{2.85}$$

Finally, note that

$$\begin{aligned} \frac{dZ}{d\alpha} &= -f(\hat{\theta})\frac{d\hat{\theta}}{d\alpha} \\ &= -f(\hat{\theta})\ \frac{R^{SB}}{\tilde{\gamma}(1+r)\alpha^2} < 0. \end{aligned} \tag{2.86}$$

□

Proof of Proposition 10. The equilibrium loan contract under perfect bank competition solves

$$\max_{d,R} \quad \pi_E(d,R)$$

subject to

$$\pi_B(d,R) \geq 0, \tag{PC}$$

$$d \leq w = 0, \tag{LL}$$

and taking into account that $\hat{\theta}(R) = (L-R)/\gamma$. In equilibrium, banks make a zero profit. Solving (PC) as an equality for d and inserting this into the target function yields

$$F(\hat{\theta}(R))L + (\gamma+1)\int_{\hat{\theta}(R)}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - c - I. \tag{2.87}$$

Thus, if (LL) is slack, the equilibrium contract maximizes the joint surplus of the entrepreneur and the bank. This is achieved for

$$R = \frac{L}{1+\gamma} = \theta^*.$$

The corresponding part financed by the entrepreneur is

$$\bar{d} = -F(\theta^*)L - \gamma \int_{\theta^*}^{\bar{\theta}} \theta f(\theta)\,\mathrm{d}\theta - [1 - F(\theta^*)]\theta^* + c + I. \tag{2.88}$$

Thus, for $\bar{d} \leq 0$, the equilibrium loan contract is $(\bar{d}, R^*)$ and the first-best allocation is implemented.

For $\bar{d} > 0$, the contract $(\bar{d}, R^*)$ is not feasible. The highest feasible d is $d = 0$. Now, the repayment needs to be increased in order to satisfy (PC). Thus, the equilibrium repayment R^C solves $\pi_B(0, R^C) = 0$. Note that $\pi_B(0, R)$ is strictly increasing in $R \leq \bar{R}$. Moreover, by Assumption 4, $\pi_B(0, \bar{R}) > 0$. As a result, there exists a unique $R^C \in (R^*, \bar{R})$ that solves $\pi_B(0, R^C) = 0$. Finally, as $\pi_E(0, R)$ is strictly decreasing in $R \in [R^*, \bar{R}]$ and $\pi_E(0, \bar{R}) = 0$, we know that $\pi_E(0, R^C) > 0$, implying that the entrepreneur accepts a loan contract $(0, R^C)$. □

Chapter 3

Preventing Reallocation – A Story of Venture and Human Capital

Abstract: This venture capital model proposes a new channel that links the growing funding competition between venture capitalists (VC) to the observed reduction of active governance: *preemptive differentiation.* In the model, a representative VC strategically reallocates human capital across competing portfolio firms to ensure the profitability of active governance. However, the relatedness of firms within the portfolio allows the individual entrepreneur to engage in anti-competitive differentiation, potentially harming rivals. As a result, an entrepreneur can preempt a human capital pull-out if the inflicted externality on the competitors is sufficiently strong. In extreme cases, the VC may find it optimal to abandon active governance by refraining from providing human capital ex ante.

Keywords: Anti-competitive practices; Corporate Governance; Venture Capital.

JEL classification: D82; D86; G24; G32; L26.

3.1 Introduction

Venture Capitalists (VC) and their remarkable positive effect on early-stage firm success rates – both on initial public offerings and acquisitions – are an ongoing topic in the finance and economic literature. One key determinant of the VCs' success story is active corporate governance, i.e., the structural involvement in portfolio firms and the provision of human capital (Gorman and Sahlman, 1989; Casamatta, 2003; Kanniainen and Keuschnigg, 2003). In overseeing business models, sharing valuable insights, and connecting young talents (Sahlman, 1990; de Carvalho *et al.*, 2008), this VC business practice has contributed to the success of some of the most innovative and profitable firms in the past decades, e.g., Amazon, Apple, Google, and Microsoft.

However, active corporate governance by VCs has significantly declined over the past years, implying that VCs are either forced or actively choose to miss out on the associated upsides (Lerner and Nanda, 2020). This development is even more puzzling considering that the scope of VCs heavily clusters around technology sectors – e.g., 39% in the software sector – which are skill and human capital intensive (National Venture Capital Association, 2023).

One potential explanation for this decline could be the increasing funding competition among VCs caused by the proliferation of mega-funds and the entrance of other investor groups. This intense competition might push VCs to offer more "founder-friendly" contracts at the expense of lesser guidance, i.e., less or dual-class stock, and thus reduced corporate governance (Aggarwal *et al.*, 2022). This notion, however, raises the question of why VCs are not choosing to compete by offering higher valuations to entrepreneurs instead of deviating from their "formula for success".

A different explanation may lie in the dropped funding costs in combination with the massive inflow of venture capital from silent, former public market investors (Chernenko *et al.*, 2021). The consequence could be that VCs cannot actively involve themselves in the invested projects – given the large size of their portfolio – and thus change their portfolio strategy as a whole. Indeed, the portfolio selection following a "spray and pray" tactic has become more popular among VCs, i.e., placing bets on many firms and counting that the few successful ones outperform to an extent that generates net profits (Ewens *et al.*, 2018). Nonetheless, studies find substantial heterogeneity in performance across private equity funds of different sizes and scopes, making it unclear what determines the VC's portfolio choice and how this relates to active corporate governance (Kaplan and Schoar, 2005; Lerner *et al.*, 2007; Phalippou and Gottschalg, 2009).

While theoretical literature addresses potential trade-offs VCs face when deciding on their optimal portfolio size and focus, active governance in terms of providing entrepreneurs with human capital – if feasible – is considered beneficial (Hsu, 2004; Kanniainen and Keuschnigg, 2004; Bernile *et al.*, 2007). Besides the direct effect of increasing the chances of venture success, another significant channel through which VCs can profit from providing human capital is the threat of reallocation. According to Fulghieri and Sevilir (2009), VCs can extract large shares of their portfolio firms' rents if these firms compete over limited human capital. The idea is that VCs can focus their portfolio – i.e., invest in ventures with overlapping markets – and use this relatedness to their advantage by threatening to reallocate human capital across ventures. Fulghieri and Sevilir (2009) further state that the threat of reallocation increases with the VC's portfolio size and degree of focus. However, given the prolonged decline in active governance in combination with the trend of large and narrow portfolios, this channel of rent extraction should be in question.

Following the concept of threatening human capital reallocation by holding a portfolio of rivaling firms, I build a financing model to propose a new channel that influences the VC's decision on human capital and may shed some light on the reduction of active governance: *preemptive differentiation*. If the VC builds a portfolio of potential rivals to threaten the entrepreneur with a human capital reallocation, the entrepreneur, in turn, has leeway for self-entrenchment. Similar to preventing foreclosure via pricing or innovation within the industrial organization literature (e.g., Zanchettin and Mukherjee, 2017), the entrepreneur could engage in business practices that create a negative externality on the rivaling portfolio to circumvent a human capital pull-out.[1]

In the model, an entrepreneur (he) requires VC financing for his project, which is either high or low in quality but unknown ex ante. The project generates some final profit and entails a differentiation decision for the entrepreneur – which resembles anti-competitive practices – at an intermediate stage, which is observable but not verifiable. The VC (she), which stands in perfect competition, is endowed with a focused portfolio and has access to limited human capital. In this framework, human capital has two purposes: First, it is used to monitor the project's quality, and second, it can increase the final profit of either the endowed portfolio or the project. The VC then decides whether to hire human capital and offers the entrepreneur a contract specifying (i) a share of the project's profit and (ii) an initial human capital level provided by the VC. However, since (effective) human capital is impossible to measure, and thus to contract over time, both parties know that the VC might shift her human capital from the project towards the endowed portfolio. Here, the essential notion of the VC's

[1] Evidence that early ventures within a VC portfolio significantly impact each other's innovation and firm success is provided by González-Uribe (2020). Related, Azar *et al.* (2018) highlight the relaxing competition effect of common ownership on market outcomes, and thus the trade-off between good governance and the hidden social cost of reduced competition.

focused portfolio comes into play. Due to the relatedness of the project to the portfolio, the entrepreneur's differentiation decision at the intermediate stage significantly affects the portfolio performance. On this account, the VC's final human capital allocation crucially depends on the inflicted externality caused by the differentiation.

In the first-best scenario, i.e., the joint surplus of the project and the endowed portfolio is maximized, the representative VC finances the entrepreneur's project, and a contract with the provision of human capital is signed. No differentiation at the intermediate stage takes place, and if the project is of low quality, the VC reallocates her human capital to boost the portfolio profit. Thus, the VC balances her portfolio performance against the project's return. However, with the introduction of frictions, the entrepreneur can limit the efficiency gains on human capital, thereby creating room for self-entrenchment. If the VC has a significant stake in the project and the negative externality of the differentiation on the rivaling portfolio is substantial, the first-best outcome is not feasible. In this case, a second-best contract is signed where the VC provides human capital, and an entrepreneur with a low-quality project engages in anti-competitive differentiation to preempt the VC's human capital pull-out. Furthermore, if the externality on the endowed portfolio is even more significant and human capital is sufficiently costly, a second-best contract is signed where no human capital is provided ex ante, and thus no active governance occurs in equilibrium.

In the extension, I investigate the persistence of inefficiencies associated with the VC providing human capital in a debt contract setting. Venture debt amplifies the contract friction between the VC and the entrepreneur, as the VC's participation in the entrepreneur's profit is now confined to the ex ante contracted repayment amount. This contract design has implications even for entrepreneurs with high-quality projects, leading to a second-best contract where anti-competitive differentiation occurs in equilibrium for at least one project quality.

This paper contributes to the existing literature by introducing the concept of *preemptive differentiation*, which connects the recent decline in active governance with the growing funding competition between VCs and the consequent loss of control over invested firms. To the best of my knowledge, this is the first paper to elucidate the reduction of active governance by considering intra-portfolio externalities, where the entrepreneur's incentive to engage in anti-competitive practices limits the VC's human capital efficiency.

First, the paper adds to the theoretical literature on the effect of active governance on VCs' portfolio creation. Kanniainen and Keuschnigg (2003) analyze the balance between VCs' active involvement and their portfolio size. In the framework, VCs provide time-costly advice and entrepreneurs determine their effort levels, creating a trade-off between the number of firms in the portfolio and the advisory effort allocated to each. Although diminishing returns to advice create an incentive for expanding the portfolio, rising managerial effort costs reduce the support available to individual firms. As entrepreneurs receive less assistance, they demand a larger profit share, eventually making further portfolio expansion unprofitable. Bernile *et al.* (2007) also establish a venture capital model with double-sided moral hazard due to effort incentives. Here, the VC maximizes the portfolio value by simultaneously choosing the profit-sharing rule between herself and the entrepreneurs and the portfolio size. By trading off larger portfolios against lower values of portfolio companies, the relation between the VC's portfolio size and the profit-sharing rule is non-monotonic: the optimal number of firms is first increasing and then decreasing in the share of the profits retained by entrepreneurs. Incorporating the portfolio's focus, Fulghieri and Sevilir (2009) show that the VC's choice of portfolio size and scope affects both the entrepreneurs' and the VC's incentives to exert effort. A small portfolio improves entrepreneurial incentives by allowing the VC to concentrate the limited human capital on fewer startups, adding more value. A large and focused portfolio is beneficial because it allows the VC to reallocate the limited resources and human capital in the case of startup failure and allows the VC to extract greater rents from the entrepreneurs. Unlike the standard literature, I abstract from any moral hazard issues related to effort incentives and instead focus on the entrepreneur's ability to exploit intra-portfolio competition.

The second connected strand of literature addresses the role of venture capital on innovation and market outcomes. For instance, Bernstein *et al.* (2016) exploit the reduction of travel time caused by new airline routes and find that this reduction leads to increased patent numbers and a higher likelihood of an IPO or acquisition, indicating that VCs' on-site involvement with their portfolio firms is a significant factor for innovation and success. Exploring the innovation exchange within VC portfolios, González-Uribe (2020) suggests that returns to innovation are higher in venture capital portfolios if VCs' bargaining power and potential conflicts of interest are low. Specifically, the data supports three exchange mechanisms inside portfolios: entrepreneurs divest innovation units, start new ventures, and reuse residual assets in other portfolio companies. Closer related to this paper is the study of Hellmann and Puri (2000), which investigates how venture capital relates to start-ups' product market strategies by utilizing a unique hand-collected database of high-tech start-ups in Silicon Valley. The study reveals that innovator firms are more likely to secure venture capital than imitator firms. Additionally, venture capital significantly shortens the time required to bring a product to market. The findings indicate that venture capital financing can shape a start-up's development trajectory, particularly influencing its product market position and strategy.

The paper is structured as follows. After introducing the model in Section 3.2, Subsection 3.2.2 outlines the necessary basic assumptions. In Subsection 3.2.3, I establish the first-best case under the provision of human capital. Section 3.3 addresses the equilibrium analysis, in which Subsection 3.3.1 shows the contract that implements first-best without present frictions. In Subsection 3.3.2, I set up the contract problem when frictions are present and derive its solutions dependent on the human capital provision. Next, Subsection 3.3.3 investigates which contract is signed in equilibrium. Subsection 3.3.4 completes the equilibrium analysis with an overview of all equilibrium contracts. Thereafter, Section 3.4 addresses the persistence of frictions in a debt contract. Finally, I conclude in Section 3.5. Numerical examples showing the existence of the equilibrium equity contracts are deferred to Appendix 3.A.

3.2 The Model

3.2.1 Players & Timing

Consider a model with three periods $t = 0, 1, 2$ and two risk-neutral agents: A cashless entrepreneur and a representative VC standing in perfect competition. The interest rate is normalized to $r = 0$.

At $t = 0$, the entrepreneur owns a project of ex ante unknown quality $i \in \{l, h\}$. Let the probability of owning a high-quality project be $p > 0$. To start the project at $t = 0$, the cashless entrepreneur requires an investment $I > 0$ from the VC. If the investment is realized, the project entails a non-verifiable differentiation decision $d_i \in \{0, 1\}$ at the beginning of $t = 1$. For simplicity, suppose this differentiation process bears no immediate costs but only impacts final profits. Specifically, the project generates the return θ_d^i with $\theta_d^h > \theta_d^l > 0$ and $\theta_0^i > \theta_1^i > 0 \;\forall\; d, i$ at $t = 2$, i.e., the project's return strictly increases with quality and decreases in the degree of differentiation.[2] Intuitively, the entrepreneur's decision on d_i can be seen as an anti-competitive business practice that comes at the expense of lower profits, e.g., market segmentation or price dumping.[3] Let the project's expected profit for a given differentiation d_i denote

$$\mu_d := p\theta_d^h + (1 - p)\theta_d^l. \tag{3.1}$$

If the investment is not realized, the entrepreneur cannot proceed with his project and receives his outside option $\underline{\mathrm{U}} = 0$.

Initially, the VC has access to limited human capital $\bar{L}$, which she can employ

[2]To cut back on the notation, the sub-subscript i connected to the differentiation d is dropped unless it requires explicit mention.

[3]Note that modeling the specific market outcome at $t = 2$ is outside the scope of this paper.

at a cost $c(L) > 0$. Furthermore, the VC is endowed with a portfolio of firms that compete with the entrepreneur's project for human capital and final profits, where the portfolio generates profits Π_d at $t = 2$. The assumption that the portfolio consists of competing firms links directly to the VC's ability to feasibly supply and reallocate human capital. Considering the VC investment focus on high-tech sectors and in line with Fulghieri and Sevilir (2009), (effective) human capital and its reallocation across ventures requires know-how, and thus firm overlap in market and product specifics.[4] However, this portfolio focus enables the entrepreneur to engage in anti-competitive business practices that affect the competing portfolio performance, where $\Pi_0 > \Pi_1$.

At $t = 0$, the VC can offer the entrepreneur an equity contract (α_L, L) in return for investment I that defines (i) an initial human capital level $L \in \{0, \bar{L}\}$ provided to the project and (ii) a share-based payment α of the project. Thus, the VC decides ex ante whether to finance the project, whether to employ human capital and, if so, its allocation between the endowed portfolio and the project. Human capital serves two purposes for the VC: Identifying the project quality and increasing final profits. Since the VC uses human capital for monitoring, note that in this framework, her choice is limited to an "all or nothing" human capital provision at $t = 0$. The argument is that a critical mass of human capital is necessary for monitoring, as otherwise, the VC could use an arbitrarily low amount to identify project qualities early.

If employed and allocated to the project at $t = 0$, the deployed human capital identifies the project's quality i for both the VC and the entrepreneur to observe at the beginning of $t = 1$. After the entrepreneur's differentiation decision d_i and before profits are realized, the VC can reallocate her human capital from the project to the portfolio, thus deviating from the contracted initial splitting. Following Fulghieri and Sevilir (2009), it is impossible to perfectly contract the VC's involvement and human capital provision throughout the project lifespan, not to mention under which preconditions these contributions would sustain or their effective quality. Therefore, let $\hat{L}_{i,d} \in \{0, L\}$ be the final human capital level provided to the entrepreneur at the end of $t = 1$ and let the project's profit θ_d^i be increased by $(1 + \hat{L}_{i,d})$ at $t = 2$. Shifting human capital from the entrepreneur towards the endowed portfolio increases the portfolio profits Π_d by $(1 + L - \hat{L}_{i,d})$.

The timeline of the model, particularly the players' actions during the three periods and the associated profits, is depicted in Figure 3.1.

[4]Evidence that fund management teams with more industry-specific human capital have a higher fraction of portfolio company exits is provided by Zarutskie (2010).

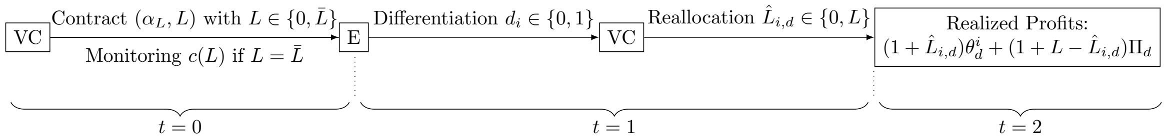

Figure 3.1: Timeline of the model.

3.2.2 Basic Assumptions

First, assume that the project has a positive net present value with

$$\mu_0 \geq I, \tag{3.2}$$

if the entrepreneur abstains from the anti-competitive differentiation. To add bite to the human capital decision of the VC, assume further that the initial provision of human capital to the endowed portfolio is not optimal where

$$c(L) > L\Pi_0. \tag{3.3}$$

Third, the entrepreneur must be incentivized to keep the human capital at his project despite suffering losses from anti-competitive practices. Thus, suppose that the return of the differentiated project with human capital exceeds the return of the undifferentiated project without human capital, i.e.,

$$(1 + L)\theta_1^i > \theta_0^i \;\forall\; i \in \{l, h\}. \tag{3.4}$$

Lastly, it is crucial to the model that the VC's overall investments are sufficiently related, such that the entrepreneur can exert a significant externality on the rivaling portfolio. On this account, assume that

$$\Pi_0 > \theta_0^l > \theta_1^l > \Pi_1, \tag{3.5}$$

where the negative effect of differentiation on the rivaling portfolio is larger than on the entrepreneur's project return.

3.2.3 First-best

In the first-best scenario, the joint surplus of the project and the endowed rivaling portfolio is maximized, which is equivalent to the VC owning the project and deciding on the differentiation d_i at the beginning of $t = 1$.

Suppose the VC invests in the project without providing human capital at $t = 0$. In that case, information about the project's quality i is only revealed once the final profit of the project is realized at $t = 2$. Furthermore, recall that in this framework, the differentiation decision on the project is purely anti-competitive: the entrepreneur is willing to suffer project losses to decrease the VC's endowed portfolio performance and thereby impact the final human capital reallocation indirectly. With no human capital provided and no early revelation of the project's quality, it is straightforward that the efficient differentiation decision is $d^{FB} = 0$. The expected surplus of the project and the endowed portfolio at $t = 0$ amounts to

$$\mu_0 + \Pi_0 - I. \tag{3.6}$$

On the other hand, suppose that the VC invests in the project and provides initial human capital $L = \bar{L}$ at $t = 0$. In that case, the project quality is identified early, and the VC may reallocate her human capital at the end of $t = 1$. Since the efficient differentiation decision remains $d_i^{FB} = 0 \;\forall\; i \in \{l, h\}$, the expected surplus generated by the project and the endowed portfolio at $t = 0$ is

$$\begin{aligned} p\left[(1 + \hat{L}_{h,0}^{FB})\theta_0^h + (1 + L - \hat{L}_{h,0}^{FB})\Pi_0\right] \\ + (1 - p)\left[(1 + \hat{L}_{l,0}^{FB})\theta_0^l + (1 + L - \hat{L}_{l,0}^{FB})\Pi_0\right] - c(L) - I, \end{aligned} \tag{3.7}$$

where $\hat{L}_{i,0}^{FB}$ denotes the project's optimal final human capital level, conditional on its quality. Recalling that $\Pi_0 > \theta_0^l$, it is self-evident that the first-best final human capital allocation denotes

$$\hat{L}_{i,0}^{FB} = \begin{cases} L\,, & \text{if } i = h, \\ 0\,, & \text{otherwise;} \end{cases} \tag{3.8}$$

such that (3.7) simplifies to

$$L\left[p\theta_0^h + (1 - p)\Pi_0\right] + \mu_0 + \Pi_0 - c(L) - I. \tag{3.9}$$

Accordingly, from a welfare optimal perspective the VC employs human capital $L = \bar{L}$ at $t = 0$ if and only if

$$L\left[p\theta_0^h + (1 - p)\Pi_0\right] \geq c(L). \tag{3.10}$$

Thus, the first-best outcome can be characterized as follows.

Observation 1 (First-best)**.** *If the project is feasible with $\mu_0 \geq I$, the representative VC finances the project, and the efficient differentiation decision is $d_i^{FB} = 0$. Furthermore, the first-best outcome is characterized by*

(i) no provision of human capital with $L = 0$ if Condition (3.10) is not satisfied,

(ii) early provision of human capital to the project with $L = \bar{L}$ if Condition (3.10) is satisfied. In addition, the project's final human capital level denotes

$$\hat{L}_{i,0}^{FB} = \begin{cases} L\ , & \text{if } i = h, \\ 0\ , & \text{otherwise}; \end{cases}$$

i.e., the shift of human capital to the rivaling portfolio occurs if the project's quality is low.

Observation 1 highlights the benefits of human capital, i.e., the VC's rationale to hire and provide human capital to the entrepreneur's project at $t = 0$. Specifically, providing human capital lets the VC verify the project's quality early, enabling its potential reallocation at the end of $t = 1$. As a result, human capital shifts to the endowed portfolio if quality l is realized and remains with the project otherwise. Formally, Condition (3.10) states that human capital is optimal when its profit-enhancing benefit conditional on the first-best reallocation decision $\hat{L}_{i,0}^{FB}$ exceeds its hiring cost. Accounting for the costly human capital constraint in (3.3), Condition (3.10) holds for sufficiently large θ_0^h which seems reasonable in the context of highly profitable "unicorn" startups.

For the remainder of the analysis, suppose that Condition (3.10) indeed holds.

3.3 Equilibrium Analysis

3.3.1 No Frictions

To analyze if and how the first-best outcome is achievable, suppose for now that the entrepreneur can (credibly) commit not to engage in anti-competitive practices against the VC's endowed portfolio, $d_i^N = 0$, and the VC can (credibly) commit only to reallocate if the project quality is low. Another interpretation is that the VC can exert control over the entrepreneur's project differentiation if l is realized. In this case, the VC's contract problem is solved analogously to the first-best scenario.

On the one hand, the VC and the entrepreneur can sign the contract $(\alpha_0, L = 0)$ at

$t = 0$. With the VC standing in perfect competition, the VC's outside option constraint

$$\alpha_0 \mu_0 + \Pi_0 - I \geq \Pi_0 \tag{3.11}$$

binds, where

$$\alpha_0 \geq \frac{I}{\mu_0} := \alpha_0^N \tag{3.12}$$

denotes the VC's project share when no human capital is provided and there is no friction present.

On the other hand, the VC and the entrepreneur can sign the contract $(\alpha_{\bar{L}}, L = \bar{L})$ that is implicitly defined by the VC's outside option constraint

$$\alpha_{\bar{L}} \left[p(1 + L)\theta_0^h + (1 - p)\theta_0^l \right] + p\Pi_0 + (1 - p)(1 + L)\Pi_0 - c(L) - I \geq \Pi_0 \tag{3.13}$$

binding, where

$$\alpha_{\bar{L}} \geq \frac{I + c(L) - (1 - p)L\Pi_0}{\mu_0 + pL\theta_0^h} := \alpha_{\bar{L}}^N \tag{3.14}$$

denotes the VC's project share when human capital is provided and there is no friction present.[5]

Since the entrepreneur retains $(1-\alpha_L)$ shares of his project, the contract $(\alpha_{\bar{L}}^N, L = \bar{L})$ implementing the first-best outcome is signed in equilibrium if and only if

$$(1 - \alpha_{\bar{L}}^N) \left[(1 + L)p\theta_0^h + (1 - p)\theta_0^l \right] \geq (1 - \alpha_0^N)\mu_0. \tag{3.15}$$

Simplifying gives

$$L \left[p\theta_0^h + (1 - p)\Pi_0 \right] \geq c(L), \tag{3.16}$$

which is equivalent to Condition (3.10) from the first-best case and holds by assumption. Therefore, the entrepreneur and the VC indeed sign the contract $(\alpha_{\bar{L}}^N, L = \bar{L})$ – i.e., human capital is agreed upon – and the first-best outcome is implemented in equilibrium.

[5] Note that even if the VC can not credibly commit to sticking to the initial human capital level, the case where the project is of quality h and the VC reallocates, $\hat{L}_{h,0}^N = 0$, can be ruled out. The reason is that the entrepreneur would anticipate bearing the cost of human capital without reaping its benefit, making the provision of human capital unfeasible ex ante.

3.3.2 Contract Problem

With the inclusion of frictions to the model – i.e., the VC can neither credibly commit to the final human capital level nor exert control over the project's differentiation – the VC now faces the following contract problem.

First, the VC and the entrepreneur can sign a contract without the provision of human capital at $t = 0$. Since the occurring frictions are only relevant in case the VC supplies the entrepreneur with human capital, it is straightforward that the VC's contract offer $(\alpha_0, L = 0)$ is equivalent to the frictionless case such that

$$\alpha_0^N = \frac{I}{\mu_0} := \alpha_0^*. \tag{3.17}$$

Note, however, that the absence of human capital provision is clearly inefficient from an ex ante perspective considering the first-best case.

To solve for the VC's contract offer $(\alpha_{\bar{L}}, L = \bar{L})$ where the entrepreneur receives $L = \bar{L}$ human capital at $t = 0$, I use backward induction.

Human Capital Reallocation

At the end of $t = 1$, the VC knows the project quality i and observes the differentiation decision d_i of the entrepreneur. Consequently, the VC may find it optimal to deviate from the project's initially contracted human capital level $L = \bar{L}$ and instead focus her human capital on the endowed portfolio. Formally, the VC chooses the project's final human capital $\hat{L}_{i,d}$ to maximize her profit at $t = 2$

$$\max_{\hat{L}_{i,d} \in \{0, L\}} \alpha_{\bar{L}}(1 + \hat{L}_{i,d})\theta_d^i + (1 + L - \hat{L}_{i,d})\Pi_d. \tag{3.18}$$

Thus, the VC decides not to reallocate her human capital and set $\hat{L} = L$ if and only if

$$\alpha_{\bar{L}}\theta_d^i \geq \Pi_d, \tag{3.19}$$

where the VC maintains the initial contracted human capital allocation if her share of the project's profit is weakly larger than the rivaling portfolio return. Note that from Condition (3.5), it follows that $\frac{\Pi_0}{\theta_0^l} > 1$, such that the VC shifts her human capital to the rivaling portfolio if no differentiation occurs at the beginning of $t = 1$.

In order to focus the attention on the non-trivial cases, only consider equilibria with the following property.

Property 1. *In equilibrium, the signed debt contract* $(\alpha_{\bar{L}}, L = \bar{L})$ *stipulates a share* $\alpha_{\bar{L}} \in (0, 1)$ *that satisfies*

$$\alpha_{\bar{L}} \geq \frac{\Pi_1}{\theta_1^l}. \tag{3.20}$$

Property 1 states that if the project is of low quality and the entrepreneur differentiates at the beginning of $t = 1$, in equilibrium, the VC's return on the project is larger than the return on her portfolio. Intuitively, this captures the premise that VCs demand significant shares of early ventures for their investment and other non-financial benefits like active governance.[6] Moreover, Property 1 implies that the VC's return on a high-quality project exceeds the return on her portfolio, irrespective of the entrepreneur's differentiation decision. To see that, recall that $\theta_d^h > \theta_d^l > 0$ and $\theta_0^i > \theta_1^i > 0 \; \forall \; d, i$; thus the VC chooses $\hat{L}_{h,d} = L$ regardless of d_h.

The optimal reallocation decision of the VC is defined as follows.

Proposition 1 (Reallocation Decision)**.** *Suppose the representative VC and the entrepreneur signed a contract* $(\alpha_{\bar{L}}, L = \bar{L})$ *at* $t = 0$*. Under Property 1, the final human capital level allocated to the project at the end of* $t = 1$ *denotes*

$$\hat{L}_{i,d}^* = \begin{cases} L \,, & \text{if } i = h \vee d_i = 1, \\ 0 \,, & \text{otherwise.} \end{cases} \tag{3.21}$$

Proposition 1 states that if the project is of low quality and the entrepreneur does not differentiate at the beginning of $t = 1$, the VC optimizes her human capital by shifting it to the rivaling portfolio. However, suppose the entrepreneur owns a project of low quality and engages in anti-competitive business practices with $d_l = 1$. In that case, the inflicted externality on the endowed portfolio is sufficiently large that the VC is better off sticking to the project's initially contracted human capital level. On the other hand, if the project is of high quality, the VC keeps her human capital at the project regardless of the entrepreneur's differentiation decision.

Differentiation Decision

At the beginning of $t = 1$, the entrepreneur learns about his project quality i. Anticipating the human capital reallocation decision $\hat{L}_{i,d}^*$ of the VC at the end of $t = 1$, the

[6]The equilibrium overview in Subsection 3.3.4 outlines the conditions for Property 1 to hold. Furthermore, Appendix 3.A shows the existence of the equity contract $(\alpha_{\bar{L}}, L = \bar{L})$ that satisfies Property 1 with a numerical example.

entrepreneur decides on the differentiation d_i that maximizes his expected payoff

$$\max_{d_i \in \{0,1\}} (1-\alpha_{\bar{L}})(1+\hat{L}^*_{i,d})\theta^i_d. \tag{3.22}$$

If the project is of high quality, the entrepreneur knows that the VC will not change the initial contracted human capital level L. Considering that the differentiation comes at the expense of lesser profit, $\theta^h_0 > \theta^h_1$, the entrepreneur has no incentive to engage in anti-competitive practices and decides not to differentiate with $d^*_h = 0$.

However, suppose the project is of quality l. In that case, the entrepreneur knows that the VC finds it optimal to pull her human capital at the end of $t = 1$ and instead use it on her endowed portfolio. To preempt this reallocation incentive of the VC, the entrepreneur can decide to differentiate with $d_l = 1$ at the beginning of $t = 1$ and cause sufficient harm to the rivaling portfolio profit. Facing the trade-off between the benefit of human capital and the harm of the business practice on his final profit, the entrepreneur differentiates with $d^*_l = 1$ if and only if

$$(1-\alpha_{\bar{L}})(1+\hat{L}^*_{l,1})\theta^l_1 \geq (1-\alpha_{\bar{L}})(1+\hat{L}^*_{l,0})\theta^l_0, \tag{3.23}$$

which simplifies to

$$(1+L)\theta^l_1 \geq \theta^l_0. \tag{3.24}$$

Since this is equivalent to Condition (3.4) and holds by assumption, the optimal differentiation decision of the entrepreneur is defined by the following.

Proposition 2 (Differentiation Decision)**.** *Suppose the representative VC and the entrepreneur signed a contract* $(\alpha_{\bar{L}}, L = \bar{L})$ *at* $t = 0$*. Under Property 1, the differentiation decision of the entrepreneur at the beginning of* $t = 1$ *denotes*

$$d^*_i = \begin{cases} 1\ , & \textit{if } i = l, \\ 0\ , & \textit{otherwise.} \end{cases} \tag{3.25}$$

Proposition 2 states that if the entrepreneur owns a project of high quality, he decides not to differentiate. However, suppose the entrepreneur owns a project of low quality. In that case, the entrepreneur engages in the anti-competitive business practice to lower the rivaling portfolio performance, thereby preventing the VC's human capital reallocation at the end of $t = 1$.

Contract with Human Capital

At $t = 0$, the VC anticipates the entrepreneur's differentiation decision d_i^* and her best-response human capital reallocation $\hat{L}_{i,d}^*$ at $t = 1$. While the entrepreneur with project quality h decides optimally from the VC's perspective with $d_h^* = 0$, the entrepreneur with project quality l differentiates with $d_l^* = 1$ to make the human capital reallocation towards the rivaling portfolio unfeasible. On this account, the VC offers the entrepreneur the contract $(\alpha_{\bar{L}}, L = \bar{L})$, implicitly defined by the VC's binding outside option constraint

$$\alpha_{\bar{L}}(1 + L)\mu_{d^*} + \mathbb{E}\left[\Pi_d \,\middle|\, d_i^*\right] - I - c(L) \geq \Pi_0, \tag{3.26}$$

such that

$$\alpha_{\bar{L}} \geq \frac{I + c(L) + (1 - p)(\Pi_0 - \Pi_1)}{(1 + L)[p\theta_0^h + (1 - p)\theta_1^l]} := \alpha_{\bar{L}}^* \tag{3.27}$$

denotes the VC's project share when human capital is provided. While standard literature proposes the threat of reallocation as a VC's tool for rent extraction, here, this commitment issue comes at the cost of the harmful preemptive differentiation of the entrepreneur. Therefore, the demanded share in equilibrium $\alpha_{\bar{L}}^*$ reflects the VC's compensation for the negative externality she might incur on her endowed portfolio. Note that $\alpha_{\bar{L}}^*$ increases with the probability of the entrepreneur owning the low-quality project, $(1 - p)$, increases with the externality magnitude on the rivaling portfolio, $(\Pi_0 - \Pi_1)$, and decreases with the profit enhancement of human capital, $(1 + L)$.

After establishing the solution to the contract problem in case the VC offers human capital, consider now the two potential contracts signed by the entrepreneur at $t = 0$.

Proposition 3 (Contracts)**.** *Suppose the project is feasible with $\mu_0 \geq I$ and Property 1 holds. Then, the entrepreneur receives investment I for his project and signs either*

(i) the contract $(\alpha_0^*, L = 0)$ at $t = 0$, where the representative VC does not provide human capital and demands the share

$$\alpha_0^* = \frac{I}{\mu_0};$$

or

(ii) the contract $(\alpha_{\bar{L}}^*, L = \bar{L})$ at $t = 0$, where the representative VC provides human

capital and demands the share

$$\alpha_{\bar{L}}^* = \frac{I + c(L) + (1-p)(\Pi_0 - \Pi_1)}{(1+L)[p\theta_0^h + (1-p)\theta_1^l]}.$$

Proposition 3 highlights the compensation for the VC in light of the arising inefficiencies when the VC can neither credibly commit to the final human capital level nor exert control over the project's differentiation. Suppose the contract $(\alpha_0^*, L = 0)$ is signed. In that case, there is the ex ante inefficiency of not providing human capital to the project. This inefficiency translates into missing out on the human capital's monitoring and profit-enhancing properties despite being welfare-optimal. On the other hand, suppose that the contract $(\alpha_{\bar{L}}^*, L = \bar{L})$ is signed. Then, there are two ex post inefficiencies associated with a low-quality project that need to be compensated for ex ante: First, the anti-competitive business practice by the entrepreneur, and second, the misallocation of human capital by the VC, both leading to lower expected profits.

3.3.3 Optimal Contract

Consider the entrepreneur's expected payoff at $t = 0$ to determine which contract occurs in equilibrium. The entrepreneur prefers VC financing with the provision of human capital over VC financing without the provision of human capital if and only if

$$(1-\alpha_{\bar{L}}^*)(1+L)\left[p\theta_0^h + (1-p)\theta_1^l\right] \geq (1-\alpha_0^*)\mu_0 \tag{3.28}$$

which is equivalent to

$$L[p\theta_0^h + (1-p)\theta_1^l] - c(L) \geq (1-p)(\theta_0^l - \theta_1^l + \Pi_0 - \Pi_1). \tag{3.29}$$

Since the VC stands in perfect competition, Condition (3.29) shows the total surplus differences – which are extracted by the entrepreneur – associated with the two contracts. The right-hand side reflects that without the provision of human capital, the signed contract $(\alpha_0^*, L = 0)$ induces the efficient differentiation decision irrespective of the entrepreneur's project quality, $d_i^* = 0$, and therefore has an advantage over the contract $(\alpha_{\bar{L}}^*, L = \bar{L})$ regarding the payoffs conditional on the probability $(1-p)$. In contrast, the left-hand side shows the benefits of human capital from the VC, i.e., the payoff-enhancing effect conditional on the potential anti-competitive differentiation decision of the entrepreneur with $d_l^* = 1$.

Corollary 1 (Optimal Contract)**.** *Suppose the project is feasible with $\mu_0 \geq I$ and Property 1 holds. Then, the representative VC and entrepreneur sign the contract*

$(\alpha^*_{\bar{L}}, L = \bar{L})$ *at* $t = 0$*, and hence human capital is provided in equilibrium if and only if*

$$L[p\theta_0^h + (1-p)\theta_1^l] - c(L) \geq (1-p)(\theta_0^l - \theta_1^l + \Pi_0 - \Pi_1).$$

Notably, the equilibrium contract $(\alpha^*_0, L = 0)$ becomes more likely with a higher probability of encountering low-quality projects and with a more significant negative differentiation externality due to firm relatedness. This result shows the VC's limitation of efficiently using her human capital – by holding a portfolio of rival firms to enable reallocation – due to the associated countervailing effect of preemptive differentiation. Furthermore, this result can relate to VCs' observed "spray and pray" portfolio tactic since monitoring, and thus the likelihood of high-quality projects plays only a subordinate role. While this intuition starkly contrasts with Fulghieri and Sevilir (2009) regarding portfolio size and focus, it coincides with the observed VC industry trends (Ewens *et al.*, 2018; Lerner and Nanda, 2020).

3.3.4 Equilibrium Overview

Now, I analyze under what preconditions which VC contract occurs in equilibrium. Figure 3.2 depicts the findings, where investment I is on the horizontal axis and the human capital cost $c(L)$ is on the vertical axis.

First, recall that Condition (3.2) holds by assumption, i.e., $I \leq \mu_0$, such that VC financing without providing human capital is always feasible.

Second, the VC's provision of human capital is only welfare-optimal if Condition (3.10) holds, i.e., if the cost of human capital is sufficiently small. The critical threshold value for the human capital cost under first-best denotes

$$c(L) \leq L\left[p\theta_0^h + (1-p)\Pi_0\right] := \bar{c}^{FB}. \tag{3.30}$$

However, the VC only offers the first-best contract $(\alpha^N_{\bar{L}}, L = \bar{L})$ if her share compensation is not too large, i.e., if Property 1 does not hold and $\alpha^N_{\bar{L}} \leq \frac{\Pi_1}{\theta_1^l}$ is satisfied. Rewriting this condition, the first-best contract stipulating the human capital provision is only signed for projects that require an investment I below the critical threshold

$$I \leq \frac{\Pi_1}{\theta_1^l}\left[\mu_0 + pL\theta_0^h\right] + (1-p)L\Pi_0 - c(L) := \bar{I}^{FB}. \tag{3.31}$$

Note that whether the critical threshold $\bar{I}^{FB}$ is smaller or larger than μ_0 is not apparent a priori. In the following, suppose that:

Assumption 1. *The externality ratio* $\frac{\Pi_1}{\theta_1^l}$ *is sufficiently small such that*

$$\bar{I}^{FB} < \mu_0. \tag{3.32}$$

In case the entrepreneur requires higher initial investment and $c(L) < \bar{c}^{FB}$ holds, the VC has too much "skin in the game" that the entrepreneur can exploit. Thus, the VC offers either the second-best contract with human capital, $(\alpha^*_{\bar{L}}, L = \bar{L})$ or the second-best contract without human capital, $(\alpha^*_0, L = 0)$. Rearranging the expected profit condition over the two potential contracts in (3.29), the second-best contract with the provision of human capital is signed if $c(L) \leq \bar{c}^{SB}$, where

$$c(L) \leq L\left[p\theta_0^h + (1-p)\theta_1^l\right] - (1-p)(\theta_0^l - \theta_1^l + \Pi_0 - \Pi_1) := \bar{c}^{SB} \tag{3.33}$$

is the critical threshold value for the human capital cost under second-best.

The equilibrium contracts can be summarized as follows.

Proposition 4 (Equilibrium Contract)**.** *Suppose the project is feasible with* $\mu_0 \geq I$ *and Assumption 1 holds. Then, the equilibrium contract at* $t = 0$

(i) provides no human capital to the project, $L = 0$*, if and only if*

$$c > \begin{cases} \bar{c}^{FB}, & \text{for } I \leq \bar{I}^{FB}, \\ \bar{c}^{SB}, & \text{for } I \in (\bar{I}^{FB}, \mu_0]; \end{cases}$$

(ii) provides (initial) human capital to the project, $L = \bar{L}$*, if and only if*

$$c \leq \begin{cases} \bar{c}^{FB}, & \text{for } I \leq \bar{I}^{FB}, \\ \bar{c}^{SB}, & \text{for } I \in (\bar{I}^{FB}, \mu_0]. \end{cases}$$

As shown in Figure 3.2, any project receives the VC's funding with the project-feasibility Condition (3.2) in play. If human capital cost is high, $c \geq \bar{c}^{FB}$, the VC offers the (efficient) contract without the provision of human capital. If human capital cost is medium or low, the VC offers the first-best contract $(\alpha^N_{\bar{L}}, L = \bar{L})$ for sufficiently small investment levels, $I \leq \bar{I}^{FB}$. If the project requires a larger investment, the VC offers the second-best contract with the supply of human capital if its cost is low, $c \leq \bar{c}^{SB}$. In this case, first-best VC funding with human capital would be efficient but is not implementable due to the entrepreneur's incentive for anti-competitive practices to prevent a human capital reallocation. Finally, for investments $I \in (\bar{I}^{FB}, \mu_0]$ and human capital cost $c \in (\bar{c}^{SB}, \bar{c}^{FB})$, VC financing with the provision of human capital

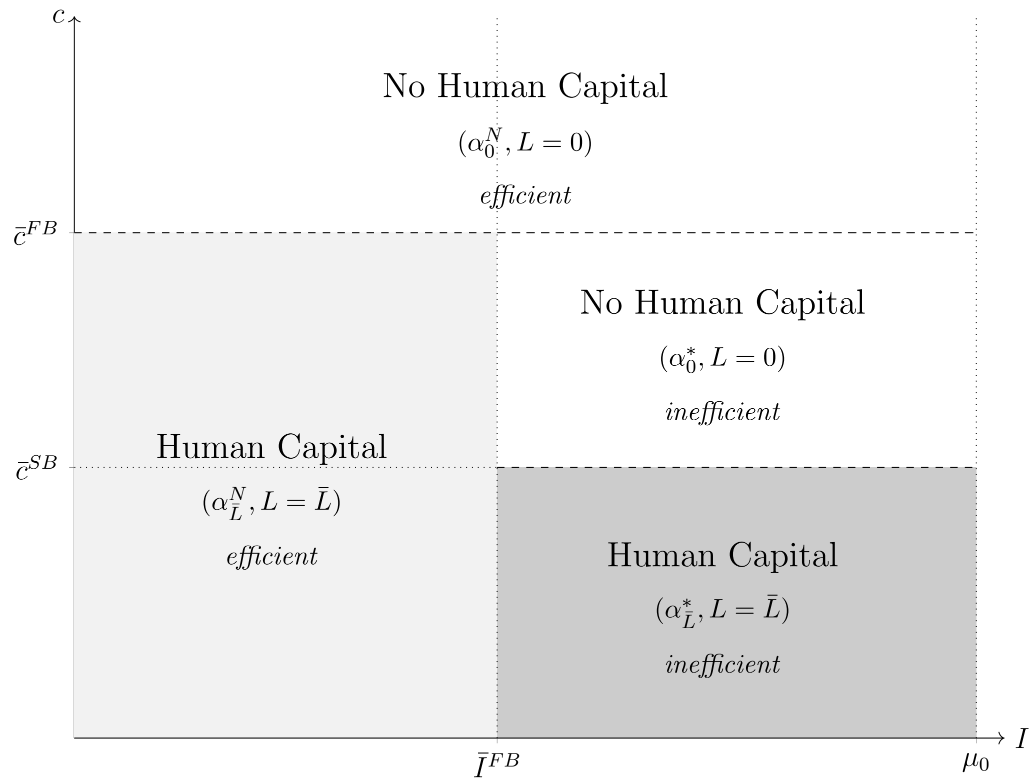

Figure 3.2: Equilibrium contracts and efficiency.

is efficient. However, the externality of the differentiation on the endowed portfolio is too large such that the second-best contract without the provision of human capital is signed in equilibrium.[7]

In summary, three inefficiencies may occur in equilibrium: On the one hand, the two associated ex post inefficiencies of the anti-competitive differentiation in combination with the forced misallocation of human capital. On the other hand, the ex ante inefficiency of not providing the entrepreneur with human capital despite its capability of monitoring and profit-enhancing being welfare optimal.

3.4 Extension: Debt Contract

After analyzing the equilibria that can arise with a share-based contract, this section investigates whether venture debt can mitigate inefficiencies. Suppose the representative VC now offers the entrepreneur a debt contract (R_L, L), where R_L is the repayment

[7]Appendix 3.A shows the existence of the equilibria for equity contract (α_L, L) via numerical examples.

made by the entrepreneur at $t = 2$. To keep the analysis tractable, we restrict our attentions to repayments with the following property:

Property 2. *Under any signed debt contract* (R_L, L)*, the entrepreneur can repay* R_L *if he owns a project of high quality and does not engage in anti-competitive differentiation, i.e.,*

$$R_L \leq \theta_0^h. \tag{3.34}$$

Under Property 2, the threat of reallocation is credible in all states of the world, i.e., the entrepreneur may not require human capital to repay the debt claim owed to the VC.

First, consider the scenario where the VC and the entrepreneur sign the contract $(R_0, L = 0)$ at $t = 0$, and thus no human capital is provided. Due to perfect competition, the VC's outside option constraint

$$p \min\{R_0, \theta_{d^*}^h\} + (1-p) \min\{R_0, \theta_{d^*}^l\} - I + p\Pi_{d^*} + (1-p)\Pi_{d^*} \geq \Pi_0 \tag{3.35}$$

binds, implicitly defining R_0. Analogous to the equity case, the entrepreneur – irrespective of his project's quality – has no incentive to differentiate, i.e., $d_i^* = 0$. Furthermore, Property 2 states that $R_0 \leq \theta_0^h$ must hold, not to mention that setting up a contract with a repayment the entrepreneur can never fulfill is unreasonable. Thus, the constraint in (3.35) simplifies to

$$pR_0 + (1-p) \min\{R_0, \theta_0^l\} - I + \Pi_0 \geq \Pi_0, \tag{3.36}$$

where

$$R_0 \geq \frac{I - (1-p) \min\{R_0, \theta_0^l\}}{p} := R_0^* \tag{3.37}$$

is the repayment contracted in equilibrium. Accordingly, the repayment is either $R_0^* = I$ if no uncertainty about the entrepreneur's ability to repay exists and $R_0^* = \frac{I-(1-p)\theta_0^l}{p}$ otherwise.

To solve for the VC's contract offer $(R_{\bar{L}}, L = \bar{L})$ where the entrepreneur receives $L = \bar{L}$ human capital at $t = 0$, I use backward induction.

Human Capital Reallocation

Suppose the VC and the entrepreneur signed the contract $(R_{\bar{L}}, L = \bar{L})$ at $t = 0$, with the entrepreneur receiving human capital initially. At the end of $t = 1$, the VC decides on the final human capital allocation that maximizes her expected profit

$$\max_{\hat{L}_{i,d} \in \{0,L\}} \quad \min\{(1 + \hat{L}_{i,d})\theta_d^i \,, R_{\bar{L}}\} + (1 + L - \hat{L}_{i,d})\Pi_d, \tag{3.38}$$

given the observed project quality i and the entrepreneur's differentiation decision d_i. Depending on the size of the investment I, three distinct cases need addressing: ex post, (i) the repayment is certain where $R_{\bar{L}} \leq \theta_d^i$, (ii) the repayment cannot be fulfilled where $R_{\bar{L}} > (1+L)\theta_d^i$, and (iii) the fulfillment of the repayment depends on the human capital decision where $(1 + L)\theta_d^i \geq R_{\bar{L}} > \theta_d^i$.

First, for $R_{\bar{L}} \leq \theta_d^i$, the entrepreneur can always repay the contracted repayment to the VC. In this case, the VC does not participate in any of the surplus generated by human capital if it remains at the entrepreneur's project. Thus, the VC shifts her human capital toward the endowed portfolio irrespective of the project's quality or differentiation, such that $\hat{L}_{i,d}^* = 0$.

Second, for $R_{\bar{L}} > (1+L)\theta_d^i$, the entrepreneur can never repay the contracted $R_{\bar{L}}$ to the VC. In this case, the VC decides to stick with the initial human capital allocation if and only if

$$(1 + L)\theta_d^i + \Pi_d \geq \theta_d^i + (1 + L)\Pi_d, \tag{3.39}$$

which simplifies to

$$\theta_d^i \geq \Pi_d. \tag{3.40}$$

Intuitively, the VC decides between the additional profits she would obtain from shifting her human capital to the endowed portfolio and the reduced losses suffered on the repayment. Similar to Proposition 1 under the equity contract, the VC's optimal human capital level provided to the project at the end of $t = 1$ denotes

$$\hat{L}_{i,d}^* = \begin{cases} L\,, & \text{if } i = h \vee d = 1, \\ 0\,, & \text{otherwise;} \end{cases} \tag{3.41}$$

since the scaling of the project exceeds that of the endowed portfolio for quality h or differentiation $d_i = 1$.[8]

[8]Note that a contract with repayment $R_{\bar{L}} > (1 + L)\theta_0^h$ is not feasible, and thus never signed ex

Finally, for $(1+L)\theta_d^i \geq R_{\bar{L}} > \theta_d^i$, the entrepreneur can only repay $R_{\bar{L}}$ and realize net profits if the human capital stays with his project. In this case, the VC decides to keep the initial human capital level on the project if and only if

$$R_{\bar{L}} + \Pi_d \geq \theta_d^i + (1+L)\Pi_d, \tag{3.42}$$

which is

$$R_{\bar{L}} \geq \theta_d^i + L\Pi_d. \tag{3.43}$$

Thus, the VC faces the decision whether to receive the full repayment $R_{\bar{L}}$ or rather incur losses on the debt repayment but receive higher profits on the endowed portfolio.

The VC's final human capital allocation can be summarized as follows.

Proposition 5 (Debt – Reallocation Decision)**.** *Suppose the representative VC and the entrepreneur signed a debt contract* $(R_{\bar{L}}, L = \bar{L})$ *at* $t = 0$. *At the end of* $t = 1$, *the final human capital level allocated to the project*

(i) for $R_{\bar{L}} \leq \theta_d^i$ *denotes*

$$\hat{L}_{i,d}^* = 0; \tag{3.44}$$

(ii) for $R_{\bar{L}} > (1+L)\theta_d^i$ *denotes*

$$\hat{L}_{i,d}^* = \begin{cases} L\,, & \text{if } i = h \vee d_i = 1, \\ 0\,, & \text{otherwise}; \end{cases} \tag{3.45}$$

(iii) for $(1+L)\theta_d^i \geq R_{\bar{L}} > \theta_d^i$ *denotes*

$$\hat{L}_{i,d}^* = \begin{cases} L\,, & \text{if } R_{\bar{L}} \geq \theta_d^i + L\Pi_d, \\ 0\,, & \text{otherwise.} \end{cases} \tag{3.46}$$

Differentiation Decision

At the beginning of $t = 1$, the entrepreneur anticipates the human capital reallocation decision of the VC, $\hat{L}_{i,d}^*$, and decides on the differentiation d_i that maximizes

$$\max_{d_i \in \{0,1\}} \quad \max\{(1+\hat{L}_{i,d}^*)\theta_d^i - R_{\bar{L}}\,, 0\}. \tag{3.47}$$

ante.

Considering Proposition 5, note that the entrepreneur is (ex ante) weakly better off choosing $d_i^* = 0$ if he would face cases (i) or (ii) at the end of $t = 1$ when instead choosing $d_i^* = 1$. In the former case, the entrepreneur can not hold onto the supplied human capital irrespective of his differentiation decision, and thus is strictly better off not engaging in anti-competitive practices. In the latter case, the limited liability constraint binds at the lower bound due to the entrepreneur's inability to repay $R_{\bar{L}}$, making him indifferent about his differentiation decision at the beginning of $t = 1$.

In case (iii), however, the entrepreneur's decision on d_i may generate net profits and impact his project's final human capital level. Furthermore, Condition (3.4) implies that the entrepreneur is strictly better off with human capital on his project given an ex ante fixed repayment $R_{\bar{L}}$. Suppose that the contracted repayment $R_{\bar{L}}$ lies in the interval $(1 + L)\theta_1^i \geq R_{\bar{L}} > \theta_1^i$. In this case, the entrepreneur anticipates the VC's final human capital allocation $\hat{L}_{i,d}^*$ and his decision on the anti-competitive practice denotes

$$d_i^* = \begin{cases} 1\,, & \text{if } R_{\bar{L}} \geq \theta_1^i + L\Pi_1, \\ 0\,, & \text{otherwise.} \end{cases} \tag{3.48}$$

The reason is that the reallocation of human capital follows the decision of the entrepreneur on the anti-competitive differentiation, i.e., if $d_i^* = 0 \Rightarrow \hat{L}_{i,0}^* = 0$ and $d_i^* = 1 \Rightarrow \hat{L}_{i,1}^* = L \;\forall\; i$. Importantly, recall that with Property 2 in place, cases (ii) and (iii), in which quality h is realized and the entrepreneur keeps human capital, $\hat{L}_{h,0}^* = L$, can be excluded.[9]

Thus, the entrepreneur's differentiation decision can be written as follows.

Proposition 6 (Debt – Differentiation Decision)**.** *Suppose the representative VC and the entrepreneur signed a debt contract $(R_{\bar{L}}, L = \bar{L})$ at $t = 0$. The entrepreneur's differentiation decision at the beginning of $t = 1$ denotes*

$$d_i^* = \begin{cases} 1\,, & \textit{if } (1 + L)\theta_1^i \geq R_{\bar{L}} \geq \theta_1^i + L\Pi_1, \\ 0\,, & \textit{otherwise.} \end{cases} \tag{3.49}$$

Proposition 6 highlights that – contrary to the equity contract – the debt contract $(R_{\bar{L}}, L = \bar{L})$ can incentivize the entrepreneur to engage in anti-competitive practices despite owning a high-quality project. Moreover, Proposition 6 captures a quite nuanced

[9]Without Property 2, there can exist repayments $R_{\bar{L}}^*$ for which $(1 + L)\theta_0^h \geq R_{\bar{L}}^* \geq \theta_0^h + L\Pi_0$ holds. Then, an entrepreneur with a high-quality project would receive human capital without differentiating, i.e., $d_h^* = 0$ and $\hat{L}_{h,0}^* = L$.

rationale of the entrepreneur to differentiate: Suppose the entrepreneur can repay the VC without the additional surplus of human capital on his project, i.e., $R_{\bar{L}} \leq \theta_0^i$ and case (iii) applies. Then, the entrepreneur differentiates to be unable to repay the debt claim without human capital, $R_{\bar{L}} > \theta_1^i$. In turn, the VC is "forced" to keep her human capital at the project, and the entrepreneur eventually ends up with higher final profits at $t = 2$, where $(1 + L)\theta_1^i > \theta_0^i$.

Contract with Human Capital

At $t = 0$, the VC anticipates the entrepreneur's differentiation decision d_i^* conditional on her final human capital allocation $\hat{L}_{i,d}^*$. Therefore, the VC offers the contract $(R_{\bar{L}}, L = \bar{L})$ where the return payment $R_{\bar{L}}$ is implicitly defined by her binding outside option constraint

$$\mathbb{E}\left[min\left\{R_{\bar{L}}\,,(1+\hat{L}_{i,d}^*)\theta_d^i\right\}\;\middle|\; d_i^*\right] + \mathbb{E}\left[(1+L-\hat{L}_{i,d}^*)\Pi_d \;\middle|\; d_i^*\right] - I - c(L) \geq \Pi_0, \quad (3.50)$$

which is equivalent to

$$\begin{aligned} p\min\left\{R_{\bar{L}}\,,(1+\hat{L}_{h,d_h^*}^*)\theta_{d_h^*}^h\right\} + (1-p)\min\left\{R_{\bar{L}}\,,(1+\hat{L}_{l,d_l^*}^*)\theta_{d_l^*}^l\right\} \\ + p(1+L-\hat{L}_{h,d_h^*}^*)\Pi_{d_h^*} + (1-p)(1+L-\hat{L}_{l,d_l^*}^*)\Pi_{d_l^*} - I - c(L) \geq \Pi_0. \quad (3.51) \end{aligned}$$

From the payoff constraint in (3.51), it is evident that the entrepreneur bears the cost of human capital, $c(L)$, regardless of the VC's final human capital allocation. Since his upside of receiving human capital materializes at the end of $t = 1$, however, the entrepreneur only benefits from signing the debt contract $(R_{\bar{L}}, L = \bar{L})$ at $t = 0$ if the human capital remains on the project. Since the reallocation of human capital follows the decision of the entrepreneur on the anti-competitive differentiation, the entrepreneur is strictly better off signing the debt contract $(R_0, L = 0)$ and forgoing the provision of human capital if he does not engage in anti-competitive differentiation. In other words, suppose the entrepreneur and the VC sign a debt contract $(R_{\bar{L}}, L = \bar{L})$ where the VC provides human capital to the project in equilibrium. Then, under Property 2, the entrepreneur is incentivized to engage in anti-competitive differentiation in at least one quality realization of the project. On this account, the analysis of determining the equilibrium repayment $R_{\bar{L}}^*$ reduces to the following three cases: (a) $R_{\bar{L}}^*$ induces $d_i^* = 1$, (b) $R_{\bar{L}}^*$ induces $d_h^* = 0$ and $d_l^* = 1$, and (c) $R_{\bar{L}}^*$ induces $d_h^* = 1$ and $d_l^* = 0$.[10]

[10]Note that without Property 2, there would be an additional case differentiation within the case (b) where $R_{\bar{L}}^* > \theta_0^h$ as well as a further case (d) where $d_i^* = 0$ might occur in equilibrium.

First, consider the case (a) in which $(1+L)\theta_1^l \geq R_{\bar{L}}^* \geq \theta_1^h + L\Pi_1$ holds, and the entrepreneur differentiates regardless of his project quality, i.e., $d_i^* = 1$ is induced.[11] With the VC's final human capital allocation denoting $\hat{L}_{i,1}^* = L$, the entrepreneur is always able to repay $R_{\bar{L}}^*$ so that no uncertainty about the repayment exists ex ante. Thus, Equation (3.51) simplifies to

$$pR_{\bar{L}} + (1-p)R_{\bar{L}} + p\Pi_1 + (1-p)\Pi_1 - I - c(L) \geq \Pi_0, \tag{3.52}$$

such that

$$R_{\bar{L}} \geq I + c(L) + \Pi_0 - \Pi_1 := R_{\bar{L}}^* \tag{3.53}$$

is the repayment demanded by the VC in equilibrium. From a profit-maximizing perspective, the VC is always prevented from the (ex ante efficient) human capital reallocation towards her portfolio. On this account, the VC demands compensation for the certain incurred externality on her portfolio, as shown in (3.53).

Second, consider the case (b) in which $\min\{\theta_1^h + L\Pi_1\,,(1+L)\theta_1^l\} \geq R_{\bar{L}}^* \geq \theta_1^l + L\Pi_1$ holds, and only the entrepreneur with a low-quality project differentiates, i.e., $d_h^* = 0$ and $d_l^* = 1$ is induced. Under Property 2, the VC's final human capital allocation denotes $\hat{L}_{h,0}^* = 0$ and $\hat{L}_{l,1}^* = L$, and no uncertainty about the entrepreneur's ability to repay $R_{\bar{L}}^*$ exists ex ante. Thus, Equation (3.51) simplifies to

$$pR_{\bar{L}} + (1-p)R_{\bar{L}} + p(1+L)\Pi_0 + (1-p)\Pi_1 - I - c(L) \geq \Pi_0, \tag{3.54}$$

where

$$R_{\bar{L}} \geq I + c(L) - p\left[(1+L)\Pi_0 - \Pi_1\right] + \Pi_0 - \Pi_1 := R_{\bar{L}}^* \tag{3.55}$$

is the repayment demanded by the VC in equilibrium. From a profit-maximizing perspective, the VC can efficiently use her human capital by reallocating it to her endowed portfolio if the project is of high quality. If the project turns out to be of low quality, however, the entrepreneur prevents the (ex ante efficient) human capital pull-out by differentiating at the beginning of $t = 1$. On this account, the VC demands compensation for the loss in efficiency of her human capital in the bad state of the world, as shown in (3.55).

Finally, consider the case (c) in which $(1+L)\theta_1^h \geq R_{\bar{L}}^* \geq \max\{\theta_1^h + L\Pi_1\,,(1+L)\theta_1^l\}$ holds, and only the entrepreneur with a high-quality project differentiates, i.e., $d_h^* = 1$ and $d_l^* = 0$ is induced. With the VC's final human capital allocation denoting $\hat{L}_{h,1}^* = L$

[11] Note that the case (a) only exists if $L(\theta_1^l - \Pi_1) \geq \theta_1^h - \theta_1^l$ is satisfied.

and $\hat{L}_{l,0}^* = 0$, the entrepreneur can only repay $R_{\bar{L}}^*$ if the project is of high quality. Thus, Equation (3.51) simplifies to

$$pR_{\bar{L}} + (1-p)\theta_0^l + p\Pi_1 + (1-p)(1+L)\Pi_0 - I - c(L) \geq \Pi_0, \tag{3.56}$$

where

$$R_{\bar{L}} \geq \frac{I + c(L) - (1-p)(\theta_0^l + L\Pi_0)}{p} + \Pi_0 - \Pi_1 := R_{\bar{L}}^* \tag{3.57}$$

is the repayment demanded by the VC in equilibrium. From a profit-maximizing perspective, the VC can efficiently use her human capital by reallocating it to her endowed portfolio if the project is of low quality. If the project turns out to be of high quality, however, the entrepreneur prevents the (ex ante efficient) human capital pull-out by differentiating at the beginning of $t = 1$. On this account, the VC demands compensation for the loss in efficiency of her human capital in the good state of the world, as shown in (3.57).

For illustration, Figure 3.3 highlights the equilibrium outcome – i.e., the differentiation decision of the entrepreneur and the final human capital allocation of the VC – dependent on the equilibrium repayment $R_{\bar{L}}^*$.

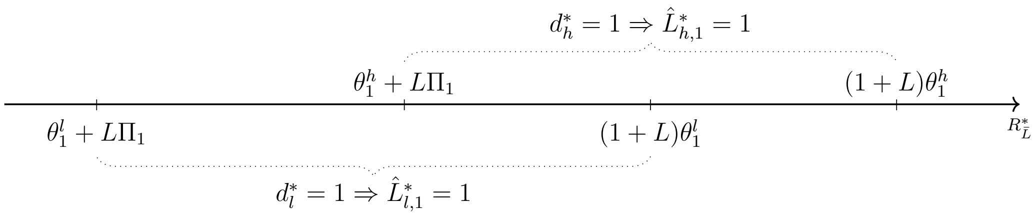

Figure 3.3: Equilibrium outcome under the debt contract ($R_{\bar{L}}^*, L = \bar{L}$) and the existence of case (a).

The following proposition summarizes the equilibrium outcome of the VC's contract offer ($R_{\bar{L}}^*, L = \bar{L}$) in which the entrepreneur receives human capital at $t = 0$.

Proposition 7 (Debt – Contract with Human Capital)**.** *Suppose that the project is feasible with $\mu_0 \geq I$ and Property 2 holds. Then, the reallocation of human capital under the debt contract ($R_{\bar{L}}^*, L = \bar{L}$) follows the decision of the entrepreneur on the anti-competitive differentiation, i.e., if $d_i^* = 0 \Rightarrow \hat{L}_{i,0}^* = 0$ and $d_i^* = 1 \Rightarrow \hat{L}_{i,1}^* = L \;\forall\; i$.*

The equilibrium can be characterized as follows:

(a) For $(1+L)\theta_1^l \geq R_{\bar{L}}^* \geq \theta_1^h + L\Pi_1$, the equilibrium repayment denotes

$$R_{\bar{L}}^* = I + c(L) + \Pi_0 - \Pi_1,$$

where the entrepreneur regardless of his project quality differentiates to prevent a human capital reallocation;

(b) *For* $\min\{\theta_1^h + L\Pi_1 \ , (1+L)\theta_1^l\} \geq R_{\bar{L}}^* \geq \theta_1^l + L\Pi_1$, *the equilibrium repayment denotes*

$$R_{\bar{L}}^* = I + c(L) - p\left[(1+L)\Pi_0 - \Pi_1\right] + \Pi_0 - \Pi_1,$$

where only the entrepreneur with a low-quality project differentiates to prevent a human capital reallocation;

(c) *For* $(1+L)\theta_1^h \geq R_{\bar{L}}^* \geq \max\{\theta_1^h + L\Pi_1 \ , (1+L)\theta_1^l\}$, *the equilibrium repayment denotes*

$$R_{\bar{L}}^* = \frac{I + c(L) - (1-p)(\theta_0^l + L\Pi_0)}{p} + \Pi_0 - \Pi_1,$$

where only the entrepreneur with a high-quality project differentiates to prevent a human capital reallocation.

[12]Proposition 7 highlights that venture debt exacerbates the contract friction compared to an equity contract. The reason lies in the VC's capped participation in the entrepreneur's profit, which results in a higher threat of human capital reallocation. This also implicates entrepreneurs with high-quality projects. Since the entrepreneur bears the cost of human capital regardless of its final allocation, any signed debt contract $(R_{\bar{L}}^*, L = \bar{L})$ where the VC provides human capital incentivizes the anti-competitive differentiation for at least one project quality. This result is consistent with the observation that traditional lenders, such as banks, are generally not known for providing entrepreneurs with human capital or strategic guidance but primarily equity investors like VCs or business angels (see Barry and Mihov, 2015).

3.5 Conclusion

In this paper, I present the concept of *preemptive differentiation*, a strategic tool for entrepreneurs to counter the VC's potential reallocation of human capital across competing portfolio firms. The model features a representative VC endowed with a portfolio of competing firms who can use her limited human capital and cash to invest in an entrepreneur's project of ex ante unknown quality. In case the VC has sufficient stake

[12]Strictly speaking, the "$\geq$" connected to the respective min and max condition is "$=$" when case (a) exists. However, this distinction is overlooked for the compactness of Proposition 7.

in the project, two second-best contracts can occur in equilibrium – depending on the magnitude of the externality and human capital cost – resulting in three inefficiencies: First, the two ex post inefficiencies of the anti-competitive differentiation combined with the forced misallocation of human capital if the second-best contract mandates the provision of human capital. Second, the ex ante inefficiency of not providing the entrepreneur with human capital despite its (welfare-optimal) potential for monitoring and profit enhancement

Analyzing the persistence of the inefficiencies associated with preemptive differentiation, I consider venture debt as an alternative contract design. With the contracted repayment being confined ex ante, the VC's profit participation in the entrepreneur's project is now capped, amplifying the contract friction. In equilibrium, any debt contract stipulating the initial provision of human capital leads the entrepreneur to engage in anti-competitive differentiation for at least one project quality. Thus, venture debt can implicate entrepreneurs regardless of their project quality.

This paper is an initial effort to explore the frictions between venture capitalists and their portfolio firms by examining intra-portfolio competition at both the financing and market stages. The model emphasizes the trade-off faced by VCs between the benefits of reallocating human capital and the potential harm caused by the entrepreneur's anti-competitive actions. To simplify the analysis, I assume a non-strategic portfolio with an exogenous relationship to the entrepreneur. Future research could build on this foundation by endogenizing the VCs' decisions regarding portfolio size and focus. This step would enable a deeper understanding of the trade-offs between portfolio size and focus, active governance, and the entrepreneurs' incentives to engage in detrimental market behaviors that could undermine mutual profits.

3.A Mathematical Appendix

3.A.1 Numerical Examples

To prove the existence of the equilibria shown in Section 3.3.4, consider the following values for the variables:

$$\theta_0^h = 3, \qquad \theta_1^h = 2,$$
$$\theta_0^l = 0.5, \qquad \theta_1^l = 0.4,$$
$$\Pi_0 = 0.6, \qquad \Pi_1 = 0.2,$$
$$p = 0.3.$$

I can now set values for investment I, human capital L, and human capital cost $c(L)$ where the contract solution (α_L, L) describes each equilibrium.

Contract $(\alpha_{\bar{L}}^N, L = \bar{L})$. Let $I = 0.5$, $L = 0.8$ and $c(L) = 0.5$.

With this parameterization, the basic assumptions – i.e., Conditions (3.2) to (3.5) – are satisfied. Furthermore, Condition (3.10) holds, and thus the provision of human capital is first-best. By setting the required investment $I = 0.5$, the VC's share compensation is sufficiently small that the entrepreneur can't preempt a human capital reallocation. Property 1 does not hold and the equilibrium contract denotes $(\alpha_{\bar{L}}^N = 0.33, L = 0.8)$. □

Contract $(\alpha_{\bar{L}}^*, L = \bar{L})$. Let $I = 1$, $L = 0.8$ and $c(L) = 0.5$.

With this parameterization, the basic assumptions – i.e., Conditions (3.2) to (3.5) – are satisfied. Furthermore, Condition (3.10) holds, and thus the provision of human capital is first-best. However, by increasing the required investment to $I = 1$, the VC's share compensation is now sufficiently large that the entrepreneur can preempt a human capital reallocation. Property 1 holds and the equilibrium contract denotes $(\alpha_{\bar{L}}^* = 0.83, L = 0.8)$. □

Contract $(\alpha_0^*, L = 0)$. Let $I = 1$, $L = 0.6$ and $c(L) = 0.5$.

With this parameterization, the basic assumptions – i.e., Conditions (3.2) to (3.5) – are satisfied. Furthermore, Condition (3.10) holds, and thus the provision of human capital is first-best. However, by reducing the human capital (effect) to $L = 0.6$, the human capital is now too costly with $c(L) > c^{SB}(L)$. Thus, no human capital is provided ex ante and the equilibrium contract denotes $(\alpha_0^* = 0.8, L = 0)$. □

Contract $(\alpha_0^N, L = 0)$. Let $I = 1$, $L = 0.6$ and $c(L) = 0.8$.

With this parameterization, the basic assumptions – i.e., Conditions (3.2) to (3.5) – are satisfied. However, by increasing the human capital cost to $c(L) = 0.8$, Condition (3.10) does not hold anymore. Thus, no provision of human capital is first-best and implementable, with the equilibrium contract denoting $(\alpha_0^N = 0.8, L = 0)$. $\square$

Chapter 4

Rationalizing Protests – From Individuals to Networks

Abstract: We propose a theory model on network goods to explain protest participation: (i) under weak network effects, the standalone benefit of individuals drives participation; (ii) under strong network effects, expected participation by others influences choices. Using survey data from the American National Election Studies, we construct a spatial autoregressive model to identify network benefits and measure standalone benefits with proxy variables. We find larger network benefits for left-leaning individuals, followed by right-leaning and moderates, with the opposite pattern of standalone benefits. Our estimates and the model's implications offer new insights into the size of current protest movements in the US.

Keywords: Social Economics; Public Protests; Network Effects.

JEL classification: A13; D72; D85.

4.1 Introduction

PUBLIC PROTESTS have been a vital force for citizens to express their discontent in the public sphere worldwide. Calling for policies, reforms, or even change of government, the act of demonstration – through peaceful protest or violent riots – drives social and political change not only in democracies but also in authoritarian regimes. For instance, rallies that later became known as the Yellow Vests Protests arose in France from late 2018 until early 2019, when diesel prices increased by approximately 23% over 12 months, as the BBC (2018) reported.[1] With the ensuing riots drawing the participation of $282,000$ protesters at its peak, the Yellow Vests Protests ultimately forced the French government to concede by announcing a halt on planned fuel tax increases and freezing electricity and gas prices for 2019. The economic damage of these civil outbursts amounted to 0.2 percentage points of quarterly growth – approx. 5 billion euro – according to Reuters (2019)[2], and was commented by the Finance Minister of France, Bruno Le Maire, with:

> "It's a catastrophe for business, it's a catastrophe for our economy." —
> *BBC, 2018*

Considering the far-reaching implications of mass mobilization like the Yellow Vests Protests, the question arises of what precise dynamics unfold behind protests.

On the individual level, one rationale for participating in a protest is deriving protest-dependent utility by either satisfying a sense of moral duty or benefiting from an underlying objective. While both causes seem plausible, two issues related to the latter must be mentioned. First, most protests are directed toward the greater good of the public, leading to the classic free-rider problem among potential participants (Olson, 1971; Apolte, 2012). The second issue is related to the paradox of voting and is present even if the free-rider problem is disregarded: With the individual's participation being non-pivotal in achieving the underlying objective of the protest, the cost of participation should outweigh the benefit of contribution (Downs, 1957). This cost is particularly substantial in the case of protests since, depending on the country, individuals might face not only minor costs of transportation and opportunity but severe repercussions like physical violence, incarceration, torture, and death.

A more recent literature focuses on the role of networks in explaining individual participation decisions in protests. Similar to the decision on a network good, like registering at a social media platform, an individual's value from protesting increases as

[1] "Yellow vest protests 'economic catastrophe' for France", BBC, December 9, 2018, `https://www.bbc.com/news/world-europe-46499996`; last accessed on May 5, 2024.

[2] "French 'Yellow Vests' protests cost 0.2 percentage points of growth - Le Maire", Reuters, February 28, 2019, `https://www.reuters.com/article/idUSKCN1QH17O/`; last accessed on May 5, 2024.

more people join (Economides, 1996). This network effect can be understood as direct and indirect benefits an individual gains from participating, which increase as the network attracts further participants. In the context of protests, direct network benefits may include peer effects through social ties (Bursztyn *et al.*, 2021), while indirect benefits may encompass increased media coverage and larger protest venues (Enikolopov *et al.*, 2020). However, the materialization of these network benefits crucially depends on others' participation and, thus, on the individual's ex ante expectation about the protest. While Carter and Carter (2020) or Truex (2019) identify focal points as a mechanism to increase individual expectations about protest size, the general problem of coordination remains.

Combining these well-known insights – (1) the individual's utility from participating is subject to protest-dependent attitudes; (2) the individual's participation is non-pivotal to achieving the protest objective; (3) networks affect the individual's participation decision while allowing for multiple equilibria due to coordination problems – we put forward a single theoretical framework and empirically test its implications. In the framework, we consider a continuum of heterogeneous individuals whose utility from protesting consists of two components: (i) the interaction between an individual's type and the standalone benefit and (ii) the network benefit that increases with the expected number of protest participants. Following the rational-expectation approach in the static model, we derive equilibrium outcomes for protest participation depending on the relation between the standalone and the network benefit. If the standalone benefit exceeds the network benefit, which we define as *weak network effects*, a unique equilibrium exists for any exogenous participation cost. If the network benefit exceeds the standalone benefit, which we define as *strong network effects*, two stable equilibria coexist for intermediate participation costs, namely the extreme outcomes of full and no participation. This result highlights the coordination problem individuals face when deciding on a protest for which the network benefit is the determinant participation factor. In the dynamic version of the model, we further show the precise adjustment processes of protest participation over discrete time, which lead to the same outcomes in the limiting case as the static model.

Deducing hypotheses from the model dynamics and underlying intuitions, we use data on up to 6264 individual respondents from the American National Election Studies (ANES) 2020 survey wave to test empirically (i) if the standalone and network benefits determine an individual's protest participation and (ii) if the standalone and network benefits vary for different protests.[3]

[3]The latter can be seen as an indirect implication of the model, as our data does not allow for matching the individual participation decision to actual protests. Instead, we focus on the composition of benefits that characterize protests and explore whether observed protest numbers align with the

The main empirical challenges consist of identifying standalone and network benefits and clustering individuals according to their political attitudes. Incorporating individuals' participation decisions within a social network into a simple OLS regression leads to endogeneity due to the inherent interdependencies among them. Individual A's protest decision affects individual B's protest decision and vice versa. Hence, regressing individual A's on individual B's protest decision leads to simultaneity in the equation, resulting in biased and inconsistent estimates. To overcome this problem, we rely on a generalized spatial two-stage least squares estimator proposed by Drukker *et al.* (2013) in which the protest decisions of an individual's social network are instrumented by their individual characteristics. The standalone benefit is measured with proxy variables about political interest, donation behavior, and consumption behavior. To elicit heterogeneities among different political groups, we rely on a clustering algorithm proposed by Kaufman and Rousseeuw (2009) and a decision rule for more granular protest groups.

The results from the baseline regression support our first hypothesis since both the standalone and network benefits are found to be driving forces behind individual protest decisions. To test our second hypothesis, we divide individuals into three clusters based on political opinions. These clusters can be interpreted as groups of individuals within the sample holding left-leaning, moderate, and right-leaning political views. We find that the standalone and network benefits are significant determinants of the protesting decision within each cluster, with varying magnitudes. Specifically, network benefits are strongest for left-leaning individuals, followed by right-leaning individuals, and weakest for moderates. As for the standalone benefit, we find the reversed ordering.

In a finer-grained specification, we pool individuals according to causes they think worthwhile protesting for and identify four relevant protest movements: Black Lives Matter (BLM), healthcare, environment, and pro-immigration.[4] Our findings show that the standalone benefit of BLM protesters is higher than for all other protests on average. Network benefits are strongest for pro-immigration protests, followed by BLM and environmental issues. Network benefits are the smallest for healthcare-related protests, even below the overall non-issue-related average. These results further confirm our second hypothesis, showing that different compositions of standalone and network benefits characterize protests. In the extension, we consider the model framework in combination with the previously measured standalone and network benefit compositions of the four movements to provide a novel explanation for their observed protest sizes.

This paper adds theoretical and empirical innovations to the literature. From a theory perspective, we argue that the individual's protest participation is non-pivotal

theory.

[4]Note that these clusters are no longer mutually exclusive.

to the success of the underlying objective and that the decision to protest is similar to consuming a network good. In this regard, our model strongly differs from the models of Acemoglu and Robinson (2001) and De Mesquita (2010) who assume protests to have a club good character. This assumption seems restrictive as resulting policy changes are rarely targeted towards protest participants only – for example, the halt on a planned fuel tax rise achieved by the Yellow Vests Protests, not only benefited the protesters but all French citizens. In contrast, our model makes no claims about the character of the protest or the eventual outcome, thus allowing for public goods and the associated free-rider problems. From an empirical point of view, we add to the literature by applying a spatial identification strategy to build a synthetic network across US citizens. To the best of our knowledge, this approach to identifying network effects is novel to the protest literature before.

The paper is structured as follows. After discussing the related literature in the following paragraphs, we introduce the framework in Section 4.2. In Subsection 4.2.1, we derive the potential equilibria of protest participation under strong and weak network effects in the static model. Next, we enrich the model by including discrete time in Section 4.2.2, deriving the precise protesting processes under weak and strong network effects. Thereafter, in Section 4.2.3, we formulate two hypotheses based on the model's implications. In Section 4.3, we introduce the empirical identification strategy used to test our hypotheses. The subsequent Section 4.4 presents the results for a baseline regression as well as for political clusters in Subsection 4.4.2 and protest-specific clusters in Subsection 4.4.3. The latter results are linked to actual protest sizes in the extension in Section 4.4.4. Finally, we conclude in Section 4.5. All proofs and additional figures and tables are deferred to Appendix 4.A.1 and Appendix 4.A.2 respectively.

Related Literature

Acemoglu and Robinson (2001, 2005) establish the first theoretical framework on the occurrence of protests by considering distribution battles among the population as the primary cause of protests and potential subsequent revolutions. Disadvantaged citizens utilize protests and the threat of revolution to pressure elites into creating a more equal distribution of wealth and income. These elites, who run the state, act under a revolution constraint that lets them strategically distribute resources. Protests and revolutions occur, according to the authors' view, when elites fail to adhere to the revolution constraint, and disadvantaged citizens respond by raising their voices via protests. However, some authors see an inconsistency between the explanation of Acemoglu and Robinson (2001, 2005) and fundamental insights from Olson's (1971) Theory of Collective Action (Apolte, 2012; Egorov and Sonin, 2024). In particular, Apolte (2012) claims that inequality among a nation's citizens is neither sufficient nor necessary for enlarged protests or revolutions. Instead, the ability to overcome the free-rider problem should explain protest events and revolutions.

The fundamental conflict between these two strands of theoretical literature emerges due to the different characterization of the underlying objective of protests: *club good* vis-à-vis *public good*. Under the notion of a club good, citizens protest to benefit from expected political changes (e.g., Acemoglu and Robinson, 2001, 2005). In contrast, under the assumption that the protest's objective is characterized as a public good, the benefits from political changes are distributed among all citizens, regardless of the individual participation decision. Consequently, individuals would decide to free-ride – i.e., not protest – if the participation cost is sufficiently high. We do not impose any characterization on the nature of protests or the outcome. Instead, the micro foundation of our model is related to the literature on the voting paradox, where the individual's action is non-pivotal to the success of the underlying objective (Downs, 1957). This approach allows us to consider an individual's choice for protesting as the choice for a single network good, where protest participation is driven by a stand-alone and a network benefit. Thus, the conception of our model is also closely related to the industrial organization literature addressing network effects (Economides, 1996; Belleflamme and Peitz, 2015).

Later developments in the protest literature acknowledge how central the coordination problem is for explaining protests. This literature focuses on informational aspects as, for example, Ellis and Fender (2011) extend the model from Acemoglu and Robinson (2001, 2005) by introducing information transmission via informational cascades based on a model proposed by Lohmann (1994). This informational exchange facilitates coordination. Another approach to overcome the coordination problem is proposed by De Mesquita (2010), who models a difference in the timing of movements. The model relies on a protest vanguard who can move first, thereby shaping the movement's trajectory. This move is observed by other citizens, who can subsequently join the protest. Both models incorporate informational exchange as a device for switching from a low-participation equilibrium to a high-participation equilibrium. However, they maintain a club good characterization of protests, which makes them prone to the above-mentioned criticism, too. In our proposed model, we incorporate the presented informational aspects via expectation formation. Especially in the dynamic specification, informational cascades are modeled explicitly, and we provide an account of the pace at which they materialize.

Lastly, one recent strand of literature discusses the formation and effects of networks in overcoming the free-rider problem. The fundamental insight is that protest participation among social peers is strategically complementary. Such effects are identified by

Bursztyn *et al.* (2021) in a field study with incentivized students in Hong Kong's anti-authoritarian protests and Enikolopov *et al.* (2020) using a natural experiment with social media data in Russia. Importantly, these approaches are capable of explaining protest participation irrespective of whether the desired outcome is a club good or public good. We aim to contribute to this literature by preserving the central role of networks in overcoming the free-rider problem while explicitly modeling the strategic complementarity and testing its resulting hypotheses with a spatial model. The spatial model is a promising technique for identifying network effects as highlighted by Souza (2024), and, to the best of our knowledge, has not been applied in the protest literature so far. Thereby, we provide complementary evidence in favor of Bursztyn *et al.* (2021) and Enikolopov *et al.* (2020) while also contributing to the question of whether protest participation is a strategic complement or substitute as discussed by Cantoni *et al.* (2019) and Shadmehr (2021).

4.2 The Framework

4.2.1 Static Model

To establish the baseline model for participating in demonstrations, we consider the textbook approach by Belleflamme and Peitz (2015) on markets for a single network good.[5] In the economy, suppose there is a continuum of heterogeneous individuals of mass 1 who derive utility from protesting with

$$U(\theta) = \theta a + vn^e - c, \tag{4.1}$$

where the individual's type θ is uniformly distributed on the interval $[0, 1]$. The utility of a representative individual to demonstrate consists of two components: (i) the interaction between an individual's type and the standalone benefit $a > 0$, which can represent but is not limited to a feeling of moral obligation, sympathy for the protest, or the benefit of the potential policy implementation to come on strike for; (ii) the network benefit $v > 0$ that, for simplicity, increases linearly with the expected number of participants n^e. Note that this network benefit can encompass direct effects (social status, associating oneself with a movement) and indirect effects (larger venues, media representation). Participating in a demonstration comes at a non-monetary cost $c > 0$, which can be seen as opportunity costs and personal repercussions. Without loss of

[5] A related version of the model is put forward by Economides (1996).

generality, assume that an individual's outside option $\underline{\text{U}}$ is normalized to zero.[6]

For a given participation cost c and an expected network size n^e, the individual being indifferent between demonstrating or not is characterized by

$$\theta a + vn^e - c \geq \underline{\text{U}}, \tag{4.2}$$

where

$$\theta \geq \frac{c - vn^e}{a} := \hat{\theta} \tag{4.3}$$

denotes the critical type threshold, i.e., individuals with types lower than $\hat{\theta}$ choose not to participate.

Ending up in a corner solution if $\hat{\theta}$ is not in the interval $[0, 1]$, the actual number of individuals demonstrating is

$$n(c, n^e) = \begin{cases} 0 & \text{if } n^e < (c-a)/v, \\ 1 - \frac{c - vn^e}{a} & \text{if } (c-a)/v \leq n^e \leq c/v, \\ 1 & \text{if } n^e > c/v. \end{cases} \tag{4.4}$$

We follow the rational-expectation approach to continue the analysis: if individuals form rational expectations in equilibrium, it must be that $n^e = n$.[7] From the demand function in (4.4), it is straightforward that a corner solution occurs if either $c > a$ (nobody participates with $n = n^e = 0$) or if $c < v$ (everyone participates with $n = n^e = 1$). It follows that both equilibria coexist if $v > a$. Furthermore, the interior equilibrium with $0 < n^e = n < 1$ is

$$n(c) = \frac{a - c}{a - v}. \tag{4.5}$$

First, suppose we are in the scenario where the standalone benefit exceeds the benefit of the network, $v < a$, which we define as *weak network effects*. In that case, the number of participants $n(c)$ is a decreasing function of the cost incurred. Thus, there is a unique participation level for any (exogenous) cost c: $n = 0$ for $c > a$, $n = (a - c)/(a - v)$ for $v \leq c \leq a$, and $n = 1$ for $c < v$.

Second, suppose we are in the scenario where the benefit of the network exceeds the

[6] In Appendix 4.A.1, we show that incorporating a non-zero outside option – which can reflect the free-rider problem – has no qualitative effect on the equilibrium outcome.

[7] The (Nash-)equilibria rely on self-fulfilling prophecies, where each individual has no incentive to change one's action when rational expectations are satisfied.

standalone benefit, $v > a$, which we define as *strong network effects*. In that case, the number of participants is an increasing function of the cost incurred. As an intuition, the marginal willingness to incur costs increases with a marginal increase in participation numbers. Furthermore, there coexist three rational-expectations equilibria for intermediate costs ($a < c < v$): $n = 0$, $n = (c - a)/(v - a)$ and $n = 1$. By the argument of a dynamic adjustment process, i.e., a marginal cost increase (decrease) drives the equilibrium towards full (no) participation, we can disregard the unstable interior equilibrium (see Rohlfs, 1974). In contrast, the two remaining equilibria depict precisely the coordination problem individuals face when deciding on a protest for which the network benefit v is the determinant participation factor. While both equilibria are stable, it is straightforward that by following the Pareto criterion, the no-participation equilibrium ($n = 0$) is Pareto-dominated: each individual would be better off coordinating to end up in the $n = 1$ equilibrium.

An example of the potential equilibria, i.e., the unique equilibrium under weak network effects and the multiple equilibria under strong network effects, for an exogenous cost $\hat{c}$ is depicted in Figure 4.1.

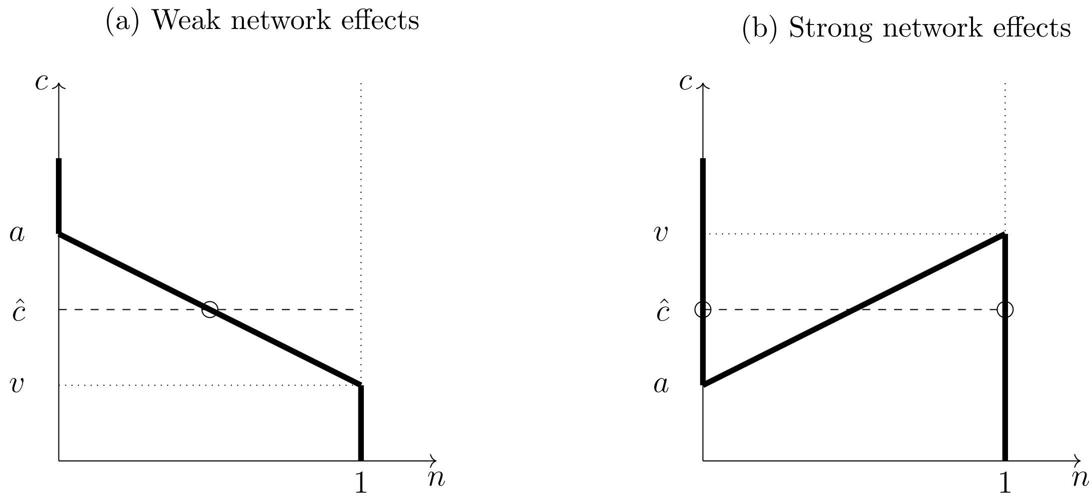

Figure 4.1: Equilibria under weak and strong network effects for given cost $\hat{c}$.

We can now characterize the following equilibria.

Proposition 1 (Static Network Effects)**.** *In equilibrium, the number of individuals participating in the demonstration* *(i) under weak network effects ($v < a$) is denoted by*

$$n(c) = \begin{cases} 0 & \text{if } c > a, \\ \frac{a-c}{a-v} & \text{if } v \leq c \leq a, \\ 1 & \text{if } c < v; \end{cases} \tag{4.6}$$

(ii) under strong network effects ($v > a$) is denoted by

$$n(c) = \begin{cases} 0 & \text{if } c > v, \\ 0 \veebar 1 & \text{if } a \leq c \leq v, \\ 1 & \text{if } c < a. \end{cases} \tag{4.7}$$

Conveying Proposition 1 to the real world, weak network effects may relate to protests about topics tied to goods (policies) and personal goals rather than social conditions and norms. Examples would be union wage talks, farmers' protests, or gun control, where the standalone benefit of the good, a, is the participant's priority, and higher private costs arguably lead to fewer participation numbers.

Examining the theoretical underpinnings of strong network effects in the context of protest participation, the BLM Movement, the 2020-2021 Belarusian Protests, and the Iranian Women's Rights Movement may be compelling examples. Despite the potential for high personal repercussions, these movements demonstrate how network effects can be powerful enough to motivate more individuals to participate (as long as the cost suffices $c \leq v$). Furthermore, the coordination problem that arises under strong network effects, with its two stable corner equilibria, provides a theoretical foundation for the concept of focal points: despite no inherent structural change, there can be an equilibrium switch from no participation ($n = n^e = 0$) to full participation ($n = n^e = 1$) when instances, e.g., the killing of George Floyd, lead to higher expectations for protest.

4.2.2 Dynamic Model

To further analyze individuals' protest participation and its underlying processes, consider now the dynamic version of the model in discrete time $t \geq 1$, where the utility from demonstrating denotes

$$U_t(\theta) = \theta a + v n_t^e - c. \tag{4.8}$$

For simplicity, assume adaptive expectations where the expected number of participants at time t is the actual number of participants from the previous period with $n_t^e = n_{t-1}$, and that the initial participation level is exogenous with $n_0 \in [0,1]$. Holding the outside option fixed and normalized to $\underline{\mathrm{U}}_t = 0 \;\forall\; t \geq 1$, the critical type threshold at time t denotes

$$\theta = \frac{c - vn_{t-1}}{a} := \hat{\theta}_t, \tag{4.9}$$

where the number of participants is $n_t = 1 - \hat{\theta}_t$.

We can then show the following.[8]

Proposition 2 (Dynamic Network Effects)**.** *The number of individuals participating in the demonstration at time $t \geq 1$ evolves according to*

$$n_t(c, n_0) = \begin{cases} 0 & \textit{if } n_o < \frac{c-a}{v}, \\ n_0 \left(\frac{v}{a}\right)^t + \left(1 - \frac{c}{a}\right) \sum\limits_{i=0}^{t-1} \left(\frac{v}{a}\right)^i & \textit{if } \frac{c-a}{v} \leq n_0 \leq \frac{c}{v}, \\ 1 & \textit{if } n_0 > \frac{c}{v}, \end{cases} \tag{4.10}$$

where in the interior, the change in the number of participation per period is defined by

$$n_t - n_{t-1} = \left(\frac{v}{a}\right)^{t-1} (n_1 - n_0) := \Delta_t. \tag{4.11}$$

Thus,

(i) under weak network effects ($v < a$), the interior solution converges to

$$\lim_{t \to \infty} n_t(c, n_0) = \frac{a - c}{a - v}; \tag{4.12}$$

(ii) under strong network effects ($v > a$), the interior solution converges to

$$\lim_{t \to \infty} n_t(c, n_0) = \begin{cases} 0 & \textit{if } n_0 < \frac{c-a}{v-a}, \\ 1 & \textit{if } n_0 > \frac{c-a}{v-a}. \end{cases} \tag{4.13}$$

Proposition 2 highlights the outcome of protest participation under weak and strong network effects subject to the adjustment process in protest participation per period. Equation (4.10) states that protests, where the initial protest size is characterized by full (no) participation, remain at this corner solution. In contrast, protests, where the initial participation number satisfies the interval $(c - a)/v \leq n_0 \leq c/v$, follow the dynamic adjustment process in Equation (4.11): Under weak network effects ($v < a$),

[8] The proof is deferred to Appendix 4.A.1.

the change in protest participation, Δ_t, decreases per period, and the protest size ultimately converges to its interior limit $(a-c)/(a-v)$. With the standalone benefit being the determinant factor of protest participation, the marginal benefit of one more individual joining the protest on the critical type $\hat{\theta}_t$ exactly outweighs the individual's participation cost. Under strong network effects ($v > a$), the change in protest participation, Δ_t, increases per period. Here, the limit outcome of the protest size crucially depends on its initial participation number n_0, where Equation (4.13) shows that the protest size drives to the full (no) participation corner if $n_0 > (c-a)/(v-a)$ (if $n_0 < (c-a)/(v-a)$). The large (small) initial protest size in connection with the strong network benefit gradually increases (decreases) the critical type threshold $\hat{\theta}_t$, eventually leading to the corner solution.

This interplay between initial protest size and network effects may very well illustrate the workings of focal points. By attracting a sufficiently large number of initial protesters, focal points, in combination with strong network effects, can spur mass mobilization.

4.2.3 Model Implications and Hypotheses

After laying out the theoretical foundation for why individuals participate in protests and the associated equilibrium outcomes on the aggregate, we now formulate two hypotheses based on the model's predictions.

Hypothesis 1. *The standalone and network benefits determine an individual's protest participation.*

Hypothesis 1 is a direct implication of the model and addresses the cornerstone of our framework: the choice for a single network good can represent an individual's rationale for participating in a protest. Therefore, the standalone and network benefits should be significant drivers in an individual's protest decision.

The second implication of the model is that the relation between standalone and network benefits determines protest size: Under the assumption that the participation cost is similar across different protests, strong network effects lead to extreme outcomes, i.e., both small and large protests, whereas weak network effects lead to medium-sized protests. Due to data limitations, however, we can not directly match the individual's protest choice to the aggregated outcome of protest size, and thus cannot test this implication directly. Instead, we follow the intuition of the model by focusing on individuals' motivation for participating in specific protests and exploring their benefit composition.

Hypothesis 2. *The standalone and network benefits vary for different protests.*

The idea behind Hypothesis 2 is that individuals' motivation for joining protests, and thus the relation between standalone and network benefits, is protest-dependent. For instance, protests related to racial issues may have a different composition of standalone and network benefits than those related to healthcare policies. By identifying and ranking these benefits across different protests, this approach allows us to assess the model's predictions on protest sizes by considering the variation in benefits. While we cannot directly infer the protests' realized equilibria from our estimates, i.e., if the benefit compositions constitute strong or weak network effects, Section 4.4.4 gives tentative insights into whether observed protest sizes, based on the previously obtained results on benefit combinations, align with the model's implications.

4.3 Empirical Strategy

The empirical part aims to find confirmatory evidence for the stylized model and deduced hypotheses from Section 4.2. There are two main empirical challenges. The first challenge is identifying the standalone benefit a and the network benefit v. This issue is resolved by employing a spatial autoregressive model for identifying network benefits in protesting and including proxy variables for the standalone benefit. The second challenge is the distinction between different political and protest clusters since it is unclear which individual is a potential participant in which protest movement. Also, individuals may participate in a variety of protest movements. Therefore, assigning an individual to a single protest movement is misleading. However, we view the variation across political and protest groups as crucial for judging the model's and hypotheses' appropriateness. Thus, there are two specifications in which we investigate political clusters and protest clusters separately. The political clusters are formed based on a clustering algorithm from Kaufman and Rousseeuw (2009), and the protest clusters are formed based on the individuals' opinions about problems in the US.

In this section, we proceed as follows. First, we introduce the ANES database. The database choice is justified based on variable requirements and an individual-level focus. Second, we argue that a spatial model including proxy variables is a good fit for collecting empirical evidence for the presented model. Next, we form political clusters and run the baseline model for each separately. Finally, we construct a model to explicitly compare the standalone and network benefits across four protest movements. This comparison allows us to assess the drivers of these movements and link them to their protest size.

4.3.1 Data

The ANES 2020 survey wave fulfills the crucial requirements for the selected empirical identification strategy. First, it contains various American individuals from the entire country and all socio-demographic backgrounds. Also, more than 6,000 individuals[9] participated in the 2020 survey wave, which is a sufficiently large sample for investigating social networks based on individual characteristics. Most importantly, the database provides us with a combination of information about (1) protest behavior, (2) socio-demographic characteristics, and (3) political attitudes. Having this information in a representative sample of the US allows us to build an empirical model of individual protest decisions.

Variable	N	Mean	Std. Dev.	Min	Pctl. 25	Pctl. 75	Max
Protest Variable							
Protest	6264	0.091	0.29	0.00	0.00	0.00	1.00
Socio-Demographic Characteristics							
Age	6264	51.60	17.02	18.00	37.00	66.00	80.00
Education	6264	4.59	2.03	1.00	3.00	6.00	8.00
Income	6264	11.90	6.73	1.00	6.00	18.00	22.00
White	6264	0.74	0.44	0.00	0.00	1.00	1.00
Black	6264	0.087	0.28	0.00	0.00	0.00	1.00
Hispanic	6264	0.087	0.28	0.00	0.00	0.00	1.00
Male	6264	0.46	0.50	0.00	0.00	1.00	1.00
Health status	6264	2.40	1.02	0.00	2.00	3.00	4.00
Married	6264	0.53	0.50	0.00	0.00	1.00	1.00
Children	6264	0.61	1.04	0.00	0.00	1.00	4.00
Personal Attitudes							
Left-right-scale	6264	5.84	2.56	1.00	4.00	8.00	10.00
Dev. Left-right-scale	6264	2.13	1.46	0.50	0.50	3.50	4.50
Religiosity	6264	2.82	1.49	1.00	1.00	4.00	5.00
Trust	6264	2.80	0.90	1.00	2.00	3.00	5.00
Longitude	6264	-91.46	15.58	-157.49	-97.33	-79.68	-69.76
Latitude	6264	37.80	5.01	21.11	35.37	41.47	61.41
Political Attitudes Towards...							
Labor Unions	5873	58.05	23.96	0	50	70	100
Conservatives	5873	54.79	27.15	0	40	75	100
Liberals	5873	49.93	28.90	0	30	70	100
Feminists	5873	58.48	26.61	0	45	85	100
Christian Fundamentalists	5873	46.73	28.54	0	30	67	100
Police	5873	71.12	24.46	0	60	85	100
Black Lives Matter	5873	52.72	35.37	0	15	85	100
Proxies standalone Benefit							
Donate	6264	0.46	0.50	0.00	0.00	1.00	1.00
Political interest	6264	2.29	1.02	1.00	2.00	3.00	5.00
Moral consumer	6264	1.99	1.15	1.00	1.00	2.00	5.00

Table 4.1: Summary statistics. The data source is the American National Election Study Wave 2020; the list contains all variables employed in the empirical analysis. Longitude and latitude of respondents' locations are summarized at the state level.

[9] After deleting those individuals with missing values.

The research project relies on five categories of data included in the ANES database. Table 4.1 presents an overview of the utilized variables and their summary statistics. The binary protest variable indicates whether the individual respondent protested at least once during the past year. This binary protest variable is going to be the dependent variable throughout this paper. The socio-demographic characteristics and personal attitudes are used as control variables and to calculate statistical proximity (network ties) between survey participants. The details of this procedure are explained below. The category political attitudes contains *thermometer* scores in which participants rate their feelings towards several societal groups (the higher the score, the more friendly the attitude). Some examples are attitudes towards conservatives, liberals, BLM protesters, and feminists. These attitudes are the foundation for analyzing political clusters separately and comparing their standalone and network benefits. The standalone benefit is proxied by three variables: donation behavior, political interest, and moral consumption choices. This choice is justified below.

4.3.2 Identification Strategy

The empirical model intends to identify the standalone and network benefits in individual decisions to protest as implied by Hypothesis 1. To identify the standalone benefit – meaning the *utility* an individual receives from protesting, excluding all social network elements – we rely on proxy variables. That is to say, we proxy the utility an individual receives merely because of protesting and expressing themselves without any social interaction before, during, or after the protest event. The first proxy is a binary variable that indicates 1 if the respondent donated money in the last 12 months to an organization concerned with a religious, social, or political issue and 0 otherwise. The decision of whether an individual donates money may provide us with information about how important an individual perceives themselves in contributing to better institutions for social purposes. The same motivation may make an individual more likely to protest for the protest's sake, not social recognition or other network benefits. The second proxy measures how much attention the respondent pays to politics and elections. The respondents answer this on a five-item scale from never (1) to always (5). The measure is supposed to grasp the importance that individuals dedicate to politics. Measuring this importance provides information about the likelihood of an individual joining a protest because of the actual cause, excluding the social component of protesting. Finally, the third proxy is a response on a five-item scale (from never: 1 to always: 5) about how often an individual bought or declined to buy a product because of moral considerations towards the company providing it. Like the other proxies, this one indicates how much an individual is adjusting their behavior because of personal opinions and values. These value-led sacrifices and adjustments to one's behavior are supposed to measure how high an individual's standalone benefit for protesting may be. As such, the standalone benefit proxies are supposed to measure one's evaluation of the act of protesting, excluding any considerations of other people.

The identification strategy for the network benefit relies on a spatial autoregressive model. As discussed by, for example, Souza (2024), a spatial model is appropriate to identify network effects among individuals. That is, if individual A is a close friend of individual B, then A's decision to go protesting should influence B's decision and vice versa. The spillover is not limited to positive reinforcement. A's decision not to go protesting should consequently make individual B less likely to join the protest. Such spillover effects are identifiable via spatial regressions (Souza, 2024). Thus, including a spatial term in a regression analysis with sufficient controls plus proxies for the standalone benefit allows us to identify the effects explored in the protest decision model from Section 4.2.

The first step in building spatial models is to grasp the social network structure within the sample. For this, we construct a measure of proximity between individuals. We rely on the concept of homophily – meaning that individuals who share similarities are more attracted to each other and, therefore, are more likely to build friendships and partnerships (McPherson *et al.*, 2001). The first empirical challenge is thus to measure how similar each individual is to all other individuals in the sample. We rely on the inverse of the Mahalanobis distance, which measures how different two individuals are based on a set of variables. The variables used to construct the Mahalanobis distance between any two individuals are a ten-item left-right scale on which individuals indicate their political attitude, religiosity, trust in others, age, education, income, the longitude of their county of residence, and the latitude of their county of residence. Unlike the Euclidean distance, the Mahalanobis distance accounts for the variables' covariances and hence does not weight highly correlated variables too strongly (Grimm and Yarnold, 2000). Before the variables enter the distance calculation, all of them are standardized.

Due to the sample size, we rely on statistical distances between individuals within our sample to build a synthetic network for the whole sample. A database containing slightly more than 6,000 individuals throughout the US is unsurprisingly too small to detect actual networks within the population. A solution is to rely on statistical distances among individuals. The distance between individual A and B, thus, does not indicate how likely it is that both have a real network tie – e.g., a friendship – but rather indicates that individual A has individuals in their network that are similar to B. On this account, each individual within the sample has a synthetic network from the whole ANES sample. This approach allows us to investigate whether an individual's synthetic network affects the individual's protest decision.

Equation (4.14) presents the regression model, including control variables, a variable for the standalone benefit, the spatial term, state fixed-effects, and an error term. The dependent variable P_i is a binary variable taking a value of 1 if individual i participated in at least one protest event during the past 12 months and 0 otherwise. The matrix X_i includes standardized control variables for sex, marriage status, children, age, race, income, education, health, extremism, religiosity, and trust. Details about these variables are available in Table 4.5 in Appendix 4.A.2. The standalone benefit S_i is proxied by three variables in different specifications. These variables are moral consumption decisions, political interest, and donation behavior. Next, the network benefit is assessed using the spatial term $W_{invdist}\, P_{-i}$. The first element is a spatial weighting matrix indicating the inverse distances between each individual i and all other individuals -i, and the second element denotes the other individuals' protest decisions P_{-i}. $\delta_{s(i)}$ denotes state fixed effects for all individuals and ϵ_i is the idiosyncratic error term.

$$P_i = \alpha\, \mathbf{X}_i + \beta\, S_i + \gamma\, \mathbf{W}_{invdist}\, P_{-i} + \delta_{s(i)} + \epsilon_i \qquad (4.14)$$

Simple OLS estimation of Equation (4.14) would lead to biased estimates due to the simultaneity between the dependent variable from individual i and all other individuals -i. Therefore, we rely on a generalized spatial two-stage least squares estimator proposed by Drukker *et al.* (2013). This estimator is based on spatial models derived earlier by Kelejian and Prucha (1998, 1999, 2010). In later specifications with multiple weighting matrices, we rely on the estimator proposed by Badinger and Egger (2013). The estimators instrument the simultaneous spatially lagged dependent variable with the remaining exogenous variables of each individual. Thereby, they allow for consistent estimation of spatial autoregressive models.

4.3.3 Identifying Political Clusters

In Hypothesis 2, we argue that the effect size of standalone and network benefits vary across protests. Considering the model dynamics, these differences in the composition of benefits may directly translate into different equilibrium outcomes for protests. The strategy for testing Hypothesis 2 relies on building subsets of protests and assessing the standalone and network benefits for them separately. There are two subsets we investigate. First, different individuals are clustered into political groups and we examine whether the protest participation determinants (i.e., standalone and network benefits) vary across these political groups. Second, individuals are clustered into potential protesters of varying protest movements, and then we assess whether the protest determinants vary across these. Both specifications contribute to testing Hypothesis 2 as they explore heterogeneities among protest determinants that provide an explanation for varying protest sizes observed for different protest movements.

The first heterogeneity analysis consists of differentiating protests according to their *political directions*. We aim to elicit differences between political groups in terms of the underlying reasons for their protest decisions – i.e., are there substantial differences in the standalone and network benefits across different political clusters? In order to come up with political clusters, we rely on a clustering algorithm by Kaufman and Rousseeuw (2009). This algorithm is based on several protest-sensitive political attitudes and forms clusters according to similarity between individuals. We rely on this algorithm to impose as little subjective clustering assumptions on the clustering process as possible.

Our approach relies on all survey respondents' attitudes towards a set of stylized groups of people within US society. These groups are listed in Table 4.1 under "Political Attitudes Towards...". These variables are scores from 0, indicating a very low opinion, to 100, indicating a very high opinion of groups like labor unions, conservatives, liberals, feminists, Christian fundamentalists, police, and BLM participants. Clustering individuals according to their attitudes towards these groups should result in groups concerned about similar political issues and with relatively similar political opinions. For example, individuals with a very low opinion of feminists and a high opinion of conservatives and Christian fundamentalists are likely to attend the same protest movements. The same is true for individuals with opposite opinions who are likely to attend another type of protest event.

The individuals are clustered according to the Partitioning Around Medoids (PAM) algorithm by Kaufman and Rousseeuw (2009). The authors built the K-medoids algorithm, which is more robust to outliers than the more common K-Means algorithm (Kassambara, 2017). A medoid is an observation that is representative of a whole cluster and all other observations sort each other around k-medoids. The algorithm determines the choice of medoids, where the clustering algorithm works as follows (Kaufman and Rousseeuw, 2009).

1. Randomly select k individuals from the sample. These are the first cluster-medoids.
2. Calculate the Euclidean dissimilarity matrix for all individuals based on the variables "Political Attitudes Towards..." from Table 4.1.
3. Assign each individual to the cluster-medoid to which it is most similar.
4. Within each cluster, check if there is an alternative medoid that minimizes the average dissimilarity coefficient of the cluster. If this is the case, this alternative medoid is the new cluster-medoid.
5. In case one or more medoids change, return to (3); otherwise, end the algorithm.

Since the ANES database contains more than 6,000 individuals, the PAM algorithm is computationally too expensive for estimation. Therefore, we rely on the Clustering Large Applications (CLARA) extension to the k-medoids algorithm which is also based on Kaufman and Rousseeuw (2009). The CLARA algorithm splits the sample into multiple subsets – in our case 50. For each subset, the k medoids are determined using the PAM algorithm. Thus, we end up with 50 sets of potential medoid-combinations. All sets are assessed using the full dataset. The remaining observations are assigned to whichever medoid is closest to them separately for each set of medoids. Then the average dissimilarity between the observations and their respective medoid is calculated. The set of medoids that minimizes this number is selected.

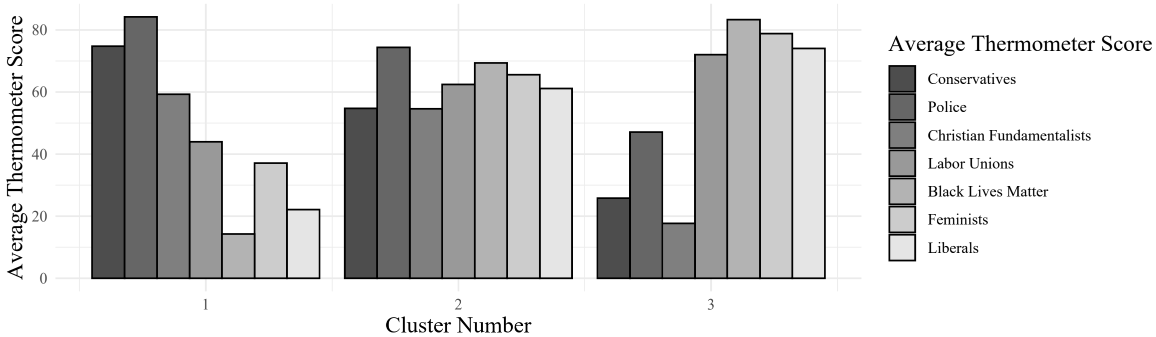

Figure 4.2: Average Thermometer Scores for political clusters. Cluster 1 includes 2,455 individuals, cluster 2 includes 2,520 individuals, and cluster 3 includes 1,897 individuals.

Applying the CLARA algorithm based on the seven political attitudes, we identify three clusters as illustrated in Figure 4.2. Individuals in cluster 1 have, on average, relatively high opinions about conservatives, the police, and Christian fundamentalists. This cluster thus seems to include individuals who align themselves with conservative ideologies in the US. The average scores for cluster 3 are roughly the opposite. Individuals in cluster 3 have, on average, relatively high opinions about liberals, feminists, BLM protesters, and labor unions. Thus, this cluster seems to include left-leaning individuals. Cluster 2 seems to collect all individuals who do not fit into both political camps. Therefore, this cluster includes individuals who deviate atypically in their opinions from left- or right-leaning individuals or scoring each group from the variables equally well. We call this cluster the "moderates" as their attitudes toward the different groups are less extreme on average.

These cluster interpretations are interesting for assessing heterogeneity in standalone and network benefits. Having individuals assigned to different clusters allows for the assessment of these effects separately. Thereby, we aim to shed light on what is driving left-wing vis-à-vis right-wing protests and whether the motivation to protest for left-leaning individuals is different from the motivation of right-leaning individuals. The estimators are obtained for the model Equation (4.15). The equation is the same as Equation (4.14), but this time, the model is estimated three times, i.e., for each cluster separately.

$$P_i = \alpha\,\mathbf{X}_i + \beta\,S_i + \gamma\,\mathbf{W}_{invdist}\,P_{-i} + \delta_{s(i)} + \epsilon_i \tag{4.15}$$

4.3.4 Identifying Protest Clusters

The last empirical specification is supposed to provide even more granular evidence by distinguishing between *protest movements*. This approach is supposed to further shed light on Hypothesis 2, which claims that there are heterogeneities among the motivation of protesters in different protests. In particular, we are interested in pooling individuals according to causes they think worthwhile protesting for and then examining how network and standalone benefits vary across these groups. Importantly, individuals may belong to multiple groups, as protest participation in one protest movement does not hinder an individual from also joining a second movement. The clusters in this specification are, therefore, no longer mutually exclusive.

The first information for building protest clusters is the awareness of the survey participants of political issues. Fortunately, the survey participants were asked what they perceived as the biggest problems in the US. Respondents were free to name multiple problems. Based on what survey respondents answered, we identified 4 topics that have been mentioned most frequently: healthcare (32.4%), race relations (30.8%), environment (17.4%), and immigration (14.9%).

Relying solely on these categories may be problematic because some topics do not directly translate to policy implications. For example, immigrant-skeptic individuals can name immigration as the biggest problem in the US today, but also pro-immigration individuals may name the same problem. Both groups, however, derive much different policy conclusions from the same issue and are, therefore, very unlikely to join the same protest or to belong to the same protest movement's group of potential protesters. Thus, we combine the naming of the problem with the thermometer scores about political groups. From the group that judges immigration to be one of the biggest problems in the US nowadays, only those who indicated a thermometer score of 50 or higher toward the group "illegal immigrants" remain in the group. All other individuals are excluded. Thus, this group represents people who see immigration as one of the most pressing issues in the country while being in favor of immigration. The same procedure is conducted for potential protesters who name race relations as one of the biggest problems in the US – only those individuals with an attitude score of at least 50 toward

BLM protesters remain in the group. The individuals are thereby grouped according to their problem awareness and their attitudes toward these protest movements. Based on individuals' survey responses, we thus assign 883 individuals to the group "Environment", 1617 individuals to the group "Black Lives Matter", 731 individuals to the group "Pro immigration", and 3789 individuals to the group "Healthcare". For each individual, four binary variables indicate (non-)membership in the respective groups. We consider an individual who is a member of one of these groups to be a potential protester for the corresponding issue.

The model equation becomes slightly more complex as indicated in Equation (4.16). First, the standalone benefit proxy, indicated by S_i, enters the equation as a single variable and in an interaction term with the protest cluster dummy variables, indicated by $\eta_{c(i)}$. The single proxy variable thereby measures the *average standalone benefit* while the interaction terms measure the *difference* between the protest cluster-specific standalone effect and the average standalone effect. The same is true for the identification of the network benefit. As in the baseline specification, $\mathbf{W}_{all}\, P_{-i}$ measures the average network benefit across all individuals. The interaction term of the spatial variable, however, measures the *difference* between the protest cluster-specific network benefit and the average network benefit.

$$P_i = \beta\, S_i + \eta_{c(i)}\, S_i + \gamma\, \mathbf{W}_{all}\, P_{-i} + \eta_{c(i)}\, \mathbf{W}_{c(i)}\, P_{-i} + \alpha\, \mathbf{X}_i + \delta_{s(i)} + \epsilon_i \tag{4.16}$$

This second extension allows for investigating whether standalone and network benefits differ across different protest movements. Fortunately, the significance level of the interaction terms directly tests whether there are statistically significant differences between the protest movement-specific effect sizes and the average effect sizes. In a later discussion, we aim to link a ranking of these effect sizes to the actual observed protest participation for the four protest movements under investigation.

4.4 Results

The following section presents the results of the empirical identification strategy. In the first step, Subsection 4.4.1 tests Hypothesis 1 by presenting the baseline results of the spatial regression for the whole sample. Next, more fine-grained evidence is collected to test for protest heterogeneities, as stated in Hypothesis 2. These heterogeneities refer to political attitudes and protest movements. Finally, actual protest numbers are descriptively presented, and potential explanations – based on the results of the previous subsection and the model implications – are verbalized.

4.4.1 Socio-demographic Networks

Table 4.2 presents the evidence collected for testing Hypothesis 1. The collection of results serves two major purposes. First, the results suggest which variables are drivers of protest participation and how they affect participation in protests across a representative sample of the US population. Additionally, the results assess the *average* standalone and network benefits that are modeled in Section 4.2. Table 4.6 in Appendix 4.A.2 presents the extensive regression output, including the full set of covariates.

Standalone Proxies:	No Standalone	Moral Consumer	Political Interest	Donate
Moral consumer		0.045***		
		(0.004)		
Political interest			0.031***	
			(0.004)	
Donate				0.040***
				(0.004)
Protest$_{-i}$	16.836***	15.439***	16.057***	15.366***
	(1.565)	(1.543)	(1.552)	(1.548)
Constant	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes
Pseudo R-Squared	0.072	0.041	0.075	0.045
Number of observations	6264	6264	6263	6257

Notes: Heteroskedasticity-robust standard errors are in brackets. *, **, and *** denote significance at the 10%-, 5%-, and 1%-level, respectively.

Table 4.2: Baseline regression output of the model presented in Equation 4.14. Full regression output including all the covariates is shown in Table 4.6 in Appendix 4.A.2. Protest$_{-i}$ indicates the spatial term. The other three variables are proxies for the standalone benefit.

The standalone benefit is assessed using three different specifications varying in the employed standardized proxy variables. The first specification contains no standalone proxy, and the following three include moral consumption, political interest, and donation behavior, respectively, as proxy variables for the standalone benefit. The effect size of the standalone benefit varies from 0.031 to 0.045. Thus, a one standard deviation increase in the standalone benefits makes protest participation 3.1 to 4.5 percentage points more likely according to the linear probability model. This finding provides some confidence for the significance of a, the standalone benefit, in individual protest decisions.

The spatial term $Protest_{-i}$ assesses the effect of the network benefit. Across all four specifications, there is statistically significant identification of its effect with small standard errors relative to the coefficient size. The latter has no economically meaningful interpretation as this variable is a weighted average of the protest participation of other individuals. A possible interpretation is that if the Mahalanobis distance-weighted average of other individuals' protest participation increases by 0.01, then an individual is 15.4 to 16.8 percentage points more likely to go protesting. The consistent statistically significant estimation of the spatial term indicates that the network benefit is a determinant of protest participation across the whole US population.

The results presented in Table 4.2 support Hypothesis 1 as both major drivers of protest participation, i.e., standalone and network benefits, are identified in the empirical specification. Potential concerns that the spatial term partly measures the included variables from the distance calculation are, from our point of view, misleading as all included variables from the distance calculation are also included as control variables in the spatial regression. In general, the results provide some confidence about the highlighted effects of the standalone and network benefits from the model and the employed empirical specification.

4.4.2 Political Networks

In the next step, we focus on Hypothesis 2 by investigating heterogeneities between different protests. According to the model, protest mobilization is a function of standalone and network benefits. Observing and measuring these effects for different protest movements is possible based on our developed empirical identification strategy. This subsection explores such heterogeneities with respect to different political directions of *potential protesters*.

A first challenge when exploring heterogeneities among different protests is defining a group of potential protesters. This step is important as the standalone and network benefits must be assessed not only for those individuals who participate in protests but also for those who decide not to participate but are exposed to both effects. Investigation requires forming groups of individuals who are *in principle* interested or motivated by similar reasons and, therefore, consider going to similar protest events. For such groups of individuals, the drivers of protest participation, i.e., standalone and network benefits, are predicted to vary given Hypothesis 2. This implication is tested in this subsection based on groups with similar political orientations.

The political groups show substantial heterogeneities in their respective standalone and network benefits. The groups are formed as explained in Section 4.3.3 and contain a right-leaning, a moderate, and a left-leaning group. Table 4.3 presents the results for each group, exhibiting large heterogeneities. The standalone benefit is highest for the moderate group, indicating that a one standard deviation shock makes an individual's protest participation 5.2 to 7.4 percentage points more likely. Right-leaning individuals have a lower standalone benefit (2.6 to 3.7 percentage points), followed by left-leaning individuals with an even lower standalone benefit (1.3 to 2.9 percentage points). In contrast, the network benefit is highest for the left-leaning group, followed by the right-leaning group, and the moderate group.

Standalone proxy:	(1) None	(2) Moral consumer	(3) Political interest	(4) Donate
Cluster 1: "right-leaning"				
Standalone proxy		0.037***	0.026***	0.029***
		(0.006)	(0.005)	(0.006)
$Protest_{-i}$	21.951***	19.774***	21.459***	20.348***
	(4.371)	(4.270)	(4.330)	(4.332)
Number of observations	2562	2562	2561	2559
Cluster 2: "moderate"				
Standalone proxy		0.066***	0.052***	0.074***
		(0.012)	(0.012)	(0.012)
$Protest_{-i}$	13.136***	12.484***	12.802***	11.909***
	(2.869)	(2.794)	(2.850)	(2.832)
Number of observations	1368	1368	1368	1367
Cluster 3: "left-leaning"				
Standalone proxy		0.029***	0.013**	0.024***
		(0.006)	(0.005)	(0.005)
$Protest_{-i}$	34.086***	31.180***	33.519***	31.790***
	(7.461)	(7.392)	(7.405)	(7.324)
Number of observations	2181	2181	2181	2178

Notes: Heteroskedasticity-robust standard errors are in brackets. *, **, and *** denote significance at the 10%-, 5%-, and 1%-level, respectively.

Table 4.3: Regression output of the model presented in Equation 4.15, estimated separately for the three political clusters. The proxy for the standalone benefit varies from columns 2 to 4 and is indicated in the respective column heading. Full list of covariates is omitted from the output.

The effect sizes of the average standalone and network benefits, a and v, are compared in Equation (4.17) and (4.18), where

$$a_{moderate} > a_{right-wing} > a_{left-wing}, \tag{4.17}$$

and

$$v_{left-wing} > v_{right-wing} > v_{moderate}. \tag{4.18}$$

Interestingly, the standalone benefit order is the opposite of the network benefit order. Left-wing protesters seem to be driven more by the network benefit than the standalone benefit as opposed to moderates, who seem to be motivated more by the standalone benefit than the network benefit. For right-leaning individuals, no clear dominance of one of the effects is observable from our results. While we cannot directly compare the coefficients of the standalone benefits with the network benefits, this finding is a first approximation. Relating our findings to the model, the opposed ordering of standalone and network benefits for left-leaning protests indicates strong network effects. It thus should translate into extreme outcomes, i.e., low or high protest sizes. Based on the same argument, protests of moderates would be medium-sized according to the benefit composition.

4.4.3 Protest Movements

The final specification collects even finer-grained evidence for Hypothesis 2 by differentiating protest movement-specific clusters. The objective is to identify and assess standalone and network benefits for four clusters of protest movements. These are BLM, pro-immigration, environment, and healthcare protests. Measuring the drivers of these protest movements contributes to (i) testing Hypothesis 2 and (ii) allowing for a tentative explanation about why some of these issues attract a large number of protesters while others do not. The cluster formation is described in Subsection 4.3.4 according to what individuals regard as the main problems in the US. Individuals may be part of no cluster or more than one cluster.

Table 4.4 displays the results indicating substantial heterogeneities within the protest-movement clusters. In terms of the standalone benefit, only BLM stands out in one of the three specifications. The interaction term “Standalone × BLM-issue” indicates the deviation from the average standalone benefit of this particular group. According to Specification (2), a one standard deviation shock in the standalone benefit increases the likelihood of BLM protesters by 6.7 percentage points, while on average, the likelihood is only increased by 3.9 percentage points. The other interaction terms are statistically insignificant, indicating that there is no significant difference between the average standalone benefit and the standalone benefit of environmental, pro-immigration, and healthcare protesters.

There are larger heterogeneities in the network benefits of individuals. The largest effect shows W_{imm}, which is the immigration-protest cluster-specific network effect difference from the average effect. The network effects of BLM and environmental protesters are also larger than the average network effect. Only the network benefit from healthcare-related topics is smaller than average, as indicated by the negative coefficient on W_{health}. All of these protest cluster-specific network effects differ from the average network effect at least at a 10% significance level.

Repeating the ordering of the average standalone and network benefits according to their effect sizes, where

$$a_{BLM} > \bar{a} \tag{4.19}$$

and

$$v_{immigration} > v_{BLM} > v_{environment} > v_{health}, \quad (4.20)$$

Equation (4.19) illustrates that the standalone benefit for BLM protesters is higher than for all other protests on average (as indicated by $\bar{a}$). Regarding the network benefit, pro-immigration protests have the largest effect, followed by BLM and environmental issues, and at the last position – even below the protest average – healthcare-related protests as shown in Equation (4.20). These results support Hypothesis 2 as they show that protests are characterized by different combinations of standalone and network benefits.

	(1) None	(2) Moral consumer	(3) Political interest	(4) Donate
Standalone		0.039***	0.030***	0.034***
		(0.008)	(0.007)	(0.007)
Standalone × Environment-issue		0.022	0.021	0.018
		(0.015)	(0.014)	(0.014)
Standalone × BLM-issue		0.027**	0.009	0.011
		(0.012)	(0.011)	(0.011)
Standalone × Immigration-issue		-0.005	0.028	0.015
		(0.026)	(0.023)	(0.026)
Standalone × Health-issue		-0.010	-0.008	0.004
		(0.010)	(0.009)	(0.009)
W_{all}	9.077***	8.189***	8.504***	7.997***
	(1.383)	(1.379)	(1.389)	(1.373)
W_{env}	0.281***	0.237***	0.238***	0.241***
	(0.070)	(0.066)	(0.068)	(0.069)
W_{blm}	0.305***	0.261***	0.299***	0.286***
	(0.062)	(0.060)	(0.062)	(0.062)
W_{imm}	0.397***	0.390***	0.393***	0.385***
	(0.122)	(0.120)	(0.121)	(0.121)
W_{health}	-0.202**	-0.157*	-0.212**	-0.177*
	(0.091)	(0.090)	(0.092)	(0.091)
Constant	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes
State dummies	Yes	Yes	Yes	Yes
Pseudo R-Squared	0.120	0.145	0.132	0.140
Number of observations	4882	4882	4882	4878

Notes: Heteroskedasticity-robust standard errors are in brackets. *, ** and *** denote significance at the 10%-, 5%- and 1%-level.

Table 4.4: Regression output of the model presented in Equation 4.16, with the effects being allowed to vary for different protest movements. The proxy for the standalone benefit varies from columns 2 to 4 and is indicated in the respective column heading. Full list of covariates is omitted from the output.

4.4.4 Extension: Linking Results to Protest Sizes

Given the results in Equations (4.19) and (4.20), we may provide a novel explanation of the actual observed protest sizes for the four protest clusters. Figure 4.3 presents the participation number and protest event count for these categories of protests for the year 2020 using the Crowd Counting Consortium database (Crowd Counting Consortium, 2021). Unsurprisingly, protests against racism are by far the largest in terms of participation and event number. Approximately 3 Million participants are counted in this protest movement, where one individual may be counted multiple times when participating in more than one BLM protest in 2020.

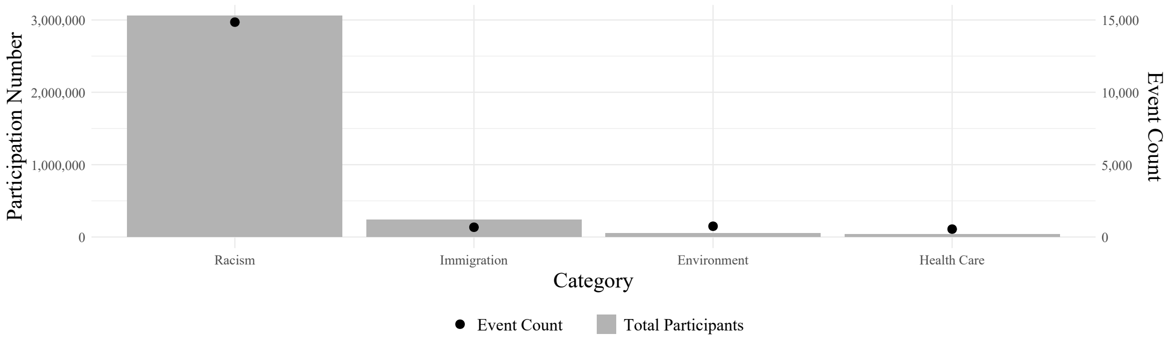

Figure 4.3: Illustration of protest size and frequency grouped by protest issues for 2020 according to the Crowd Counting Consortium database.

Based on the model implications and the empirical results, we expect anti-racism, pro-immigration, and environmental protests to show a high or low participation rate since relatively large network benefits characterize all three movements – which should constitute strong network effects. Looking at the BLM protest and the extensive participation numbers associated with it, this dynamic aligns with the theory. We suggest that this protest ended up in the full- instead of the no-participation equilibrium due to the murder of George Floyd, which arguably marked a focal point and solved the coordination problem by attracting a sufficient mass of initial protesters – as shown to be the critical factor in the dynamic model. A large body of literature on protest movements highlights the role of focal points for protest coordination (Truex, 2019; Carter and Carter, 2020; Ketchley and Barrie, 2020). In contrast, the relatively low number of protesters associated with immigration and environmental issues are, according to our model framework and the estimated benefit composition, not to be explained by a lack of network benefits but rather attributed to an absence of coordination. Hence, protests about these topics may eventually achieve extensive participation once a major event acts as a focal point, thereby shaping individuals' expectations of protest sizes and encouraging their participation.

The setting for healthcare-related protests looks different. Even though healthcare was named one of the most pressing problems in the US by most participants in the ANES survey, the number of protesters advocating for policy changes to improve healthcare remains very low. Considering the results from Subsection 4.4.3, these low participation numbers could be driven by the fundamentally different composition of standalone and network benefits that might constitute weak network effects. Protests on healthcare topics seem to be too unattractive for individuals to participate in, and thus only a small fraction of potential protesters actively engage in demonstrations.

The explanations provided in this subsection are by no means conclusive and are not supposed to be conclusive in the first place. Protests' dynamics depend on factors not accounted for in the model, e.g., crowding-out dynamics. However, we believe the results indicate that a simple model that explains protest mobilization by solely relying on standalone and network benefits as well as participation expectations already provides valuable insights into explaining protest events and the emergence of protest movements. Additionally, the results provide an insightful intuition of the driving factors of four major US issues today and present one explanation of the phenomenons in question.

4.5 Conclusion

This paper provides a theory model on protesting and empirically tests the model's implications. First, we establish a new framework for individuals' protest decisions by adopting a model on a single network good by Belleflamme and Peitz (2015). In contrast to standard literature, we argue that the individual's protest participation is non-pivotal to the success of the underlying protest objective and that the individual's utility derived from protesting consists of two components: (i) the interaction between an individual's type and the standalone benefit, and (ii) the network benefit which increases linearly with the expected number of further participants. We then derive equilibrium outcomes where the relation between the standalone benefit and the network benefit determines the aggregated outcome: Under *weak network effects*, a unique equilibrium exists for any exogenous participation cost. Under *strong network effects*, two stable equilibria coexist for intermediate participation costs, namely the extreme outcomes of full and no participation, which illustrates the coordination problem of protests.

The results from the empirical investigation speak in favor of the model dynamics. Using a spatial auto-regressive model, we identify the standalone and network benefits as drivers of individual protest participation across all individuals in the US sample. Furthermore, we measure the effect size for different political directions and protest movements. The findings suggest network benefits generally drive mobilization in BLM, pro-immigration, and left-leaning protests. In contrast, protests by moderates and those related to health care seem to be driven by standalone benefits. In the extension, we contribute to the current literature by providing a tentative explanation for actual observed protest sizes, contextualizing the previously measured combination of standalone and network benefits with the model's implications.

4.A Appendix

4.A.1 Mathematical Appendix

Outside Option $\underline{U}$. First, suppose that an individual's outside option is type-dependent with

$$\underline{\mathrm{U}} = \theta \underline{\mathrm{u}}, \tag{4.A.21}$$

where θ is uniformly distributed on the interval $[0, 1]$. Then, the critical type threshold denotes

$$\hat{\theta} := \frac{c - vn^e}{a - \underline{\mathrm{u}}}, \tag{4.A.22}$$

where it is straightforward to see that the type-dependent outside option only affects the level and slope of the function $n(c)$.

Second, suppose that an individual's outside option scales with the expected number of participants, i.e.,

$$\underline{\mathrm{U}} = \phi n^e \tag{4.A.23}$$

with base utility $\phi > 0$. Then, the critical type threshold denotes

$$\hat{\theta} := \frac{c - (v - \phi)n^e}{a}, \tag{4.A.24}$$

where we can readily see that the scaling outside option, ϕn^e, only affects the steepness of the function $n(c)$.

With the above cases being variable transformations, it is without loss of generality to normalize an individual's outside option to $\underline{\mathrm{U}} = 0$. □

Proof of Proposition 2. Recall the utility function $U_t(\theta) = \theta a + vn_t^e$, where $n_t^e = n_{t-1}$ and the initial participation level $n_0 \in [0, 1]$. Recall further that with an outside option $\underline{\mathrm{U}}_t = 0$, the critical type threshold at time t denotes $\hat{\theta}_t = (c - vn_{t-1})/a$.

At $t = 1$, the number of participants at a protest denotes

$$n_1(c, n_0) = \begin{cases} 0 & \text{if } n_0 < \frac{c-a}{v}, \\ 1 - \frac{c - vn_0}{a} & \text{if } \frac{c-a}{v} \leq n_0 \leq \frac{c}{v}, \\ 1 & \text{if } n_0 > \frac{c}{v}, \end{cases} \tag{4.A.25}$$

where we can readily see that $n_1(c, n_0) \geq n_0$ if $n_0 \geq (c - a)/(v - a)$ and that this property holds for any time $t \geq 1$, i.e., $n_t(c, n_0) \geq n_{t-1}(c, n_0)$ if $n_{t-1} \geq (c-a)/(v-a)$.

Lemma 1. *For any time $t \geq 1$, the number of participants weakly increases if n_0 is sufficiently large, i.e.,*

$$n_t(c, n_0) \geq n_{t-1}(c, n_0) \quad \textit{if} \quad n_0 \geq \frac{c-a}{v-a}, \tag{4.A.26}$$

Considering the corner intervals in (4.A.25), Lemma 1 implies that if the initial participation satisfies $n_0 \geq c/v$ (if $n_0 < (c-a)/v$), the number of participants remain in the corner solution with $n_t = 1$ ($n_t = 0$) for any time $t \geq 1$.

We can now focus on the interior where $(c-a)/v \leq n_0 \leq c/v$. The number of participants at time $t \geq 1$ can be written as

$$n_t(c, n_0) = n_0 \left(\frac{v}{a}\right)^t + \left(1 - \frac{c}{a}\right) \sum_{i=0}^{t-1} \left(\frac{v}{a}\right)^i, \tag{4.A.27}$$

and the change in the number of participants per period denotes $\Delta_t = \left(\frac{v}{a}\right)^{t-1} (n_1 - n_0)$.
(i) Under weak network effects ($v < a$), Equation (4.A.27) describes a geometric series in the limiting case in t, and converges to its supremum with $\lim_{t\to\infty} n_t(c, n_0) = (a-c)/(a-v)$.
(ii) Under strong network effects ($v > a$), the change in the number of participants, Δ_t, implies that in the limiting case in t, the participation number, n_t, always ends up in a corner solution. From Lemma 1, it must follow that $\lim_{t\to\infty} n_t(c, n_0) = 0$ if $n_0 < (c-a)/(v-a)$ and $\lim_{t\to\infty} n_t(c, n_0) = 1$ if $n_0 > (c-a)/(v-a)$, which finishes the proof. □

4.A.2 Figures and Tables Appendix

Variable	Question
Protest Variable	
Protest	During the past 12 months, have you joined in a protest march, rally, or demonstration, or have you not done this in the past 12 months? *1. yes; 0. no.*
Socio-Demographic Characteristics	
Age	What is the month, day and year of your birth? *1. 17 - 24; 2. 25 - 34; 3. 35 - 44; 4. 45 - 54; 5. 55 - 64; 6. 65 - 74; 7. 75 - 99 and over.*
Education	What is the highest degree that you have earned? *1. 8 grades or less; 2. 9-12 grades; 3. 12 grades, diploma or equivalency; 4. 12 grades, diploma or equivalency plus non-academic training; 5. Some college, no degree; 6. BA level degrees; advanced degrees incl. LLB.*
Income	Please mark the answer that includes the income of all members of your family during the past 12 months before taxes. *Scale from 1. to 28. indicating income from 0-10k USD to >250k USD.*
White	Are you White? *1. yes; 0. no.*
Black	Are you Black? *1. yes; 0. no.*
Hispanic	Are you Hispanic? *1. yes; 0. no.*
Male	Are you Male? *1. yes; 0. no.*
Health status	Self-evaluation of R health. *1. excellent; 2. very good; 3. good; 4. fair; 5. poor.*
Married	Are you married? *1. yes; 0. no.*
Children	How many children are there under 18 years old in this family (household)? *0. None; 1. One; 2. Two; 3. Three or more*
Personal Attitudes	
Left-right-scale	Where would you place yourself on a scale from 0 to 10 where 0 means the left and 10 means the right?
Dev. Left-right-scale	Absolute number of 5 - previous answer
Religiosity	Do you consider religion to be an important part of your life, or not? *1. Yes, important 2. Little to no importance.*
Trust	Generally speaking, how often can you trust other people? *1. Always; 2. Most of the time; 3. About half the time; 4. Once in a while; 5. Never*
Longitude	Longitude of the respondent's county.
Latitude	Latitude of the respondent's county.
Political Attitudes Towards...	
Labor Unions	Feelings towards some of [these groups]. *Scale 0 to 100.*
Conservatives	Feelings towards some of [these groups]. *Scale 0 to 100.*
Liberals	Feelings towards some of [these groups]. *Scale 0 to 100.*
Feminists	Feelings towards some of [these groups]. *Scale 0 to 100.*
Christian Fundamentalists	Feelings towards some of [these groups]. *Scale 0 to 100.*
Police	Feelings towards some of [these groups]. *Scale 0 to 100.*
Black Lives Matter	Feelings towards some of [these groups]. *Scale 0 to 100.*
Proxies Standalone Benefit	
Donate	During the past 12 months, have you ever given money to a political, social, or religious organization, or have you not done this in the past 12 months? *1. yes; 0. no.*
Political interest	How interested would you say you are in politics? *1. Not at all interested; 2. Not very interested; 3. Somewhat interested; 4. Very interested.*
Moral consumer	In the past 12 months, how often have you either bought or declined to buy a certain product or service because of the social or political values of the company that provides it? *1. Never; 2. Once in a while; 3. About half the time; 4. Most of the time; 5. All the time.*

Table 4.5: Variable description. Questions from the American National Election Survey 2020.

	(1)	(2)	(3)	(4)
Male	0.002	0.002	-0.003	0.002
	(0.004)	(0.004)	(0.004)	(0.004)
Age	0.003	-0.000	-0.008	-0.004
	(0.006)	(0.005)	(0.006)	(0.005)
White	-0.004	-0.003	-0.004	-0.007*
	(0.004)	(0.004)	(0.004)	(0.004)
Income	0.005	0.003	0.003	0.000
	(0.004)	(0.004)	(0.004)	(0.004)
Education	-0.015***	-0.017***	-0.018***	-0.019***
	(0.005)	(0.005)	(0.005)	(0.005)
Left-Right-Scale Dev.	0.037***	0.028***	0.032***	0.034***
	(0.004)	(0.004)	(0.004)	(0.004)
Health	0.006	0.005	0.006	0.005
	(0.004)	(0.004)	(0.004)	(0.004)
Married	-0.011***	-0.013***	-0.011***	-0.013***
	(0.004)	(0.004)	(0.004)	(0.004)
Children	-0.012***	-0.011***	-0.012***	-0.011***
	(0.004)	(0.004)	(0.004)	(0.004)
Religiosity	-0.011**	-0.011***	-0.009**	0.004
	(0.004)	(0.004)	(0.004)	(0.005)
Trust	0.003	0.003	0.001	0.000
	(0.004)	(0.004)	(0.004)	(0.004)
Moral consumer		0.045***		
		(0.004)		
Political interest			0.031***	
			(0.004)	
Donate				0.039***
				(0.004)
Protest_{-i}	16.836***	15.439***	16.057***	15.366***
	(1.565)	(1.543)	(1.552)	(1.548)
Constant	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes
Pseudo R-Squared	0.072	0.041	0.075	0.045
Number of observations	6264	6264	6263	6257

Notes: Heteroskedasticity-robust standard errors are in brackets. *, **, and *** denote significance at the 10%-, 5%-, and 1%-level, respectively.

Table 4.6: Full baseline regression output of the model presented in Equation 4.14, including all the covariates. Protest_{-i} indicates the spatial term. The other three variables are proxies for the standalone benefit.

Chapter 5

Conclusion

In this dissertation, I studied the strategic decision-making of economic actors through three distinct papers, each applying core concepts of microeconomic theory to scenarios in economics and finance. Chapter 2 analyzes the relationship between interest rates and zombie firms, providing insights into the short- and long-run effects of monetary policies on banks' lending behavior. In doing so, the paper fills the research gap in determining a precise mechanism for how interest rates affect zombie lending. Chapter 3 examines the role of human capital provision in venture capital portfolios, highlighting the potential for market-based externalities through entrepreneurial actions. By introducing the concept of *preemptive differentiation*, the paper is the first to account for intra-portfolio competition at the market stage and how this approach might help explain the reduction of active corporate governance in the VC industry. Lastly, Chapter 4 adopts a micro-founded framework on network goods to model protest participation and employs a spatial autoregressive model to test its dynamics. Both the theoretical and the empirical approaches are novel in the existing literature, with the results highlighting the significance of individual attitudes and network effects and offering a tentative explanation for observed protest sizes in the US.

This work contributes to a deeper understanding of strategic behavior and how individual incentives and decision-making can harm welfare. I hope my research offers directions for future work on zombie lending, venture capital, and public protests, as well as instills fresh perspectives on the broad application areas of microeconomic theory.

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