Essays on Private Antitrust Enforcement

Dissertation
zur Erlangung des Grades eines Doktors der Wirtschaftswissenschaft
der Rechts- und Wirtschaftswissenschaftlichen Fakultät
der Universität Bayreuth

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Tag der mündlichen Prüfung: 03.04.2020

Für meine Eltern

Acknowledgements

In the course of writing my doctoral thesis I was delighted to benefit from the support of many people. I would like to take this opportunity to express my gratitude to them.

First and foremost I am indebted to my Ph.D. supervisor Stefan Napel. His lecture on industrial organization sparked my interest in the subject, his motivation during my master thesis encouraged me to start my Ph.D. and his great support while writing my thesis strongly contributed to the successful performance. Without Stefan Napel, this book would certainly never have been written. I am also particularly grateful to Fabian Herweg for not only kindly agreeing to become my second supervisor but also for giving me helpful comments for many of my research activities.

This thesis has highly benefited from academic stimulation by Maarten Pieter Schinkel and Hartmut Egger. Their critical discussion improved my work significantly. My colleagues Alexander Mayer, Lars Just and Richard Franke have been important contact points whenever help was needed. Lars Just, who supervised my bachelor thesis, proofread parts of the book together with Richard Franke. Alexander Mayer was not only a perfect colleague, he also became a good friend and motivator. Many thanks also go to Heidi Rossner-Schöpf for her invaluable organizational assistance.

I also want to thank Nicole Grüner and Johanna Rosenbeck who provided a tremendous amount of emotional support and constantly motivated me to carry on.

Finally, I am particularly grateful to my parents and my sister for always believing in me and to my adorable niece Leonie Müller. In particular, my parents did not only enable me to study and to do my Ph.D., their trust in me also motivated me to constantly strive for the best.

Abstract

This thesis studies the economic impact of private antitrust enforcement on competition and on cartel members' compensation payments. After Chapter 1's short introduction to the topic, Chapter 2 investigates the effects of compulsory compensation for *umbrella losses* on competition. These losses arise when the price increase by the cartel results in a general price increase in the market concerned, i.e., customers who bought a product produced by a cartel outsider also had a damage. Chapters 3 and 4 analyse the internal allocation of compensation payments between cartel members in the EU.

More specifically, Chapter 2 disproves the conventional wisdom that giving more customers a legal entitlement to compensation leads to more cartel deterrence and enhances competition. In a Bertrand-Edgeworth model it is shown that more stringent compensation provisions can have the opposite effect when the size of formed cartels is endogenous. In particular, compulsory compensation for umbrella losses deters small cartels, which have limited influence on the market price, and inadvertently stimulates formation of big encompassing cartels with significant price influence.

Detected wrongdoers are commonly *jointly and severally liable* towards customers who successfully claimed for compensation, i.e., they may be forced to compensate victims on behalf of all. In the EU, they are internally liable in proportion to their "*relative responsibility*" for the harm. Chapter 3 operationalizes a firm's relative responsibility by evaluating counterfactual damages had one or more cartelists rejected collaboration. Basic normative requirements – in particular causality – call for aggregation of counterfactual overcharges via the *Shapley value*. Damage allocations for linear market environments are characterized and bounds on payment obligations are established. Several ad hoc suggestions for deducing relative responsibility, e.g., from market shares or profits, are evaluated. The chapter also provides a new decomposition of the Shapley value which can be useful for other applications.

The applicability of the heuristics discussed in Chapter 3 is however limited for two reasons. First, they do not provide a good approximation of the Shapley value independently of firms' characteristics. Second, none of the heuristics reflects the cartelists' relative responsibility. Chapter 4 resolves this obstacle by arguing that *simple games*, used in game-theoretic analysis of voting, can provide a workable approximation of the crucial causal links that define responsibility. Moreover, numerical examples show the *Shapley-Shubik index* of simple games to reflect EU law better than ad hoc heuristics for a variety of linear market models.

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Chapter 1

Introduction

1.1 Motivation and Scope

Competition is impeded when rival firms coordinate their strategies. Agreements how to set prices, quantities or other decision variables are forbidden in many jurisdictions. They not only decrease consumer surplus but can also have negative effects on other market players. The European Commission estimated the annual social costs for 'hardcore cartels' in the EU to approximately €25–65 billion (see European Commission 2013a). Deterring firms from illegal conduct is therefore an important task for policymakers. Public and private antitrust enforcement institutions have been established in order to sustain competition.

How *public* antitrust rules should be designed to achieve this goal has been a key question in the literature for long. Many policy recommendations of economists have already been implemented into national law. The most prominent example is the introduction of leniency programs in the EU and the US with the aim to increase a cartel's detection probability. The great success of leniency programs is emphasized by many competition authorities (see, e.g., Hammond 2010 or Bundeskartellamt 2016).[1]

In the US, there is also a long and successful history of *private* antitrust enforcement (see Clayton Act 1914, §4, §16). More and more other countries have therefore extended their private antitrust rules, e.g., the EU, Japan or Australia. In some aspects, legal rules among countries are comparable. For example, several countries established rules that (i) allow litigants to reach out-of-court settlements, (ii) make

[1]See Bryant and Eckard (1991) and Combe et al. (2008) for an economic discussion of leniency programs.

tortfeasors *jointly* liable towards third parties and (iii), at least partially, exclude leniency applicants from joint liability.[2]

The economic discussion of private antitrust enforcement has, however, received far less attention than the public one. This is surprising with respect to at least three points. First, the deterrent effect of private antitrust enforcement is generally accepted by legal and economic scholars. Second, although influential competition authorities around the world seek for effective competition, designs of legal rules differ. A question which arises is which legal environment indeed promotes effective competition? Third, private antitrust enforcement is a highly dynamic part of competition policy. Thus, policy recommendations have to be established and legal rules have to be transferred into economic concepts in an ongoing process. These three points are now discussed in more detail.

(1) The deterrent effect of private antitrust enforcement is frequently highlighted by competition economists. For instance, Lande and Davis (2008) showed in a sample of forty cartel cases that "... private litigation provides more than four times the deterrence of the criminal fines".[3] Deterrence arising from private antitrust enforcement in the EU increased since the European Commission passed Directive 2014/104/EU (frequently referred to as the "*Directive on Antitrust Damages Actions*") which contains several rules to simplify private antitrust actions. High compensation claims in private antitrust enforcement may become the norm in the EU. Several courts throughout the EU already handle claims for compensation of overcharge losses, aggregating to several billions, caused by a long-lasting violation of antitrust law in the European truck market.[4]

(2) The design of legal rules can differ between countries – and even their interpretation within a country. This, per se, would not be surprising if the legislative objectives differed between countries or specific regions. But the goal of *effective competition* is the same and omnipresent in the legal and economic literature. The point is that opinions on how effective competition can be achieved differ. These differences can be driven by ideological aspects (e.g., is an over-deterrence of antitrust infringements possible?) but also by methodological aspects (e.g., what are the channels of influence?).

[2] *Jointly liable* means that harmed customer can sue any cartel member for any desired share of the total damage no matter whether this cartel member sold the product or not.

[3] See Werden et al. (2012) for a critical discussion.

[4] See, e.g., LG Hannover Az. 18 O 8/17, LG Stuttgart Az. 45 O 11/17, Spanish commercial Court-Juzgados Delo Mercantil 151/2019 and 118/2019.

For instance, in the US, the legal interpretation of private antitrust rules depends highly on the competent court, like the handling of cases in which customers suffer *umbrella losses* (i.e., they paid a price overcharge due to collusion although they bought the product from a non-cartel member; see 596 F.2d 573 3rd. Cir. 1979 and 600 F.2d 1148 5th Cir. 1979 for varied assessments). In the light of effective competition, some scholars ask the US Supreme Court to follow the Court of Justice of the European Union (CJEU) in obligating firms to compensate customers who suffered umbrella losses (see Blair and Durrance 2018). These scholars only consider one channel of influence: *cartel deterrence*.[5]

A prominent example where the legal norm differs between European and US private antitrust law is the internal liability of cartel members after compensation has been paid jointly (external liability). In the US, there is generally no internal compensation among joint tortfeasors (see Texas Industries, Inc. v. Radcliff Materials, Inc., 451 U.S. 630, 1981). By contrast, the EU Directive on Antitrust Damages Actions states clearly that a cartel member is liable according to his *relative responsibility* for the harm.

(3) The derivation of policy implications and the "translation" of legal rules into economic concepts are important and ongoing tasks. Thus, after the appropriate legal and economic concept is figured out, guidelines for lawyers on how to easily apply this concept have to be developed. An economic model which perfectly reflects the legal rule is not useful if data requirements or mathematical burdens too severely limit its applicability.

This thesis addresses points (1)–(3) and focuses on questions which are outstanding due to their economic and legal scope. Chapter 2 analyses how compensation for umbrella losses influences effective competition. Thus, the existing literature (see, e.g., Blair and Maurer 1982 or Blair and Durrance 2018) is extended to an important topic. It is shown that an increasing number of customers who are allowed to bring a lawsuit before the court can lead to more encompassing cartels: the price overcharge can increase. Chapters 3 and 4 deal with the economic operationalization of the *relative responsibility* norm for the internal allocation of compensation payments between jointly and severally liable cartel members. It is argued that the *Shapley value*, a prominent solution concept in cooperative game theory, applies best from a theoretical perspective.

[5]Note that the notions of *effective competition* and *cartel deterrence* cannot be used synonymously as increasing cartel deterrence for partial cartels does not necessarily lead to fewer cartels forming or to lower observed market prices.

1.2 Structure

All chapters are designed to be self-contained and can be read in any sequence. This comes at the cost of some overlap. Chapters 3 and 4 are based on joint work with Stefan Napel.

1.2.1 Compensation for Umbrella Losses: Cartel Deterrence and Cartel Size

Chapter 2 considers compensation for *umbrella losses* and how effective competition is affected when all (direct) customers who suffered harm caused by a cartel agreement can reclaim antitrust damages, no matter whether the product was produced by a cartel member or by an outsider.

Umbrella pricing arises when the price choice of a best-responding cartel outsider is increasing in the price chosen by cartel members. Then, also customers who bought a product from a firm which acted competitively had a damage caused by the cartel: the *umbrella loss*. To achieve effective competition, the CJEU established in 2014 that former cartel members also have to compensate these losses (see CJEU C-557/12 2014). Thus, all customers have *legal standing*, that is, have the right to bring a corresponding lawsuit before the court.

The argument made by legal and economic proponents of an entitlement to compensation for umbrella losses is that a larger potential number of suing customers increases *cartel deterrence* since net expected collusive profits decrease (see, e.g., CJEU C-557/12 2014 or Blair and Durrance 2018). Needless to say, this is indeed a key argument for allowing all customers to have legal standing when effective competition is the aim. However, this argument can only be the final judgment when cartel size is exogenous; an assumption which can hardly be satisfied.

Extending legal standing to customers who bought from non-cartel members will influence cartel *size* and therefore the *market price* when endogenous cartel formation is considered. The reason is that compensation for umbrella losses influences a specific cartel's attractiveness because profits of both the cartel members and the non-cartel members depend on whether all customers have legal standing or not.

In a Bertrand-Edgeworth model related to Bos and Harrington (2010) we discuss how compensation for umbrella losses influences cartel deterrence and cartel size in a dynamic market environment. This allows to challenge the policy recommendation that all customers should have legal standing when effective competition is desired.

For illustrative reasons, consider a market with three symmetric but capacity-constrained firms A, B and C that produce a homogeneous good and set prices. Regarding a firm's capacity, assume that no firm is large enough to influence the competitive market price on its own. Then, competitive profits of firms are zero when per unit production costs are constant. Let us also presume that the capacity of a single firm is sufficiently small to ensure that a partial cartel of two firms can profitably increase their prices above the competitive level. In fact, when a partial cartel of two firms was formed (e.g., let firms A and B be the wrongdoers), their customers will have paid an overcharged price. However, also firm C's customers bought the product at a price which was above the competitive level: firm C's competitive best response to increased prices by firms A and B is to increase its price under the umbrella of the cartel. Hence, it made a positive profit compared to zero profits received under competition. Thus, all customers have had a damage caused by illegal coordination of just two firms.

If firm C also joined the cartel, competition among all firms would be eliminated and cartel members would set the monopoly price. Then, the worst market outcome is reached from a customer's but also from a total welfare perspective: the price overcharge damage and the deadweight loss are highest. Additionally, when all firms jointly infringe competition law, the question how a compensation for umbrella losses influences the market outcome becomes irrelevant since there are no umbrella losses which could be compensated.

Whether a stable cartel exists, what size this cartel has and how it can influence the market price, will depend on whether firms have to compensate for umbrella losses or not. Cartel deterrence, when only firms A and B coordinate their strategies, is obviously strengthened when all customers are allowed to bring a lawsuit before the court, because expected compensation payments increase. We additionally show that members of a partial cartel will decrease their prices when all customers have legal standing. Thus, conditional on that a partial cartel has formed, prospective customers benefit from a compensation for umbrella losses: deterrence increases and the damage caused by a potential cartel decreases.

However, decreasing market prices when firms A and B have to compensate for umbrella losses also leads to decreasing profits for firm C, since the price-raising effect caused of the cartel is softened. This lowers the attractiveness of operating outside the cartel and inadvertently stimulates firm C to join the partial cartel; so the industry-wide cartel can successfully operate (assuming that firms are sufficiently patient). Then firms set the joint profit maximizing price, that is, the monopoly price. Hence, the argument that all harmed customers should have legal standing to achieve effective competition has to be viewed with caution.

The opposite effects of an extended legal standing, that is, decreasing prices and increasing cartel deterrence given a cartel has already formed versus an increasing cartel size when endogenous cartel formation is considered, are discussed and proven to hold true in a dynamic market environment with n firms. Whether the (average) price overcharge of a cartel will increase or decrease when firms have to compensate for umbrella losses cannot be answered unambiguously. This new and highly relevant observation hasn't been acknowledged in the literature so far. It could help to improve policy recommendations.

1.2.2 Shapley Apportioning of Cartel Damages by Relative Responsibility

Chapter 3 considers damage allocation between jointly and severally liable cartel members when a firm's contribution share should depend on its *relative responsibility* for the harm.

We start by discussing several properties that a responsibility-based allocation should satisfy, assuming that organizational roles of cartel members are symmetric. It is, for example, reasonable to require that all compensation payments have to be allocated among detected cartel members and that different currencies or increasing interest payments should not change the share that a former cartel member must contribute. In particular, to be in line with relative responsibility, three properties are central. First, when a firm joins a cartel but a customer's damage stays unchanged, no matter which (partial) cartel is considered (this firm is termed *null player* in the field's literature), it is convincing to argue that it bears no responsibility. Null players should be excluded from compensating antitrust victims. Second, when one firm can be replaced by another firm and a customer's damage stays unchanged for every counterfactual damage scenario, both firms bear the same relative responsibility and should have to contribute equally. Most importantly, a cartel member's compensation payments should depend on its ability to influence market prices: when a firm joins a partial cartel and a customer's damage increases strongly, this firm bears huge responsibility for the resulting total. The *Shapley value*, introduced by Shapley (1953b), is the unique value which satisfies all these and several other desirable properties.

All counterfactual market scenarios enter the Shapley value, that is, damages of all partial cartels that could form have to be known. Thus, when a market consists of three firms A, B and C and the industry-wide cartel formed, not only the competitive and the collusive outcomes have to be known, also damages caused by partial cartels $\{A, B\}$, $\{A, C\}$ and $\{B, C\}$ enter the Shapley value. To simplify the derivation of a firm's compensation payments, Chapter 3 introduces a new decomposition of the Shapley value, based on incremental contributions. Its interpretation can be illustrated by determining firm A's contribution payments: it starts with an equal 'per head' allocation of cartel damages. Firm A's contribution share then decreases when the damage caused by the partial cartel $\{B, C\}$, exceeds the *average* damage caused by a partial cartel of two firms, i.e., $\{A, B\}$ and $\{A, C\}$. In general, only average damages by partial cartels of the same size enter the Shapley value when the new decomposition is used. Thus, it is not necessary to determine damages in all counterfactual market scenarios. This decomposition could be of general interest whenever costs, benefits, etc. have to be allocated between firms or contractual partners in general.

One can explicitly determine the Shapley value in a linear market environment with differentiated substitutes by characterizing a firm's average damage for a given cartel size s. This allows to derive general bounds between which the Shapley value of a specific firm lies. The firm which has sold the product must usually contribute more than an equal share of the compensation, even under mild asymmetry of firms.

Since all relevant market parameters have to be estimated to determine the Shapley value exactly, we compare an allocation based on Shapley shares with ad hoc heuristics suggested by legal practitioners. We can infer that the heuristic which is closest to the Shapley value depends on the kind of asymmetry between firms. If, e.g., firms differ in their size, an allocation based on competitive sales is most adequate; when firms differ in their efficiency, an allocation based on competitive revenues fits best. No heuristic outperforms all others – in particular, even the closest heuristic is in some cases far off.

Last, the chapter argues briefly that a generalization of the Shapley value – the *weighted Shapley value* – can be used to take organizational roles of the cartel's firms into account. Thus, a responsibility-based allocation of cartel damages is feasible even if a cartel member's contribution share does not solely depend on market parameters. It can, e.g., be desirable to increase a ringleader's contribution share or to exclude a leniency applicant from compensation payments to correctly reflect relative responsibility.

1.2.3 Simple Games and Cartel Damage Proportioning

In Chapter 4 we introduce a new heuristic to approximate a cartel member's contribution share when damage should be allocated by relative responsibility. Then, as discussed in Chapter 3, the Shapley value should be used to determine a firm's compensation payments. This heuristic has two advantages compared to an ad-hoc damage allocation based on market shares or profits for example. First, it reflects cartel members' relative responsibility. Second, it continuously outperforms ad hoc heuristics with respect to the accuracy of the approximation in the market model with linear demand and cost functions introduced in Subsection 3.3.1.

To set up this heuristic, we first discuss *simple games* in a cartel damage context. Simple games are a special case of cooperative games and characterized by two properties. First, the worth (or damage) of each (partial) cartel is either 0 or 1. Second, the damage caused by a partial cartel weakly increases from zero to one when more and more firms join the cartel.

We therefore normalize the damage caused by a (partial) cartel to 1 if the damage caused by this cartel is large; it is normalized to zero if a cartel's damage is relatively small by comparison. This dichotomous approximation regarding a cartel's damage can be represented very compactly by the set of *minimal winning coalitions (MWC)*. A MWC contains *only* firms which are needed to cause a unit damage; damage would be zero if one firm left a MWC.

A given set of MWCs determines a *dichotomous damage scenario (DDS)*. With three firms four distinct DDS are feasible; for five or less firms we enumerate *all* 179 DDS which can arise. We argue that the specification of a DDS to approximate the underlying market scenario is much easier than a full-blown merger simulation analysis. For example, the precise estimation of a firm's production costs is not needed to derive the appropriate DDS. Determining the Shapley-Shubik (power) index for a specific DDS gives a damage allocation which reflects relative responsibility.

We evaluate how a heuristic based on DDS performs in the linear market model discussed in Subsection 3.3.1. We analyse two claim scenarios. First, we assume that only one customer who bought one product unit acts against former cartel members. Second, we assume that all or a fixed share of all customers act against former cartel members. In both scenarios, we infer that a heuristic based on approximating the cartel market by a DDS frequently outperforms ad hoc heuristics. In particular, in the first claim scenario we show for a class of numerical examples that only an allocation based on DDS always comes close to a firm's Shapley share evaluated in the original market. In the second claim scenario, only the heuristic based on DDS comes close to a responsibility-based allocation of cartel damages *independently* of the considered asymmetry between firms.

Chapter 2

Compensation for Umbrella Losses: Cartel Deterrence and Cartel Size

In most jurisdictions, victims of antitrust infringements have a right to act against detected cartel members and to reclaim damages caused by a cartel. Firms can anticipate that they will have to compensate suing customers if their cartel should be detected. Thus, compensation payments not only redistribute money ex post from wrongdoers to harmed customers but play a role in deterring cartels ex ante. Private antitrust enforcement has therefore become an important regulatory instrument in the US, EU, Japan and elsewhere.

An important aspect when discussing private antitrust enforcement is the *legal standing* of cartel victims: who is legally entitled to be compensated by former cartel members? Is it only customers who bought a product produced by a cartel member, or are customers who bought a product produced by outsiders eligible to bring a lawsuit before the court, too? This question is answered differently in the US and the EU. In the US, there is no final decision of the Supreme Court yet. Whether only customers who bought products produced by cartel members are entitled to compensation crucially depends on the competent court (see 596 F.2d 573 3rd. Cir. 1979 and 600 F.2d 1148 5th Cir. 1979 for conflicting views and Blair and Durrance 2018 for an overview of more recent cases). By contrast, the Court of Justice of the European Union (CJEU) recently established that also customers who suffered a loss from *umbrella pricing* have legal standing (see CJEU C-557/12 2014 for a landmark judgement and 6 U 204/15 Kart (2) for a relevant application). Umbrella pricing refers to the fact that also best-responding outsiders raise their prices in reaction to price increases by cartel participants.[1]

The CJEU endorsed an expanded legal standing of antitrust victims by reference to antitrust goals and in order to further effective competition. It concluded that "[t]he right of any individual to claim compensation for such a loss [umbrella pricing] actually strengthens the working of the European Union competition rules, since it discourages agreements or practices, frequently covert, which are liable to restrict or distort competition, thereby making a significant contribution to the maintenance of effective competition in the European Union" (see CJEU C-557/12 2014, recital 23). Similar arguments are acknowledged in the US. For instance, Leon Higginbotham Jr., former judge at the U.S. Court of Appeals for the Third Circuit, already noted in 1979 (judgement 596 F.2d 573 3rd. Cir.) that "[a]llowing standing [for umbrella pricing] would also encourage [private] enforcement, and thereby deter violation, of the antitrust laws."

The legal standing of antitrust victims clearly has great economic importance but its effects on cartel behavior has been investigated only by a comparatively small literature. Blair and Maurer (1982) stated: "[i]t is obvious that the prospect of recovery by purchasers from noncolluding competitors should have a greater deterrent effect than recovery limited to direct purchasers, assuming a constant probability of detection." Blair and Durrance (2018) concluded that awarding compensation for umbrella losses "... further deters illegal price-fixing behavior".

These quotes are intuitively very appealing. However, they have a common shortcoming. The legal standing of victims of antitrust infringements will not only affect cartel deterrence as such: *stable cartel sizes* may depend on how umbrella victims are treated.

Compared to the baseline case of no compensation whatsoever, expected collusive profits decrease if customers who bought a product produced by a cartel member are entitled to reclaim losses. Requiring compensation for umbrella losses is an additional financial burden on firms' profits *if and only if a partial cartel operates*: there are no umbrella effects if an industry-wide cartel operates. This changes the relative attractiveness and stability of partial vs. industry-wide cartels. When markets involve three or more firms, the deterrence effects of compensation – notably the legal standing of umbrella victims – are therefore more subtle than the conventional wisdom articulated by above quotes.

Building on a Bertrand-Edgeworth model investigated by Bos and Harrington

[1]See Inderst et al. (2014) for a detailed discussion on umbrella pricing and Holler and Schinkel (2017) for a correction.

(2010), we first show for symmetric firms and *given coalition size s* that cartel sustainability is decreasing with an extended legal standing: deterrence is strengthened and the economic intuition in the quotations applies.

Second, for *endogenous coalition size s*, we show that this beneficial effect of extended legal standing to umbrella victims may however be reversed. Compensation payments for umbrella losses imply that small partial cartels are disproportionately harshly burdened, since the number of suing cartel victims is large compared to the product units sold by the cartel. For given detection probabilities, expected collusive profits for small coalitions of firms hence decrease more strongly when all customers are given legal standing. This makes it more probable that larger cartels form, as we illustrate in Section 2.1 with a numerical example and show in a dynamic market environment in Section 2.3.

The first and only paper so far that has investigated how (public) antitrust enforcement affects cartel size is by Bos and Harrington (2015). Their key finding is that cartel size can either increase or decrease in a penalty which is proportional to a cartel member's collusive profit.[2] The present study introduces a detailed analysis of private antitrust enforcement (which can lead to disproportionate changes in a cartel member's profit) and clarifies the interaction of cartel size and cartel deterrence. In particular, for a specific discount factor $\delta \in (0, 1)$, we first determine all dynamically *sustainable* (partial) cartels (i.e., after a cartel has formed, cartel members adhere to the agreed behavior); among those, we select *stable* coalition sizes (i.e., cartel formation is considered). This allows to derive and to compare, for any discount factor δ, the size of a formed cartel depending on whether victims who suffered umbrella losses have legal standing or not.

The remainder of this chapter is structured as follows. After presenting an illustrative example, we introduce the model in Section 2.2. Section 2.3 discusses how compensation payments affect the market outcome given a cartel of size s has formed. Assuming that cartel formation is endogenous, we then show in Section 2.4 that allowing all customer to bring a lawsuit before the court can have adverse effects on

[2]They identify three main factors how antitrust enforcement influences cartel size. First, decreasing collusive profits when expected fines increase can discourage firms from joining a cartel. Second, since the collusive price is shown to be weakly decreasing when firms are penalised, more firms which coordinate their strategies are needed to ensure sufficiently high collusive profits. Third, since they assume that the detection probability depends on the capacity controlled by the cartel, it follows that expected fines increase when cartels are more encompassing. While the first and the third factor imply a decreasing coalition size, the second factor can lead to increasing coalition sizes with a more severe antitrust regime. The overall effect is ambiguous and will depend on the magnitude of these factors.

the market outcome. Before concluding we discuss results with another numerical example in Section 2.5. All proofs are collected in Appendix B.

2.1 Illustration in Static Market Environment

Consider a standard Bertrand oligopoly with n firms and linear demand $D(p) = 10-p$. Production costs c are normalized to zero. Let each firm be constrained to a capacity of k with $(n-1)k \geq D(0) > 0$. Then, it is well-known that a Nash equilibrium exists where all firms price at costs.[3]

Now suppose that the symmetric firms can make binding agreements to form a single cartel of size $s \leq n$.[4] Such cartel will be uncovered with probability α. In this case, the antitrust authority imposes fines of $\tau \geq 0$ times each cartel member's profit. Additionally, the share $\beta \geq 0$ of all customers who have legal standing successfully reclaims overcharge losses. These compensation payments are allocated equally among cartel members.

All firms simultaneously set prices. If a cartel of s firms operates, its participants choose a price $p_s \geq 0$ which maximizes cartel members' expected profits. Suppose that a best-responding non-cartel member (referred to as a *free rider*), which correctly anticipates that a cartel operates, increases its price under the "umbrella" of the cartel. In particular, free riders are assumed to marginally undercut the cartel price as long as $p_s > 0$.[5] Thus, all customers essentially would pay p_s; however, customers with a high willingness to pay are assumed to be served first (i.e., efficient rationing is assumed). Then, cartel members only make profits if residual demand $D_s^R(p_s) := \max\{10 - p_s - (n-s)k, 0\}$ is strictly positive. All free riders produce at capacity while cartel members have unused capacity because aggregated cartel capacities exceed residual demand. Since firms are symmetric it is assumed that each cartel member supplies $D_s^R(p_s)/s$.

If $\beta = 0$, i.e., firms do *not* have to compensate customers, a cartel member's profit is

$$\pi_s^N(p_s) := p_s D_s^R(p_s)/s - \alpha\tau p_s D_s^R(p_s)/s = (1-\alpha\tau)p_s D_s^R(p_s)/s.$$

Let p_s^N be the price which maximizes $\pi_s^N(p_s)$. The profit of a free-riding non-cartel firm

[3] Depending on the price grid it is unique.

[4] The assumption that firms can make binding agreements is relaxed in Section 2.2.

[5] This leads to the unique static Nash equilibrium in the model introduced by Bos and Harrington (2010).

is $\pi_s^{FN}(p_s^N) := p_s^N k$. Let $\mathcal{I}_s^N(p_s^N) := \pi_s^N(p_s^N)/\pi_{s-1}^{FN}(p_{s-1}^N)$ measure a firm's incentive to leave coalition s given that remaining $s-1$ firms still coordinate strategies: for $\mathcal{I}_s^N(p_s^N) > 1$, a cartel member strictly prefers to stay in coalition s.

If $\beta > 0$ and legal standing is restricted to customers who directly bought a product produced by a cartel member, denoted as *home* customers, a cartel member's expected compensation payment is $\alpha\beta p_s D_s^R(p_s)/s$. Its expected profit is

$$\pi_s^H(p_s) := \pi_s^N(p_s) - \alpha\beta p_s D_s^R(p_s)/s = (1-\alpha\tau-\alpha\beta)p_s D_s^R(p_s)/s.$$

The profit maximizing price when only home customers have legal standing is therefore independent of compensation payments, that is, it equals p_s^N as long as $1-\alpha\tau-\alpha\beta > 0$. A free rider's profit is $\pi_s^{FH}(p_s^N) = \pi_s^{FN}(p_s^N) = p_s^N k$. As above, we define $\mathcal{I}_s^H(p_s^N) := \pi_s^H(p_s^N)/\pi_{s-1}^{FH}(p_{s-1}^N)$.

When *all* customers have legal standing, a cartel member's expected compensation payment is $\alpha\beta p_s D(p_s)/s$; its expected profit is

$$\pi_s^A(p_s) := \pi_s^N(p_s) - \alpha\beta p_s D(p_s)/s = (1-\alpha\tau)p_s D_s^R(p_s)/s - \alpha\beta p_s D(p_s)/s.$$

This is no longer proportional to $\pi_s^N(p_s)$ and hence compensation payments generally affect the profit maximizing cartel price for $s \neq n$. Let p_s^A be the price that maximizes $\pi_s^A(p_s)$. As for the case with no compensation at all, we define $\pi_s^{FA}(p_s^A) := p_s^A k$ and $\mathcal{I}_s^A(p_s^A) := \pi_s^A(p_s^A)/\pi_{s-1}^{FA}(p_{s-1}^A)$ to measure a firm's incentive to leave coalition s.

Now consider a numerical example with $n = 5$, $k = 3$, $\alpha = 1/5$, $\tau = 1/2$ and $\beta = 1$. Resulting prices p_s^N under "no" or "home" compensation and p_s^A under compensation of "all", profits and $\mathcal{I}_s^N(p_s^N)$, $\mathcal{I}_s^H(p_s^N)$ and $\mathcal{I}_s^A(p_s^A)$ are listed in Table 2.1 for all $s \leq n$.[6]

	p_s^N	π_s^N	π_s^{FN}	$\mathcal{I}_s^N$	π_s^H	$\mathcal{I}_s^H$	p_s^A	π_s^A	π_s^{FA}	$\mathcal{I}_s^A$
$s=1$	0	0	0	–	0	–	0	0	0	–
$s=2$	**0.5**	**0.11**	1.5	∞	**0.09**	∞	0	**0**	0	–
$s=3$	2	1.2	6	**0.8**	0.93	**0.62**	**1.14**	0.31	3.43	∞
$s=4$	3.5	2.76	10.5	0.46	2.15	0.36	3.07	1.65	9.21	0.48
$s=5$	5	4.5	–	0.43	3.5	0.33	5	3.5	–	0.38

Table 2.1 Market outcomes and a firm's incentive to leave coalition s in corresponding regimes $i \in \{N, H, A\}$

When firms have to compensate for umbrella losses and a partial cartel of given size $s < n$ has formed, cartel members take the increasing number of customers that

[6]Numbers in all tables are rounded to two decimal places.

are entitled to bring a lawsuit before the court into account by reducing the cartel price. Then, profits of cartel and non-cartel members decrease. All customers benefit from an extended legal standing.

However, to see that the compensation regime can influence cartel formation, first, consider a coalition of size 2. Table 2.1 illustrates that expected coalition profits (writtenv in bold) are only (strictly) positive when outside customers have no legal standing. Hence, antitrust enforcement deters a coalition of size 2 from coordinating strategies when umbrella losses have to be compensated. Second, consider a coalition of size 3 with $\mathcal{I}_3^A(p_3^A) > 1 > \mathcal{I}_3^N(p_3^N) > \mathcal{I}_3^H(p_3^N)$. Firms have a strict incentive to leave this coalition when umbrella losses remain uncompensated (a partial cartel of size 2 will form since $\mathcal{I}_s^j < 1$ with $j \in \{N, H\}$ and $s \geq 3$), but, crucially, staying in a coalition of size 3 is preferable and stable (i.e., firms have no incentive to build a larger coalition) when all customers have legal standing. Then, the resulting market price when umbrella losses have to be compensated is more than twice as high as the market price when these losses stay uncompensated. In the following, we will show that such – at first glance – counterintuitive effect is a robust phenomenon and that the example's results generalize to a dynamic market environment.

2.2 Model

We adopt a simplified version of the Bertrand-Edgeworth competition model introduced by Bos and Harrington (2010). Public and private antitrust enforcement are added below.

2.2.1 Partial Collusion and Capacity Constrained Firms

The model investigated by Bos and Harrington (2010) allows the analysis of *partial* cartel formation in a dynamic framework. This is highly useful to discuss how policy instruments affect cartel size and therefore the market price. In order to focus on the analysis of compensation payments, we adopt a simplified version of their model.[7]

Symmetric and capacity constrained firms $i \in \mathcal{N} := \{1, \ldots, n\}$ produce homogeneous goods and compete in an infinitely repeated price game. Firms set prices simultaneously. Each firm's capacity k is exogenously given. Hence we consider

[7]The two main simplifying assumptions are symmetry of firms and a linear market environment. Bos and Harrington (2010) assume that the demand function is twice continuously differentiable and decreasing in the market price and that firms differ in size.

firms of equal sizes. Past moves are assumed to become public information. Firms' common discount factor is $\delta \in (0, 1)$. The payoff of each firm is the expected present value of its profit stream.

We normalize constant marginal costs c to zero and assume the linear market demand function $D(p) := a - bp$ with $a, b > 0$.[8] The monopoly profit $pD(p)$ is strictly concave; let p^m denote the price which maximizes it, i.e., $p^m = \arg\max pD(p)$. When capacity constraints bind, residual demand is allocated efficiently, i.e., consumers with a high willingness to pay are served first (see Kreps and Scheinkman 1983).

Firms simultaneously set prices and then each firm produces to meet demand up to capacity. If a set of firms $\mathcal{T} \subseteq \mathcal{N}$ charges a common price, it is assumed that demand of each single firm $i \in \mathcal{T}$ is positive when the total demand for these firms is positive. Additionally, if aggregated capacity of all firms in $\mathcal{T}$ exceeds their aggregated demand, then each single firm has excess capacity.

The following assumptions impose upper and lower bounds on a single firm's capacity. They simplify the analysis while guaranteeing that at least sufficiently inclusive partial cartels can earn profits when firms do not have to compensate for umbrella losses (see Prop. 2.1).[9]

$$k < D(p^m) \quad \text{and} \quad k \geq \frac{D(0)}{n-1} = \frac{a}{n-1}. \tag{A1}$$

The first part of assumption (A1) states that monopoly demand exceeds each firm's capacity.Hence, the firm that chooses the lowest price produces at capacity, if the market price does not exceed p^m. With $k \geq a/(n-1)$, the standard Bertrand outcome is reached when all firms compete.

Non-negative price choices are assumed to be discrete, that is, they differ by increments of $\epsilon > 0$. Then two symmetric stage game Nash equilibria ($p = 0$ and $p = \epsilon$) exist. When ϵ is sufficiently small, competitive prices essentially equal costs and therefore zero.

As in Bos and Harrington (2010) we assume that only a single cartel operates. Roles of its members are identical in that no ringleaders are needed to lead or instigate the union. In addition, we introduce public antitrust enforcement by assuming that

[8]We will show that prices of firms in the competitive and in the collusive equilibrium coincide. Thus, each firm earns the margin $m = p - c$. Substituting $p = m + c$ in $D(p)$ gives $\tilde{D}(m) = a - b(m + c) = \tilde{a} - bm$ with $\tilde{a} = a - bc$. Maximizing with respect to p or with respect to m does not affect our analysis. Section 2.5 considers a numerical example with $c > 0$.

[9] More general assumptions regarding capacities of firms are made in Compte et al. (2002). They however assume that only the industry-wide cartel can form.

a cartel generates hard evidence which could be found by the antitrust authority if it audits the industry with probability $\alpha \in (0, 1)$ per period. Bryant and Eckard (1991) estimated the annual probability of getting indicted by federal authorities in the US to be at most between 13% and 17%. Combe et al. (2008) found comparable results in an European sample. As in Katsoulacos et al. (2015) we assume that α does not depend on a cartel's price choice.

When the cartel is detected, the competition authority imposes a fine with probability one: each firm has to pay share $\tau \geq 0$ of its one period profit as in Bos and Harrington (2015).[10] We suppose that former cartel members are not under special observation, i.e., they may directly enter a convicted cartel again.

2.2.2 Equilibrium Price and Cartel Sustainability

To simplify the analysis, Bos and Harrington (2010) imposed the following three properties on equilibrium strategy profiles. First, a stationary collusive outcome is reached. Second, when a firm deviates, former cartel members play Nash reversion strategies, i.e., deviations are punished by a permanent return to the static Nash equilibrium. Last, cartel members choose their strategy profile without conditioning on past behavior by non-cartel members. This induces non-cartel members to maximize their current profit in equilibrium. In particular, they take the common prices set by the cartel members as given, i.e., all firms are aware of the existence of a cartel.

Acting against cartel members is superfluous if the cartel has not caused any damage. Hence, only (partial) cartels which earn positive profits, i.e., which cause damage, are relevant and collusive prices are necessarily positive.

We now assume that a cartel of size $s < n$ operates and profitably chooses a price $p_s^N \geq 0 + 2\epsilon$.[11] The superscript N is used in the following when firms do not have to compensate. A best-responding free rider prices at $p_s^N - \epsilon$ in the unique static Nash equilibrium, as proven by Bos and Harrington (2010). The intuition is as follows: first, assume that free riders match or even outbid p_s^N. If a single free rider prices at $p' > p_s^N$, its demand is zero since $(n-1)k > D(p')$ by assumption (A1). Duplicating a cartel member's strategy however leads to positive profits. If a group of free riders chooses any price $p' > p_s^N$, each single free rider prefers to undercut

[10] Profits equal revenues with $c = 0$. In the EU, fines are indeed primarily based on revenues of the last business year (see European Commission 2006). Including time-dependent fines would be possible by assuming that the share of the one period profit a firm has to pay increases each year by some factor κ, i.e., the share would increase to $(1 + \kappa t)\tau$ in period t.

[11] Whenever there is no risk of misinterpretation we denote a partial cartel of size s as "coalition s".

p'. The trivial price effect is dominated by a non-trivial capacity effect, since free riders in sum have unused capacity (see assumption (A1)) and from the assumption on demand allocation when several prices are identical, it follows that each single firm has unused capacity. This argument extends to the case when free riders meet the cartel price. Hence, free riders undercut p_s^N. Second, positive profits for cartel members require that residual demand, supplied by the cartel is positive although free-riding non-cartel firms' price choice is $p' < p_s^N$. Thus, with efficient rationing, it is needed that $D(p_s^N) > (n-s)k$. Then, free riders will produce at capacity by pricing at $p_s^N - \epsilon$. Any price below $p_s^N - \epsilon$ reduces an outsider's profit. With $\epsilon \approx 0$ the market price essentially is p_s^N.

Residual demand served by the cartel aggregates to $D_s^R(p_s^N) := D(p_s^N) - (n-s)k$. The cartel in sum has unused capacity because $D_s^R(p_s^N) < sk$ by assumption (A1). How to allocate $D_s^R(p_s^N)$ among symmetric firms is far less critical compared to the general case of asymmetric firms: the allocation proportional to capacities, used by Bos and Harrington (2010), coincides with an allocation by heads. Hence, the expected current profit of a cartel member under "no-compensation" is

$$\pi_s^N(p_s^N) := p_s^N(1-\alpha\tau)D_s^R(p_s^N)/s = p_s^N D_s^{nN}(p_s^N). \tag{2.1}$$

The term $D_s^{nN}(p_s^N) := (1-\alpha\tau)D_s^R(p_s^N)/s$ denotes the *net demand* of a cartel member with $\beta = 0$, that is, its residual demand after subtracting expected fines. Thus, to ensure positive collusive profits, net demand has to be positive for $p_s^N > \epsilon$. Since firms are not under special observation after the cartel was uncovered, profit maximization implies that firms immediately return to anticompetitive behavior in case of detection. So, discounted collusive profits are $V_s^N(p_s^N) := \pi_s^N(p_s^N)/(1-\delta)$.

The expected current profit of a free-riding non-cartel firm is

$$\pi_s^{FN}(p_s^N) := (p_s^N - \epsilon)k \approx p_s^N k; \tag{2.2}$$

its discounted profit is $V_s^{FN}(p_s^N) := \pi_s^{FN}(p_s^N)/(1-\delta)$.

Next, *dynamic cartel sustainability* for $D_s^R(p_s^N) > 0$ is discussed. A cartel is dynamically sustainable iff discounted collusive profits are at least as large as discounted deviation profits (see Friedman 1971). Latter are only positive in the period of defection since Nash reversion strategies are played. Maximizing one-shot deviation profits implies to slightly undercut the market price. Then, the dissident produces at capacity. Since non-cartel members price at $p_s^N - \epsilon$, a deviating firm will not choose a price below $p_s^N - 2\epsilon$ because sold quantity stays unchanged with $k < D(p_s^N - 2\epsilon)$. If it is capacity constrained by meeting the price of non-cartel members, it will prefer to do so. All prices essentially equal p_s^N with $\epsilon \approx 0$.

The question whether dissidents should be fined in case of cartel detection in the deviation period or not is answered differently in the literature. Whereas, e.g., Aubert et al. (2006) or Buccirossi and Spagnolo (2007) assumed that a deviating firm is fined, other authors, e.g., Motta and Polo (2003) or Katsoulacos et al. (2015) excluded dissidents from penalty payments. Following the assumption that only firms that belong to the cartel in the period of detection are fined, yields a deviation profit of[12]

$$V_s^{DN}(p_s^N) := p_s^N k + \frac{\delta}{1-\delta}0. \tag{2.3}$$

A cartel is sustainable if the *dynamic incentive compatibility constraint* (DICC)

$$V_s^N(p_s^N) \geq V_s^{DN}(p_s^N) \Leftrightarrow \frac{1}{1-\delta} p_s^N (1-\alpha\tau) D_s^R(p_s^N)/s \geq p_s^N k \tag{DICC}$$

holds. Solving inequality (DICC) for the discount factor yields

$$\delta \geq \frac{sk - (1-\alpha\tau)D_s^R(p_s^N)}{sk} = 1 - \frac{(1-\alpha\tau)D_s^R(p_s^N)}{sk} =: \delta_s^N(p_s^N). \tag{2.4}$$

With $1 > \alpha\tau$ and $D_s^R(p_s^N) < sk$ it is necessary and sufficient that $D_s^R(p_s^N)$ is positive to conclude that $0 < \delta_s^N(p_s^N) < 1$. Thus, each coalition that has a positive net demand is sustainable for some discount factors. Since $\delta_s^N(p_s^N)$ is increasing in p_s^N, there is no price which allows sustainability of coalition s if

$$\delta \leq \delta_s^N(0) = 1 - \frac{(1-\alpha\tau)(a - k(n-s))}{sk} =: \delta_s^{mN}. \tag{2.5}$$

δ_s^{mN} is referred to as the *minimal* discount factor.

When coalition s has formed, cartel members choose the same profit maximizing cartel price $p_s^N > 0$. Two cases, depending on whether the DICC is binding or not, have to be distinguished. First, assume that δ is sufficiently large so that the DICC is

[12] Assuming that a deviating firm is fined but allowing firms to apply for immunity (where an immunity recipient's fine is reduced to zero) leads to $V_s^{DN}(p_s^N)$ since a deviating firm's best strategy is to apply for immunity in the introduced model (see Aubert et al. 2006). The same applies for compensation payments where the immunity recipient is excluded from liability.

not binding. Then, maximizing the strictly concave collusive value $V_s^N(p_s^N)$ yields

$$\frac{\partial V_s^N(p_s^N)}{\partial p_s^N} = a - 2bp_s^N - (n-s)k = 0 \Leftrightarrow p_s^N = \frac{a-(n-s)k}{2b} =: p_s^{MN}. \tag{2.6}$$

p_s^{MN} is strictly increasing in the coalition size s. Substituting $p_s^N = p_s^{MN}$ in $\delta_s^N(p_s^N)$ yields

$$\delta_s^N(p_s^{MN}) = 1 - \frac{(1-\alpha\tau)(a-(n-s)k)}{2sk} =: \delta_s^N. \tag{2.7}$$

If $\delta \geq \delta_s^N$, each cartel member chooses the price p_s^{MN}. δ_s^N is referred to as the *non-binding* discount factor. A cartel member's expected current profit at p_s^{MN} is $\pi_s^N(p_s^{MN}) = (1-\alpha\tau)(a-k(n-s))^2/(4bs)$. Rewriting assumption (A1) gives $a/(n-1) \leq k < D(p_n^{MN}) = a/2$. Thus, at least four firms are needed to satisfy assumption (A1).

When $\delta \in (\delta_s^{mN}, \delta_s^N)$ the DICC binds but a price $p_s^N > 0$ where profits are positive is still feasible. Define p_s^{DN} as the highest price that satisfies the DICC when firms do not have to compensate. It is given by

$$p_s^{DN} := \frac{(a-kn)(1-\alpha\tau) + ks(\delta-\alpha\tau)}{b(1-\alpha\tau)}. \tag{2.8}$$

p_s^{DN} is linearly increasing in the discount factor. A necessary condition for $p_s^{DN} > 0$ is $\delta > \alpha\tau$, since the first summand of the numerator in equation (2.8) is negative when assumption (A1) is satisfied. Hence, whenever p_s^{DN} is positive, it is increasing in the coalition size. A cartel which is sustainable chooses the price $p_s^N := \min\{p_s^{DN}, p_s^{MN}\}$. A price above p_s^{DN} would violate sustainability, a price above p_s^{MN} cannot be profit maximizing since $V_s^N(p_s^N)$ is strictly concave. Thus, p_s^N maximizes a firm's collusive value while satisfying its DICC.

2.2.3 Private Antitrust Enforcement

We assume that some fraction $\beta \in [0,1]$ of harmed individuals who have legal standing successfully reclaim one year's overcharge damages after the cartel was discovered.[13] Since competitive prices equal marginal costs and therefore zero, the overcharge damage coincides with the market price. A cartel's aggregated expected

[13] A critical discussion on this assumption is given in Appendix A. There, we additionally argue that co-defendants will expect that each firm has to contribute the same share on the total compensation no matter whether firms are internally liable as in the EU or whether a rule of no contribution applies as in the US.

compensation payment is $\alpha\beta p_s^H D_s^R(p_s^H)$ when only *home* customers have legal standing. Expected compensation payments are $\alpha\beta p_s^A D(p_s^A)$ when *all* customers can reclaim overcharge damages. Prices p_s^H and p_s^A will be derived in Subsection 2.3.2. We assume that each cartel member has to bear the same share of the total compensation. Thus, a cartel member's expected compensation payments are $\alpha\beta p_s^H D_s^R(p_s^H)/s$ respectively $\alpha\beta p_s^A D(p_s^A)/s$.

In summary, a firm's expected current collusive payoff depends on the legal standing of antitrust victims as follows

$$\pi_s^H(p_s^H) := p_s^H(1-\alpha(\tau+\beta))D_s^R(p_s^H)/s = p_s^H D_s^{nH}(p_s^H) \tag{2.9}$$

$$\pi_s^A(p_s^A) := p_s^A\Big((1-\alpha\tau)D_s^R(p_s^A) - \alpha\beta D(p_s^A)\Big)/s = p_s^A D_s^{nA}(p_s^A) \tag{2.10}$$

with $D_s^{nH}(p_s^H) := (1-\alpha(\tau+\beta))D_s^R(p_s^H)/s$ and $D_s^{nA}(p_s^A) := \Big((1-\alpha\tau)D_s^R(p_s^A) - \alpha\beta D(p_s^A)\Big)/s$. Introducing private antitrust enforcement is normally equivalent to an increase in τ when only home customers have legal standing.[14] This ceases to be true when all customers have legal standing.

A firm's deviation payoff – but not its profit maximizing strategy – depends on whether a deviating firm must contribute to compensation or not in case of cartel detection. We assume that a deviating firm is excluded from compensation payments (see fn. 12). Thus, a deviating firm earns the profit $V_s^{Dj}(p_s^j) := (p_s^j - x)k \approx p_s^j k$ with $x \in \{\epsilon, 2\epsilon\}$ and $j \in \{H, A\}$.

2.3 Exogenous Coalition Size s

We now assume that coalition size s is exogenously given. First, we derive a necessary condition for positive net demand; then, we determine the smallest coalition size that satisfies this condition. Second, we discuss cartel pricing and cartel sustainability.

[14]There are two reasons why this equivalence could not be given. First, according to Thaler (1985), decision makers prefer *integrated losses* compared to *segregated losses* since the *value function* is convex for losses. That is, firms prefer high payments at once compared to fractionated payments. Second, an argument which is closely related to the first one, but which builds on rational profit maximizing behavior, is that bad news in the media over a longer period of time substantially influence the reputation and therefore the profit of a firm.

2.3.1 Positive Net Demand

Static collusive profits can only exceed static competitive profits if the cartel can profitably charge a price $p_s^i \geq 0 + 2\epsilon$ with $i \in \{N, H, A\}$ (recall that costs are $c = 0$, superscript i is used in the following for $i \in \{N, H, A\}$). For this, it is necessary that the cartel could at least serve a positive net demand at $p_s^i = 2\epsilon$: market demand is decreasing in the market price and free-riding non-cartel firms always undercut the cartel price as long as $p_s^i \geq 2\epsilon$. Thus, when cartel members choose the price 2ϵ but outsiders still have enough capacity to serve total demand $D(\epsilon)$, there is no price $p_s^i \geq 2\epsilon$ which satisfies $D_s^{ni}(p_s^i) > 0$. Hence, a necessary condition for a positive net demand is $D_s^{ni}(2\epsilon) \approx D_s^{ni}(0) > 0$.

Net demand for $p_s^j = 2\epsilon \approx 0$ with $j \in \{H, A\}$, $s = n$ and $\beta > 0$ is positive if $D_n^{nH}(0) = D_n^{nA}(0) = (1 - \alpha(\tau + \beta))D(0)/n > 0$. This always holds if

$$1 - \alpha(\tau + \beta) =: e > 0. \tag{A2}$$

When condition (A2) is violated, no coalition $s \leq n$ can make profits. In the following, we assume that the *enforcement parameter e* is strictly positive. This implies that $1 > \alpha\tau$ since $\beta \geq 0$.

When $e > 0$ and capacities are bounded by assumption (A1), some coalitions of size $s < n$ may serve a positive net demand; this was illustrated in Section 2.1. Which partial cartels actually satisfy $D_s^{ni}(2\epsilon) > 0$ depends on the aggregated outside capacity: the larger the individual capacity k, the more encompassing a cartel must be to have positive net demand. Recall that k is bounded by assumption (A1), that is, $k \in [a/(n-1), a/2)$.

Lemma 2.1 derives a general condition for $D_s^{ni}(2\epsilon) > 0$ for $s < n$ and $p_s^i = 2\epsilon \approx 0$.[15]

Lemma 2.1. *Let $e > 0$ and $\mu := (1 - \alpha(\beta + \tau))/(1 - \alpha\tau)$. Coalition $s < n$ has positive net demand for $p_s^i = 2\epsilon \approx 0$ iff*

(i) $k < \frac{a}{n-s} =: k_s^{PN}$ *when umbrella losses stay uncompensated and*

(ii) $k < \frac{\mu a}{n-s} =: k_s^{PA}$ *when umbrella losses have to be compensated,*

with $k_s^{PN} \geq k_s^{PA}$.

[15]The condition ensures positive collusive profits when discount factors are sufficiently high.

From $\beta > 0$ follows that $\mu \in (0,1]$. Since μ is only relevant when cartel members have to compensate outside customers, it will be referred to as the *umbrella coefficient*.

The larger s and the smaller k, the looser are the constraints in Lemma 2.1 since outside capacity decreases. Additionally, from $k_s^{PN} \geq k_s^{PA}$ follows that the condition for net demand being positive for $p_s^i \approx 2\epsilon$ becomes more restrictive when umbrella losses have to be compensated: to ensure positive net demand, aggregated outside capacity has to decrease when expected compensation payments increase.

Lemma 2.1 allows to partition the interval $k \in [a/(n-1), a/2)$ into several subintervals. In particular, given capacity level k, Proposition 2.1 derives the smallest coalition size, denoted by $\underline{s}$, which can serve a positive net demand.

Proposition 2.1. *Let $e > 0, k \in [a/(n-1), a/2)$ and $\mu > 1/2$. The capacity of the smallest coalition $\underline{s} < n$ which has positive net demand for $p_s^i = 2\epsilon \approx 0$ satisfies*

(i) $k_{\underline{s}-1}^{PN} \leq k < k_{\underline{s}}^{PN}$ *when umbrella losses stay uncompensated and*

(ii) $\max\{a/(n-1), k_{\underline{s}-1}^{PA}\} \leq k < \min\{k_{\underline{s}}^{PA}, a/2\}$ *when umbrella losses have to be compensated.*

At least coalition $n-2$ satisfies $D_s^{nj}(2\epsilon) > 0$ with $j \in \{N, H\}$; at least coalition $n-1$ satisfies $D_s^{nA}(2\epsilon) > 0$.

For $k_{\underline{s}-1}^{PN} \leq k < k_{\underline{s}}^{PN}$ with $2 \leq \underline{s} \leq n-2$, coalition $\underline{s}$ is the smallest coalition which has positive net demand when umbrella losses remain uncompensated. A similar result obtains when all customers have legal standing. Capacity bounds then also depend on the umbrella coefficient and on the constraints imposed by assumption (A1). From the assumptions on the capacity bounds also follow that at least a coalition of $n-2$ firms has positive net demand when outside customers have no legal standing. This is not true when legal standing is extended to all customers since expected compensation payments increase. In particular, if private antitrust enforcement has a rather high weighting compared with public antitrust enforcement (that is, μ is sufficiently small) and firm-specific capacity is rather large (that is, even one cartel outsider supplies a huge share of total demand), only the industry-wide cartel would have positive net demand. However, $\mu > 1/2$ ensures that at least coalition $n-1$ has positive net demand when all customers have legal standing.

Whether small coalitions have positive net demand or not therefore crucially depends on the legal standing of antitrust victims. The conditions for net demand of small coalitions being positive when firms have to compensate for umbrella losses are tighter since $\mu \in (0,1]$. The reason is that the potential number of suing customers

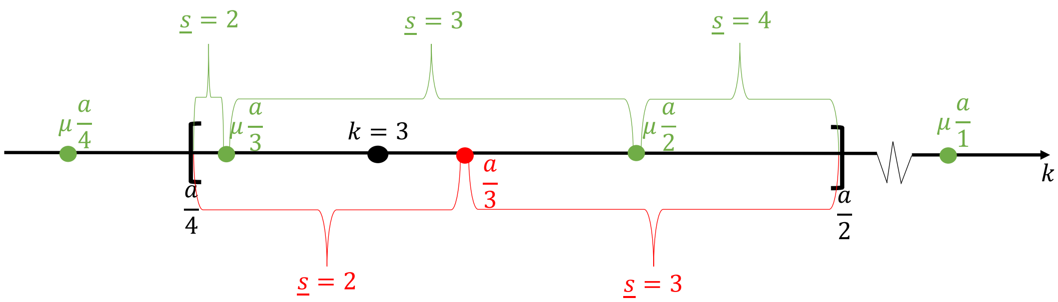

Figure 2.1 Minimal cartel size $\underline{s}$ for positive net demand in "home" (red) and "all" (green) regimes

increases. Thus, net demand is more likely to be negative. This is illustrated in Figure 2.1 for the example discussed in Section 2.1, that is, for $n = 5$ and $\mu = 7/9$.[16] When only home customers have legal standing, coalitions of size 2 or 3 are the smallest coalitions which have positive net demand (depending on the capacity level k). With an extended legal standing to all customers, more firms are needed to satisfy $D_{\underline{s}}^{nA}(2\epsilon) > 0$ for many values of k. Damage caused by coalition s is zero if $k \geq k_s^{PN}$ resp. if $k \geq k_s^{PA}$. In Subsection 2.4.2 we will however show that given capacity level k only the smallest or the next lager coalition that has positive net demand can form stable coalitions when firms are patient.

2.3.2 Cartel Pricing and Cartel Sustainability

We next show how an extended legal standing affects cartel pricing and cartel sustainability, given a partial cartel of size s has already formed. Recall here that a cartel is dynamically *sustainable* iff discounted collusive profits are at least as large as discounted deviation profits, i.e., the DICC has to be satisfied. Defining the minimal discount factor δ_s^{mj}, the non-binding discount factor δ_s^j and the optimal cartel price p_s^j with $j \in \{H, A\}$ analogously to δ_s^{mN}, δ_s^N and p_s^N (see Section 2.2), we show:

Proposition 2.2. *For a given coalition size s, extended legal standing leads to*

(i) (weakly) increasing deterrence: $\delta_s^{mA} \geq \delta_s^{mH} \geq \delta_s^{mN}$; $(\delta_s^A \geq \delta_s^H \geq \delta_s^N)$ *and to*

(ii) (weakly) decreasing market prices: $p_s^N \geq p_s^H \geq p_s^A$.

Moreover, discount factors δ_s^{mi} *and* δ_s^i *are decreasing and* p_s^i *is increasing in the coalition size s.*

[16] For illustrative reasons, we only compare the two cases $j \in \{H, A\}$ with $\beta > 0$.

For a fixed $s < n$, we see that optimal cartel prices p_s^i decrease and minimal discount factors δ_s^{mi} and non-binding discount factors δ_s^i increase when firms have to compensate for umbrella losses since expected compensation payments increase.[17] Moreover, large cartels are rather easy to sustain and their members are able to charge high prices.

Figure 2.2 illustrates the optimal prices for all coalitions which have positive net demand for the example discussed in Section 2.1. When the industry-wide cartel forms (coloured blue), umbrella losses do not occur. Since net demand of a coalition of size 2 is not positive for $p_2^A \approx 2\epsilon$, no $\delta_2^{mA} < 1$ exists.

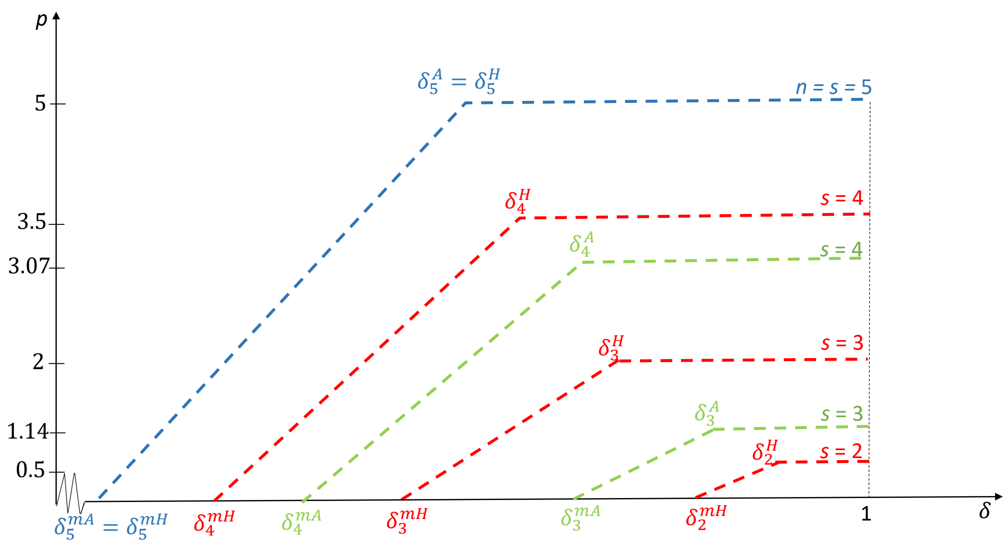

Figure 2.2 Cartel pricing and cartel sustainability given coalition size s in "home" (red) and "all" (green) regimes

For the case when coalition size is exogenous, Propositions 2.1 and 2.2 confirm intuition of the related literature, that is, cartel deterrence is increasing when more customers are entitled to compensation (see, e.g., Blair and Durrance 2018). Moreover, prices are decreasing with an extended legal standing to outside customers. Given a cartel has formed, a more severe compensation rule is unambiguously good. However, the legal standing of antitrust victims influences the generally endogenous size of stable cartels as is discussed next.

[17] Note that no common definition of cartel deterrence exists. Generally, it is assumed that deterrence increases when critical discount factors ($\delta_s^i(p_s^i)$) increase. We will mainly consider minimal discount factors to discuss cartel deterrence: with $\delta < \delta_s^{mi}$ a cartel is deterred from coordinating strategies.

2.4 Endogenous Coalition Size s

We first discuss the concepts used to identify stable (partial) cartels. Next, stable coalition sizes for large discount factors, that is, all coalitions satisfying $D_s^{ni}(2\epsilon) > 0$ can price without considering the DICC, are derived. Last, it is shown that the stable coalition size if firms have to compensate for umbrella losses is at least as large as the coalition size if they do not have to compensate these losses for all $\delta > \delta_n^{mA}$.

2.4.1 $\mathcal{A}$-stability

Escrihuela-Villar (2008, 2009) and Bos and Harrington (2010, 2015) discuss models of partial cartel formation in a dynamic market environment. They use the same concepts to identify stable cartels, namely adaptations of the stability concepts introduced by D'Aspremont et al. (1983).[18] We follow this literature here.

We discuss cartel stability for different values of $\delta \in (0, 1)$ and $k \in [a/(n-1), a/2)$ given $\beta \geq 0$ and $\mu > 1/2$. Then, the umbrella coefficient μ is sufficiently large to ensure that at least coalition $n-1$ satisfies $D_s^{nA}(2\epsilon) > 0$.

Firms simultaneously decide whether to join the cartel or not. As usual in the literature on cartel formation among symmetric firms, we do not address the question which firms actually join the conspiracy (see, e.g., D'Aspremont et al. 1983, Donsimoni et al. 1986 or Escrihuela-Villar 2008, 2009).

A first requirement for cartel stability is that coalition s has to be sustainable for $p_s^i = 2\epsilon$, i.e., $\delta \geq \delta_s^i(2\epsilon) > \delta_s^i(\epsilon) \approx \delta_s^{mi}$ and therefore $\delta > \delta_s^{mi}$. Then, cartel members can choose a price where their profits and profits of free riders are positive. This excludes coalitions that do not affect prices, i.e., do not cause damage.

To identify internally and externally stable cartels among coalitions that are sustainable for $p_s^i = 2\epsilon$, it is sufficient to consider static member and free rider profits, since $\pi_s^i(p_s^i)$ and $\pi_s^{Fi}(p_s^i)$ are only scaled by $1/(1-\delta)$ to derive $V_s^i(p_s^i)$ and $V_s^{Fi}(p_s^i)$. Internal cartel stability requires that no cartel member has an incentive to leave whereas external cartel stability states that free rider wants to join the coalition.[19] Formally, a

[18] D'Aspremont et al. (1983) investigate *internal* and *external* cartel stability in a static market environment. They conclude that a cartel is stable if and only if both conditions are satisfied. This stability concept can easily be adapted to a dynamic setting.

[19] Other approaches to select stable coalitions were, e.g., introduced by Bernheim et al. (1987), Bernheim and Whinston (1987), Ray and Vohra (1999) or Diamantoudi (2005).

cartel of size $s > 1$ is said to be internally stable if

$$\mathcal{I}_s^i(k,\delta) := \pi_s^i(p_s^i) - \pi_{s-1}^{Fi}(p_{s-1}^i) \geq 0 \tag{INT}$$

and a cartel of size $s < n$ is said to be externally stable if

$$\mathcal{E}_s^i(k,\delta) := \pi_{s+1}^i(p_{s+1}^i) - \pi_s^{Fi}(p_s^i) \leq 0. \tag{EXT}$$

For $s = 1$ internal and for $s = n$ external cartel stability is assured. Coalitions which satisfy internal and external stability and which are additionally dynamically sustainable for $p_s^i = 2\epsilon$ are denoted as $\mathcal{A}$*-stable*.[20] Note that coalitions which cannot influence the market price can nevertheless satisfy stability defined as in D'Aspremont et al. (1983): as shown in Lemma 2.1, cartel profits can only be positive when aggregated outside capacity is sufficiently small. Thus, when coalition $s + 1$ does not control enough capacity to satisfy $\pi_{s+1}^i(p_{s+1}^i) > 0$ it directly follows that $\pi_s^i(p_s^i) = 0$ and therefore $\mathcal{I}_s^i(k,\delta) = 0$ and $\mathcal{E}_s^i(k,\delta) = 0$.

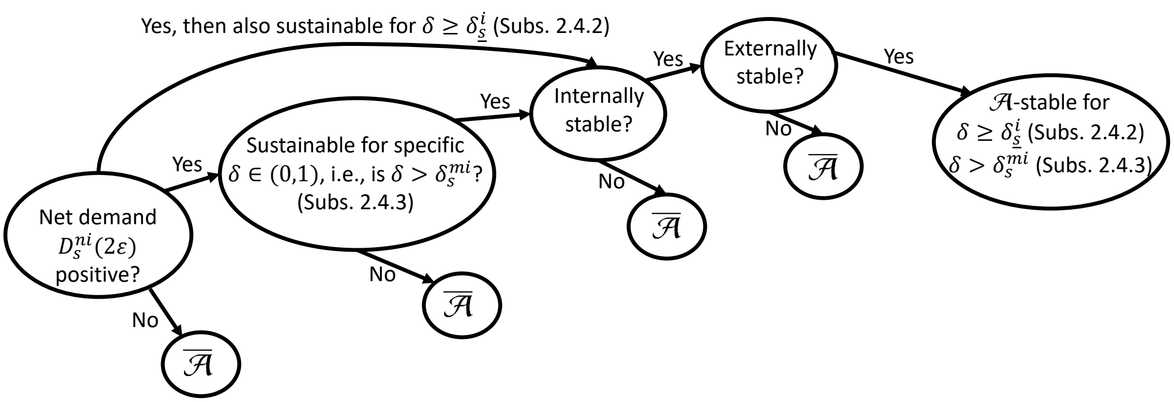

Figure 2.3 How to derive $\mathcal{A}$-stable coalition sizes

Figure 2.3 illustrates the different test steps, to identify an $\mathcal{A}$-stable coalition size. Coalition s is not $\mathcal{A}$-stable (denoted by "$\overline{\mathcal{A}}$") whenever one step is not met. In the first step we select coalitions which have positive net demand, that is, coalitions which satisfy $D_s^{ni}(2\epsilon) > 0$ (see Lemma 2.1). All coalitions that have positive net demand are dynamically sustainable for p_s^{Mi} if $\delta \geq \delta_{\underline{s}}^i$. Among those coalitions, we identify internally and externally stable cartels to derive $\mathcal{A}$-stable cartel sizes for $\delta \geq \delta_{\underline{s}}^i$ in Subsection 2.4.2. Positive net demand of coalition s does however not

[20] An alternative way to exclude coalitions which cannot influence the market price is to assume that inequality (INT) is strict as in Bos and Harrington (2010) or Escrihuela-Villar (2008, 2009).

ensure that this coalition is also dynamically sustainable for any $\delta \in (0,1)$; dynamic sustainability only applies for $\delta > \delta_s^{mi}$. In Subsection 2.4.3 we compare the coalition size of an $\mathcal{A}$-stable cartel for different compensation regimes depending on $\delta \in (0,1)$.

2.4.2 Cartel Size when Firms are Patient

We start by discussing cartel stability for discount factors which are sufficiently large to ensure that each coalition that has positive net demand can choose the price p_s^{Mi} where the DICC is not binding. Then, neither collusive nor outside profits depend on the discount factor δ.[21] Static collusive profits for $\beta > 0$, which are given by

$$\pi_s^H(p_s^{MH}) = \frac{e[a-k(n-s)]^2}{4bs} \quad \text{and} \quad \pi_s^A(p_s^{MA}) = \frac{[ae-k(n-s)(1-\alpha\tau)]^2}{4bse}, \tag{2.11}$$

are positive when coalition s has a positive net demand.

Lemma 2.2 identifies internally stable cartels. The capacity where firms are indifferent between being a cartel member of coalition $\underline{s}+1$ or an outsider assuming that $\underline{s}$ firms form a cartel is denoted by $\tilde{k}_{\underline{s}}^i$.

Lemma 2.2. *Let $\delta \geq \delta_{\underline{s}}^i$, $e > 0$, $\mu > 1/2$ and $k \in [a/(n-1), a/2)$.*
(a) The unique internally stable cartel which makes profits consists of $\underline{s}$ firms, if and only if

(i) $k < a/(n - x_{\underline{s}}^{NA}) =: \tilde{k}_{\underline{s}}^N$ *when firms do not have to compensate,*

(ii) $k < a/(n - x_{\underline{s}}^{H}) =: \tilde{k}_{\underline{s}}^H$ *when only home customers have legal standing,*

(iii) $k < \mu a/(n - x_{\underline{s}}^{NA}) =: \tilde{k}_{\underline{s}}^A$ *when all customers have legal standing,*

with

$$x_{\underline{s}}^{NA} := \frac{\alpha\tau(\underline{s}+1) - \sqrt{\underline{s}^2 - 1 + 2\alpha\tau(\underline{s}+1)}}{-1+\alpha\tau};$$

and

$$x_{\underline{s}}^{H} := \frac{\alpha(\underline{s}+1)(\tau+\beta) - \sqrt{\underline{s}^2 - 1 + 2\alpha(\underline{s}+1)(\tau+\beta)}}{-1+\alpha\beta+\alpha\tau}.$$

(b) In addition, a coalition of $\underline{s}+1$ firms is internally stable if and only if $k \geq \tilde{k}_{\underline{s}}^i$.

[21] Assuming that firms can sign binding contracts in a static model yields similar results.

Among those coalitions which earn profits, internal stability either permits a single cartel of size $\underline{s}$ or two internally stable cartels exist. The largest coalition that could be internally stable when umbrella losses stay uncompensated has size $n-1$; coalition n could be internally stable when all customers have legal standing (see Prop. 2.1).

The internal stability condition for coalition $\underline{s}+1$ gets tighter, i.e., is satisfied for fewer values of k, when only home customers have legal standing compared to no compensation at all: outside profits do not depend on private antitrust enforcement whereas collusive profits decrease when firms have to compensate home customers. When all customers have legal standing, collusive profits further decrease but also outside profits are strictly lower for $\beta > 0$ and $s < n$ since the market price is decreasing when all customers are allowed to bring a lawsuit before the court. Which effect dominates can be analysed by comparing critical capacities $\tilde{k}^i_{\underline{s}}$ where firms are indifferent between staying in coalition $\underline{s}$ or joining coalition $\underline{s}+1$. We prove that $x^H_s > x^{NA}_s$ when $\beta > 0$ and that $\tilde{k}^i_{\underline{s}}$ is unique. Then, it is easily checked that $\tilde{k}^H_{\underline{s}} > \tilde{k}^N_{\underline{s}} > \tilde{k}^A_{\underline{s}}$ when $\beta > 0$: the first inequality follows since $x^H_s > x^{NA}_s$; the second one since $\mu < 1$. Thus, the decrease in outside profits more than offsets the decrease in collusive profits when umbrella losses have to be compensated. Hence, the condition for internal stability of coalition $\underline{s}+1$ is satisfied also for smaller values of k.

Selecting externally stable cartels among coalitions $\underline{s}$ and $\underline{s}+1$ gives

Proposition 2.3. *Let $\delta \geq \delta^i_{\underline{s}}$, $e > 0$, $\mu > 1/2$ and $k \in [a/(n-1), a/2)$. The largest coalition which is internally stable is also externally stable and $\mathcal{A}$-*stable *since $\pi^i_{\underline{s}}(p^{Mi}_{\underline{s}}) > 0$. Coalitions $\underline{s}$ and $\underline{s}+1$ are both $\mathcal{A}$-stable when $k = \tilde{k}^i_{\underline{s}}$.*

For $k \neq \tilde{k}^i_{\underline{s}}$ $\mathcal{A}$-stability determines a unique cartel size. This is consistent with the related literature on cartel formation in a static market environment (see, e.g., Donsimoni et al. 1986 or Shaffer 1995).[22] The unique $\mathcal{A}$-stable coalition size for $k < \tilde{k}^i_{\underline{s}}$ is $\underline{s}$; only coalition $\underline{s}+1$ operates if $k > \tilde{k}^i_{\underline{s}}$.

Only the two smallest coalitions which have positive net demand for capacity level k, that is, coalitions $\underline{s}$ and $\underline{s}+1$, can form. Which coalition actually forms depends on a firm's capacity level k and it is more likely that larger coalitions form if cartel members have to compensate for umbrella losses.

Figure 2.4 illustrates the $\mathcal{A}$-stable coalition sizes depending on the capacity level k for the example discussed in Section 2.1. Since $\tilde{k}^A_{\underline{s}} < \tilde{k}^H_{\underline{s}}$, it is easy to see that the

[22] In Donsimoni et al. (1986) two stable cartel sizes can exist depending on the number of operating firms. Then, one of the two cartels is necessarily industry-wide.

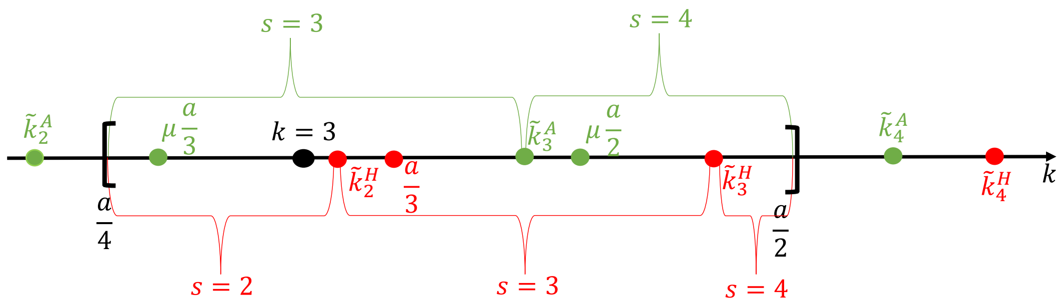

Figure 2.4 $\mathcal{A}$-stable coalition sizes given capacity k for patient firms in "home" (red) and "all" (green) regimes

coalition size when firms have to compensate for umbrella losses is larger than the coalition size when only home customers have legal standing, for many capacity levels k (e.g., for $k = 3$ as in the discussed example.)

Two important insights can be derived. First, small potential cartels that can earn positive profits would actually form for $\delta \geq \delta^i_{\underline{s}}$. Proposition 2.1 however established that it is less likely that profits of small coalitions are positive when firms have to compensate for umbrella losses. Thus, an $\mathcal{A}$-stable cartel's size generally increases for $\delta \geq \delta^i_{\underline{s}}$ when also outside customers are entitled to compensation and the umbrella coefficient μ is sufficiently small. Second, since $\tilde{k}^H_{\underline{s}} > \tilde{k}^N_{\underline{s}} > \tilde{k}^A_{\underline{s}}$, it is less likely that coalition $\underline{s}$ forms (i.e., fewer values of k allow for stability of coalition $\underline{s}$) when firms have to compensate for umbrella losses even if $D^{ni}_{\underline{s}}(2\epsilon) > 0$ is satisfied for the same coalition size s (see Figure 2.4). Although prices for a given coalition size satisfy $p^{MA}_s \leq p^{MH}_s = p^{MN}_s$, the ranking of prices and market performance can be reversed if $\mathcal{A}$-stable coalitions with endogenous size s are considered (as in the discussed example).

2.4.3 Coalition Size for $\delta \in (0, 1)$

Finally, we discuss cartel formation in a dynamic market environment that cannot be reduced to a static one, i.e., the case $\delta \in (0, 1)$.

To see how the size of an $\mathcal{A}$-stable cartel depends on the discount factor, we first recall and combine results of Propositions 2.2 and 2.3. From Proposition 2.2 follows that cartel agreements are easier to sustain with an increasing coalition size. For $\delta \in (\delta^{mi}_n, \delta^{mi}_{n-1}]$ *only* the industry-wide cartel is sustainable for $p^i_s = 2\epsilon$ and thus can earn positive expected collusive profits. Hence, it is the unique coalition which is $\mathcal{A}$-stable since external stability is assured and internal stability is easily derived.

Proposition 2.2 additionally states that cartel deterrence is increasing when firms have to compensate. If the discount factor is restricted to $\delta \in (\delta_n^{mN}, \delta_n^{mj}]$ with $j \in \{H, A\}$ and harmed customers have legal standing, not even the industry-wide cartel is sustainable for $p_s^j = 2\epsilon$: increasing cartel deterrence leads to effective competition, that is, the competitive price is reached.

With $\delta > \delta_n^{mA}$ results change. At least the industry-wide cartel is sustainable for $p_s^i = 2\epsilon$ no matter whether firms have to compensate or not; it is the unique sustainable coalition whenever δ is sufficiently close to the minimal discount factors $\delta_n^{mA} = \delta_n^{mH}$ and firms have to compensate. Further increasing discount factors do not only lead to more coalitions which are sustainable, also prices when the DICC is binding increase (see equations (2.8) and (2.20)). This influences the attractiveness of coalition n. For $\delta \geq \delta_{\underline{s}}^i$ and $\tilde{k}_n^A > a/2$ only coalitions $\underline{s}$ and $\underline{s} + 1 < n$ can be $\mathcal{A}$-stable, as shown in Proposition 2.3. Then, for $\delta \in (0, 1)$, at least two different coalitions satisfy $\mathcal{A}$-stability.[23]

To show that coalition size is weakly increasing with an extended legal standing to outside customers for $\delta \geq \delta_n^{mA}$, we evaluate the discount factor $\tilde{\delta}_s^{Ii}$ which satisfies $\pi_s^i(p_s^i) - \pi_{s-1}^{Fi}(p_{s-1}^i) = 0$. This discount factor makes a member of a cartel indifferent between staying in or leaving coalition s. We will derive that $\tilde{\delta}_s^{Ii}$ is unique for $\delta < \delta_{\underline{s}}^i$. The coalition size of an $\mathcal{A}$-stable cartel then evolves with increasing discount factors as follows: for $\delta \in (\delta_n^{mi}, \tilde{\delta}_n^{Ii})$ only the industry-wide cartel satisfies $\mathcal{A}$-stability; for $\delta \in (\tilde{\delta}_n^{Ii}, \tilde{\delta}_{n-1}^{Ii})$ only coalition $n - 1$ does. This repeats until the $\mathcal{A}$-stable coalition size for $\delta \geq \delta_{\underline{s}}^i$ is reached. A sufficiently small change of δ changes the $\mathcal{A}$-stable coalition size by at most one since $\pi_s^i(p_s^i) - \pi_{s-1}^{Fi}(p_{s-1}^i)=0$ and $\pi_s^i(p_s^i) - \pi_{s-2}^{Fi}(p_{s-2}^i)=0$ cannot be simultaneously satisfied because a free rider's profit is increasing in the coalition size s.

We now define s_δ^i as the coalition size of an $\mathcal{A}$-stable cartel given discount factor δ and prove that $s_\delta^A > s_\delta^j$ with $j \in \{N, H\}$ when $\delta > \delta_n^{mA}$. To achieve this, three steps are needed.

Step I: Three disjunct intervals in which $\tilde{\delta}_s^{Ii} \in (\delta_s^{mi}, 1)$ can lie are identified. They correspond to three different constraint situations and modes of cartel operation. First, for $\tilde{\delta}_s^{Ii} \in (\delta_s^{mi}, \delta_s^i)$, the DICC is binding for coalitions s and $s - 1$. Second, the DICC is only binding for coalition $s - 1$, i.e., $\tilde{\delta}_s^{Ii} \in (\delta_s^i, \delta_{s-1}^i)$. Last, the dynamic sustainability conditions are non-binding for $\tilde{\delta}_s^{Ii} \geq \delta_{s-1}^i$. Interval bounds depend

[23] For $\tilde{k}_n^A < k < a/2$ only coalition n is $\mathcal{A}$-stable when firms have to compensate for umbrella losses for all $\delta \in (\delta_n^{mA}, 1)$. Then, we can directly conclude that coalition size is weakly increasing for $\delta > \delta_n^{mA}$ when all customers have legal standing.

on the private antitrust regimes; they are weakly increasing with an extended legal standing (except for the upper bound of interval 3, see Prop. 2.2). Intervals when home (all) customers have legal standing are illustrated in Figure 2.5.

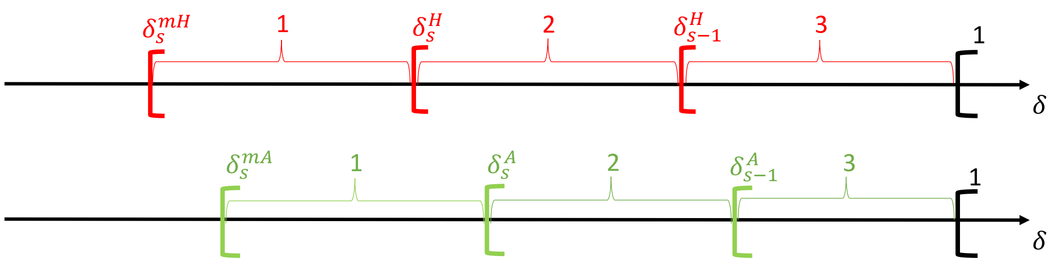

Figure 2.5 Intervals in which $\tilde{\delta}_s^{\mathcal{I}j}$ with $j \in \{H, A\}$ can lie in "home" (red) and "all" (green) regimes

Step II: We show that $\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}j}$ with $j \in \{N, H\}$ if $\tilde{\delta}_s^{\mathcal{I}i}$ lies in interval 1 or 2. This is illustrated in Figure 2.6 and stated in Lemmata 2.3 and 2.4.

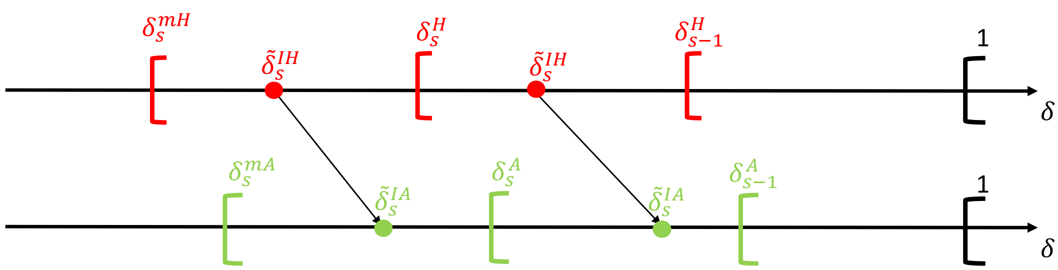

Figure 2.6 $\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}H}$ given $\tilde{\delta}_s^{\mathcal{I}j}$ with $j \in \{H, A\}$ lies in interval 1 or 2 in "home" (red) and "all" (green) regimes

LEMMA 2.3. *Let $\tilde{\delta}_s^{\mathcal{I}i} \in (\delta_s^{mi}, \delta_s^i)$. The critical discount factors that make a firm indifferent can be ranked such that $\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$.*

LEMMA 2.4. *Let $\tilde{\delta}_s^{\mathcal{I}i} \in [\delta_s^i, \delta_{s-1}^i)$. The critical discount factors that make a firm indifferent can be ranked such that $\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}H}$ and $\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}N}$. Additionally, $\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$ iff $k < \frac{a}{n-x_s}$ and $\tilde{\delta}_s^{\mathcal{I}H} \leq \tilde{\delta}_s^{\mathcal{I}N}$ iff $k \geq \frac{a}{n-x_s}$ with*

$$x_s := \frac{s\alpha\beta + 2s\alpha\tau - 2\sqrt{s^2 - 2s + s\alpha\beta + 2s\alpha\tau}}{-2 + \alpha\beta + 2\alpha\tau}.$$

Step III: The final step is split into three parts. We first show that for a specific capacity level k, $\tilde{\delta}_s^{\mathcal{I}i}$ can only lie in interval 3 for one of the cases $i \in \{N, H, A\}$. However, when

$\tilde{\delta}_s^{\mathcal{I}i}$ lies in interval 3, firms are indifferent between staying in coalition s or leaving this coalition for all $\delta \in [\delta_{s-1}^i, 1)$. Second, we argue that the indifferent discount factor is unique whenever $\tilde{\delta}_s^{\mathcal{I}i}$ does not lie in interval 3. Last, we prove that the number (1, 2 or 3) of the interval in which $\tilde{\delta}_s^{\mathcal{I}i}$ lies is weakly increasing when outside customers have legal standing. Then, we can conclude:

Proposition 2.4. *Let $\delta > \delta_n^{mA} = \delta_n^{mH}$, $e > 0$, $k \in [a/(n-1), a/2)$ and $\mu > 1/2$. The $\mathcal{A}$-stable cartel size depends on the standing of antitrust victims with $s_\delta^A \geq s_\delta^H$ and $s_\delta^A \geq s_\delta^N$. The ordering of s_δ^H and s_δ^N additionally depends on firms' capacities.*

Table 2.2 summarizes results for the four possible scenarios that $\tilde{\delta}_s^{\mathcal{I}A}$ either lies in one of the intervals 1, 2 or 3, or that no $\tilde{\delta}_s^{\mathcal{I}A}$ exists. When $\tilde{\delta}_s^{\mathcal{I}j}$ with $j \in \{N, H\}$ both exist, we can conclude from Proposition 2.3 that only $\tilde{\delta}_s^{\mathcal{I}N}$ can lie in interval 3. The distinction between capacity levels $k < a/(n - x_s)$ and $k > a/(n - x_s)$ follows from Lemma 2.4; it is relevant when the DICC is only binding for coalition $s-1$. Combining the ranking of discount factors that make firms indifferent, which is depicted in Table 2.2, with the result that lower bounds of intervals are increasing with an extended legal standing, allows the conclusion that critical discount factors $\tilde{\delta}_s^{\mathcal{I}i}$ are increasing when all customers have legal standing. From an increasing critical discount factor $\tilde{\delta}_s^{\mathcal{I}i}$ directly follows that coalition size of an $\mathcal{A}$-stable cartel is nondecreasing when compensation for umbrella losses is awarded.

	both DICC binding	DICC binding for $s-1$	no DICC binding
$k \in [\frac{a}{n-1}, \frac{a}{2})$	$\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$	–	–
$k < \frac{a}{n-x_s}$	$\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$	$\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}\mathbf{H}} > \tilde{\delta}_s^{\mathcal{I}\mathbf{N}}$	–
$k > \frac{a}{n-x_s}$	$\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$	$\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}\mathbf{N}} > \tilde{\delta}_s^{\mathcal{I}\mathbf{H}}$	–
$k < \frac{a}{n-x_s}$	$\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$	$\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$	$\tilde{\delta}_s^{\mathcal{I}A}$
$k > \frac{a}{n-x_s}$	$\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$	$\tilde{\delta}_s^{\mathcal{I}N} > \tilde{\delta}_s^{\mathcal{I}H}$	$\tilde{\delta}_s^{\mathcal{I}A}$
$k < \frac{a}{n-x_s}$	$\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$	$\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$	$\tilde{\delta}_s^{\mathcal{I}N}$
$k > \frac{a}{n-x_s}$	$\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$	$\tilde{\delta}_s^{\mathcal{I}N} > \tilde{\delta}_s^{\mathcal{I}H}$	$\tilde{\delta}_s^{\mathcal{I}N}$

Table 2.2 Ranking of critical discount factors $\tilde{\delta}_s^{\mathcal{I}i}$

Thus, the legal standing of antitrust victims will influence cartel formation. Note however that it need not be the case that cartel size when only *home* customers have legal standing is larger than in the baseline scenario of no compensation: when the critical discount factor $\tilde{\delta}_s^{\mathcal{I}j}$ with $j \in \{N, H\}$ lies in interval 3, coalition size can either increase or decrease with more customers having legal standing. The reason is that collusive profits are decreasing when firms have to compensate their home customers and the DICC is not binding. Thus, incentives to stay in coalition $\underline{s} + 1$ decrease. However, also the market price decreases when the DICC is binding and firms have to compensate home customers. Hence, the profit of an outsider decreases when coalition $\underline{s}$ forms; firms have a stronger incentive to stay in coalition $\underline{s}+1$. These ambiguous effects can lead to increasing or decreasing coalition sizes as in Bos and Harrington (2015).

An unambiguous conclusion can be stated when all customers have legal standing: coalition size of an $\mathcal{A}$-stable cartel is (weakly) larger than for compensation only of home customers or no compensation at all. However, the market price may decrease or increase with a more severe antitrust regime because even a large cartel's price can be comparatively small when all customers have legal standing. Latter is driven by the *disproportional* burdening of small coalitions that would have to compensate a huge number of customers: even if the sustainability condition is not binding, partial cartels will take a compensation for umbrella losses into account by reducing their prices.

Discount factors that imply indifference and allow for two $\mathcal{A}$-stable cartel sizes are illustrated in Figure 2.7 as dotted vertical lines for Section 2.1's example. Thick lines depict an $\mathcal{A}$-stable cartel's profit-maximizing price path given discount factor δ, depending on whether outside customers have legal standing or not. (Dashed lines are adopted from Figure 2.2.) One can see that the coalition size of an $\mathcal{A}$-stable cartel when firms have to compensate for umbrella losses is at least as large as the coalition size of an $\mathcal{A}$-stable cartel when only home customers have legal standing for all $\delta \in (0, 1)$. The effect of an extended legal standing on the market price is however ambiguous. These conclusions are now illustrated with a numerical example, assuming constant positive rather than zero per-unit production costs.

2.5 Application

The numerical example considered in Section 2.1 is adapted supposing that firms operate in a dynamic environment with positive per-unit production costs $c = 2$. Recall that $n = 5$, $k = 3$, $\alpha = 1/5$, $\tau = 1/2$ and $\beta = 1$. Market demand is $D(p) = 10 - p$. Table 2.3 summarizes prices, cartel members' and free riders' profits when the DICC is non-binding, that is, when $\delta > \delta^i_s$, and the corresponding minimal discount factors

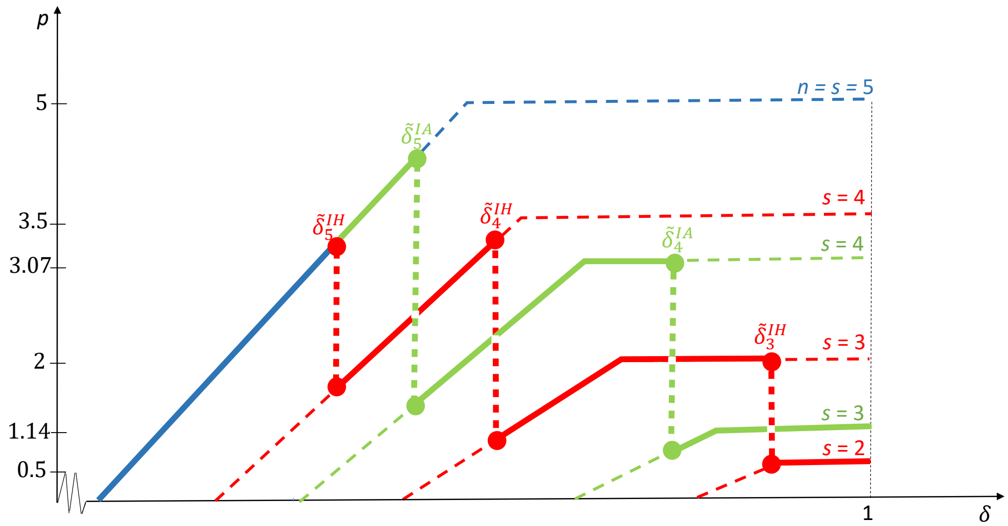

Figure 2.7 $\mathcal{A}$-stable coalition sizes for $\delta \in (0,1)$ in "home" (red) and "all" (green) regimes

δ_s^{mi} for $s \geq 2$ (recall, $p_s^{MN} = p_s^{MH}$ and $\pi_s^{FN}(p_s^{MN}) = \pi_s^{FH}(p_s^{MN})$).

	p_s^{MN}	π_s^N	π_s^{FN}	δ_s^{mN}	π_s^H	δ_s^{mH}	p_s^{MA}	π_s^A	π_s^{FA}	δ_s^{mA}
$s = 2$	0	0	–	–	0	–	0	0	–	–
$s = 3$	3	0.3	3	0.8	0.23	0.84	2.14	0.01	0.43	0.98
$s = 4$	4.5	1.41	7.5	0.63	1.09	0.71	4.07	0.75	6.21	0.76
$s = 5$	6	2.88	–	0.52	2.24	0.63	6	2.24	–	0.63

Table 2.3 Market outcomes

Table 2.3 first illustrates that the smallest coalition which can serve a positive net demand has size 3 in all compensation regimes. Second, deterrence, that is, minimal discount factors δ_s^{mi}, are (weakly) increasing with an extended legal standing. Third, prices, when the DICC is not binding, are strictly lower when all customers have legal standing compared to regimes N or H for $s < n$.

Figure 2.8 additionally includes prices when the DICC is binding. This allows to illustrate the price path as firm's common discount factor δ is increased assuming a cartel of size s has already formed with **no compensation**, a compensation for home resp. all customers with $s < n$ and a compensation for all customers with $s = n$ (blue).[24] The arrows on the δ-axis for $s < n$ are separated into two intervals: increasing deterrence, that is, greater δ is required to render the given cartel size dynamically sustainable when firms have to compensate home customers is coloured red; when

[24]The origin of the price is set equal to 2 – the marginal costs.

all customers have legal standing, deterrence is further increasing. This is illustrated by the green arrow. The vertical arrows at $\delta = 1$ illustrate the decreasing price given coalition size $s < n$ when the DICC does not bind and firms have to compensate for umbrella losses.

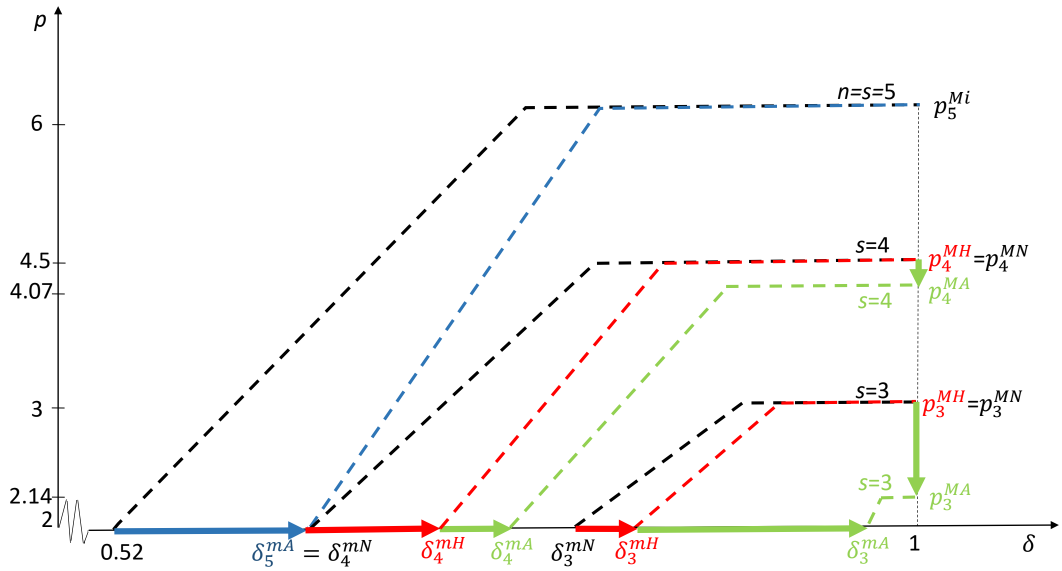

Figure 2.8 Price paths given a (partial) cartel of size s has formed

Next assume that coalition size s is endogenous and that $\delta \geq \delta^i_{\underline{s}}$. Then, it is a coalition of size 3 which is the unique $\mathcal{A}$-stable coalition when compensation payments are not awarded to victims who suffered umbrella losses. However, a partial cartel of four firms is internally stable when firms have to compensate for umbrella losses since $\pi_4^A(p_4^{MA}) - \pi_3^{FA}(p_3^{MA}) > 0$; external stability of coalition 4 is given since $\pi_5^A(p_5^{MA}) - \pi_4^{FA}(p_4^{MA}) < 0$ (see Table 2.3). Additionally, a coalition of four firms is the unique coalition size with satisfies internal and external stability simultaneously. Thus, a partial cartel of four firms is the unique $\mathcal{A}$-stable coalition size.[25] The market price for the $\mathcal{A}$-stable coalition size increases by more than 30% if all customers have legal standing.

With decreasing discount factors, that is, $\delta < \delta^i_{\underline{s}}$, also larger coalitions can be $\mathcal{A}$-stable. Which coalitions form depends on the indifferent discount factors $\tilde{\delta}_s^{\mathcal{I}i}$ which are summarized in Table 2.4. Ranking indifferent discount factors by size yields

[25] Following Diamantoudi (2005) and assuming that a cartel of size s' is internally (externally) stable if the decision to be outsider (insider) would start a sequence ending with an $\mathcal{A}$-stable cartel where profits of outsiders (insiders) are smaller than $\pi^i_{s'}(p^{Mi}_{s'})$ $(\pi^{Fi}_{s'}(p^{Mi}_{s'}))$, leads to the same $\mathcal{A}$-stable coalition sizes in this example. Stability results therefore do not necessarily hinge on the concepts introduced

	$\tilde{\delta}_s^{IN}$	$\tilde{\delta}_s^{IH}$	$\tilde{\delta}_s^{IA}$
$s = 5$	0.69	0.75	0.80
$s = 4$	0.85	0.87	–
$s = 3$	–	–	–

Table 2.4 Indifferent discount factors $\tilde{\delta}_s^{Ii}$

$\tilde{\delta}_5^{IN} < \tilde{\delta}_5^{IH} < \tilde{\delta}_5^{IA}$ and $\tilde{\delta}_4^{IN} < \tilde{\delta}_4^{IH}$.[26] Thus, the coalition size when customers who suffered an umbrella loss have legal standing is at least as large as the coalition size when they are not allowed to bring a lawsuit before the court, whenever at least the industry-wide cartel is dynamically sustainable.

In Figure 2.9 we combine Figure 2.8 and Table 2.4 and illustrate the ambiguous effects of an extended legal standing. Bold lines depict the price paths of $\mathcal{A}$-stable cartels as we consider possible discount factors of firms. Indifferent discount factors subdivide the δ-axis into six segments. In segment *I*, prices unambiguously increase when firms do not have to compensate because even the industry-wide cartel is not dynamically sustainable for a wide range of discount factors when firms have to compensate; if it is dynamically sustainable, prices will only slightly increase. The opposite is true in segment *II*: a coalition of four firms is $\mathcal{A}$-stable in regime *N* whereas the industry-wide cartel is still $\mathcal{A}$-stable when firms have to compensate. The positive effect of a decreasing coalition size now dominates when firms do not have to compensate. Then, the market price decreases when private antitrust enforcement is prohibited. In segment *III*, all market prices differ. Highest prices can be observed when all customers have legal standing; lowest prices when only home customers have legal standing. Almost the same applies in segment *VI* although prices in regimens *N* and *H* coincide for some discount factors. Prices when all customers have legal standing can also be lowest as in segment *IV* or intermediate in between the price when only home customers have legal standing (above) and the price when no customer has legal standing (below) as in segment *V*.

How an extended legal standing effects social welfare can be directly assessed by comparing corresponding market prices since a compensation only for overcharge losses is simply a redistribution of money between firms and customers. Thus, the higher the market price, the higher the deadweight loss (in legal terms denoted as loss of profits), which is hardly ever reclaimed. Whether pro- or anticompetitive effects

by D'Aspremont et al. (1983).

[26]Since coalition 4 is $\mathcal{A}$-stable when all customers have legal standing for $\delta \geq \delta_{\underline{s}}^{A}$, no $\tilde{\delta}_4^{IA} \in (0,1)$ exists.

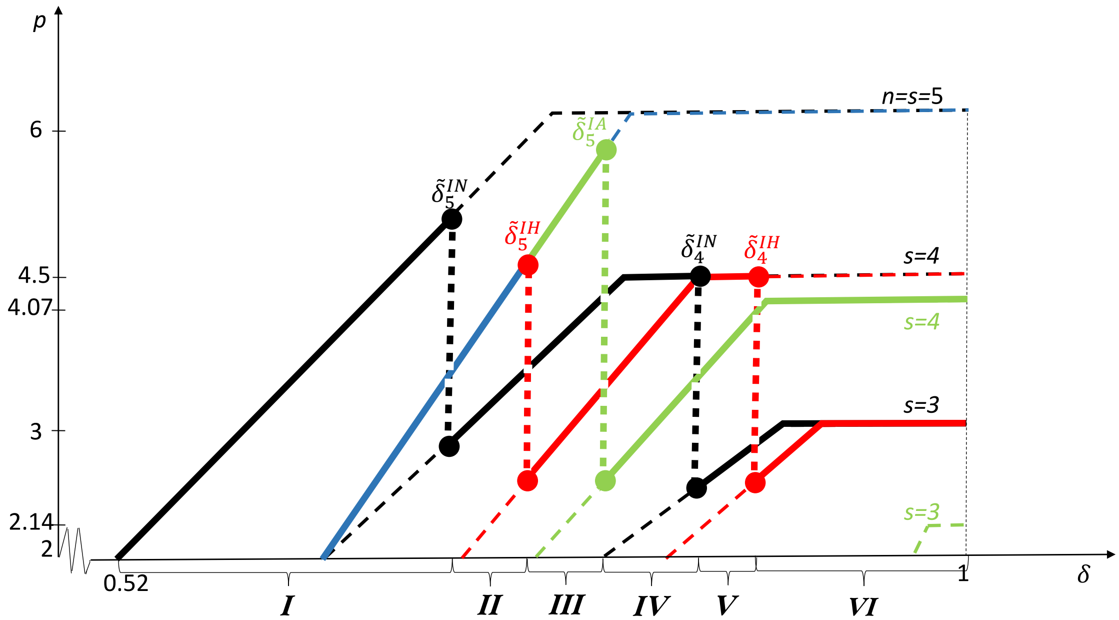

Figure 2.9 $\mathcal{A}$-stable coalitions' price paths

of an extended legal standing dominate on the welfare for $\delta \in (0, 1)$, can be analysed for specific distributions of δ. Following Katsoulacos et al. (2015) and assuming that δ is uniformly distributed[27] on $(0, 1)$ yields

$$\overline{m}^N = 0.68; \quad \overline{m}^H = 0.28 \ (-58.82\%); \quad \overline{m}^A = 0.69 \ (+1.47\%)$$

with $\overline{m}^N$ as the average mark-up when firms do not have to compensate and $\overline{m}^H$ and $\overline{m}^A$ as the average mark-up (increase) when home or all customers have legal standing, respectively. The ranking $\overline{m}^A > \overline{m}^N > \overline{m}^H$ extends to the case where $c = 0$ with $\overline{m}^N = 0.92$, $\overline{m}^H = 0.62$ (-32.61%) and $\overline{m}^A = 1.02$ $(+10.87\%)$. Thus, a regime where only home customers have legal standing leads to the highest social welfare in the considered example for $\delta \in (0, 1)$.

The analysis of consumer surplus (CS) requires more detailed consideration. A regime which leads to rather high prices but which ensures compensation of all customers can be preferable from an aggregated customer perspective compared to regimes N and H even if prices in these regimes are lower. The increasing deadweight loss (which is born by the customers) when the market price increases can be offset by high compensation payments when umbrella losses have to be compensated. In

[27] The distribution of δ will depend on the considered market. However, since one legal rule should apply for all markets, it is a natural assumption to start with a uniformly distributed discount factor.

particular, with $\beta = 1$, customers who (*i*) bought a product produced by a free rider and (*ii*) did not decrease their demand, will always prefer a rule of compensation for all, although the market price might increase.

In the considered example, all customers successfully reclaim overcharge damages, that is, $\beta = 1$. This gives compensation regime A a good shot with respect to CS. Indeed, a compensation of all customers leads in each segment of Figure 2.9 to a higher CS compared to no compensation at all (CS is evaluated for the average price in each segment). Additionally, except for segment *III*, CS when all customers have legal standing exceeds CS when only home customers have legal standing. However, in segment *III*, compensation regime H outperforms regime A and regime N; but regime H performs worst in segment V.

The considered example clearly advocates a compensation only of home customers when social welfare should be maximized. However, when the maximization of consumer surplus is the objective, a compensation of all customers is preferable for a wide range of discount factors. Results crucially hinge on the size of β and on the distribution of δ. In particular, β will be smaller than one in most real-life cases. Moreover, in many cartel cases in the EU (e.g., in the canned vegetables or in the beauty product market), it is highly likely that only big players act against former cartel members since individual damages are rather small. Thus, arguing that all customers should have legal standing to increase CS although this implies increasing prices, could harm small customers.

2.6 Concluding Remarks

This chapter shows that the coalition size of a stable cartel depends on whether firms have to compensate for umbrella losses or not. Our model predicts that cartel size tends to increase if private antitrust enforcement also applies to umbrella effects. Future empirical analysis may detect whether the legal certainty about cartel members having to compensate umbrella victims, which was established by judgement CJEU C-557/12 for the EU, has indeed induced more encompassing cartels. How coalition size and therefore the market price depends on the right for outside customers to bring an action before the court also matters to recent discussion on the legal standing of cartel victims in the US. Authors who urge the Supreme Court to take a decision and to extend legal standing to outside customers only take cartel deterrence for given cartel size into account (see Blair and Durrance 2018).

We have shown that coalition size may increase and hence a market price increase is possible because of extended legal standing. This has been shown in a symmetric setup but we have no reason to believe that this will be different under mild asymmetry of the firms. Still, an extension to firms that differ in their capacities would be a natural extension of our analysis. Bos and Harrington (2010) provide important insights into cartel formation among asymmetric-sized firms. For instance, they show that a stable cartel must include the largest firms in the market, since these firms' influence on the resulting market price is greatest. The respective bigger incentive of large firms to coordinate their strategies in asymmetric cases should in our view not change results of this chapter.

The same applies for the detection probability, which is assumed to be independent of the price or the size of the cartel. No matter whether the cartel price or the cartel size drives detection probabilities, stable coalition sizes would decrease since firms are more likely to leave rather large cartels (the effect is either direct since larger cartels are easier to detect or indirect since the market price is increasing in the cartel size). Only if the detection probability depends on the cartel size, results would be systematically affected because the coalition size is shown to be larger when all customers have legal standing whereas prices can either increase or decrease with a more severe antitrust regime. However, although smaller coalitions can form for a wider range of discount factors when detection probabilities would depend on the market price or the cartel size, there is no reason to believe that the main conclusion of this chapter is reversed: the coalition size tends to increase when all customers would have legal standing.

Consideration of different concepts of cartel stability would also be useful as theoretical robustness check. The related approach by Diamantoudi (2005) probably leads to similar results (see fn. 25) but complex settings as in Ray and Vohra (1999) might change our findings. How an expanded legal standing of cartel victims affects the size of stable cartels when multiple partial cartels can form is left for future research.

That private antitrust enforcement effects market prices plays a crucial role in antitrust policy. With the exception of Bos and Harrington (2015), the relevant studies however do not consider how enforcement rules affect cartel size – which is a main determinant of market prices. A compensation for umbrella effects reduces a cartel member's profit more the smaller the coalition size. This is opposite to the policy recommendation by Bos and Harrington (2015) which suggest policies that should be "*. . . progressively more aggressive for more inclusive cartels*". However, even a policy where rather small cartels are disproportionately harshly burdened can be beneficial from a welfare perspective when increasing cartel deterrence and decreasing market prices outweigh negative effects caused by increasing coalition sizes.

A central policy recommendation can by deduced: courts should not argue that all customers should have legal standing to strengthen *effective competition* just because cartel deterrence is increasing. How a compensation for umbrella losses affects competition will depend on the specific market, on firms' discount factors, on characteristics of plaintiffs, etc. A legal rule, which should maximize social welfare, may favour compensation only of home customers as in the discussed example.

2.7 Appendix A Private Antitrust Enforcement

We will first discuss the assumptions made to simplify the incorporation of private antitrust enforcement. Second, we clarify the assumption that firms will expect to bear an equal share of the total compensation payments.

The assumption that share β of harmed customers who have legal standing successfully reclaim annual overcharge damages simplifies the analysis in three ways. First, firms only have to compensate the damage for one single period.[28] Second, we only consider the price overcharge damage. Depending on national law, plaintiffs can also reclaim loss of profits. This damage arises when the product was not bought because it became too expensive. However, a compensation for loss of profits comes with many practical hurdles, e.g., how to estimate its size. It plays an almost negligible role in legal practice (see Laborde 2017). Third, the probability of winning the lawsuit is assumed to not depend on characteristics of claimants. In particular, we assume for the case with legal standing of all customers that customers who bought a product produced by non-cartel members have the same winning probability as customers who bought a product produced by a cartel member. Increasing standing is therefore directly transferred to increasing aggregated expected compensation payments. Providing causal evidence that there was a damage caused by a cartel, although the product was not produced by a cartel member, may however be burdensome when products are rather bad substitutes. This assumption should be

[28] A time variable could be included to take cartel duration into account. Then, compensation payments would increase in the time a cartel operated. There is however no reason to believe that increasing compensation payments over time will change results of this chapter.

unproblematic with homogeneous goods (see Blair and Durrance 2018).

Next, we will argue that firms will expect to pay an equal share of the total compensation. Harmed customers who have legal standing can act against any former cartel member since co-defendants are jointly liable towards third parties in most jurisdictions, e.g., in the EU, US, Japan or Australia. This means that they can sue any cartel member for the total or any desired share of the total damage no matter whether this firm has produced the good bought by the customer or not. Which firm actually has to bear the compensation (internal liability) will be settled afterwards. Different concepts to allocate compensation payments among joint tortfeasors exist in the EU and the US.

In the EU, the European Commission established a rule of contribution. The applicable Directive 2014/104/EU states that each cartel member has to contribute according to its *relative responsibility* for the harm (Article 11). How to economically quantify this norm is discussed in more detail in Chapter 3, by Schwalbe (2013) and by Napel and Oldehaver (2015). The only reasonable internal allocation rule in our model is an allocation per heads since firms are symmetric even on a customer level and all firms are assumed to have the same role in the cartel.

In the US, firms do not have to contribute (see Texas Industries, Inc. v. Radcliff Materials, Inc., 451 U.S. 630, 1981). The firm that is sued has to bear the whole compensation, that is, firms are not internally liable. This however does not mean that firms expect to pay the whole compensation: incentives of customers to act against particular cartel members are in our model the same ex ante. Thus, the probabilities that a given firm has to compensate all customers are identical among cartel members. For risk-neutral firms, this is equivalent to the case where all customers which have legal standing split equally between detected cartel members.[29]

No matter whether firms have to contribute or not, they expect that compensation payments are allocated equally among cartel members. The expected collusive value of each cartel member therefore decreases symmetrically in compensation payments and all firms choose the same profit-maximizing price.

[29] See Easterbrook et al. (1980) for a detailed discussion on expected liability shares.

2.8 Appendix B Proofs

Proof of Lemma 2.1.
Part (i): Outside customers have no legal standing. Then, net cartel demand for $p_s^j = 2\epsilon \approx 0$ with $j \in \{N, H\}$ and $s < n$, i.e., $D_s^{nN}(0) = (1-\alpha\tau)D_s^R(0)/s$ and $D_s^{nH}(0) = eD_s^R(0)/s$, is positive if the enforcement parameter e is positive and if

$$D_s^R(0) > 0 \Leftrightarrow k < \frac{a}{n-s}. \tag{2.12}$$

Part (ii): All customers have legal standing. Then, net cartel demand for $p_s^A = 2\epsilon \approx 0$ and $s < n$ is positive if

$$D_s^{nA}(0) = [(1-\alpha\tau)(a-(n-s)k) - \alpha\beta a]/s > 0 \Leftrightarrow k < \mu\frac{a}{n-s} \tag{2.13}$$

with $\mu := (1-\alpha(\tau+\beta))/(1-\alpha\tau)$. Given (A2), i.e., $e = 1-\alpha(\tau+\beta) > 0$, and $\beta \geq 0$ it follows that $\mu \in (0,1]$. That umbrella coefficient $\mu \in (0,1]$ implies $\frac{a}{n-s} \geq \frac{\mu a}{n-s}$. ■

Proof of Proposition 2.1.
Part (i): Consider $k < a/(n-(\underline{s}-1))$. Condition ($i$) of Lemma 2.1 then implies that a coalition of size $\underline{s}-1$ would also have positive net demand. By contrast, net demand of coalition $\underline{s}$ would be zero for $k \geq a/(n-\underline{s})$ since condition (i) of Lemma 2.1 would then be violated. Hence, $a/(n-(\underline{s}-1)) \leq k < a/(n-\underline{s})$ when coalition $\underline{s}$ is the smallest coalition which could supply a positive net demand.

To see how capacity constraints imposed by assumption (A1) influence coalition size $\underline{s}$, first recall that positivity of $D_s^{nj}(2\epsilon)$ with $j \in \{N, H\}$ is only given when outside capacity is sufficiently small, i.e., given coalition size s, capacity level k has to be sufficiently small. An increasing capacity level k can be offset with a larger coalition size s to satisfy $D_s^{nj}(2\epsilon) > 0$. Assumption (A1) first requires that firms are not too small, i.e., $k \geq a/(n-1)$. However, with $k \approx a/(n-1)$, a coalition of size 2, that is, the smallest possible coalition size which could form, satisfies $D_s^{nj}(2\epsilon) > 0$, since $a/(n-(\underline{s}-1)) \geq a/(n-1)$ with $\underline{s} \geq 2$. Second, firms are not allowed to be too large, that is, $k < a/2$.[30] Hence, we can conclude that no coalition of size $n-1$ or n could be the smallest coalition which has positive net demand even if $k \approx a/2$ since $a/(n-\underline{s}) \leq a/2$

[30]The industry-wide cartel price, given the DICC is non-binding, will be shown to be independent of the legal standing of antitrust victims (see equation (2.17)). Thus, the monopoly output is $a/2$.

implies $\underline{s} \leq n-2$.

Part (*ii*): Umbrella losses have to be compensated. When capacity constraints introduced in (A1) are not binding, Lemma 2.1 determines interval bounds of the capacity of coalition $\underline{s}$ as in (*i*). However, depending on the size of μ, it can be true that $\mu a/(n-(\underline{s}-1)) < a/(n-1)$ resp. that $\mu a/(n-\underline{s}) > a/2$. Then, capacity bounds are binding and the capacity of coalition $\underline{s}$ satisfies $\max\{a/(n-1), \mu a/(n-(\underline{s}-1))\} \leq k < \min\{\mu a/(n-\underline{s}), a/2\}$.

For $k \approx a/(n-1)$, it is no longer true that coalition $s=2$ must have positive net demand since a necessary condition for $D_s^{nA}(2\epsilon) > 0$ is $\mu a/(n-s) > a/(n-1) \Leftrightarrow \mu > (n-s)/(n-1)$. Thus, if the umbrella coefficient is sufficiently small, coalition $s=2$ has no positive net demand. Additionally it is no longer true that at least coalition $n-2$ has positive net demand for $k \approx a/2$. The smallest coalition size s for $k \approx a/2$ satisfies $\max\{a/(n-1), \mu a/(n-(\underline{s}-1))\} \leq k < a/2 \leq \mu a/(n-\underline{s})$. The last inequality holds if $\mu \geq (n-\underline{s})/2$. That some capacity level k satisfies $a/2 > k \geq \max\{a/(n-1), \mu a/(n-(\underline{s}-1))\}$ follows if $a/2 > \mu a/(n-(\underline{s}-1)) \Leftrightarrow \mu < (n-\underline{s}+1)/2$.[31] This inequality is always satisfied for $\underline{s} < n$ and $\beta > 0$. Thus, a coalition has positive net demand if $\mu > \max\{(n-s)/(n-1), (n-s)/2\} = (n-s)/2$ for $n \geq 4$. Hence, with $\mu > 1/2$, it follows that at least coalition $n-1$ has positive net demand. ■

Proof of Proposition 2.2.
Part (*i*): When only home customers have legal standing, the DICC is

$$V_s^H(p_s^H) \geq V_s^{DH}(p_s^H) \Leftrightarrow \delta \geq 1 - \frac{e(a-bp_s^H-k(n-s))}{ks} =: \delta_s^H(p_s^H). \tag{2.14}$$

With legal standing extended to outside customers, DICC requires

$$V_s^A(p_s^A) \geq V_s^{DA}(p_s^A) \Leftrightarrow \delta \geq \frac{k(n-\alpha\tau(n-s)) - e(a-bp_s^A)}{ks} =: \delta_s^A(p_s^A) \tag{2.15}$$

to sustain the agreement. With $p_s^H = 0$ resp. $p_s^A = 0$ minimal discount factors δ_s^{mj} are

$$\delta_s^H(0) = 1 - \frac{e(a-k(n-s))}{ks} =: \delta_s^{mH}; \quad \delta_s^A(0) = \frac{k(n-\alpha\tau(n-s)) - ea}{ks} =: \delta_s^{mA}. \tag{2.16}$$

Arranging minimal discount factors δ_s^{mN} (see equation (2.5)), δ_s^{mH} and δ_s^{mA} by size yields $\delta_s^{mA} \geq \delta_s^{mH} \geq \delta_s^{mN}$. This ranking is strict for $s < n$ and $\beta > 0$.

[31] It directly follows that $a/2 > a/(n-1)$ with $n \geq 4$.

To derive non-binding discount factors δ_s^j with $j \in \{H, A\}$, we first have to determine profit maximizing prices when the DICC is non-binding. This yields

$$p_s^{MH} := \frac{a - k(n-s)}{2b}; \qquad p_s^{MA} := \frac{ae - k(n-s)(1-\alpha\tau)}{2be} \tag{2.17}$$

with $p_s^{MN} = p_s^{MH} \geq p_s^{MA}$ and $D(p_n^{MH}) = D(p_n^{MA}) = a/2$. Substituting p_s^{Mj} into $\delta_s^j(p_s^j)$ with $j \in \{H, A\}$ gives

$$\delta_s^H(p_s^{MH}) = 1 - \frac{e(a-(n-s)k)}{2ks} =: \delta_s^H; \quad \delta_s^A(p_s^{MA}) = \frac{k(n+s-\alpha\tau(n-s))-ea}{2ks} =: \delta_s^A. \tag{2.18}$$

For $\delta \geq \delta_s^H$ ($\delta \geq \delta_s^A$) firms price at p_s^{MH} (p_s^{MA}). Ranking these discount factors by size yields $\delta_s^A \geq \delta_s^H \geq \delta_s^N$ with $\beta \geq 0$ and $s \leq n$.

To see that δ_s^{mH}, δ_s^{mA}, δ_s^H and δ_s^A are decreasing in the coalition size, consider

$$\Delta\delta_s^{mH} := \delta_s^{mH} - \delta_{s-1}^{mH} = \frac{e(a-kn)}{k(s-1)s} < 0; \quad \Delta\delta_s^H := \delta_s^H - \delta_{s-1}^H = \frac{\Delta\delta^{mH}}{2} < 0;$$
$$\Delta\delta_s^{mA} := \delta_s^{mA} - \delta_{s-1}^{mA} = \frac{ea - kn(1-\alpha\tau)}{k(s-1)s} < 0; \quad \Delta\delta_s^A := \delta_s^A - \delta_{s-1}^A = \frac{\Delta\delta^{mA}}{2} < 0. \tag{2.19}$$

Denominators are always positive for $s \geq 2$. Numerators are negative when assumptions (A1) and (A2) are satisfied.[32]

Part (*ii*): Suppose that the DICC is binding, that is, $\delta \in (\delta_s^{mj}, \delta_s^j)$ with $j \in \{H, A\}$. Then, the profit maximizing cartel price is

$$p_s^{DH} := \frac{(a-kn)e + ks(\delta - \alpha(\beta+\tau))}{be}; \quad p_s^{DA} := \frac{\big(ae - kn(1-\alpha\tau)\big) + ks(\delta - \alpha\tau)}{be} \tag{2.20}$$

with $p_s^{DN} \geq p_s^{DH} \geq p_s^{DA}$. These prices can only be positive if $\delta > \alpha(\beta+\tau)$ resp. if $\delta > \alpha\tau$ since the first addends of the numerators in (2.20) are negative by assumption (A1).

The optimal cartel price $p_s^i := \min\{p_s^{Di}, p_s^{Mi}\}$ is increasing in the coalition size s since p_s^{Di} and p_s^{Mi} are increasing in s. Additionally, p_s^i is (weakly) decreasing with an extended legal standing: $p_s^N \geq p_s^H$ follows since $p_s^{MN} = p_s^{MH} > p_s^{DN} \geq p_s^{DH}$ and discount factors can be ranked such that $\delta_s^N \leq \delta_s^H$. Thus, if firms do not have to compensate, they choose p_s^{MN} also for values of δ where p_s^{DH} is chosen if firms have to compensate. Following the same approach gives $p_s^H \geq p_s^A$ with $p_s^{MH} \geq p_s^{MA} \geq p_s^{DA}$, $p_s^{DH} \geq p_s^{DA}$ and

[32]From $\Delta\delta_s^{mH} < 0$ directly follows $\delta_s^{mN} - \delta_{s-1}^{mN} < 0$ since $e > 0$ implies $(1-\alpha\tau) > 0$. The same applies for $\Delta\delta_s^H < 0$ and $\delta_s^N - \delta_{s-1}^N < 0$.

$\delta_s^H \leq \delta_s^A$. ■

Proof of Lemma 2.2.
We first substitute $\pi_s^i(p_s^{Mi})$ and $\pi_{s-1}^{Fi}(p_{s-1}^{Mi})$ in equation (INT):

$$\text{Part } (i): \quad \mathcal{I}_s^N(k) = \frac{(1-\alpha\tau)}{sb}\left(\frac{a-k(n-s)}{2}\right)^2 - \frac{a-kn+k(s-1)}{2b}k \tag{2.21}$$

$$\text{Part } (ii): \quad \mathcal{I}_s^H(k) = \frac{e}{sb}\left(\frac{a-k(n-s)}{2}\right)^2 - \frac{a-kn+k(s-1)}{2b}k \tag{2.22}$$

$$\text{Part } (iii): \quad \mathcal{I}_s^A(k) = \frac{\left(ae-k(n-s)(1-\alpha\tau)\right)^2}{4bse} - \frac{ae-k(n-(s-1))(1-\alpha\tau)}{2be}k. \tag{2.23}$$

Deriving second derivatives of $\mathcal{I}_s^i(k)$ with respect to k yields

$$\frac{\partial^2 \mathcal{I}_s^N(k)}{\partial k^2} = \frac{2s(1+(n-s)\alpha\tau)+(1-\alpha\tau)(n-s)^2}{2sb} \tag{2.24}$$

$$\frac{\partial^2 \mathcal{I}_s^H(k)}{\partial k^2} = \frac{2s(n+1-s)+e(n-s)^2}{2sb} \tag{2.25}$$

$$\frac{\partial^2 \mathcal{I}_s^A(k)}{\partial k^2} = \frac{(1-\alpha\tau)[2s(1+(n-s)\alpha\tau)+(n-s)^2(1-\alpha\tau)]}{2bse}. \tag{2.26}$$

Equ. (2.24)–(2.26) are strictly positive for $e > 0$ and $s \leq n$. $\mathcal{I}_s^i(k)$ is therefore strictly convex in the capacity.

The following part considers the case that $\max\{a/(n-1), \mu a/(n-(\underline{s}-1))\} = \mu a/(n-(\underline{s}-1))$ and that $\min\{\mu a/(n-\underline{s}), a/2\} = \mu a/(n-\underline{s})$. When this affects results, it is explicitly highlighted. Then, capacities are bounded by $k \in [a/(n-(\underline{s}-1)), a/(n-\underline{s}))$ when outside customers have no legal standing and by $k \in [\mu a/(n-(\underline{s}-1)), \mu a/(n-\underline{s}))$ when all customers have legal standing and coalition $\underline{s}$ is the smallest coalition which could serve a positive net demand (see Proposition 2.1). Thus, capacities for coalition $\underline{s}$ can be written as $k = a/(n-x)$ resp. as $k = \mu a/(n-x)$ with $x \in [\underline{s}-1, \underline{s})$. Capacity level k increases in x.

Coalition $\underline{s}$ is internally stable since its collusive value is positive, whereas the outside profit of coalition $\underline{s}-1$ is zero. Firms therefore have no incentive to leave coalition $\underline{s}$.

We next show, that also a coalition of size $\underline{s}+1$ is internally stable for some values of k. Substituting $k = a/(n-x)$ resp. $k = \mu a/(n-x)$ in equations (2.21)–(2.23) yields[33]

$$\mathcal{I}_s^N(x) = \frac{a^2[x^2 - (s-2)s - (s-x)^2\alpha\tau]}{4bs(n-x)^2} \tag{2.27}$$

$$\mathcal{I}_s^H(x) = \frac{a^2[x^2 e - s^2(1+\alpha\beta+\alpha\tau) + 2s(1+x\alpha(\beta+\tau))]}{4bs(n-x)^2} \tag{2.28}$$

$$\mathcal{I}_s^A(x) = \mu\frac{a^2[x^2 - (s-2)s - (s-x)^2\alpha\tau]}{4bs(n-x)^2} = \mu\mathcal{I}_s^N(x). \tag{2.29}$$

It is sufficient to prove that $\mathcal{I}_s^N(x)$ is positive to conclude that $\mathcal{I}_s^A(x)$ is positive since $\mu > 0$ by assumption (A2). Substituting $s = \underline{s} + 1$ in equations (2.27) and (2.28) and inserting the lower bound for the capacity which still allows coalition $\underline{s}$ to earn positive profits, i.e., $x = \underline{s} - 1$ in $k = a/(n-x)$, gives the internal stability condition for a coalition of size $\underline{s} + 1$ when capacities are $k = a/(n - (\underline{s} - 1))$, that is,

$$\mathcal{I}_{\underline{s}+1}^N(\underline{s}-1) = \frac{a^2(1-\underline{s}-2\alpha\tau)}{2b(\underline{s}+1)(n-\underline{s}+1)^2} < 0; \quad \mathcal{I}_{\underline{s}+1}^H(\underline{s}-1) = \frac{a^2(1-\underline{s}-2\alpha(\beta+\tau))}{2b(\underline{s}+1)(n-\underline{s}+1)^2} < 0. \tag{2.30}$$

Both denominators are positive and numerators are negative for all $\underline{s} \geq 2$. Hence, internal stability is violated. Inserting the upper bound for x, that is, $x = \underline{s}$, yields

$$\mathcal{I}_{\underline{s}+1}^N(\underline{s}) = \frac{a^2(1-\alpha\tau)}{4b(\underline{s}+1)(n-s)^2} > 0; \quad \mathcal{I}_{\underline{s}+1}^H(\underline{s}) = \frac{a^2 e}{4b(\underline{s}+1)(n-\underline{s})^2} > 0 \tag{2.31}$$

with $n \geq s$ and $e > 0$; coalition $\underline{s} + 1$ is internally stable for $x = \underline{s}$.

Since $\mathcal{I}_{\underline{s}+1}^j(k)$ is strictly convex in k, a unique $k > a/(n-(\underline{s}-1))$ resp. $k > \mu a/(n-(\underline{s}-1))$ satisfies $\mathcal{I}_{\underline{s}+1}^j(k) = 0$ since $\mathcal{I}_{\underline{s}+1}^j(\underline{s}-1) < 0$. Solving $\mathcal{I}_{\underline{s}+1}^j(x) = 0$ with $j \in \{N, H\}$ for x gives

$$x_{\underline{s}1}^{NA} = \frac{\alpha\tau\phi_s - \sqrt{\underline{s}^2 - 1 + 2\alpha\tau\phi_s}}{-1+\alpha\tau}; \quad x_{\underline{s}2}^{NA} = \frac{\alpha\tau\phi_s + \sqrt{\underline{s}^2 - 1 + 2\alpha\tau\phi_s}}{-1+\alpha\tau}; \tag{2.32}$$

$$x_{\underline{s}1}^{H} = \frac{\alpha\phi_s(\tau+\beta) - \sqrt{\underline{s}^2 - 1 + 2\alpha\phi_s(\tau+\beta)}}{-1+\alpha\beta+\alpha\tau}; \quad x_{\underline{s}2}^{H} = \frac{\alpha\phi_s(\tau+\beta) + \sqrt{\underline{s}^2 - 1 + 2\alpha\phi_s(\tau+\beta)}}{-1+\alpha\beta+\alpha\tau}$$

with $\phi_s := \underline{s}+1$. We can directly exclude $x_{\underline{s}2}^{NA}$ and $x_{\underline{s}2}^{H}$ from the following analysis, since denominators are negative whereas numerators are positive, leading to negative

[33]Similar equations can be stated allowing unit costs to be constant but positive with $(a - bc)^2$ instead of a^2.

values of x which contradicts assumption (A1). To simplify notation, the '1' in subscripts of $x^{NA}_{\underline{s}1}$ and $x^{H}_{\underline{s}1}$ is omitted in the following analysis.

The only step of the proof where capacity bounds of coalition $\underline{s}$ matter when firms have to compensate for umbrella losses is the following: since $\mathcal{I}^{A}_{\underline{s}+1}(k)$ is strictly convex in k, coalition $\underline{s}+1$ is internally stable for all relevant values of $k \in [a/(n-1), \mu a/(n-\underline{s}))$ if $a/(n-x^{NA}_{\underline{s}}) < a/(n-1)$. Internal stability of coalition $\underline{s}+1$ is violated for any relevant capacity level $k \in [\mu a/(n-(\underline{s}-1)), a/2)$ with $a/2 \leq a/(n-x^{NA}_{\underline{s}})$. Then, only coalition $\underline{s}$ is internally stable. This however does not influence the analysis since $k = a/(n-x^{NA}_{\underline{s}})$ only partitions the interval $k \in [\mu a/(n-(\underline{s}-1)), \mu a/(n-\underline{s}))$ without proving existence of coalition $\underline{s}$ or $\underline{s}+1$.

We next prove that $x^{H}_{\underline{s}} \geq x^{NA}_{\underline{s}}$. First, note that $x^{H}_{\underline{s}} = x^{NA}_{\underline{s}}$ with $\beta = 0$. Second, $x^{H}_{\underline{s}}$ can be shown to be increasing in β: taking the first derivative of $x^{H}_{\underline{s}}$ for β yields

$$\frac{\partial x^{H}_{\underline{s}}}{\partial \beta} = \frac{\alpha\phi_s\left[\underline{s} + \alpha(\beta+\tau) - \sqrt{\phi_s(\underline{s}-1+2\alpha(\beta+\tau))}\right]}{(-1+\alpha(\beta+\tau))^2\sqrt{\phi_s(\underline{s}-1+2\alpha(\beta+\tau))}}. \tag{2.33}$$

The denominator is always positive with $\underline{s} \geq 2$. The numerator would be zero with $\alpha(\beta+\tau) = 1$ and is decreasing in $y := \alpha(\beta+\tau)$ since

$$\frac{\partial(\underline{s} + y - \sqrt{\phi_s(\underline{s}-1+2y)})}{\partial y} = 1 - \frac{\phi_s}{\sqrt{\phi_s(\underline{s}-1+2y)}} < 0. \tag{2.34}$$

Inequality (2.34) is satisfied for $y < 1$ and $\underline{s} \geq 2$. Hence, we can conclude that the numerator of equation (2.33) is always positive since $y < 1$ as long as assumption (A2) holds. Thus, $x^{H}_{\underline{s}}$ is increasing in β and we can conclude that $x^{H}_{\underline{s}} \geq x^{NA}_{\underline{s}}$.

We next show that no cartel of size $s \geq \underline{s}+2$ is internally stable when coalition $\underline{s}$ is the smallest partial cartel that has a positive net demand. Starting with a coalition of size $s = \underline{s}+2$ and determining $\mathcal{I}^{N}_{\underline{s}+2}(\underline{s})$ and $\mathcal{I}^{H}_{\underline{s}+2}(\underline{s})$ yields

$$\mathcal{I}^{N}_{\underline{s}+2}(\underline{s}) = -\frac{a^2(\underline{s}+2\alpha\tau)}{2b(\underline{s}+2)(n-\underline{s})^2} < 0; \quad \mathcal{I}^{H}_{\underline{s}+2}(\underline{s}) = -\frac{a^2(\underline{s}+2\alpha(\beta+\tau))}{2b(\underline{s}+2)(n-\underline{s})^2} < 0. \tag{2.35}$$

With increasing capacity levels k, more firms coordinating their strategies are needed to allow for internal stability. Since coalition $\underline{s}+2$ is not even internally stable for the upper bound of k, internal stability is violated for any $k \in [a/(n-(\underline{s}-1)), a/(n-\underline{s}))$.

No coalition of size $s > \underline{s}+2$ can be internally stable since $\mathcal{I}^{N}_{s}(x)$ and $\mathcal{I}^{H}_{s}(x)$ are decreasing in s. This follows since

$$\mathcal{I}^N_{s+1}(x) - \mathcal{I}^N_s(x) = -\frac{a^2\Big(2+3s+s^2+x^2+(2+s^2+3s-x^2)\alpha\tau\Big)}{4b(s+1)(s+2)(n-x)^2} < 0 \tag{2.36}$$

$$\mathcal{I}^H_{s+1}(x) - \mathcal{I}^H_s(x) = -\frac{a^2\Big((2+3s+s^2)(1+\alpha(\beta+\tau))+x^2e\Big)}{4b(s+1)(s+2)(n-x)^2} < 0 \tag{2.37}$$

for any coalition size s with $n \geq s > x$ and $e > 0$. Thus, we can conclude: $\mathcal{I}^j_s(k) < 0$ with $j \in \{N, H\}$ for all $s \geq \underline{s}+2$ and $k \in [a/(n-(\underline{s}-1)), a/(n-\underline{s}))$.■

Proof of Proposition 2.3.
As already pointed out by D'Aspremont et al. (1983), a coalition s is externally *stable* when coalition $s+1$ is internally *unstable*. Thus, if *only* coalition $\underline{s}$ is internally stable, that is, if $k < \tilde{k}^i_{\underline{s}}$, we can conclude that this coalition is the unique which is $\mathcal{A}$-stable.

Now assume that coalitions $\underline{s}$ and $\underline{s}+1$ are both internally stable, that is, $k \geq \tilde{k}^i_{\underline{s}}$. When $k > \tilde{k}^i_{\underline{s}}$ *only* coalition $\underline{s}+1$ is $\mathcal{A}$-stable: first, coalition $\underline{s}+2$ is internally unstable, i.e., coalition $\underline{s}+1$ is externally stable.[34] Second, since firms have a strict incentive to stay in coalition $\underline{s}+1$, inequality (INT) requires $\pi^i_{\underline{s}+1}(p^i_{\underline{s}+1}) - \pi^{Fi}_s(p^i_s) > 0$. Thus, external stability of coalition $\underline{s}$, that is, $\pi^i_{\underline{s}+1}(p^i_{\underline{s}+1}) - \pi^{Fi}_s(p^i_s) \leq 0$, cannot be satisfied. When $k = \tilde{k}^i_{\underline{s}}$ firms are indifferent between staying in coalition $\underline{s}+1$ or leaving this coalition. Then, both coalitions of size $\underline{s}$ and $\underline{s}+1$ satisfy $\mathcal{A}$-stability. ■

Proof of Lemma 2.3.
Let $\tilde{\delta}^{\mathcal{I}i}_s \in (\delta^{mi}_s, \delta^i_s)$. Internal stability conditions, assuming that the DICC binds for coalition $s > 2$, that is, prices are given by (2.8) and (2.20), yield

$$\mathcal{I}^N_s(k,\delta) = \frac{k[k(\delta(n+1-\alpha\tau(n-s)) - s\delta^2 - \alpha\tau) - a\delta(1-\alpha\tau)]}{b(1-\alpha\tau)} \tag{2.38}$$

$$\mathcal{I}^H_s(k,\delta) = \frac{k[a\alpha\delta(\beta+\tau) - a\delta + k\delta(n+1-s\delta) - k\alpha(1+\delta(n-s))(\beta+\tau)]}{be} \tag{2.39}$$

$$\mathcal{I}^A_s(k,\delta) = \frac{k[a\alpha\delta(\beta+\tau) - a\delta - k\alpha\tau + k\delta(1+n-s\delta-(n-s)\alpha\tau)]}{be}. \tag{2.40}$$

[34] When firms have to compensate for umbrella losses and coalition $\underline{s}+1 = n$ is internally stable, it is by definition also externally stable.

Taking second derivatives of (2.38)–(2.40) with respect to δ gives

$$\frac{\partial^2 \mathcal{I}_s^N(k,\delta)}{\partial\delta^2} = -\frac{2k^2 s}{b(1-\alpha\tau)} < 0; \qquad \frac{\partial^2 \mathcal{I}_s^H(k,\delta)}{\partial\delta^2} = \frac{\partial^2 \mathcal{I}_s^A(k,\delta)}{\partial\delta^2} = -\frac{2k^2 s}{be} < 0. \tag{2.41}$$

Internal stability conditions are strictly concave in δ. Thus, since equations (2.38)–(2.40) are positive when coalition $s-1$ is not sustainable, that is, when $\delta \in (\delta_s^{mi}, \delta_{s-1}^{mi}]$, it remains to show that $\mathcal{I}_s^t(k,\delta) - \mathcal{I}_s^j(k,\delta) > 0$ to conclude that there are *unique* values $\tilde{\delta}_s^{\mathcal{I}t} > \delta_{s-1}^{mt}$ and $\tilde{\delta}_s^{\mathcal{I}j} > \delta_{s-1}^{mj}$ such that $\tilde{\delta}_s^{\mathcal{I}t} > \tilde{\delta}_s^{\mathcal{I}j}$ with $t \neq j$.[35]

In order to do that we first define

$$\Delta\mathcal{I}_s^1(k,\delta) := \mathcal{I}_s^A(k,\delta) - \mathcal{I}_s^H(k,\delta) = \frac{k^2\alpha\beta(1+n\delta-s\delta)}{be} \tag{2.42}$$

$$\Delta\mathcal{I}_s^2(k,\delta) := \mathcal{I}_s^H(k,\delta) - \mathcal{I}_s^N(k,\delta) = \frac{k^2\alpha\beta(\delta-1)(1-s\delta)}{b(1-\alpha\tau)e} \tag{2.43}$$

$$\Delta\mathcal{I}_s^3(k,\delta) := \mathcal{I}_s^A(k,\delta) - \mathcal{I}_s^N(k,\delta) = \frac{k^2\alpha\beta[-\alpha\tau+\delta(1+n-s\delta-n\alpha\tau+s\alpha\tau)]}{b(1-\alpha\tau)e}. \tag{2.44}$$

$\Delta\mathcal{I}_s^1(k,\delta) > 0$ follows from $n \geq s$, $e > 0$ and $\delta \in (0,1)$. Thus, $\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}H}$.

To ensure that $\Delta\mathcal{I}_s^2(k,\delta)$ is positive, it is additionally needed that $\delta > 1/s$. For $\delta = 1/s$ follows $\mathcal{I}_s^H(k,1/s) - \mathcal{I}_s^N(k,1/s) = 0$. Whenever $\tilde{\delta}_s^{\mathcal{I}H}$ is larger than $1/s$ we can conclude that $\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$. Substituting $\delta = 1/s$ in $\mathcal{I}_s^N(k,\delta)$ yields $\mathcal{I}_s^N(k,1/s) = k(kn-a)/bs$ which is positive by assumption (A1). Thus, the value of $\mathcal{I}_s^N(k,1/s)$ at the intersection of $\mathcal{I}_s^N(k,1/s)$ and $\mathcal{I}_s^H(k,1/s)$ is positive and no $\tilde{\delta}_s^{\mathcal{I}H} \leq 1/s$ can exist. Hence, $\Delta\mathcal{I}_s^2(k,\tilde{\delta}_s^{\mathcal{I}H})$ is positive and $\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$.

Last, we show that $\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}N}$. Since the denominator of $\Delta\mathcal{I}_s^3(k,\delta)$ is positive for $e > 0$, it remains to show that the factor in squared brackets of the numerator in (2.44) is also positive. First, recall that $\delta > \alpha\tau$ is needed to conclude that firms can charge positive prices when all customers have legal standing and the DICC is binding. Thus,

$$-\alpha\tau + \delta(1+n-s\delta-n\alpha\tau+s\alpha\tau) \geq -\alpha\tau + \delta + \delta n(1-\delta) > 0. \tag{2.45}$$

Substituting $s = n$ gives the weak inequality which follows since the bracketed factor on δ is decreasing in the coalition size with $\delta > \alpha\tau$. The strict inequality follows from $\delta > \alpha\tau$ and $1 > \delta$. ■

[35] Another way to prove that $\tilde{\delta}_s^{\mathcal{I}t} > \tilde{\delta}_s^{\mathcal{I}j}$ would be to show that $\mathcal{I}_s^j(k,\tilde{\delta}_s^{\mathcal{I}t}) < 0$ for $\tilde{\delta}_s^{\mathcal{I}t} > \delta_{s-1}^{mt}$. Since $\tilde{\delta}_s^{\mathcal{I}t}$ is not unique, this complicates the proof.

Proof of Lemma 2.4.
If $\tilde{\delta}_s^{\mathcal{I}i} \in (\delta_s^i, \delta_{s-1}^i]$ a firm leaves coalition s when the DICC only binds for coalition $s-1$. Substituting $\pi_s^i(p_s^i) = \pi_s^i(p_s^{Mi})$ and $\pi_{s-1}^{Fi}(p_{s-1}^i) = p_{s-1}^{Di}k$ in equation (INT) yields

$$\mathcal{I}_s^N(k,\delta) = \frac{(1-\alpha\tau)(a-k(n-s))^2}{4bs} - \frac{(a-kn)(1-\alpha\tau)+k(s-1)(\delta-\alpha\tau)}{b(1-\alpha\tau)}\cdot k \tag{2.46}$$

$$\mathcal{I}_s^H(k,\delta) = \frac{e(a-k(n-s))^2}{4bs} - \frac{(a-kn)e+k(s-1)(\delta-\alpha(\beta+\tau))}{be}\cdot k \tag{2.47}$$

$$\mathcal{I}_s^A(k,\delta) = \frac{e(a-k(n-s)/\mu)^2}{4bs} - \frac{\big(ae-kn(1-\alpha\tau)\big)+k(s-1)(\delta-\alpha\tau)}{be}\cdot k. \tag{2.48}$$

Internal stability conditions are therefore linearly decreasing in δ since the collusive value does not depend on the discount factor but the outside profit is linearly increasing in the price, which is linearly increasing in δ. Thus, there is a *unique* discount factor from which internal stability of coalition s is violated.

We start by showing that $\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}H}$. To achieve this, we define $\Delta\mathcal{I}_s^{HA}(k) := \mathcal{I}_s^H(k,\delta) - \mathcal{I}_s^A(k,\delta)$ which is

$$\Delta\mathcal{I}_s^{HA}(k) = \frac{k\alpha\beta\Big[2a(n-s)e - k\Big(\alpha(\beta+2\tau)(2ns-n^2-s^2)+2(n^2+2s-s^2)\Big)\Big]}{4bse}. \tag{2.49}$$

$\Delta\mathcal{I}_s^{HA}(k)$ does not depend on the discount factor. Hence, slopes of $\mathcal{I}_s^H(k,\delta)$ and $\mathcal{I}_s^A(k,\delta)$ in δ are the same, i.e., they are parallel. The difference is negative since the denominator is positive with $e > 0$ and the term in square brackets of the numerator is negative

$$2a(n-s)e - k\Big(\alpha(\beta+2\tau)(2ns-n^2-s^2)+2(n^2+2s-s^2)\Big) \tag{2.50}$$
$$\leq k[2(n-1)(n-s)e - \alpha(\beta+2\tau)(2ns-n^2-s^2) - 2(n^2+2s-s^2)]$$
$$= k[(s^2\alpha\beta+2s^2+2s^2\alpha\tau) - (n^2\alpha\beta+2ns+2ns\alpha\tau) + 2(-n(1-\alpha\beta-\alpha\tau)-s(1+\alpha\beta+\alpha\tau))]$$
$$\leq 2k[-ne - s(1+\alpha\beta+\alpha\tau)] < 0 \tag{2.51}$$

Expression (2.50) is increasing in a. Larger values of a for given k loosen the upper bound of assumption (A1) but tighten the lower one. Inserting the largest a which satisfies assumption (A1)'s lower bound, that is, $a = (n-1)k$, yields the first inequality. The second inequality follows since $(n^2\alpha\beta + 2ns + 2ns\alpha\tau) \geq (s^2\alpha\beta + 2s^2 + 2s^2\alpha\tau)$ for $n \geq s$. The strict inequality is obvious. Since $\mathcal{I}_s^H(k,\delta)$ and $\mathcal{I}_s^A(k,\delta)$ are both linearly decreasing in δ and $\mathcal{I}_s^A(k,\delta) > \mathcal{I}_s^H(k,\delta)$ we can conclude: $\tilde{\delta}_s^{\mathcal{I}A} > \tilde{\delta}_s^{\mathcal{I}H}$.

Next, we show that $\tilde{\delta}_s^{IA} > \tilde{\delta}_s^{IN}$. To achieve this, we first solve $\mathcal{I}_s^N(k,\delta) = 0$ for δ. This yields the unique indifferent discount factor $\tilde{\delta}_s^{IN}$. Then, by substituting $\delta = \tilde{\delta}_s^{IN}$ in $\mathcal{I}_s^A(k,\delta)$, we show that $\mathcal{I}_s^A(k,\tilde{\delta}_s^{IN}) > 0$. It can only be true that $\mathcal{I}_s^A(k,\tilde{\delta}_s^{IN}) > 0$ if $\tilde{\delta}_s^{IA} > \tilde{\delta}_s^{IN}$. Starting with the derivation of $\tilde{\delta}_s^{IN}$ yields

$$\mathcal{I}_s^N(k,\delta) = \frac{(a-k(n-s))^2(1-\alpha\tau)}{4bs} - \frac{k((a-kn)(1-\alpha\tau)+k(s-1)(\delta-\alpha\tau))}{b(1-\alpha\tau)} = 0 \tag{2.52}$$

$$\Leftrightarrow \tilde{\delta}_s^{IN} = \frac{2ak(1-\alpha\tau)(\alpha\tau(n-s)-n-s)-(a^2+k^2n^2)(1-\alpha\tau)^2-k^2s\Big(4\alpha\tau-s(1+\alpha\tau)^2-2n(1-\alpha^2\tau^2)\Big)}{4k^2(s-1)s}.$$

Substituting $\tilde{\delta}_s^{IN}$ for δ in $\mathcal{I}_s^A(k,\delta)$ gives

$$\mathcal{I}_s^A(k,\tilde{\delta}_s^{IN}) = \frac{a\alpha\beta[2(k(n+s)-a)(1-\alpha\tau)+a\alpha\beta]}{4bse}. \tag{2.53}$$

The denominator as well as the numerator are positive with $e > 0$, $a\alpha\beta > 0$, and $k(n+s)-a > 0$, which is ensured by assumption (A1). Hence, $\mathcal{I}_s^A(k,\tilde{\delta}_s^{IN}) > 0$.

Last, we set out the capacity condition under which $\tilde{\delta}_s^{IH} > \tilde{\delta}_s^{IN}$. Substituting $\delta = \tilde{\delta}_s^{IN}$ in $\mathcal{I}_s^H(k,\delta)$ yields

$$\mathcal{I}_s^H(k,\tilde{\delta}_s^{IN}) = \frac{\alpha\beta\Big(a\big(\alpha(\beta+2\tau(1+2k(n-s)))-2(a-2kn)\big)-k^2(\alpha(\beta+2\tau(2ns-n^2-s^2))+2(n^2+2s-s^2))\Big)}{4bse}. \tag{2.54}$$

The sign of the numerator depends on the capacity level k. The interval in which capacities can lie is given by assumption (A1), i.e., $k \in [a/(n-1), a/2)$. Substituting $k = a/(n-x)$ with $x \in [1, n-2)$ in $\mathcal{I}_s^H(k,\tilde{\delta}_s^{IN})$ yields

$$\mathcal{I}_s^H(x,\tilde{\delta}_s^{IN}) = \frac{a^2\alpha\beta\Big[s^2(2+\alpha(\beta+2\tau))-2s(2+x\alpha(\beta+2\tau))-x^2(2-\alpha(\beta+2\tau))\Big]}{4bs(n-x)^2e}. \tag{2.55}$$

The denominator is positive with $e > 0$. The sign of the numerator depends on the sign of the factor in square brackets $[\cdot]$. The lower bound of capacity level k is reached for $x = 1$; inserting $x = 1$ into $[\cdot]$ yields

$$\alpha(\beta+2\tau)(s^2+1-2s)+2(s^2-2s-1) \tag{2.56}$$

A necessary and sufficient condition for (2.56) being positive is $s > 2$. Note that no $\tilde{\delta}_2^{Ii} < 1$ can exist since competitive profits would be zero when a former cartel member leaves a coalition of size 2. Hence, when $x = 1$, it follows that $\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$.

Last, we derive the capacity level that satisfies $\mathcal{I}_s^H(x, \tilde{\delta}_s^{\mathcal{I}N}) = 0$. Solving the bracketed term of the numerator in (2.55) towards x gives

$$x_{s1} = \frac{s\alpha\beta + 2s\alpha\tau + 2\sqrt{s^2 - 2s + s\alpha\beta + 2s\alpha\tau}}{-2 + \alpha\beta + 2\alpha\tau}; \quad x_{s2} = \frac{s\alpha\beta + 2s\alpha\tau - 2\sqrt{s^2 - 2s + s\alpha\beta + 2s\alpha\tau}}{-2 + \alpha\beta + 2\alpha\tau}.$$

Note that x_{s1} is negative since the denominator is smaller than zero with assumption (A2) and the numerator is positive. This contradicts assumption (A1). Since $\mathcal{I}_s^N(k, \delta)$ and $\mathcal{I}_s^H(k, \delta)$ are both linearly decreasing in δ, we can conclude that from $\mathcal{I}_s^H(x, \tilde{\delta}_s^{\mathcal{I}N}) > 0$, which is satisfied for $x \in [1, x_{s2})$, follows that $\tilde{\delta}_s^{\mathcal{I}H} > \tilde{\delta}_s^{\mathcal{I}N}$. When $x \in [x_{s2}, n-2)$ it follows that $\mathcal{I}_s^H(x, \tilde{\delta}_s^{\mathcal{I}N}) \leq 0$ and therefore that $\tilde{\delta}_s^{\mathcal{I}N} \geq \tilde{\delta}_s^{\mathcal{I}H}$. ■

Proof of Proposition 2.4.
Step III.1: Let $\tilde{\delta}_s^{\mathcal{I}i} \geq \delta_{s-1}^i$. When a firm is indifferent between leaving or staying in coalition s at $\delta = \delta_{s-1}^i$ it is also indifferent for all $\delta > \delta_{s-1}^i$ since neither internal nor external stability then depends on δ. Thus, from Proposition 2.3 follows that coalition size $\underline{s}$ or $\underline{s} + 1$ is reached since only these coalitions can be $\mathcal{A}$-stable for sufficiently large discount factors. We can conclude that for a specific capacity level k, only one indifferent discount factor can lie in interval 3: capacity levels which satisfy indifference between coalitions $\underline{s}$ and $\underline{s} + 1$ for $\delta \geq \delta_{\underline{s}}^i$ differ, i.e., $\tilde{k}_{\underline{s}}^A < \tilde{k}_{\underline{s}}^N < \tilde{k}_{\underline{s}}^H$. Thus, with $k = \tilde{k}_{\underline{s}}^A$, $\tilde{\delta}_{\underline{s}+1}^{\mathcal{I}A}$ will exist but coalition $\underline{s}$ is also $\mathcal{A}$-stable when firms do not have to compensate for umbrella losses; $\tilde{\delta}_{\underline{s}+1}^{\mathcal{I}j}$ with $j \in \{H, N\}$ will lie in interval 1 or 2. With $k > \tilde{k}_{\underline{s}}^A$, $\tilde{\delta}_{\underline{s}+1}^{\mathcal{I}A}$ will not exist but coalition $\underline{s}$ may form when outside customers do not have legal standing.

Step III.2: We next prove uniqueness of $\tilde{\delta}_s^{\mathcal{I}i} \in (\delta_s^{mi}, \delta_{s-1}^i)$. Collusive profits of coalition s are continuously increasing when δ lies in interval 1. Also profits of non-cartel members, given that coalition $s - 1$ operates, are strictly increasing in δ as long as coalition $s-1$ is dynamically sustainable. Since the internal stability condition $\mathcal{I}_s^i(k, \delta)$ is strictly concave in the discount factors δ *if* δ lies in interval 1 and positive for $\delta \approx \delta_s^{mi}$, there is a unique discount factor $\tilde{\delta}_s^{\mathcal{I}i}$, which satisfies $\mathcal{I}_s^i(k, \delta) = 0$ for $\tilde{\delta}_s^{\mathcal{I}i} \in (\delta_s^{mi}, \delta_s^i)$ (see proof of Lemma 2.3). Additionally, note that $\mathcal{I}_s^i(k, \delta_s^i) > 0$ when $\tilde{\delta}_s^{\mathcal{I}i}$ does not lie in interval 1. With further increasing discount factors, that is, interval 2 is relevant, a cartel member's collusive profit of coalition s no longer depends on the discount factor, since the DICC is not binding. Outside profits however continuously increase until δ^i_{s-1} (see proof of Lemma 2.4). Thus, starting from $\mathcal{I}^i_s(k, \delta^i_s) > 0$, incentives to leave coalition *s continuously* increase with increasing discount factors. Hence, if $\tilde{\delta}^{\mathcal{I}i}_s \in (\delta^{mi}_s, \delta^i_{s-1})$ exists, it is unique.

Step III.3: In Step *III.1* we already showed that $\tilde{\delta}^{\mathcal{I}j}_s$ with $j \in \{N, H\}$ will lie in interval 1 or 2 when $\tilde{\delta}^{\mathcal{I}A}_s$ lies in interval 3. Next, assume that $\tilde{\delta}^{\mathcal{I}A}_s$ lies in interval 1. From uniqueness of $\tilde{\delta}^{\mathcal{I}i}_s \in (\delta^{mi}_s, \delta^i_{s-1})$ and from $\tilde{\delta}^{\mathcal{I}A}_s > \tilde{\delta}^{\mathcal{I}H}_s > \tilde{\delta}^{\mathcal{I}N}_s$ when $\tilde{\delta}^{\mathcal{I}i}_s$ lies in interval 1 (see Lemma 2.3) follows that $\tilde{\delta}^{\mathcal{I}j}_s$ with $j \in \{N, H\}$ also lies in interval 1.[36] Finally, when $\tilde{\delta}^{\mathcal{I}A}_s$ lies in interval 2, $\tilde{\delta}^{\mathcal{I}j}_s$ with $j \in \{N, H\}$ will exist; additionally, $\tilde{\delta}^{\mathcal{I}j}_s$ is unique (see Step *III.2*). From $\tilde{\delta}^{\mathcal{I}A}_s > \tilde{\delta}^{\mathcal{I}j}_s$ with $j \in \{N, H\}$ when $\tilde{\delta}^{\mathcal{I}A}_s$ lies in interval 2 (see Lemma 2.4) then follows that interval 1 can be relevant for the cases $j \in \{N, H\}$. Since lower bounds of interval 2 are increasing with an extended legal standing (see Step *I*), it follows that $\tilde{\delta}^{\mathcal{I}A}_s > \tilde{\delta}^{\mathcal{I}j}_s$ with $j \in \{N, H\}$. ■

[36] $\tilde{\delta}^{\mathcal{I}j}_s$ with $j \in \{H, N\}$ exists when $\tilde{\delta}^{IA}_s$ exists since the coalition size of an $\mathcal{A}$-stable cartel for $\delta \geq \delta^i_{\underline{s}}$ is weakly larger when firms have to compensate for umbrella losses compared to the case where they do not have to compensate these losses (see Prop. 2.1 and 2.3).

Chapter 3

Shapley Apportioning of Cartel Damages by Relative Responsibility

Cartels are illegal because they generally harm customers and suppliers of the involved firms and possibly others. Victims have for long had a right to compensation but the pertinent legal hurdles used to be high. Annually up to 23.3 billion euro of damages have remained unclaimed from EU-wide cartels according to the European Commission (SWD/2013/203/Final, recital 67). This was a main reason in 2014 for the European Commission to revise rules and establish a *Directive on Antitrust Damages Actions* (2014/104/EU). The position of plaintiffs has since improved and some big cases are pending – e.g., against the air cargo, elevator, or truck cartels.

Two provisions for the compensation of cartel victims in the Directive motivate this chapter. First, the members of a cartel are liable *jointly and severally*. An injured party can sue *any* cartel member for the full amount of its damages; if courts confirm the claim, the defendant must compensate the plaintiff on behalf of the entire cartel. This is regardless of whether the plaintiff made its purchases from the sued firm or other ones. Similar provisions apply in Australia, Japan, and the US.

Second, the sued cartel member is entitled to *internal redress*. Such a rule of contribution existed in the EU before but details differed across member states. It contrasts with the no contribution rule in federal US antitrust cases and somewhat opaque rules in Australia and Japan.

The goal of this chapter is to operationalize the contribution norm established by the European Union in its Directive 2014/104/EU in an economically sensible way.[1]

[1] The issue of how alternative norms, such as the no contribution rule in the US, affect incentives for cartel formation, whistleblowing, settlements, etc. is here left aside. See, for instance, Landes and Posner (1980), Polinsky and Shavell (1981) or Hviid and Medvedev (2010).

We focus on the question: how are damages of a cartel to be apportioned among its potentially heterogeneous members? According to the Directive, cartelists' internal obligations in compensating any external claimant must reflect *"... their relative responsibility for the harm caused by the infringement of competition law"* (Article 11(5)). The Directive is not specific on how "relative responsibility" should be quantified but leaves doors open by stating that "... determination of that share [of compensation] as the relative responsibility of a given infringer, and the relevant criteria such as turnover, market share, or role in the cartel, is a matter for the applicable national law, while respecting the principles of effectiveness and equivalence" (recital 37).

This chapter focuses on the economic quantification of relative responsibility but this is based on the canonical causal conception of legal and moral responsibility for damages. See Feinberg (1970, p. 195f) for a classical discussion of its three parts: firstly, the defendant was at fault in acting. This clearly applies if, for instance, firm i's manager illegally coordinated its commodity production with competitors over dinner, violating antitrust laws. Secondly, the faulty act caused the harm: these conversations resulted in a price increase for the customer. And, finally, the faulty aspects of the act were relevant to its causal connection to the harm: illegal coordination by the managers – not, perhaps, just the reaction of commodity speculators to observing them dine together – caused the increase. All three parts call for appropriate verification in practical applications.

If legal responsibility of the infringers for compensation has been affirmed, a systematic approach is warranted to determine individual contributions. A key reason is that asymmetry of cartel members can translate very differently into asymmetric turnover, market shares, etc. Picking one ad hoc criterion to determine contributions rather than another involves a high degree of arbitrariness and contrasts with Article 11's explicit reference to relative responsibility.[2] One can do better.

In particular, one can specify several properties that contributions ought to satisfy: first, in order to account for the Directive's responsibility criterion, a firm should contribute to compensating a given customer only if this customer's damages would have been lower had the firm refused to participate in the cartel. Second, contribution levels ought to be determined by how much lower the respective damage would have been if the firm (and possibly some others) had stayed legal. If cartel membership of two firms had identical effects on harm then, third, both should contribute the same

[2]Complexity of the issue and disagreement on ad hoc criteria are explicitly cited in the US Supreme Court's no contribution ruling (cf. Texas Industries, Inc. v. Radcliff Materials, Inc., 451 U.S. 637–38, 1981).

to its remedy. Finally, a victim's full compensation should be apportioned in a way that neither depends on the unit of account nor on whether multiple damages are dealt with separately or jointly.

These properties translate into mathematical conditions that are well-known in cooperative game theory – as null player, marginality, symmetry, efficiency and linearity axioms. Classical results by Shapley (1953b) and Young (1985) imply that the *Shapley value* of an appropriately defined game is the right way to split external compensation obligations internally among the offenders.

The Shapley value is typically used for allocating costs and profits in joint ventures.[3] The corresponding rationale extends to joint liabilities. This has recently been taken up by Dehez and Ferey (2013, 2016) and Huettner and Karos (2017) for *sequential liability games*. These games reflect incremental harm caused by chronologically ordered acts of negligence; they differ from joint liability by cartels in that they always have a non-empty core. Use of the Shapley value for the allocation of cartel damages has been proposed by Schwalbe (2013) and Napel and Oldehaver (2015) to law audiences. This chapter is the first to analyze its quantitative aspects and how firms' internal contributions relate to industry parameters.

Shapley contributions proportion overcharges according to individual abilities of the participating firms to influence prices. We analyze how the resulting damage shares are linked to demand and costs in linear market environments. We derive bounds on a firm's differing responsibility for its own overcharges and those of other cartel members. We also compare Shapley apportionments to ad hoc ones based on market shares or profits. Such divisions have been suggested by law practitioners but tend to be at odds with relative responsibility.

We start our analysis with a more detailed description of the damage apportionment problem (Section 3.1). We present the Shapley value in Section 3.2 along with a useful way to calculate it in the given context. Section 3.3 specializes the analysis to linear market environments. For these, heuristic apportionment rules are compared to the Shapley benchmark. Section 3.4 discusses leniency and ringleader provisions.

3.1 Cartel Damages and Relative Responsibility

Antitrust victims have a right to be compensated. This means they should be put in the position that they would have been in without the infringement. Cartel

[3]See Moretti and Patrone (2008) for an overview.

customers usually suffer two types of damage. The first is the visible loss due to higher prices (*damnum emergens*): each unit that was purchased involved an *overcharge damage*. Further harm relates to deadweight losses: customers who would have made (additional) purchases, and thus would have enjoyed surplus had prices been competitive, failed to do so. This is acknowledged as *lucrum cessans* in the legal literature but has played little role in practice yet.

We concentrate on overcharge damages and will assume that they were caused by a hardcore cartel that fixed quantities, sales areas, or prices of differentiated goods. This leaves aside other kinds of infringements, deadweight losses and general equilibrium effects (see Eger and Weise 2015). The suggested approach can be generalized, however. Changes in prices could, e.g., be replaced by lost downstream profits or indirect utility of consumers. The scale invariance condition that will be introduced below allows to deal naturally with the payment of interest, which is an essential part of compensation (cf. Directive 2014/104/EU, recital 12).

Cartel overcharges are often determined on the basis of a monthly but-for estimation because they can change over time (see, e.g., Bernheim 2002, 2008). Our presentation adopts a static perspective but, with sufficient data, it would be desirable to apportion damages based on monthly estimations, too. A firm's relative responsibility may have varied as other firms joined or left the cartel as well as when demand or costs changed the scope to influence prices.

A cartel member i having responsibility for damages of a given claimant k requires that k's damages are causally linked to i's cartel membership, i.e., their scale, scope or distribution would have differed without i's illegal action. Identifying the causal links between anticompetitive conduct and harm is generally fraught with difficulty (see, e.g., Lianos 2015). What makes economic analysis of responsibility for cartel damages particularly interesting, however, is that even symmetric cost and demand structures may generate asymmetric links to harm suffered by a specific victim. Namely, price effects of individual cartel membership in a but-for test differ across cartelists as long as own-price and cross-price elasticities of the respective demands differ.

As an example, consider n otherwise identical firms on a Salop circle. Think of cement plants that are equally spaced on the shores of an unshippable lake. They sold their cement at inflated prices to local construction companies around the lake. Their cartel was busted and a customer of firm i sues. Firm i's and another firm j's relative responsibilities for this customer's damages are tied to the counterfactual price that the customer would have paid had i or respectively j refused to participate.

Unless transportation costs are zero, and thus all products perfect substitutes, cartel membership of the northernmost vendor has smaller effect on overcharges faced by customers in the south than does membership of southern vendors, and vice versa (see, e.g., Levy and Reitzes 1992). The closer two firms are located and hence the more intensely they would have competed in the absence of the cartel, the greater the price effect of their collusion.

So counterfactual prices that the suing customer would have paid if i or if j had not joined the cartel, but just best-responded to its practices, vary according to i's and j's locations. Differential effects of cartel membership imply differential responsibilities for a specific customer's damage; hence different obligations for compensation. Formally, i and j need not be symmetric players with respect to individual damages even though they have symmetric roles in the market at large.

Of course, a symmetric market structure implies that obligations which i and j have in compensating each others' customers are the same. Mutual claims cancel out if all constructors sue, or if equal measures of them do everywhere. However, they do *not* cancel in almost all other situations – e.g., if just some construction companies in the south go to court. A general analysis hence requires that responsibility be allocated to the cartel members for the price overcharge on each single product in the cartel portfolio. Asymmetric market structures make a focus on individual products even more important.

A sound procedure for apportioning damages matters also for umbrella losses.[4] These arise to victims that purchased from cartel outsiders at prices whose elevated level derived from the infringement. Umbrella losses can be reclaimed from cartels in the EU (CJEU C-557/12 2014). Since compensation is not linked to transactions with a cartelist, apportionment based on one of the typically conflicting notions of market shares – by sales, revenues or profits; in the cartel period, before, thereafter – would be even more ad hoc than for damages to cartel customers.

Finally, note that apportionment is an issue also if litigants settle. In the EU, an injured party's claim after settling with a co-defendant "…should be reduced by the settling infringer's share of the harm caused to it, regardless of whether the amount of the settlement equals or is different from the relative share of the harm that the settling co-infringer inflicted upon the settling injured party. That relative share should be determined in accordance with the rules otherwise used to determine

[4] See Inderst et al. (2014) for a general discussion on umbrella losses and Holler and Schinkel (2017) for a correction. In Chapter 2 we discuss how a compensation for umbrella losses affects the market price.

the contributions among infringers" (Directive 2014/104/EU, recital 51). The settling defendant's relative responsibility thus affects litigants and all co-infringers.

3.2 The Shapley Value as a Tool for Apportioning Damages

We propose several conditions that a rule for apportioning damages in line with relative responsibilities should satisfy. It turns out that all are verified by the Shapley value while any other proportioning suggestion would violate at least one.

3.2.1 Preliminaries

In order to formalize sensitivity to individual responsibility and other desirable properties of an apportioning rule, we adopt some terminology from the theory of TU games. The latter describe situations in which *transferable utility* (TU), such as a surplus or cost, is to be divided among *players* from a given set $N = \{1, \dots, n\}$. In our context, the players are the firms involved and N is the detected cartel.[5]

For every subset or *coalition* $S \subseteq N$ of players who might cooperate with one another, a real number $v(S)$ generally captures the positive or negative *worth* which cooperation by S creates and which may be shared arbitrarily. In our context, $v(S)$ describes damage inflicted if firms $i \in S$ coordinate their actions while firms $j \in N \setminus S$ maximize their respective profits in competitive fashion. Mapping $v\colon 2^N \to \mathbb{R}$ is known as the *characteristic function* of *TU game* (N, v).

For strict subsets of N, $v(S)$ reflects a counterfactual. This is necessary: responsibility is driven by the fact that overcharges would have differed from the observed damage $v(N)$ if conducts had differed, i.e., if some firms had stayed out. Directive 2014/104/EU explicitly acknowledges a role for counterfactual scenarios: "... quantifying harm means assessing how the market in question would have evolved had there been no infringement. This assessment implies a comparison with a situation which is by definition hypothetical ..." (recital 46). Defining $v(S)$ for every set $S \subseteq N$ extends this logic from *quantifying harm* to *quantifying contributions to harm*.

[5]The relevant market may comprise firms $j \notin N$ which did not partake in the cartel. They need not contribute to compensations and matter as exogenous co-determinants of damage rather than players.

Naturally, $v(S) = 0$ if the set S of collaborators is either empty ($S = \emptyset$) or comprises but a single firm, i.e., if $\#S = 1$. For other coalitions $S \subset N$, an estimate $v(S)$ is needed. Intertemporal variation in cartel participation may help obtaining it but simulation analysis is likely to be the best option. Market simulation is rather well-established in merger control.[6] There, parameters of a structural model of price or quantity competition are estimated based on pre-merger observables; these generate equilibrium predictions for when a subset of firms merge and internalize mutual externalities, just as cartel members do. Analogous analysis of cartel behavior is comparatively rare and its use for the estimation of function v is more tedious: many scenarios rather than just a single proposed merger need to be evaluated. The model's calibration could, however, draw not only on pre-cartel (like pre-merger) observables but also observations during and after the cartel's operation. Former members may have an interest to disclose cost information if they expect lower contributions than under an ad hoc apportionment. Sensitive cost or demand data could be pooled by a trusted intermediary (auditing or law firms) in order to reach a settlement on apportionment.

We take no stance here on how sophisticated estimates $v(S)$ ought to be in practice. For instance, the analysis of a hypothetical scenario with a sub-cartel $S \neq N$ may consider the question of whether S satisfies suitable stability conditions, and put $v(S) = 0$ if not. The illustrations below will keep things simple. Just note that each number $v(S)$ with $i \notin S$ reflects a scenario for how the market might have evolved if there had been no infringement *by firm i*. It is both possible that firm $j \neq i$ would then have joined the cartel anyhow ($j \in S$) or that it would have stayed legal too ($j \notin S$). These scenarios need not have equal probability. But all partial cartels $S \subseteq N \setminus \{i\}$ are, in principle, relevant in assessing i's contribution to the situation which calls for compensation, hence i's relative responsibility.[7]

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[6] COMP/M.5644–Kraft Foods/Cadbury or COMP/M.5658–Unilever/Sara Lee are key cases in Europe; prominent US cases include Dept. of Justice, Final Judgement: U.S. v. Kimberly-Clark Corp. and Scott Paper Co., 1995. See Budzinski and Ruhmer (2010) for a survey, Weinberg (2011) or Knittel and Metaxoglou (2011) for critical discussions.

[7] It is also conceivable that several partial cartels would have formed if i had refused to join. This could be accommodated by considering (extensions of the Shapley value to) *partition functions* V from the set of partitions $\mathcal{P} = \{P_1, \ldots, P_r\}$ of N (satisfying $\bigcup_l P_l = N$ and $P_l \cap P_k = \emptyset$ for any $l, k \in \{1, \ldots, r\}$) to estimated damages $V(\mathcal{P})$ instead of characteristic function v. See Ray and Vohra (1999).

3.2.2 Desirable Properties of Responsibility-Based Allocations

With damages in the factual cartel scenario and related counterfactuals described by (N, v), a *damage apportioning rule* is a mapping Φ from any conceivable cartel damage problem (N, v) to a vector $\Phi(N, v) \in \mathbb{R}^n$. Such a mapping is referred to as a *value* in the context of general TU games. The main restriction that the cartel context imposes is that $v(\{i\}) = 0$ for all $i \in N$. As prices of substitute goods are usually higher for bigger cartels (see Davidson and Deneckere 1984, Deneckere and Davidson 1985), we can take v to be monotonic in S but it will generally not be convex nor superadditive.[8] The i-th component $\Phi_i(N, v)$ denotes the part of the compensation for damages $v(N)$ which cartel member $i \in N$ must contribute.

That an apportioning rule reflects relative responsibilities can be translated into several formal properties of a rule Φ. The first one is straightforward. Suppose that participation or not of a particular firm i would never have made a difference to the damage in question, i.e., removing player i if $i \in S$ or adding player i if $i \notin S$ does not change $v(S)$. If i's conduct has no effect on damage the conditions are not met for i being responsible (see Feinberg 1970). Hence, no responsibility-based obligations to contribute follow. Technically, a player i for whom $v(S) = v(S \setminus \{i\})$ for every $S \subseteq N$ is known as a *null player*. The first requirement for rule Φ to be based on relative responsibility hence is the so-called *null player property*:

$$\Phi_i(N, v) = 0 \text{ whenever } i \text{ is a null player in } (N, v). \qquad \text{(NUL)}$$

Presumably, the supply and demand conditions in real markets are rarely compatible with a convicted cartel member being a null player, but (NUL) conducts a valid thought experiment. It also formalizes a certain robustness to misspecification of the relevant market. For instance, a large cartel may have caused damage in several regions with independent costs and demand. If a firm is accidentally included as 'player' in a region where it had no role, (NUL) ensures it need not contribute there.

As responsibility derives from the causal link between cartel membership and the suffered harm, another straightforward requirement is that i's damage share $\Phi_i(N, v)$

[8] Convexity and therefore also superadditivity are however natural assumptions when cartel formation is considered. To ensure that (i) existing coalition members have an incentive to accept a new cartel member and (ii) a new cartel member simultaneously has an incentive to join a coalition, games have to be superadditive. That most illegal agreements cover a huge share of the market suggests that incentives for joining a coalition increase as the coalition grows, i.e., convexity. In particular, many cartels, as the truck cartel, consist of all big players.

should be determined by this link – and this link alone. Namely, presuming that v correctly describes factual damages as well as the relevant counterfactuals, $\Phi_i(N, v)$ shall be a function *only* of the differences $v(S) - v(S \setminus \{i\})$ that i makes to $S \subseteq N$. These differences are also called i's *marginal contributions* in (N, v). The corresponding formal property of *marginality*, introduced by Young (1985), demands that i's shares in two apportioning problems (N, v) and (N, v') ought to coincide whenever i's marginal contributions do:[9]

$$\Phi_i(N, v) = \Phi_i(N, v') \text{ whenever } v(S) - v(S \setminus \{i\}) = v'(S) - v'(S \setminus \{i\}) \text{ holds for all } S \subseteq N. \tag{MRG}$$

Marginality does not pin down *how* $\Phi_i(N, v)$ should depend on the differences that i makes to various coalition S. For instance, imposing (MRG) does not imply (NUL); the properties formalize different aspects of Φ reflecting firms' responsibilities.

A third such property refers to situations in which the roles of firms i and j in determining damages $v(S)$ are perfectly symmetric to another. Formally, players i and j are called *symmetric* if $v(S \cup \{i\}) = v(S \cup \{j\})$ for every coalition $S \subseteq N \setminus \{i, j\}$. When adding i to a sub-cartel S has the same damage implications as adding j whenever S previously contained neither, their responsibilities are the same. So Φ should also satisfy *symmetry*:

$$\Phi_i(N, v) = \Phi_j(N, v) \text{ whenever } i \text{ and } j \text{ are symmetric in } (N, v). \tag{SYM}$$

Irrespective of whether a damage apportionment reflects responsibility of the involved players or alternative normative criteria, it is desirable that individual contributions of all firms $i \in N$ add up to $v(N)$. In the context of TU games, this condition is referred to as *efficiency* of a value:

$$\sum_{i \in N} \Phi_i(N, v) = v(N). \tag{EFF}$$

Symmetry and efficiency imply that both participants in any 2-firm cartel must contribute $v(N)/2$. Responsibilities are equal even if the firms are asymmetric in size, costs, etc. because exit by either would have restored competition.

Scale invariance is another natural requirement: firms' shares should not depend

[9] Note that *criminal sanctions* follow different principles than civil law obligations to victims or co-offenders. We are concerned only with the latter. The former seek to punish and deter; they may well differ for (N, v) and (N, v') even if the respective marginal contributions are identical.

on whether damages are expressed in US dollar or euro, nor on whether they already include interest payments. So multiplying all numbers $v(S)$ by an exchange rate or interest factor $\lambda > 0$ should re-scale firms' contributions by the same factor. Moreover, if the same cartel N caused damages to suing customers in several markets – reflected by a characteristic function v^1 for market 1, by v^2 for market 2, etc. – then the total contribution of firm $i \in N$ should not depend on whether the apportioning rule is applied to damages v^l in one market l at a time, or in one go to the total $v = v^1 + v^2 + \ldots$ Different 'markets' could here refer to different plaintiffs or subsidiaries of the same plaintiff, to different products in the cartel's portfolio, or distinct quantities of the same product.[10] Additivity combines with scale invariance to the *linearity* condition:

$$\Phi(N, \lambda \cdot v + \lambda' \cdot v') = \lambda \cdot \Phi(N, v) + \lambda' \cdot \Phi(N, v') \qquad \text{(LIN)}$$

for any scalars $\lambda, \lambda' \in \mathbb{R}$ and any characteristic functions v, v'.

3.2.3 Shapley Value and Decomposition by Average Damage Increments

The above properties are more than is needed in order to conclude that the apportioning rule should have a particular form:

Shapley-Young Theorem *The following statements about a damage apportioning rule Φ are equivalent:*

(I) Φ *satisfies (NUL), (SYM), (EFF) and (LIN).*

(II) Φ *satisfies (MRG), (SYM) and (EFF).*

(III)

$$\Phi_i(N, v) = \varphi_i(N, v) := \sum_{S \subseteq N} \frac{(s-1)!(n-s)!}{n!} \cdot \left[v(S) - v(S \setminus \{i\}) \right] \qquad (3.1)$$

where $s = \#S$ denotes the cardinality of coalition S.

$\varphi(N, v)$ is known as the *Shapley value* of (N, v). Equivalence of (I) and (III) was established by Shapley (1953b); equivalence of (II) and (III) by Young (1985). See, e.g., Maschler et al. (2013, ch. 18) for an excellent exhibition.[11] Even though formula (3.1)

[10] Additivity applies also to different *types* of damages described by v and v' – for instance, overcharge damages and deadweight losses.

[11] Shapley's proof indeed extends to the class of damage apportionment problems: cartels in which $i \in T \subseteq N$ produce perfect substitutes with competitive price $p^* = 0$ and cartel price $p^C = 1$ while $j \notin T$ operate in unrelated markets define the required carrier games (N, u_T).

may look unwieldy, the weight $(s-1)!(n-s)!/n!$ on i's marginal contribution to a given coalition S is the logical consequence of the desirable properties listed above.

An equivalent way of writing eq. (3.1) is

$$\varphi_i(N,v) = \frac{v(N)}{n} + \frac{1}{n}\sum_{s=1}^{n-1}\left[\bar{v}^{i}(s) - \bar{v}^{\not i}(s)\right]. \tag{3.2}$$

Here

$$\bar{v}^{i}(s) := \binom{n-1}{s-1}^{-1} \sum_{\substack{S\ni i\\ \#S=s}} v(S) \quad \text{and} \quad \bar{v}^{\not i}(s) := \binom{n-1}{s}^{-1} \sum_{\substack{S\not\ni i\\ \#S=s}} v(S) \tag{3.3}$$

denote the *average damages* caused by coalitions of size s which *include* firm i and which *exclude* firm i, respectively. Abbreviating $\kappa(s) := (s-1)!(n-s)!/n! = \frac{1}{n}\cdot\binom{n-1}{s-1}^{-1}$, this follows from

$$\begin{aligned}\varphi_i(N,v) &= \sum_{S\subseteq N}\kappa(s)\cdot\left[v(S)-v(S\setminus\{i\})\right] = \sum_{\substack{S\subseteq N\\ S\ni i}}\kappa(s)v(S) - \sum_{\substack{S\subseteq N\\ S\not\ni i}}\kappa(s+1)v(S) \\ &= \kappa(n)v(N) + \sum_{s=1}^{n-1}\left[\sum_{\substack{S\ni i\\ \#S=s}}\kappa(s)v(S) - \sum_{\substack{S\not\ni i\\ \#S=s}}\kappa(s+1)v(S)\right] = \frac{v(N)}{n} + \frac{1}{n}\sum_{s=1}^{n-1}\left[\bar{v}^{i}(s)-\bar{v}^{\not i}(s)\right].\end{aligned} \tag{3.4}$$

Equation (3.4) can be simplified further because any degenerate 'cartel' of size $s = 1$ leaves prices constant, i.e., $\bar{v}^{i}(1) = \bar{v}^{\not i}(1) = 0$ for each $i \in N$.

In summary, we must use the *Shapley apportioning rule*

$$\varphi_i(N,v) = \frac{v(N)}{n} + \frac{1}{n}\sum_{s=2}^{n-1}\left[\bar{v}^{i}(s) - \bar{v}^{\not i}(s)\right] \tag{3.5}$$

if we deem properties (LIN), (EFF), (SYM), (NUL) and (MRG) desirable. Apportionment by relative responsibility of the infringers thus means: start out with *equal shares per head*; then add an n-th of the *average size-specific damage increments* that arise due to a given firm i's participation.

The addition accounts for asymmetric effects on harm that arise even in symmetric market environments (cf. Section 3.1). The decomposition in eq. (3.5) provides a useful perspective on φ_i and can simplify its calculation: symmetries among cartel members reduce the sum of 2^n differences in eq. (3.1) to less than n ones in (3.5). This extends when asymmetries are such that i's damage increments for specific coalition sizes s can be written as a function of 'aggregate asymmetry' among the other firms (see Subsection 3.3.3). The calculation also simplifies if cartels of size s below some threshold $\bar{s}$ are unstable (Bos and Harrington 2010) or if exchangeability of the firms implies all damage increments $\bar{v}^i(s) - \bar{v}^{k}(s)$ are zero. For instance, the second summand in (3.5) vanishes in undifferentiated Bertrand or Cournot oligopoly with symmetric firms or whenever $n = 2$; then equal shares follow.

3.3 Apportionment in Linear Market Environments

Shapley apportioning requires estimates of counterfactual damages for all conceivable partial cartels. This amplifies the analytical and empirical challenges associated already with estimating a litigant's damages. Four main approaches to quantifying a customer's harm are discussed in the literature (see, e.g., Inderst et al. 2013 or European Commission 2013b): the pure forecasting or cartel dummy variable versions of the regression approach, determination of but-for prices by default cost mark-ups, financial performance comparisons, and simulation analysis of a structural market model. The latter has been applied to cartel cases relatively rarely (see, e.g., Roos 2006) but is well-established in merger control. It gives rather straightforward estimates of but-for prices for partial cartels and thus counterfactual damages.

We illustrate this here for situations in which the costs and demand for differentiated goods are described, in acceptably good approximation, by linear functions. Parameter restrictions in analogy to, e.g., the proportionality condition of Epstein and Rubinfeld (2001) could reduce the data requirements in practical cases sufficiently to be applicable. If the producers of differentiated products face at most one kind of asymmetry, closed-form expressions for the Shapley shares can be derived via eq. (3.5).[12] This is often impossible in other applications of the Shapley value. The parametric solutions allow to derive distinct upper and lower bounds on the responsibility-based contribution by a firm to harm of its own and of other firms' customers, respectively. It is possible to compare Shapley apportionments to those implied by heuristic rules with numerical methods and, hence, to assess the degrees to which, e.g., cartel-period market shares could be relied on as proxies for relative responsibility in applications.

[12] Quadratic costs do not change the findings much: eq. (3.16) below then involves cost parameter γ but remains independent of a. Later expressions get significantly more unwieldy, however.

3.3.1 Model

We focus on a cartel by $n \geq 3$ suppliers where each firm $i \in N = \{1, \ldots, n\}$ produces a single good.[13] Firm i's costs are given by

$$C_i(q_i) = \gamma_i q_i \text{ for } \gamma_i \geq 0. \tag{3.6}$$

Demand at price vector $p = (p_1, \ldots, p_n)$ is described by

$$D_i(p) = a_i - d_i \cdot p_i + \sum_{j \in N \setminus \{i\}} b_{ij} \cdot p_j \;\text{ for } a_i > \gamma_i,\; d_i > 0, \text{ and } b_{ij} > 0 \text{ for all } j \neq i. \tag{3.7}$$

We presume $D_i(\gamma) > 0$, i.e., demand is positive when all firms price at cost. The corresponding parameter restriction is $a + (n-1)b\gamma > d\gamma$ in the symmetric case where $\gamma_i = \gamma$, $a_i = a$, $d_i = d$ and $b_{ij} = b$ for all $i \neq j \in N$. Firms set prices simultaneously à la Bertrand. If some group $S \subseteq N$ of them forms a cartel, outsiders $j \notin S$ best-respond to the anticipated decisions of insiders.

Members of $S \subseteq N$ maximize the sum of their profits

$$\Pi_S(p) = \sum_{i \in S} (p_i - \gamma_i) D_i(p) \tag{3.8}$$

with corresponding first-order conditions

$$\frac{\partial \Pi_S(p)}{\partial p_j} = D_j(p) + \sum_{i \in S} (p_i - \gamma_i) \frac{\partial D_i(p)}{\partial p_j} \text{ for all } j \in S. \tag{3.9}$$

Analogous expressions hold if j is a cartel outsider. It is sufficient for existence and uniqueness of a Nash equilibrium that a uniform increase of all prices or a unilateral increase of any single price decreases the industry's aggregated demand.[14] Formally, this requires $\sum_{j=1}^{n} \partial D_i / \partial p_j < 0$ and $\sum_{j=1}^{n} \partial D_j / \partial p_i < 0$, i.e., we will assume

$$\alpha_i := d_i / \sum_{j \neq i} b_{ij} > 1 \text{ and } d_i > \sum_{j \neq i} b_{ji} \text{ for all } i \in N. \tag{3.10}$$

---

[13] If we have a *multi-product* firm and all its prices are determined either competitively or cooperatively then we would simply consider overcharges $\Delta p_l = p_l^C - p_l^B$, $l \in P$, for a set of products P which no longer coincides with the set of players N. If, in contrast, the conduct decision is made autonomously for each l by distinct departments of the firm, then each should be included as a player in N.

[14] See Vives (1999, Sec. 6.2) and Federgruen and Pierson (2011, Cor. 4.6).

In the symmetric case, this simplifies to $\alpha := \frac{d}{(n-1)b} > 1$.

Products are relatively good substitutes when α_i is small; then price increases by one firm significantly raise profits for other firms. The cartel internalizes this externality; the price p_i set by cartel member i will be the higher, the smaller α_i.

For any given $S \subseteq N$, the (unique) Nash equilibrium p^S summarizes equilibrium prices p_i^S of all products $i \in N$ assuming firms in S coordinate and the remaining ones act competitively. See, e.g., Davis and Garcés (2009, ch. 8).

3.3.2 Symmetric Case

In the symmetric case the cartel price evaluates to

$$p^C := p_i^N = \left(\frac{a}{d-(n-1)b} + \gamma\right)\Big/2 \tag{3.11}$$

for each differentiated product $i \in N$.[15] Corresponding competitive Bertrand prices are

$$p^B := p_i^\varnothing = \frac{a + d\gamma}{2d-(n-1)b} \text{ for all } i \in N. \tag{3.12}$$

This implies cartel overcharges of

$$\Delta p = p^C - p^B = \frac{a/d - \gamma(1-\frac{1}{\alpha})}{4\alpha - 6 + 2/\alpha} \quad \text{with} \quad \alpha = \frac{d}{(n-1)b} > 1 \tag{3.13}$$

for each product $i \in N$. They are homogeneous of degree one in (a, γ) and strictly decreasing in differentiation parameter α as well as in unit costs γ. Overcharges vanish if demand becomes independent, i.e., $\lim_{\alpha\to\infty} \Delta p = 0$.

If there is a partial cartel S of size $s = 2, \dots, n-1$, equilibrium prices are

$$p_i^S = \begin{cases} \dfrac{a(2d+b) + \gamma\Big(2d^2 + bd(3-2s) + b^2(ns - n - s^2 + 1)\Big)}{4d^2 - 2(n+s-3)bd + b^2\eta_s} & \text{if } i \in S, \\[2ex] \dfrac{a(2d - sb + 2b) + \gamma\Big(2d^2 - bd(s-2) - b^2(s^2 - s)\Big)}{4d^2 - 2(n+s-3)bd + b^2\eta_s} & \text{if } i \notin S \end{cases} \tag{3.14}$$

with $\eta_s = s(n-s) - 2(n-1) \geq -(n-1)$.[16] Comparing the price p_h^S of the "home" product $h \in N$ paid by a suing customer in case that the respective producer h is part

[15] All calculations underlying Section 3.3 are in Appendix B.

[16] Static stability of the industry-wide cartel requires that the degree of differentiation is not too

of a cartel with s members, i.e., for $h \in S$, to the respective price p_h^S if h is not, i.e., for $h \notin S$, yields[17]

$$\bar{v}^h(s) - \bar{v}^{\not h}(s) = p^h(s) - p^{\not h}(s) = \frac{b(s-1)\Big(a + (n-1)b\gamma - d\gamma\Big)}{4d^2 - 2(n+s-3)bd + b^2\eta_s} > 0. \tag{3.15}$$

Inserting this into eq. (3.5) gives the Shapley allocation in absolute terms. Division by $v(N) = \Delta p$ yields h's share as an explicit function of the model's parameters:

$$\rho_h^* := \frac{\varphi_h(N,v)}{v(N)} = \frac{1}{n} + \frac{n-1}{n} \sum_{s=2}^{n-1} \frac{(s-1)\cdot(4\alpha^2 - 6\alpha + 2)}{4\alpha^2(n-1)^2 - 2(n+s-3)(n-1)\alpha + \eta_s}. \tag{3.16}$$

One can see that, in the symmetric case, the common unit cost γ and demand intercept a have no effect on h's share. It is determined only by the degree of differentiation, i.e., ratio $\alpha = d/(n-1)b$ of own and cross-price parameters. We can deduce $\rho_h^* > 1/n$ for any $\alpha > 1$ directly from $\bar{v}^h(s) - \bar{v}^{\not h}(s) > 0$ and eq. (3.5).

If the degree of differentiation α is low, discipline by all cartel members is especially important for maintaining an overcharge. In the limit, each firm's participation is essential and affects damage equally:

$$\lim_{\alpha\to 1} \rho_h^* = \frac{1}{n} \quad \text{and} \quad \lim_{\alpha\to 1} \rho_j^* = \frac{1}{n} \text{ for } j \neq h. \tag{3.17}$$

This generalizes to non-linear settings: for perfect substitutes and identical technology each firm has identical influence on the price and, by (SYM), must contribute the same.

If, in contrast, firms produce highly differentiated goods, eq. (3.16) yields

$$\lim_{\alpha\to\infty} \rho_h^* = \frac{1}{n} + \frac{1}{n(n-1)} \sum_{s=2}^{n-1}(s-1) = \frac{1}{2}. \tag{3.18}$$

So seller h must cover up to half of the compensation for its overcharges.[18] Checking

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large for $n > 3$. This is no concern, however, in the derivation of bounds on contributions. See Deneckere and Davidson (1985), Weikard (2009) and Federgruen and Pierson (2011) on cartel profits under price competition and their relation to internal vs. external stability.

[17] The three factors in the numerator are strictly positive. Invoking $s \leq n-1$ and $\eta_s \geq -(n-1)$ first, and $d > (n-1)b$ next, the denominator can be bounded below by $2d[2d - 2(n-2)b] - b^2(n-1) > 2d[2(n-1)b - 2(n-2)b] - bd = 3bd > 0$. Hence $p^h(s) - p^{\not h}(s) > 0$.

[18] Recall however that Δp vanishes as $\alpha \to \infty$.

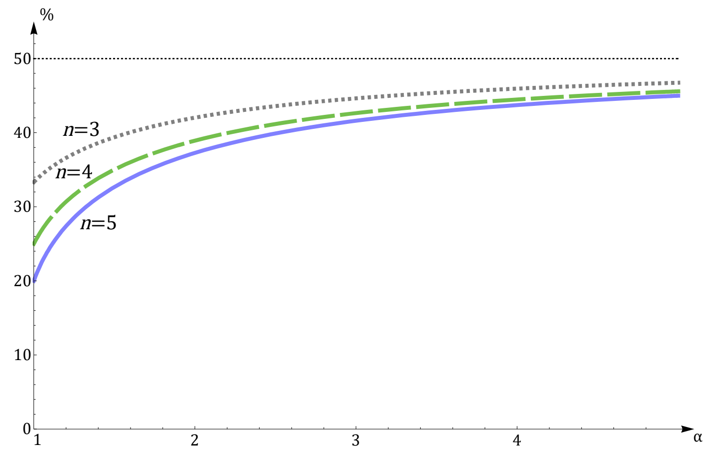

Figure 3.1 Share ρ_h^* of overcharge damages on product h attributed to its vendor ($a = 10, d = 2, \gamma = 0, b = 2/(n-1)\alpha$)

that ρ_h^* is strictly increasing in α yields:

Proposition 3.1. *Suppose $n \geq 3$ firms are symmetric in the linear market environment defined by equations (3.6), (3.7) and (3.10). If v reflects damages to a customer of firm $h \in N$, then*

$$\varphi_i(N, v) \in \begin{cases} \left(\frac{v(N)}{n}, \frac{v(N)}{2}\right) & \text{if } i = h, \\ \left(\frac{v(N)}{2(n-1)}, \frac{v(N)}{n}\right) & \text{if } i \neq h. \end{cases} \tag{3.19}$$

Figure 3.1 illustrates the behavior of $\varphi_h(N, v)$ for intermediate degrees of differentiation. That firm h's share strictly exceeds $1/n$ extends to symmetric differentiated goods with more general non-linear cost and demand structures (see Appendix A).

3.3.3 Asymmetric Case

Bounds for the symmetric case provide guidance for mildly asymmetric markets by continuity. But when firms are sufficiently heterogeneous, it is possible that the producer of a good h will be assigned a *smaller* share of compensation than the competitors, i.e., $\varphi_h(N, v) < v(N)/n$. This happens when the cross-price effects involving firm h are sufficiently smaller than those between other cartel members. We can, e.g., have three firms such that demands of firm 1 and 2 involve high mutual cross-price reactions b_{12} and b_{21}, while there are only small linkages b_{i3} and b_{3i} with firm 3 ($i \neq 3$). Firm 3's cartel participation contributes to the overcharges on p_1, p_2 and p_3 if all parameters are positive. But a significant increase of p_3 would have occurred even if firm 3 had not been part of the cartel and had just best-responded. This part of Δp_3 is caused by price increases on goods 1 and 2, which are mostly driven by shutting down competition between firms 1 and 2, not firm 3. Hence the former bear greater responsibility for Δp_3 than the latter.[19] Asymmetry in cross-price effects does not come with useful bounds.

Asymmetry in the demand parameters a_i or costs γ_i can be dealt with, though calculations become very tedious. Supposing $\gamma = 0$ and that firm-specific intercepts a_i are the only asymmetry, one can for instance compute

$$\Delta p_h = p_h^C - p_h^B = \frac{b(n-1)[b(3d+2b-bn)a_h + (2d^2+b^2n-b^2)\bar{a}_{-h}]}{2(d+b)(2d+b)(d+b-bn)(2d+b-bn)} \tag{3.20}$$

as the cartel's price increase for product h. It rises in the saturation level a_h of firm h's demand as well as in the average saturation quantity $\bar{a}_{-h} := \sum_{i \neq h} a_i/(n-1)$ of firms $i \neq h$. The corresponding Shapley value of firm h in the apportioning of Δp_h is

$$\varphi_h = \frac{\Delta p_h}{n} + \frac{1}{n}\sum_{s=2}^{n-1} \frac{b(s-1)[b(6d+b(s+4-n))a_h + (4d^2+\tau_s b^2)\bar{a}_{-h}]}{2(d+b)(2d+b)(4d^2-(2n-6+2s)db+\eta_s b^2)} \tag{3.21}$$

with $\tau_s := (n-s-2)$ and $\eta_s := s(n-s) - 2(n-1)$. The implied damage share of firm h can, after suitable algebraic manipulation, be written as a function of $\alpha = \frac{d}{b(n-1)}$ and $\bar{a}_{-h}/a_h$ as follows

$$\rho_h^* = \frac{1}{n} + \frac{1}{n(n-1)}\sum_{s=2}^{n-1} \frac{(s-1)\Big[6\alpha(n-1)+(s+4-n)+\Big(4\alpha^2(n-1)^2+\tau_s\Big)\frac{\bar{a}_{-h}}{a_h}\Big]\cdot(\alpha-1)(2\alpha-1)}{\Big(4\alpha^2(n-1)-(2n-6+2s)\alpha+\frac{\eta_s}{n-1}\Big)\cdot\Big[(3\alpha+\frac{2-n}{n-1})+(2\alpha^2(n-1)+1)\frac{\bar{a}_{-h}}{a_h}\Big]}. \tag{3.22}$$

Ratio $\bar{a}_{-h}/a_h$ relates the market sizes of firm h and its competitors: a large ratio means firm h is comparatively small, a ratio close to zero that h's market is big.

It can be checked that ρ_h^* is strictly decreasing in $\bar{a}_{-h}/a_h$. From that follows

$$\rho_h^* \leq \lim_{\bar{a}_{-h}/a_h \to 0} \rho_h^* = \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1}\frac{(s-1)}{(n-1)} \cdot \frac{\Big[6\alpha(n-1)+(s+4-n)\Big]\cdot(\alpha-1)(2\alpha-1)}{\Big(4\alpha^2(n-1)-(2n-6+2s)\alpha+\frac{\eta_s}{n-1}\Big)\cdot\Big(3\alpha+\frac{2-n}{n-1}\Big)}. \tag{3.23}$$

The right-most fraction, with terms involving α, is maximal for $s = n-1$. This

[19] For instance, assuming $a_i = 10, d_i = 3, \gamma_i = 0$ for $i \in \{1,2,3\}, b_{12} = b_{21} = 2, b_{13} = b_{23} = b_{31} = b_{32} = 0.5$ and considering $v(N) = \Delta p_3$, the Shapley shares evaluate to $\rho_1^* = \rho_2^* \approx 35.1\% > \rho_3^* \approx 29.8\%$.

maximum can be shown to be strictly increasing in α. It is hence bounded by its limit as $\alpha \to \infty$, which evaluates to 1. This gives

$$\rho_h^* \leq \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1} \frac{s-1}{n-1} = \frac{1}{2} \tag{3.24}$$

as an upper bound. $1/n \leq \rho_h^*$ follows from considering $\bar{a}_{-h}/a_h \to \infty$ and $\alpha \to 1$.

So if only demand parameters a_i vary *and* we focus on firm h's share then the same bounds obtain as under symmetry. Things differ for a firm $j \neq h$, however. The key determinant of j's share in Δp_h is $\tilde{a}_{-h,j} := \sum_{i \in N \setminus \{h,j\}} a_i/(n-2)$, the average demand intercept of firms other than h and j. If $a_j \gg \tilde{a}_{-h,j}$ then j is the only large competitor of h and both end up splitting Δp_h about 50 : 50. If conversely the market size of j is negligible compared to that of h's other competitors (i.e., $a_j \ll \tilde{a}_{-h,j}$) then j is basically a null player.

These observations are summarized by the following analogue to Proposition 3.1:

Proposition 3.2. *Suppose $n \geq 3$ firms are symmetric except for the demand intercepts $a_1, \ldots, a_n$ in the linear market environment defined by equations (3.6), (3.7) and (3.10) with $\gamma = 0$. If v reflects damages to a customer of firm $h \in N$, then*

$$\varphi_i(N, v) \in \begin{cases} \left(\frac{v(N)}{n}, \frac{v(N)}{2}\right) & \text{if } i = h, \\ \left(0, \frac{v(N)}{2}\right) & \text{if } i \neq h. \end{cases} \tag{3.25}$$

Bounds concerning competitors $i \neq h$ of the suing customer's seller are now so wide that they are unlikely to be of practical help.

The same bounds apply to firms which are symmetric in all but technology. This is illustrated in Figure 3.2. It considers two low-cost and two high-cost producers with common parameters $a = 10$, $d = 2$, and $b = \frac{2}{3\alpha}$. No matter whether the selling firm has (a) low costs $\gamma_i = 1$ or (b) high costs $\gamma_j = 5$, it bears between 25% and 50% of overcharges on its product, and always the greatest share. In the former case, the share of the other low-cost firm increases in differentiation and approaches 50% for $\alpha \to \infty$. The share of a high-cost firm in overcharges on the product of a competitor falls in α and vanishes.

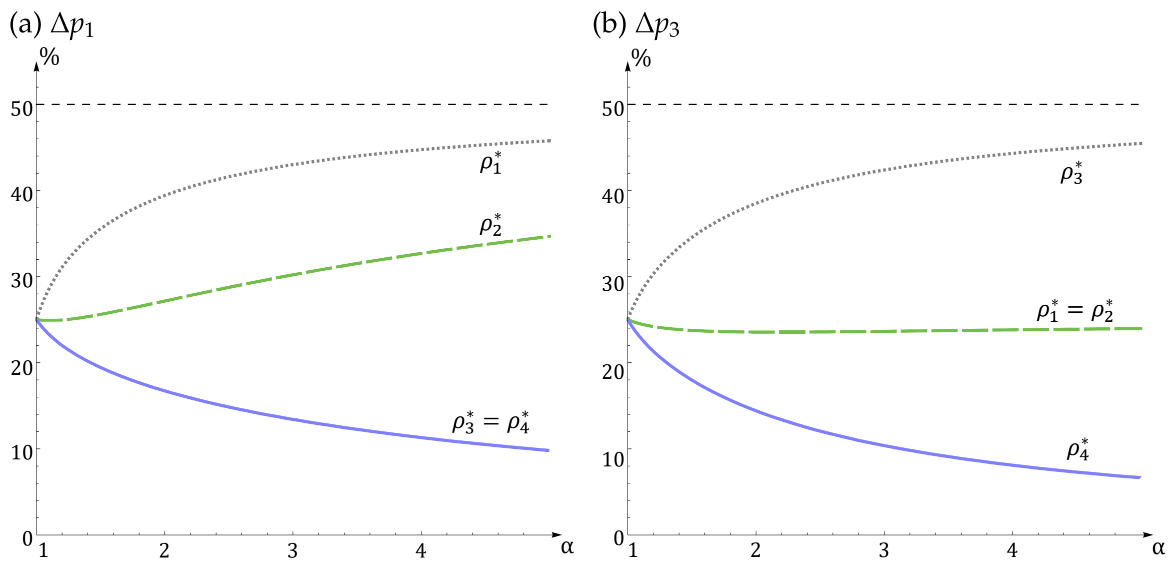

Figure 3.2 Shares ρ^* for cost leaders $i = 1, 2$ and laggards $j = 3, 4$

3.3.4 Comparison to Heuristic Apportioning

A reliable heuristic could save the complications of calibrating a structural model. Perhaps market shares, which are much easier to obtain, are a good proxy for whose cartel participation contributed how much to damages, at least under some identifiable circumstances? If yes, should we use sales or revenues? From the cartel or competitive regime? Or perhaps better use a profit measure?

We address these questions by doing some *in vitro* comparisons. Specifically, we consider Shapley apportioning under a range of parameter choices and numerically compare deviations from this benchmark for several apportioning heuristics.

We adopt an aggregate perspective here and suppose that *every* harmed customer goes after the cartel. Then the total overcharge damage

$$D := \sum_{i \in N} q_i^C \cdot \Delta p_i \tag{3.26}$$

will either be allocated according to the Shapley value $\varphi(N, v^j)$ for each individual product j, or according to some heuristic. Firm i's aggregate Shapley payments are

$$\Phi_i := \sum_{j \in N} \varphi_i(N, v^j) = \sum_{j \in N} q_j^C \cdot \Delta p_j \cdot \rho_i^*(N, v^j). \tag{3.27}$$

Absolute values of over or under-payments relative to Φ_i are summed across firms and normalized to give an index of *aggregate mis-allocation of damages*

$$M^{\rho} = \sum_{i \in N} \mid \Phi_i - H_i^{\rho} \mid \Big/ D \tag{3.28}$$

where H_i^{ρ} denotes aggregate payments by firm i according to heuristic shares ρ. M^{ρ} is proportional to the expected mis-allocation of compensation for a unit purchase by a randomly drawn customer, for a customer who made purchases from all firms in proportion to their cartel sales, or when all customers go after the cartel with identical positive probabilities.[20] Considering M^{ρ} rather than over and under-payments at the product-specific level gives heuristics a good shot: differential responsibilities for own and other firms' customers can net out across products. In particular, an equal distribution per head, by market shares, or by profits all yield zero aggregate mis-allocation for symmetric environments.

We hence focus on asymmetric configurations and report on six distinct variations of the example underlying Figure 3.2. The baseline parameters are $\gamma = 1, a = 10, d = 2$ and $b = d/3\alpha$; we break symmetry for one parameter at a time. Several variations which we tried, e.g., with six firms instead of four, yielded similar patterns.

The two top panels of Figure 3.3 consider heterogeneity in firm-specific market sizes a_i. Panel (a) involves two large and two small firms; in panel (b) all differ. An *equal per head allocation* ρ^0 non-surprisingly performs well when differentiation is very low. It soon loses out to allocating damages in proportion to *market shares based on competitive sales* ρ^4 and to *market shares based on cartel sales* ρ^2. Market shares determined by *cartel revenues* ρ^1 or *competitive revenues* ρ^3 produce high mis-allocations at all levels of differentiation. Only apportioning in proportion to *cartel profits* ρ^5 is worse.

Panels (c) and (d) assume an intermediate and a big cost asymmetry between firms 1 and 2 vs. firms 3 and 4. The deviations from the Shapley payments, aggregated for each firm across all four overcharges, is significantly higher with the bigger asymmetry in (d). The kink which is visible in panel (c) for ρ^3 – or ρ^2 in (e) – results from cancellation of product-specific deviations at the firm level when these initially have opposite signs but switch to same sign. Revenue-based market shares ρ^1 or ρ^3 and sales-based competitive market shares ρ^4 all perform well.

[20]The latter may be a plausible a priori assumption. It suggests that cartel members should pool their obligations in a kind of trust in order to save on transaction costs. Symmetry of firms would call for symmetric shares in the trust. The analysis in this section shows, however, that no simple market or profit share rule applies to funding the trust under asymmetry.

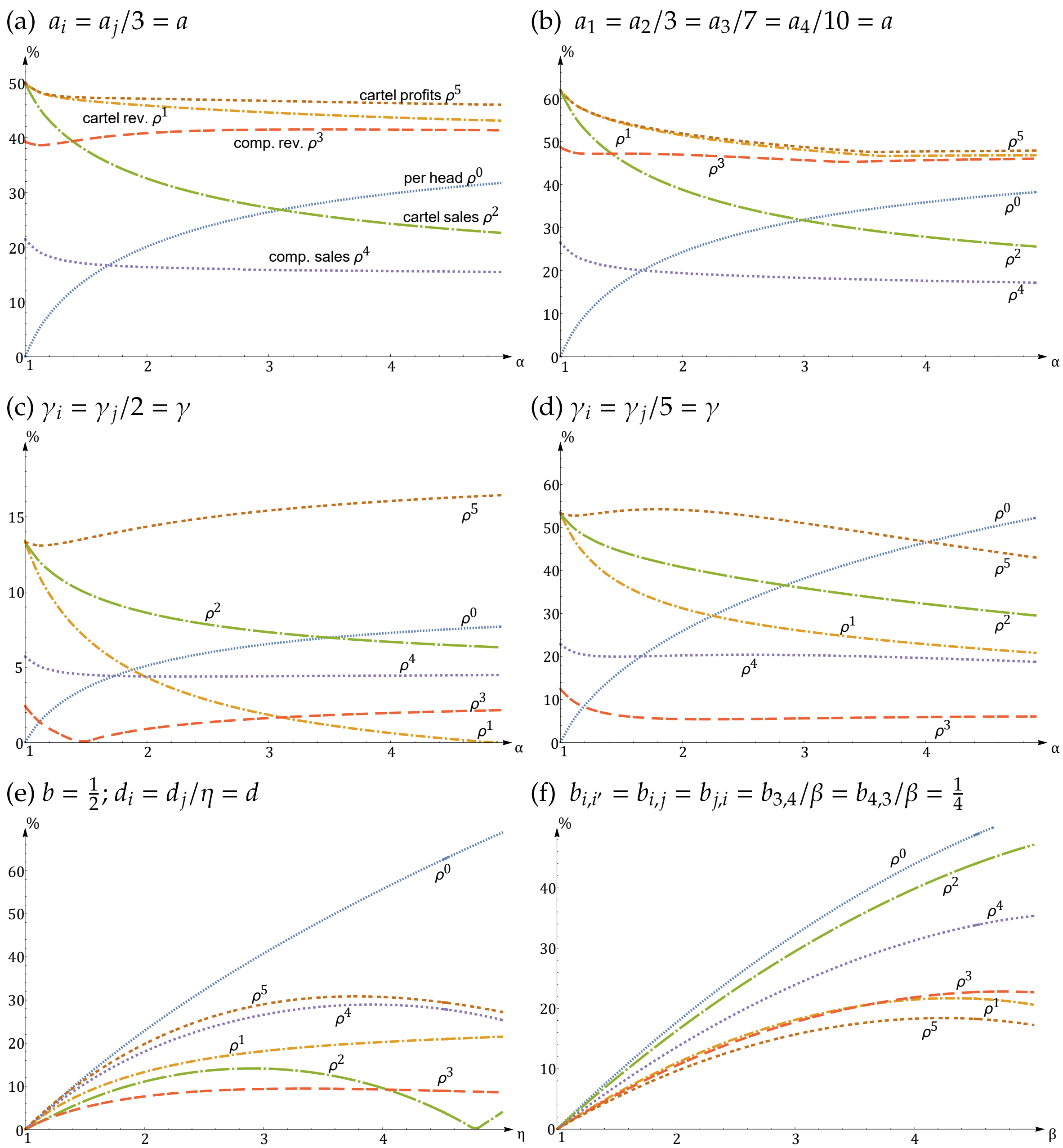

Figure 3.3 Mis-allocation M^ρ by different heuristics considering $i = 1, 2$ and $j = 3, 4$

Panel (e) assumes firms 3 and 4 face bigger own-price elasticities than firms 1 and 2. Market shares based on cartel sales or competitive revenues are close to the Shapley value, as far as aggregate payments to all victims are concerned. The final panel (f) assumes heterogeneity in cross-price effects: firms 1 and 2 face a fixed cross-price parameter of 1/4, competition between firms 3 and 4 is more intense by some factor β. Somewhat unexpectedly, after investigating five environments in which its ranking was consistently low, apportioning by cartel profits ρ^5 comes closest to representing a short-cut to the exact Shapley payments.

Overall, there is *no* heuristic which always outperforms the others. Those based on market shares – preferably sales-based for heterogeneity in a_i, otherwise revenue-based – tend to score better than a profits-based division; but panel (f) provides an exception to the rule. Generally, when firms produce close substitutes and hence ratio $\alpha = d/(n-1)b$ is close to 1, an equal division by heads performs well. This comes with the warning that all figures consider *aggregate* mis-allocation. If firm-specific fractions of customers seek compensation for their harm, the picture looks much worse.

3.4 Ring Leaders and Leniency Applicants

Regarding the conduct of a firm, we have so far discriminated only between being a member of the cartel or competing with it. There are at least two cases where the specific roles of firms require attention.

First, some members may have acted as *ringleaders* of the cartel and therefore bear greater responsibility for inflicted harm. The legal literature points to the role of *leader in an infringement*, i.e., in organizing the operations of an existing cartel, and of *instigator of an infringement* by particularly furthering the establishment or enlargement of a cartel (see EC Case T-15/02 (14)). A cartel's success and therefore the harm it causes are known to vary in its organizational characteristics (see, e.g., Davies and De 2013). Attributions of relative responsibility based only on a model of cost and demand likely understate a ringleader's due share.

Second, former offenders that cooperate with the authorities often enjoy reduced obligations. Leniency applicants tend to attract the main damage cases because they were first to admit wrong; liability exemptions partly offset this effect and raise the attractiveness of coming clean just like immunity from criminal charges and fines does (e.g., detrebling of damages in the US). This is appreciated in Directive 2014/104/EU (see recital 38) and Article 11(4) makes the leniency provision that ". . . an immunity recipient is jointly and severally liable as follows: (a) to its direct or indirect purchasers or providers; and (b) to other parties only where full compensation cannot be obtained from the other undertakings that were involved in the same infringement of competition law."

The restricted role of an immunity recipient in compensating victims can be incorporated into the proposed approach rather easily when only one firm is granted immunity or when the analysis is restricted to almost positive games.[21] The key modification is to replace symmetry condition (SYM). This leads to the use of *weighted Shapley values*, which were first suggested by Shapley (1953a).

A prominent axiomatic characterization by Kalai and Samet (1987) relaxes symmetry to requiring merely that $v(S)$ is distributed in a consistent way whenever S is a so-called "partnership"; that is, when members of S make contributions to coalitions of other players only if they are together, i.e., $v(R \cup T) = v(R)$ for any strict subset $T \subset S$ and any $R \subseteq N \setminus S$. The members of a partnership are symmetric to another in terms of their marginal contributions. If surplus or costs must be split asymmetrically, for reasons not reflected by v, at least there should be no inconsistency between a two-step allocation – first to partnerships in their entirety, then internally – or one directly to individual members.

Including this requirement, while dropping symmetry, turns out to impose a non-negative vector $\omega = (\omega_1, \ldots, \omega_n)$ of weights. This modifies the symmetric Shapley value φ to φ^ω such that the shares of players $i \in T$ in any carrier game (N, u_T) over $T \subseteq N$ (where $u_T(S) = 1$ if $T \subseteq S$ and 0 otherwise) are proportional to their weights, i.e., $\varphi_i^\omega(N, u_T) = \omega_i / \sum_{j \in T} \omega_j$ if $i \in T$ and 0 otherwise.

The leniency rules in Article 11(4) can therewith be accommodated as follows:[22] (i) use φ^ω with $\omega = (1, \ldots, 1)$, i.e., the standard Shapley value φ, for allocating any overcharge damages (N, v^l) which have accrued on goods produced by leniency recipient $l \in N$; (ii) by contrast use $\varphi^{\tilde{\omega}}$ with $\tilde{\omega} = (1, \ldots, 1, 0, 1, \ldots, 1)$ where $\tilde{\omega}_l = 0$ when overcharges by l's competitors are concerned. This can be generalized to the case of multiple immunity recipients $L \subset N$: use $\varphi^{\tilde{\omega}}$ with $\tilde{\omega}_i = 1$ if $i \notin L$ or overcharges Δp_i are concerned, and $\tilde{\omega}_i = 0$ otherwise.[23]

The same kind of extension can account for elevated responsibilities that derive from ringleader positions. Namely, use $\varphi^{\tilde{\omega}}$ with $\tilde{\omega}_r = \kappa > 1$ for any ringleader $r \in R \subset N$, and $\tilde{\omega}_i = 1$ for conventional cartel members $i \notin R$. The appropriate value

[21] The set of almost positive games is a subset of the class of convex games (see Derks et al. 2000). For almost positive games, higher (lower) weights also imply higher (lower) contribution shares. This is not generally true when a game is not almost positive and more than one firm's weight has to be scaled up or down (see Owen 1968).

[22] Same applies to the liability restriction in Article 11(2) for *small or medium-sized enterprises*.

[23] We simplify here. Potential divisions by zero are avoided by actually working with a lexicographic *weight system*, consisting of strictly positive weights and an ordered partition of N into classes $N_1, \ldots, N_m$. Members of N_p receive zero when a carrier T also involves members of N_q with $p < q$. See Kalai and Samet (1987). Also see Nowak and Radzik (1995) on axiomatizing φ^ω based on (MRG) and (EFF).

for κ – or possibly different levels $\kappa_1 \geq \kappa_2 \geq \ldots > 1$ when there existed multiple ringleaders – depends on how pronounced the respective leadership or instigating role was. This is outside the scope of our setup. Criminal rulings and fines that precede civil-law proceedings may provide a reference point.

3.5 Concluding Remarks

We have argued in this chapter that a non-arbitrary way of apportioning contributions in line with relative responsibilities exists. The co-infringers are to start out with equal shares of any compensation payment; these shares then are to be corrected in a well-defined way for greater or smaller-than-average effects on the damage in question. This follows from translating the legal stipulation that contributions track relative responsibilities for harm into the marginality property of Young (1985) and other natural requirements.

The requirements' satisfaction makes the Shapley value the right instrument for allocating cartel damages. This chapter investigated what its use entails and how it relates to ad hoc approaches. We focused on linear cost and demand in Section 3.3 in order to obtain general parametric expressions. Quadratic costs can be handled analogously, but yield very unwieldy results; numerical analysis could cope with more general non-linear environments.

The merits of the damage apportionments that result from applying the Shapley value clearly depend on the quality of its input, i.e., the description of counterfactual damage scenarios by characteristic function v. Reaching a reasonable level of agreement on it among the former cartel members or in court is bound to be difficult. That, however, is no unsurmountable hurdle. Estimating v essentially means calibrating and simulating a structural model of the relevant market. The pertinent trade-offs between tractability and temporal, spatial, or other details are known, e.g., from merger simulation and also arise when quantifying harm in the first place.

Former cartel members may, of course, settle internal redress claims as they please. Market or profit share heuristics provide a tempting short-cut. But – see Subsection 3.3.4 – they are generally inconsistent with one another and are unreliable proxies of relative responsibility. If one tries to operationalize the redress norm in Directive 2014/104/EU in a systematic way, the Shapley value should be invoked.

3.6 Appendix A Symmetric Differentiated Substitutes

Customers pay a firm-specific price $p_i = p^i(S)$ for purchases from i when cartel S is formed and this generally depends on the composition of S rather than just its size. We will consider a particularly well-behaved environment with $n \geq 3$ firms

Let the profits Π_i of every firm $i \in N$ be a smooth, strictly concave function of a profile $y = (y_1, \ldots, y_n)$ of 'actions' of all firms with $\partial\Pi_i/\partial y_i\big|_{y=0} > 0$. These actions could be price choices, production levels, choices on the geographic radius of operation, etc. We presume that the associated prices $p = (p_1, \ldots, p_n)$ are smooth functions of y, too, and if $\partial p_i/\partial y_i$ is positive (negative) then the same should go for the sign of the externality $\partial\Pi_j/\partial y_i$ that firms exert on each other.[24] Specifically, we think of goods as differentiated substitutes and require

$$\frac{\partial\Pi_j}{\partial y_i} \cdot \frac{\partial p_i}{\partial y_i} > 0 \text{ for all } i \neq j \in N \tag{3.29}$$

for the relevant range of actions. For instance, if firm i's output choice y_i negatively affects its own price p_i, we assume it also has a negative effect on any competitor's profits Π_j. If i's action is its price, i.e., $p_i(y) \equiv y_i$, then Π_j increases in y_i.

A coalition $S \neq \emptyset$ chooses $(y_i)_{i\in S}$ to maximize $\Pi_S(y) = \sum_{i\in S} \Pi_i(y)$ for given actions $y_{-S} = (y_j)_{j\notin S}$ of outsiders. If S is a singleton, this corresponds to individual profit maximization by all, implying the competitive benchmark prices $p_1^*, \ldots, p_n^*$. We assume that a unique, interior profit maximizer exists for each non-empty $S \subseteq N$. So, for any fixed cartel S, reaction functions $R_S(y_{-S})$ and $(R_j(y_{-j}))_{j\notin S}$ are well-defined by the first-order conditions

$$\frac{d\Pi_i}{dy_i} = \frac{\partial\Pi_i}{\partial y_i} \qquad = 0 \;\text{ if } i \notin S, \tag{3.30}$$

$$\frac{d\Pi_S}{dy_i} = \sum_{j\in S} \frac{\partial\Pi_j}{\partial y_i} = 0 \;\text{ if } i \in S. \tag{3.31}$$

We further specialize this to *strongly symmetric* situations in which profits Π_i and prices p_i depend identically on i's own action y_i for each $i \in N$ and identically also on any respective action y_j by a firm $j \neq i$. Formally, for each $i \neq j$ and every permutation

[24]Without an externality, competitive and cartel behavior would not differ and no harm arise.

$\varrho\colon N \to N$ with $\varrho(i) = j$ and $\varrho(j) = i$

$$p_i(y_1, \ldots, y_n) \equiv p_j(y_{\varrho(1)}, \ldots, y_{\varrho(n)}) \text{ and } \Pi_i(y_1, \ldots, y_n) \equiv \Pi_j(y_{\varrho(1)}, \ldots, y_{\varrho(n)}). \tag{3.32}$$

One can, e.g., think of equal measures of customers with a favorite product i to whom all varieties $j \neq i$ are identically imperfect substitutes. This assumes greater symmetry than the Salop model.[25] In particular, cross effects on prices and profits are identical for all firms. The first-order condition (3.31) for a cartel member $i \in S$ then simplifies to

$$\frac{d\Pi_S}{dy_i} = \frac{\partial \Pi_i}{\partial y_i} + (s-1)\frac{\partial \Pi_j}{\partial y_i} = 0. \tag{3.33}$$

The only asymmetry is that i's own actions may affect p_i and Π_i differently from the actions of $j \neq i$. We will suppose own actions have bigger effects and therefore

$$\left|\frac{\partial p_i}{\partial y_i}\right| > \left|\frac{\partial p_i}{\partial y_j}\right|. \tag{3.34}$$

The inequality is trivially satisfied for price competition. Otherwise it formalizes that inverse demand responds more to changes of the quantity, delivery range, etc. of the product in question than that of others.

We assume that for any fixed cartel S, the simultaneous best-response behavior by it and any outsiders $j \in N \setminus S$ determine a unique type-symmetric Nash equilibrium profile $y^*(S) = (y_1^*(S), \ldots, y_n^*(S))$ where $y_i^*(S) \equiv y^C(S)$ if $i \in S$, and $y_i^*(S) \equiv y^O(S)$ if $i \notin S$. We will drop the argument S below when the reference is clear. Sufficient conditions for such an equilibrium to exist can be found in Section 3.3.

The first-order conditions (3.30) and (3.33) cannot simultaneously be satisfied for $s > 1$ if $y^C = y^O$: $\partial \Pi_j / \partial y_i \neq 0$ implies either $y^C > y^O$ or $y^C < y^O$ in equilibrium. The former holds if the externality is positive, the latter if it is negative.

For specificity, suppose quantity competition with a negative externality $\partial \Pi_j / \partial y_i < 0$ and $\partial p_i / \partial y_i < 0$ for a moment. The key observation then will be that $y^C < y^O$ translates into higher prices for the goods sold by cartel members. This implies that for a cartel S of a fixed size s, firm i's prices – and hence its customers' damages – depend on whether i is an element of S or not. In particular, if v describes the damages of a customer of good i then $\bar{v}^i(s) > \bar{v}^{\not i}(s)$.

To see this formally, let $S = \{1, \ldots, s\}$ w.l.o.g. and consider the straight line L which

[25] There, some permutation ϱ with $\varrho(i) = j$ and $\varrho(j) = i$ satisfies (3.32), not every such permutation.

connects profile $\hat{y} = (y^O, y^C, \ldots, y^C, y^O, \ldots, y^O, y^C)$ to $\hat{\hat{y}} = (y^C, y^C, \ldots, y^C, y^O, \ldots, y^O, y^O)$ in the space of output choices. L can be parameterized by

$$r(t) = (\underbrace{y^O - t, y^C, \ldots, y^C}_{s \text{ terms}}, \underbrace{y^O, \ldots, y^O, y^C + t}_{n-s \text{ terms}}) \tag{3.35}$$

with $t \in [0, y^O - y^C]$, i.e., we simultaneously decrease firm 1's action and increase firm n's action by identical amounts as we move along L. The gradient $\nabla p_n = \left(\frac{\partial p_n}{\partial y_1}, \ldots, \frac{\partial p_n}{\partial y_n}\right)$ of function p_n can be used in order to evaluate the price change caused by switching from $\hat{y}$ to $\hat{\hat{y}}$. In particular, the gradient theorem for line integrals (see, e.g., Protter and Morrey 1991, Thm. 16.15) yields

$$p_n(\hat{\hat{y}}) - p_n(\hat{y}) = \int_L \nabla p_n \, dr = \int_0^{y^O - y^C} \nabla p_n(r(t)) \cdot r'(t) \, dt \tag{3.36}$$

$$= \int_0^{y^O - y^C} \left(\frac{\partial p_n}{\partial y_1}, \ldots, \frac{\partial p_n}{\partial y_n}\right)\Bigg|_{y=r(t)} \cdot \Big(-1, 0, \ldots, 0, 1\Big) \, dt \tag{3.37}$$

$$= \int_0^{y^O - y^C} \left[\frac{\partial p_n(r(t))}{\partial y_n} - \frac{\partial p_n(r(t))}{\partial y_1}\right] dt < 0. \tag{3.38}$$

The inequality follows from own actions having bigger effects than a competitor's actions: (3.34) entails $\frac{\partial p_n}{\partial y_n} < \frac{\partial p_n}{\partial y_1}$ when $\partial p_n / \partial y_n < 0$. The strong symmetry of the considered setting (see condition (3.32)) then implies

$$p^1(s) := p_1(y^C, y^C, \ldots, y^C, y^O, \ldots, y^O, y^O) = p_n(y^O, y^C, \ldots, y^C, y^O, \ldots, y^O, y^C) \tag{3.39}$$
$$= p_n(\hat{y}) > p_n(\hat{\hat{y}}) = p_n(y^C, y^C, \ldots, y^C, y^O, \ldots, y^O, y^O) := p^{\not n}(s).$$

That is, the price $p^1(s)$ of good 1 when its producer is one of s symmetric cartel members exceeds the price $p^{\not n}(s)$ of good n when firm n is *not* part of a cartel with s members.

By symmetry, we have $p^{\not 1}(s) = p^{\not n}(s)$ and $p^1(s) = p^n(s)$. So we can conclude $p^1(s) > p^{\not 1}(s)$ from (3.39) for $1 < s < n$.[26] The same applies to any other firm, too – for instance, the plaintiff's 'home' firm $h \in N$ from which its disputed purchases were made:

$$p^h(s) > p^{\not h}(s) \;\text{ for any }\; s = 2, \ldots, n-1. \tag{3.40}$$

The average per-unit damage to h's customer in scenarios where h behaves anti-

[26] Recall that there is no well-defined partial cartel for $s = 1$ or n.

competitively is

$$\bar{v}^h(s) = p^h(s) - p^*_h \tag{3.41}$$

where p^*_h is h's price in the competitive benchmark (identical across firms). The price of firm h does not depend on the specific $s-1$ firms with which h colludes, and neither does the damage. Analogously, the per-unit damage when firm h behaves competitively but s others collude is

$$\bar{v}^{\not h}(s) = p^{\not h}(s) - p^*_h. \tag{3.42}$$

Inequality (3.40) then yields

$$\bar{v}^h(s) - \bar{v}^{\not h}(s) = p^h(s) - p^{\not h}(s) > 0 \text{ for any } s = 2, \ldots, n-1. \tag{3.43}$$

So all summands in the Shapley value's correction term in equation (3.5) are positive. It follows that the 'home' firm's share in compensating overcharges on its own sales must *strictly exceed* $1/n$; that of others must consequently be less than $1/n$.

This extends to other interpretations of variables $y_1, \ldots, y_n$, notably price competition: inequalities (3.40) and hence (3.43) also follow when positive externalities $\partial\Pi_j/\partial y_i > 0$ and $\partial p_i/\partial y_i > 0$ are concerned. The cartel members choose $y^C(S) > y^O(S)$ for any fixed S; the reversed orientation as we integrate from $t = 0$ to $y^O - y^C < 0$ in (3.38) and the reversed sign of integrand $\partial p_n/\partial y_n - \partial p_n/\partial y_1$ cancel. In summary, we have:

Proposition 3.3. *Let $n \geq 3$ firms be strongly symmetric in the sense of (3.32) and let assumptions (3.29) and (3.34) be satisfied by smooth own and cross-effects of firms' actions. If v reflects damages to a customer of firm $h \in N$, then*

$$\varphi_i(N, v) \begin{cases} > \frac{v(N)}{n} & \text{if } i = h, \\ < \frac{v(N)}{n} & \text{if } i \neq h. \end{cases} \tag{3.44}$$

Simple rules of thumb like distributing damages on a per-head basis or according to market shares, profits, etc. will allocate exactly $1/n$-th of compensation payments to all producers if they are symmetric. Proposition 3.3 shows that this generally clashes with a responsibility-based allocation. Only if identical numbers of customers of all firms act against the cartel, each $h \in N$ is the 'home' producer equally often and asymmetric responsibilities for overcharges Δp_h perfectly net out. Otherwise, responsibility of vendor h is underestimated and that of its collaborators $j \neq h$

overestimated.

3.7 Appendix B
Calculations Underlying Section 3.3

We first consider symmetric firms; the case of firms that differ in demand saturation parameter a_i is addressed afterwards.

3.7.1 Symmetric Firms

3.7.1.1 Price Overcharge

We assume $\alpha := \frac{d}{(n-1)b} > 1$, i.e., $d > (n-1)b$, in order to ensure existence and uniqueness of a symmetric Nash equilibrium (see (3.47), (3.50), (3.58) and (3.59) below). The profit maximization problem (PMP) of firm i is

$$\max_{p_i} \; \pi_i = \Big(a - dp_i + b \sum_{l=1 \neq i}^{n} p_l\Big)p_i - \gamma\Big(a - dp_i + b \sum_{l=1 \neq i}^{n} p_l\Big) \tag{3.45}$$

if all firms act competitively. The FOC yields

$$\frac{\partial \pi_i}{\partial p_i} = a - 2dp_i + b \sum_{l=1 \neq i}^{n} p_l + \gamma d = 0. \tag{3.46}$$

Solving for a symmetric Nash equilibrium gives the unique Bertrand price

$$p^B = \frac{a + \gamma d}{2d - (n-1)b}. \tag{3.47}$$

$D_i(\gamma) > 0$ ensures $p^B > \gamma$. If the industry-wide cartel forms, the coalition profit is

$$\begin{aligned} \pi_N = {} & \Big(a - dp_i + b \sum_{t=1 \neq i}^{n} p_t\Big)p_i + \sum_{t=1 \neq i}^{n} \Big(a - dp_t + b \sum_{r=1 \neq t}^{n} p_r\Big)p_t \\ & -\gamma\Big(a - dp_i + b \sum_{t=1 \neq i}^{n} p_t\Big) - \gamma \sum_{t=1 \neq i}^{n} \Big(a - dp_t + b \sum_{r=1 \neq t}^{n} p_r\Big). \end{aligned} \tag{3.48}$$

The FOC gives

$$\frac{\partial \pi_i}{\partial p_i} = a - 2dp_i + 2b \sum_{t=1 \neq i}^{n} p_t + \gamma d - \gamma(n-1)b = 0 \quad \text{for each } i \in N. \tag{3.49}$$

So, invoking symmetry, the grand coalition has each firm charge cartel price

$$p^C = \frac{a + \gamma(d-(n-1)b)}{2(d-(n-1)b)} = \left(\frac{a}{d-(n-1)b} + \gamma\right)\Big/2. \tag{3.50}$$

The resulting cartel overcharge per unit is

$$\begin{aligned}
\Delta p &= p^C - p^B \qquad (3.51)\\
&= \frac{(a+\gamma(d-(n-1)b))(2d-(n-1)b) - (2d-2(n-1)b)(a+\gamma d)}{(2d-2(n-1)b)(2d-(n-1)b)}\\
&= \frac{a[2d-(n-1)b-2d+2(n-1)b] + \gamma[(d-(n-1)b)(2d-(n-1)b) - d(2d-2(n-1)b)]}{(2d-2(n-1)b)(2d-(n-1)b)}\\
&= \frac{a[(n-1)b] + \gamma[(n-1)^2b^2 - (n-1)bd]}{(2d-2(n-1)b)(2d-(n-1)b)} = \frac{(n-1)b[a-\gamma(d-(n-1)b)]}{4d^2 - 6(n-1)bd + 2(n-1)^2b^2}. \qquad (3.52)
\end{aligned}$$

3.7.1.2 Shapley Value of Firm i

W.l.o.g. let members of $S = \{1, \ldots, s\}$ coordinate their actions in case of a partial cartel. The remaining firms $s+1, \ldots, n$ act competitively. Then the PMP of firm $i \in S$ is

$$\begin{aligned}
\max_{p_i} \pi_i = &\Big(a - dp_i + b \sum_{t=1 \neq i}^{s} p_t + b \sum_{l=s+1}^{n} p_l\Big)p_i + \sum_{t=1 \neq i}^{s} \Big(a - dp_t + b \sum_{r=1 \neq t}^{s} p_r + b \sum_{l=s+1}^{n} p_l\Big)p_t\\
&- \gamma\Big(a - dp_i + b \sum_{t=1 \neq i}^{s} p_t + b \sum_{l=s+1}^{n} p_l\Big) - \sum_{t=1 \neq i}^{s} \gamma\Big(a - dp_t + b \sum_{r=1 \neq t}^{s} p_r + b \sum_{l=s+1}^{n} p_l\Big).
\end{aligned} \tag{3.53}$$

If firm $j \notin S$ acts competitively its PMP is

$$\max_{p_j} \ \pi_j = \Big(a - dp_j + b \sum_{t=1}^{s} p_t + b \sum_{l=s+1 \neq j}^{n} p_l\Big)p_j - \gamma\Big(a - dp_j + b \sum_{t=1}^{s} p_t + b \sum_{l=s+1 \neq j}^{n} p_l\Big). \tag{3.54}$$

Let $p_i^s \geq 0$ denote the price of firm i in case it is part of the cartel and $p_j^{\not s} \geq 0$ the price of firm j in case this firm is not among the s cartelists. We can focus on symmetric strategies among insiders and outsiders respectively. The FOC then yields

$$\frac{\partial \pi_i}{\partial p_i^s} = a - 2dp_i^s + 2(s-1)bp_i^s + (n-s)bp_j^{\not s} + \gamma d - \gamma(s-1)d = 0 \tag{3.55}$$

$$\frac{\partial \pi_j}{\partial p_j^{\not s}} = a - 2dp_j^{\not s} + sbp_i^s + (n-s-1)bp_j^{\not s} + \gamma d = 0. \tag{3.56}$$

Solving for p_i^s and $p_j^{\not s}$ gives the best response functions

$$R_i^s(p_j^{\not s}) = \frac{a + (n-s)bp_j^{\not s} + \gamma d - \gamma(s-1)b}{2d - 2(s-1)b} \quad \text{and} \quad R_j^{\not s}(p_i^s) = \frac{a + sbp_i^s + \gamma d}{2d - (n-s-1)b}. \tag{3.57}$$

Solving $p_i^s = R_i^s(R_j^{\not s}(p_i^s))$ yields the symmetric equilibrium price of any firm i that is part of the cartel:

$$p_i^s = \frac{a + (n-s)b\frac{a+sbp_i^s+\gamma d}{2d-(n-s-1)b} + \gamma d - \gamma(s-1)b}{2d - 2(s-1)b}$$

$$\Leftrightarrow p_i^s = \frac{a(2d+b) + \gamma[2d^2 + bd(3-2s) + b^2(ns - n - s^2 + 1)]}{4d^2 - (2n - 6 + 2s)bd + b^2(2 - 2n + sn - s^2)}. \tag{3.58}$$

Analogously, the equilibrium price of a firm j which is not part of a cartel with s members is

$$p_j^{\not s} = \frac{a + sb\frac{a(2d+b)+\gamma(2d^2+bd(3-2s)+b^2(ns-n-s^2+1))}{4d^2-(2n-6+2s)bd+b^2(2-2n+sn-s^2)} + \gamma d}{2d - (n-s-1)b}$$

$$\Leftrightarrow p_j^{\not s} = \frac{a(2d - sb + 2b) + \gamma[2d^2 - bd(s-2) - b^2(s^2 - s)]}{4d^2 - (2n - 6 + 2s)bd + b^2(2 - 2n + sn - s^2)}. \tag{3.59}$$

Symmetry implies $p_j^{\not s} = p_i^{\not s}$. The price effect of a given firm i being part of a cartel with

s members rather than not therefore is

$$\begin{aligned}
p_i^s - p_i^{\not s} &= \frac{a(2d+b)+\gamma[2d^2+bd(3-2s)+b^2(ns-n-s^2+1)]}{4d^2-bd(2n-6+2s)+b^2(2-2n+sn-s^2)} \\
&\quad - \frac{a(2d-sb+2b)-\gamma[2d^2-bd(s-2)-b^2(s^2-s)]}{4d^2-bd(2n-6+2s)+b^2(2-2n+sn-s^2)} \\
&= \frac{a[sb-b]+\gamma[bd-sbd+b^2ns-b^2n+b^2-sb^2]}{4d^2-bd(2n-6+2s)+b^2(2-2n+sn-s^2)} = \frac{b(s-1)(a-(b+d-bn)\gamma)}{4d^2-2bd(n-3+s)+b^2(2+n(s-2)-s^2)}.
\end{aligned} \tag{3.60}$$

Fixing a particular firm $i = h$ as the "home firm" from which a suing customer made purchases, the corresponding Shapley value is

$$\varphi_h(N, v) = \frac{\Delta p}{n} + \frac{1}{n}\sum_{s=2}^{n-1}[p_i^s - p_i^{\not s}] \tag{3.61}$$

$$= \frac{\Delta p}{n} + \frac{1}{n}\sum_{s=2}^{n-1}\frac{b(s-1)(a-\gamma(d-(n-1)b))}{4d^2-2bd(-3+n+s)+b^2(2+n(s-2)-s^2)} \tag{3.62}$$

by Theorem 2. Firm h's resulting share $\rho_h^* = \varphi_h(N, v)/\Delta p$ in overcharge compensations on its own sales is

$$\begin{aligned}
\rho_h^* &= \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1}\frac{\frac{b(s-1)(a-(d-(n-1)b)\gamma)}{4d^2-2bd(-3+n+s)+b^2(2+n(s-2)-s^2)}}{\frac{b(n-1)[a-(d-(n-1)b)\gamma]}{4d^2-6(n-1)bd+2(n-1)^2b^2}} \\
&= \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1}\frac{(s-1)(4d^2-6(n-1)bd+2(n-1)^2b^2)}{(n-1)[4d^2-2bd(-3+n+s)+b^2(2+n(s-2)-s^2)]}.
\end{aligned} \tag{3.63}$$

Substituting $d = \alpha(n-1)b$ and rearranging gives

$$\begin{aligned}
\rho_h^* &= \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1}\frac{(s-1)(4(\alpha(n-1)b)^2-6(n-1)b\alpha(n-1)b+2(n-1)^2b^2)}{(n-1)[4(\alpha(n-1)b)^2-2b\alpha(n-1)b(-3+n+s)+b^2(2+n(s-2)-s^2)]} \\
&= \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1}\frac{(s-1)(n-1)^2b^2(4\alpha^2-6\alpha+2)}{(n-1)b^2[4\alpha^2(n-1)^2-2(n-1)(-3+n+s)\alpha+s(n-s)-2(n-1)]} \\
&= \frac{1}{n} + \frac{n-1}{n}\sum_{s=2}^{n-1}\frac{(s-1)\cdot(4\alpha^2-6\alpha+2)}{4\alpha^2(n-1)^2-2(n+s-3)(n-1)\alpha+\eta_s}
\end{aligned} \tag{3.64}$$

with $\eta_s := s(n-s) - 2(n-1)$.

3.7.1.3 Monotonicity of ρ_h^*

Firm h's Shapley share, written yet more concisely as

$$\rho_h^* = \frac{1}{n} + \frac{n-1}{n} \sum_{s=2}^{n-1} \frac{(s-1)\cdot(4\alpha^2 - 6\alpha + 2)}{4\alpha^2(n-1)^2 - \lambda_s\alpha + \eta_s} \tag{3.65}$$

with $\eta_s = s(n-s) - 2(n-1)$ and $\lambda_s = 2(n+s-3)(n-1)$, strictly increases in α if

$$f(\alpha) := \frac{4\alpha^2 - 6\alpha + 2}{4\alpha^2(n-1)^2 - \lambda_s\alpha + \eta_s} \tag{3.66}$$

strictly increases in α. The first derivative with respect to α is

$$\frac{\partial f(\alpha)}{\partial \alpha} = \frac{(8\alpha-6)(4\alpha^2(n-1)^2 - \lambda_s\alpha + \eta_s) - (4\alpha^2 - 6\alpha + 2)(8\alpha(n-1)^2 - \lambda_s)}{(4\alpha^2(n-1)^2 - \lambda_s\alpha + \eta_s)^2}. \tag{3.67}$$

The denominator is always positive. Hence, it remains to show that also the numerator is positive. Denoting this numerator by $g(\alpha)$ and inserting λ_s and η_s yields

$$\begin{aligned} g(\alpha) :=\ & (8\alpha-6)(4\alpha^2n^2 - 8\alpha^2n + 4\alpha^2 - 2n^2\alpha + 8n\alpha - 6\alpha - 2sn\alpha + 2s\alpha + sn - s^2 - 2n + 2) \\ & - (4\alpha^2 - 6\alpha + 2)(8\alpha n^2 - 16\alpha n + 8\alpha - 2n^2 + 8n - 6 - 2sn + 2s) \\ & = 16\alpha^3n^2 - 32\alpha^3n + 16\alpha^3 - 8n^2\alpha^2 + 32n\alpha^2 - 24\alpha^2 - 8sn\alpha^2 + 8s\alpha^2 + 4sn\alpha - 4s^2\alpha \\ & - 8n\alpha + 8\alpha - 12\alpha^2n^2 + 24\alpha^2n - 12\alpha^2 + 6n^2\alpha - 24n\alpha + 18\alpha + 6sn\alpha - 6s\alpha \\ & - 3sn + 3s^2 + 6n - 6 - 16\alpha^3n^2 + 32\alpha^3n - 16\alpha^3 + 4n^2\alpha^2 - 16n\alpha^2 + 12\alpha^2 + 4\alpha^2sn \\ & - 4s\alpha^2 + 24\alpha^2n^2 - 48\alpha^2n + 24\alpha^2 - 6\alpha n^2 + 24\alpha n - 18\alpha - 6\alpha sn + 6\alpha s - 8\alpha n^2 \\ & + 16\alpha n - 8\alpha + 2n^2 - 8n + 6 + 2sn - 2s \\ & = 8n^2\alpha^2 - 8n\alpha^2 - 4(sn\alpha^2 + s\alpha^2 + sn\alpha - s^2\alpha) + 8n\alpha - 8n^2\alpha + sn + 3s^2 - 2n + 2n^2 - 2s \\ & = 4\alpha^2 \underbrace{(2n^2 - 2n - sn + s)}_{(a)} + 4\alpha(sn - s^2 + 2n - 2n^2) + n(s - 2 + 2n) + s(3s-2). \end{aligned} \tag{3.68}$$

Note that (a) is strictly positive: it decreases in s, since $sn > s$. Inserting $s = n - 1$ gives

$$2n^2 - 2n - sn + s \geq 2n^2 - 2n - (n-1)n + n - 1 = n^2 - 1 > 0. \tag{3.69}$$

This and $\alpha > 1$ imply that the first derivative of $g(\alpha)$ with respect to α is larger than zero:

$$\begin{aligned}\frac{\partial g(\alpha)}{\partial \alpha} &= 8\alpha(2n^2 - 2n - sn + s) + 4sn - 4s^2 + 8n - 8n^2 \\ &\geq 8(2n^2 - 2n - sn + s) + 4sn - 4s^2 + 8n - 8n^2 = 2n^2 - 2n - ns + 2s - s^2 \\ &\geq 2n^2 - 2n - n(n-1) + 2(n-1) - (n-1)^2 = 3n - 3 > 0. \end{aligned} \tag{3.70}$$

The first '≥' in equation (3.70) directly uses (3.69) and $\alpha > 1$, the last '≥' follows since $(2n^2 - 2n - ns + 2s - s^2)$ is decreasing in s for $n > 2$ and $s \geq 2$; therefore $s = n - 1$ minimizes the expression.

Since function $g(\alpha)$ is strictly increasing in α, for all $\alpha > 1$

$$\begin{aligned} g(\alpha) &\geq 4(2n^2 - 2n - sn + s) + 4(sn - s^2 + 2n - 2n^2) + n(s - 2 + 2n) + s(3s - 2) \\ &= 2n^2 - 2n + ns + 2s - s^2 \geq 2n^2 > 0. \end{aligned} \tag{3.71}$$

The last '≥' in equation (3.71) follows since $(-2n + ns + 2s - s^2) \geq 0$. This follows from

$$-2n + ns + 2s - s^2 = (n-s)(s-2) \geq 0 \text{ for } n \geq s \geq 2. \tag{3.72}$$

So, overall, we can conclude that $f(\alpha)$ has a positive first derivative for $\alpha > 1$. Hence, ρ_h^* strictly increases as products become more differentiated.

3.7.2 Asymmetric Firms

Assume $\gamma = 0$ and that firms differ only in a_i. Again let $d > (n-1)b$.

3.7.2.1 Price Overcharge

If all firms $i \in \{1, \ldots, n\}$ act competitively the profit of firm i is

$$\pi_i = \Big(a_i - dp_i + b \sum_{l=1 \neq i}^{n} p_l\Big)p_i. \tag{3.73}$$

Taking first derivatives gives the FOC

$$\begin{aligned}
\frac{\partial \pi_1}{\partial p_1} &= a_1 - 2dp_1 + b\sum_{l=2}^{n} p_l = 0 \\
\frac{\partial \pi_2}{\partial p_2} &= a_2 - 2dp_2 + bp_1 + b\sum_{l=3}^{n} p_l = 0 \\
&\vdots \\
\frac{\partial \pi_n}{\partial p_n} &= a_n - 2dp_n + bp_1 + b\sum_{l=2}^{n-1} p_l = 0.
\end{aligned} \tag{3.74}$$

The FOC of firm i can also be written as

$$\frac{\partial \pi_i}{\partial p_i} = a_i - 2dp_i + b(n-1)\bar{p}_{-i} = 0 \tag{3.75}$$

with $\bar{p}_{-i} = \sum_{l=1 \neq i}^{n} p_l/(n-1)$. Solving for p_i gives the best response function of firm i

$$R_i(\bar{p}_{-i}) = \frac{a_i + b(n-1)\bar{p}_{-i}}{2d}. \tag{3.76}$$

The remaining $(n-1)$ FOC can be added to

$$\sum_{l=1\neq i}^{n} \frac{\partial \pi_l}{\partial p_l} = \sum_{l=1\neq i}^{n} a_l - 2d\sum_{l=1\neq i}^{n} p_l + (n-1)bp_i + (n-2)b\sum_{l=1\neq i}^{n} p_l = 0. \tag{3.77}$$

Dividing by $(n-1)$ yields

$$\bar{a}_{-i} - 2d\bar{p}_{-i} + bp_i + (n-2)b\bar{p}_{-i} = 0 \tag{3.78}$$

with $\bar{a}_{-i} = \sum_{l=1\neq i}^{n} a_l/(n-1)$. Solving for $\bar{p}_{-i}$ gives

$$\bar{p}_{-i} = \frac{\bar{a}_{-i} + bp_i}{2d - b(n-2)}. \tag{3.79}$$

Solving $p_i = R_i(\bar{p}_{-i})$ for p_i then yields

$$p_i = \frac{a_i + b(n-1)\frac{\bar{a}_{-i}+bp_i}{2d-b(n-2)}}{2d} \Leftrightarrow p_i^B = \frac{2a_i d - (n-2)a_i b + b(n-1)\bar{a}_{-i}}{(2d+b)u} \tag{3.80}$$

with $u = 2d + b - bn$ for $i \in N$.
For an industry-wide cartel, the coalition profit is

$$\pi_N = \Big(a_i - dp_i + b\sum_{j=1\neq i}^{n} p_j\Big)p_i + \sum_{t=1\neq i}^{n}\Big(a_t - dp_t + b\sum_{r=1\neq t}^{n} p_r\Big)p_t. \tag{3.81}$$

This corresponds to an encompassing multiproduct monopolist. Taking first derivatives gives the FOC

$$\frac{\partial \pi_N}{\partial p_i} = a_i - 2dp_i + 2b\sum_{t=1\neq i}^{n} p_t = 0 \qquad \forall\ i \in N. \tag{3.82}$$

The FOC for a fixed product i can also be written as

$$a_i - 2dp_i + 2b(n-1)\bar{p}_{-i} = 0 \tag{3.83}$$

with $\bar{p}_{-i} = \sum_{t=1\neq i}^{n} p_t/(n-1)$. Solving for p_i yields

$$p_i = \frac{a_i + 2b(n-1)\bar{p}_{-i}}{2d}. \tag{3.84}$$

Adding the remaining $(n-1)$ FOC and dividing by $(n-1)$ gives

$$\bar{a}_{-i} - 2d\bar{p}_{-i} + 2b(n-2)\bar{p}_{-i} + 2bp_i = 0 \Leftrightarrow \bar{p}_{-i} = \frac{\bar{a}_{-i} + 2bp_i}{2d - 2b(n-2)}. \tag{3.85}$$

Substituting this for $\bar{p}_{-i}$ in equation (3.84) yields

$$p_i = \frac{a_i + 2b(n-1)\frac{\bar{a}_{-i}+2bp_i}{2d-2b(n-2)}}{2d} \Leftrightarrow p_i^C = \frac{a_i d - (n-2)a_i b + b(n-1)\bar{a}_{-i}}{2(b+d)x} \tag{3.86}$$

with $x = d + b - bn$.

It follows that the resulting cartel overcharge is

$$\begin{aligned}
\Delta p_i &= p_i^C - p_i^B \\
&= \frac{(a_i d-(n-2)a_i b+b(n-1)\bar{a}_{-i})(2d+b)u-(2a_i d-(n-2)a_i b+b(n-1)\bar{a}_{-i})2(b+d)x}{2(b+d)(d+b-bn)(2d+b)(2d+b-bn)} \\
&= \frac{a_i[(d-(n-2)b)(2d+b)u+((n-2)b-2d)2(b+d)x]}{2(b+d)(d+b-bn)(2d+b)(2d+b-bn)} \\
&\quad + \frac{\bar{a}_{-i}[b(n-1)(2d+b)u-b(n-1)2(b+d)x]}{2(b+d)(d+b-bn)(2d+b)(2d+b-bn)}.
\end{aligned} \tag{3.87}$$

The bracketed factor on a_i in the numerator, after substituting x and u, can be simplified to

$$\begin{aligned}
&(d-(n-2)b)(2d+b)(2d+b-bn)+((n-2)b-2d)2(b+d)(d+b-bn) \\
=&2b^3+9b^2d+12bd^2+4d^3-3b^3n-9b^2dn-6bd^2n+b^3n^2+2b^2dn^2 \\
&-4b^3-12b^2d-12bd^2-4d^3+6b^3n+12b^2dn+6bd^2n-2b^3n^2-2b^2dn^2 \\
=&3b^3n+3b^2dn-3b^2d-2b^3-b^3n^2=b^2(n-1)(3d+2b-bn).
\end{aligned} \tag{3.88}$$

Similarly the bracketed factor on $\bar{a}_{-i}$ is

$$\begin{aligned}
&b(n-1)(2d+b)(2d+b-bn)-b(n-1)2(b+d)(d+b-bn) \\
=&-b^3-4b^2d-4bd^2+2b^3n+6b^2dn+4bd^2n-b^3n^2-2b^2dn^2 \\
&-(-2b^3-4b^2d-2bd^2+4b^3n+6b^2dn+2bd^2n-2b^3n^2-2b^2dn^2) \\
=&b^3-2bd^2-2b^3n+2bd^2n+b^3n^2=b(n-1)(2d^2+b^2n-b^2).
\end{aligned} \tag{3.89}$$

Inserting both factors back into equation (3.87) finally gives

$$\Delta p_i = \frac{b(n-1)[b(3d+2b-bn)a_i+(2d^2+b^2n-b^2)\bar{a}_{-i}]}{2(d+b)(2d+b)(d+b-bn)(2d+b-bn)}. \tag{3.90}$$

3.7.2.2 Shapley Value of Firm *i*

As above, let firms $S = \{1, \ldots, s\}$ coordinate their actions while the remaining firms $s+1, \ldots, n$ act competitively. First, the price of a cartel outsider given coalition size s

will be derived. W.l.o.g. focus on outsider firm n. Profits for S and the outsiders are

$$\begin{aligned}
\pi_S &= \sum_{t=1}^{s} \Big(a_t - dp_t + bp_n + b \sum_{r=1 \neq t}^{s} p_r + b \sum_{l=s+1}^{n-1} p_l\Big) p_t \\
\pi_{s+1} &= \Big(a_{s+1} - dp_{s+1} + bp_n + b \sum_{t=1}^{s} p_t + b \sum_{l=s+2}^{n-1} p_l\Big) p_{s+1} \\
&\vdots \\
\pi_n &= \Big(a_n - dp_n + b \sum_{t=1}^{s} p_t + b \sum_{l=s+1}^{n-1} p_l\Big) p_n. \qquad (3.91)
\end{aligned}$$

Taking first derivatives gives the FOC

$$\begin{aligned}
\frac{\partial \pi_S}{\partial p_1} &= a_1 - 2dp_1 + bp_n + 2b \sum_{t=2}^{s} p_t + b \sum_{l=s+1}^{n-1} p_l = 0 \\
&\vdots \\
\frac{\partial \pi_S}{\partial p_s} &= a_s - 2dp_s + bp_n + 2b \sum_{t=1}^{s-1} p_t + b \sum_{l=s+1}^{n-1} p_l = 0 \\
\frac{\partial \pi_{s+1}}{\partial p_{s+1}} &= a_{s+1} - 2dp_{s+1} + bp_n + b \sum_{t=1}^{s} p_t + b \sum_{l=s+2}^{n-1} p_l = 0 \\
&\vdots \\
\frac{\partial \pi_n}{\partial p_n} &= a_n - 2dp_n + b \sum_{t=1}^{s} p_t + b \sum_{l=s+1}^{n-1} p_l = 0. \qquad (3.92)
\end{aligned}$$

The FOC of firm n can be rewritten as

$$\frac{\partial \pi_n}{\partial p_n} = a_n - 2dp_n + bs\bar{p}^s + b(n-s-1)\bar{p}^{\not s}_{-n} \qquad (3.93)$$

with $\bar{p}^s = \sum_{t=1}^{s} p_t / s$ and $\bar{p}^{\not s}_{-n} = \sum_{l=s+1}^{n-1} p_l / (n-s-1)$. Solving for p_n gives the best response function of firm n:

$$R_n(\bar{p}^s; \bar{p}^{\not s}_{-n}) = \frac{a_n + bs\bar{p}^s + b(n-s-1)\bar{p}^{\not s}_{-n}}{2d}. \qquad (3.94)$$

The FOC for all $i \in S$ can be added to

$$\sum_{t=1}^{s} \frac{\partial \pi_S}{\partial p_t} = \frac{\partial \pi_1}{\partial p_1} + \ldots + \frac{\partial \pi_s}{\partial p_s} = \sum_{t=1}^{s} a_t - 2d \sum_{t=1}^{s} p_t + sbp_n + 2b(s-1)\sum_{t=1}^{s} p_t + sb \sum_{l=s+1}^{n-1} p_l = 0. \tag{3.95}$$

Dividing by s gives:

$$\bar{a}^s - 2d\bar{p}^s + bp_n + 2b(s-1)\bar{p}^s + b(n-s-1)\bar{p}^{\not s}_{-n} = 0$$
$$\Leftrightarrow \bar{p}^s(p_n; \bar{p}^{\not s}_{-n}) = \frac{\bar{a}^s + bp_n + b(n-s-1)\bar{p}^{\not s}_{-n}}{2d - 2b(s-1)} \tag{3.96}$$

with $\bar{a}^s = \sum_{t=1}^{s} a_t/s$. Doing the same for all $i \in \{s+1, \ldots, n-1\}$ gives:

$$\sum_{s+1}^{n-1} \frac{\partial \pi_l}{\partial p_l} = \frac{\partial \pi_{s+1}}{\partial p_{s+1}} + \ldots + \frac{\partial \pi_{n-1}}{\partial p_{n-1}} \tag{3.97}$$
$$= \sum_{l=s+1}^{n-1} a_l - 2d \sum_{l=s+1}^{n-1} p_l + (n-s-1)bp_n + b(n-s-1)\sum_{t=1}^{s} p_t + (n-s-2)b \sum_{l=s+1}^{n-1} p_l = 0. \tag{3.98}$$

Dividing by $(n-s-1)$ then yields

$$\bar{a}^{\not s}_{-n} - 2d\bar{p}^{\not s}_{-n} + bp_n + bs\bar{p}^s + (n-s-2)b\bar{p}^{\not s}_{-n} = 0 \Leftrightarrow p^{\not s}_{-n}(p_n; \bar{p}^s) = \frac{\bar{a}^{\not s}_{-n} + bp_n + bs\bar{p}^s}{2d - b(n-s-2)} \tag{3.99}$$

with $\bar{a}^{\not s}_{-n} = \sum_{l=s+1}^{n-1} a_l/(n-s-1)$.

It remains to solve for p_n. Inserting $p^{\not s}_{-n}(p_n; \bar{p}^s)$ into $R_n(\bar{p}^s; \bar{p}^{\not s}_{-n})$ resp. $\bar{p}^s(p_n; \bar{p}^{\not s}_{-n})$ gives

$$R_n(\bar{p}^s; p^{\not s}_{-n}(p_n; \bar{p}^s)) = \frac{a_n + bs\bar{p}^s + b(n-s-1)\frac{\bar{a}^{\not s}_{-n} + bs\bar{p}^s + bp_n}{2d-(n-s-2)b}}{2d}$$
$$\Leftrightarrow R_n(\bar{p}^s) = \frac{a_n(b(-2+n-s) - 2d) + b(\bar{a}^{\not s}_{-n}(1-n+s) - s(b+2d)\bar{p}^s)}{(b+2d)(-2d+b(-1+n-s))}; \tag{3.100}$$

$$\bar{p}^s(p_n; p^{\not s}_{-n}(p_n; \bar{p}^s)) = \frac{\bar{a}^s + bp_n + b(n-s-1)\frac{\bar{a}^{\not s}_{-n} + bs\bar{p}^s + bp_n}{2d-(n-s-2)b}}{2d - 2(s-1)b}$$
$$\Leftrightarrow \bar{p}^s(p_n) = \frac{b(b+2d)p_n + \bar{a}^{\not s}_{-n}b(-1+n-s) + \bar{a}^s(2d + b(2-n+s))}{4d^2 - 2bd(-4+n+s) + b^2(4+n(-2+s) - s - s^2)}. \tag{3.101}$$

Inserting $\bar{p}^s(p_n)$ into $R_n(\bar{p}^s)$ and noting that firm n can be replaced by any firm $i \notin S$

gives

$$
\begin{aligned}
p_i^{\not s} &= \frac{a_i(b(-2+n-s)-2d) + b\left(\bar{a}_{-i}^{\not s}(1-n+s) - s(b+2d)\frac{b(b+2d)p_i + \bar{a}_{-i}^{\not s} b(-1+n-s) + \bar{a}^s(2d+b(2-n+s))}{4d^2-2bd(-4+n+s)+b^2(4+n(-2+s)-s-s^2)}\right)}{(b+2d)(-2d+b(-1+n-s))} \\
&= \frac{a_i(4d^2-2bd(n+s-4)+b^2\mu_s) + \bar{a}^s b(b+2d)s + \bar{a}_{-i}^{\not s} b(b(s-2)-2d)(1-n+s)}{(2d+b)[4d^2-(2n-6+2s)bd+b^2(2-2n+sn-s^2)]}
\end{aligned}
\tag{3.102}
$$

with $\mu_s = 4 - 2n + sn - s^2 - s$.

Next, the price of a cartel insider given coalition size s will be derived. W.l.o.g focus on insider $i = 1$. The profit functions in equation (3.91) can also be written as

$$
\begin{aligned}
\pi_S &= \Big(a_1 - dp_1 + b\sum_{t=2}^{s} p_t + b\sum_{l=s+1}^{n} p_l\Big)p_1 + \sum_{t=2}^{s}\Big(a_t - dp_t + b\sum_{r=1\neq t}^{s} p_r + b\sum_{l=s+1}^{n} p_l\Big)p_t \\
\pi_{s+1} &= \Big(a_{s+1} - dp_{s+1} + bp_1 + b\sum_{t=2}^{s} p_t + b\sum_{l=s+2}^{n} p_l\Big)p_{s+1} \\
&\ \ \vdots \\
\pi_n &= \Big(a_n - dp_n + bp_1 + b\sum_{t=2}^{s} p_t + b\sum_{l=s+1}^{n-1} p_l\Big)p_n.
\end{aligned}
\tag{3.103}
$$

Respective FOC are given by (3.92) above which we can also write as

$$
\begin{aligned}
\frac{\partial \pi_S}{\partial p_1} &= a_1 - 2dp_1 + 2b\sum_{t=2}^{s} p_t + b\sum_{l=s+1}^{n} p_l = 0 \\
&\ \ \vdots \\
\frac{\partial \pi_S}{\partial p_s} &= a_s - 2dp_s + 2bp_1 + 2b\sum_{t=2}^{s-1} p_t + b\sum_{l=s+1}^{n} p_l = 0 \\
\frac{\partial \pi_{s+1}}{\partial p_{s+1}} &= a_{s+1} - 2dp_{s+1} + bp_1 + b\sum_{t=2}^{s} p_t + b\sum_{l=s+2}^{n} p_l = 0 \\
&\ \ \vdots \\
\frac{\partial \pi_n}{\partial p_n} &= a_n - 2dp_n + bp_1 + b\sum_{t=2}^{s} p_t + b\sum_{l=s+1}^{n-1} p_l = 0.
\end{aligned}
\tag{3.104}
$$

Rearranging the FOC for product 1 gives

$$\begin{aligned}\frac{\partial \pi_S}{\partial p_1} &= a_1 - 2dp_1 + 2(s-1)b\bar{p}^s_{-1} + b(n-s)\bar{p}^{\not s} = 0\\ \Leftrightarrow R_1(\bar{p}^s_{-1};\bar{p}^{\not s}) &= \frac{a_1 + 2(s-1)b\bar{p}^s_{-1} + b(n-s)\bar{p}^{\not s}}{2d}\end{aligned} \tag{3.105}$$

with $\bar{p}^s_{-1} = \sum_{t=2}^{s} p_t/(s-1)$ and $\bar{p}^{\not s} = \sum_{l=s+1}^{n} p_l/(n-s)$. The FOC for products $2,\ldots,s$ can be added to

$$\begin{aligned}\sum_{t=2}^{s}\frac{\partial \pi_S}{\partial p_t} &= \frac{\partial \pi_2}{\partial p_2} + \ldots + \frac{\partial \pi_s}{\partial p_s}\\ &= \sum_{t=2}^{s} a_t - 2d\sum_{t=2}^{s} p_t + 2(s-1)bp_1 + 2b(s-2)\sum_{t=2}^{s} p_t + (s-1)b\sum_{l=s+1}^{n-1} p_l = 0.\end{aligned} \tag{3.106}$$

Dividing by $(s-1)$ and solving for $\bar{p}^s_{-1}$ yields

$$\begin{aligned}\bar{a}^s_{-1} - 2d\bar{p}^s_{-1} + 2bp_1 + 2b(s-2)\bar{p}^s_{-1} + b(n-s)\bar{p}^{\not s} = 0&\\ \Leftrightarrow \bar{p}^s_{-1}(p_1;\bar{p}^{\not s}) = \frac{\bar{a}^s_{-1} + 2bp_1 + b(n-s)\bar{p}^{\not s}}{2d - 2b(s-2)}&\end{aligned} \tag{3.107}$$

with $\bar{a}^s_{-1} = \sum_{t=2}^{s} a_t/(s-1)$. Doing the same for all $i \notin S$ gives

$$\begin{aligned}\sum_{s+1}^{n}\frac{\partial \pi_l}{\partial p_l} &= \frac{\partial \pi_{s+1}}{\partial p_{s+1}} + \ldots + \frac{\partial \pi_n}{\partial p_n}\\ &= \sum_{l=s+1}^{n} a_l - 2d\sum_{l=s+1}^{n} p_l + (n-s)bp_1 + b(n-s)\sum_{t=1}^{s} p_t + (n-s-1)b\sum_{l=s+1}^{n} p_l = 0.\end{aligned} \tag{3.108}$$

Dividing by $(n-s)$ yields

$$\begin{aligned}\bar{a}^{\not s} - 2d\bar{p}^{\not s} + bp_1 + b(s-1)\bar{p}^s_{-1} + (n-s-1)b\bar{p}^{\not s} = 0&\\ \Leftrightarrow \bar{p}^{\not s}(p_1;\bar{p}^s_{-1}) = \frac{\bar{a}^{\not s} + bp_1 + b(s-1)\bar{p}^s_{-1}}{2d - (n-s-1)b}&\end{aligned} \tag{3.109}$$

with $\bar{a}^{\not s} = \sum_{l=s+1}^{n} a_l/(n-s)$.

Next, solve for p_1. Inserting $p^{\not s}(p_1; \bar{p}^s_{-1})$ into $R_1(\bar{p}^s_{-1}; \bar{p}^{\not s})$ resp. $\bar{p}^s_{-1}(p_1; \bar{p}^{\not s})$ then gives

$$\begin{aligned} R_1(\bar{p}^s_{-1}; p^{\not s}(p_1; \bar{p}^s_{-1})) &= \frac{a_1 + 2(s-1)b\bar{p}^s_{-1} + b(n-s)\frac{\bar{a}^{\not s} + b(s-1)\bar{p}^s_{-1} + bp_1}{2d - b(n-s-1)}}{2d} \\ \Leftrightarrow R_1(\bar{p}^s_{-1}) &= \frac{a_1(2d + b(1-n+s)) - b[\bar{a}^{\not s}(s-n) - \bar{p}^s_{-1}(s-1)(4d + b(2-n+s))]}{(b+2d)(2d - b(n-s))} \end{aligned} \tag{3.110}$$

and

$$\begin{aligned} \bar{p}^s_{-1}(p_1; p^{\not s}(p_1; \bar{p}^s_{-1})) &= \frac{\bar{a}^s_{-1} + 2bp_1 + b(n-s)\frac{\bar{a}^{\not s} + bp_1 + b(s-1)\bar{p}^s_{-1}}{2d - (n-s-1)b}}{2d - 2b(s-2)} \\ \Leftrightarrow \bar{p}^s_{-1}(p_1) &= \frac{\bar{a}^s_{-1}(2d + b(1-n+s)) + b[\bar{a}^{\not s}(n-s) + p_1(4d + b(2-n+s))]}{4d^2 - 2bd(n-5+s) + b^2(4 + n(s-3) + s - s^2)}. \end{aligned} \tag{3.111}$$

Inserting $\bar{p}^s_{-1}(p_1)$ into $R_1(\bar{p}^s_{-1})$ yields

$$\begin{aligned} p^s_i &= \frac{a_i(2d + b(1-n+s))}{(b+2d)(2d - b(n-s))} \\ &\cdot \frac{-b\left[\bar{a}^{\not s}(s-n) - \frac{\bar{a}^s_{-i}(2d + b(1-n+s)) + b[\bar{a}^{\not s}(n-s) + p^s_i(4d + b(2-n+s))]}{4d^2 - 2bd(n-5+s) + b^2(4 + n(s-3) + s - s^2)}(s-1)(4d + b(2-n+s))\right]}{(b+2d)(2d - b(n-s))} \\ &= \frac{a_i(4d^2 - 2bd(n+s-5) + b^2\kappa_s) + 2\bar{a}^{\not s}b(b+d)(n-s) + \bar{a}^s_{-i}b(s-1)(4d + b(2-n+s)))}{2(b+d)[4d^2 - (2n-6+2s)bd + b^2(2 - 2n + sn - s^2)]} \end{aligned} \tag{3.112}$$

with $\kappa_s = sn - 3n - s^2 + s + 4$ for $i = 1$ as well as any other $i \in S$.

The average of coefficients a_i for cartel members resp. non-cartel members in (3.102) and (3.112) can depend on the specific i considered and on coalition size s. It is possible that $\bar{a}^s \neq \bar{a}^{\not s} \neq \bar{a}^s_{-i} \neq \bar{a}^{\not s}_{-i}$ depending on the specific asymmetries between firms. Proposition 1 however establishes that only the average price for product i given coalition size s is needed in order to determine the Shapley value of firm i. To derive the average price of product i for all $S \ni i$ with coalition size s, one can enumerate all such coalitions S with $i \in S$ and add the resulting prices p^s_i. The same applies for all $S \not\ni i$ and $p^{\not s}_i$. In these sums, all firm-specific saturation quantities occur equally many times. Dividing by the cardinalities $\#\{S : i \in S\}$ and $\#\{S : i \notin S\}$ it turns out that only the average coefficient $\bar{a}_{-i} = \sum_{j \neq i}^{n} a_j/(n-1)$ matters and we obtain

$$
\begin{aligned}
\bar{p}_i^s =& \frac{a_i(4d^2 - 2bd(n+s-5) + b^2\kappa_s) + \bar{a}_{-i}b(2d(n+s-2) - b(2+n(s-3)+s-s^2))}{(2d+2b)[4d^2-(2n-6+2s)bd+b^2(2-2n+sn-s^2)]} \\
\bar{p}_i^{\not s} =& \frac{a_i(4d^2 - 2bd(n+s-4) + b^2\mu_s) + \bar{a}_{-i}b(2d(n-1) + b(s^2-2-n(s-2)))}{(2d+b)[4d^2-(2n-6+2s)bd+b^2(2-2n+sn-s^2)]}.
\end{aligned}
\tag{3.113}
$$

The denominators of $\bar{p}_i^s$ and $\bar{p}_i^{\not s}$ differ only in the first factor. In order to compute $\bar{p}_i^s - \bar{p}_i^{\not s}$ it is convenient to define

$$
\begin{aligned}
A_i :=& (4d^2 - 2bd(n+s-5) + b^2(sn-3n-s^2+s+4))(2d+b) \\
& - (4d^2 - 2bd(n+s-4) + b^2(4-2n+sn-s^2-s))2(b+d) \\
=& 4b^3+18b^2d+24bd^2+8d^3-3b^3n-8b^2dn-4bd^2n+b^3s-4bd^2s+b^3ns+2b^2dns-b^3s^2 \\
& -2b^2ds^2 - [8b^3+24b^2d+24bd^2+8d^3-4b^3n-8b^2dn-4bd^2n-2b^3s-6b^2ds \\
& -4bd^2s+2b^3ns+2b^2dns-2b^3s^2-2b^2ds^2] \\
=& -4b^3-6b^2d+b^3n+3b^3s+6b^2ds-b^3ns+b^3s^2 = b^2(s-1)(6d+b(s+4-n))
\end{aligned}
\tag{3.114}
$$

and similarly

$$
\begin{aligned}
A_{-i} :=& b[(2d(n+s-2)-b(2+n(s-3)+s-s^2))(2d+b)-(2d(n-1)+b(s^2-2-n(s-2)))2(b+d)] \\
=& b[-2b^2-8bd-8d^2+3b^2n+8bdn+4d^2n-b^2s+4d^2s-b^2ns-2bdns+b^2s^2+2bds^2 \\
& -[-4b^2-8bd-4d^2+4b^2n+8bdn+4d^2n-2b^2ns-2bdns+2b^2s^2+2bds^2]] \\
=& b[2b^2-4d^2-b^2n-b^2s+4d^2s+b^2ns-b^2s^2] = b(s-1)(4d^2+b^2(n-s-2)).
\end{aligned}
\tag{3.115}
$$

With these terms we can first move to a common denominator and then simplify to

$$
\begin{aligned}
p_i^s - p_i^{\not s} =& \frac{a_i \cdot A_i + \bar{a}_{-i} \cdot A_{-i}}{(2d+2b)(2d+b)[4d^2-(2n-6+2s)db+b^2(2-2n+sn-s^2)]} \\
=& \frac{b(s-1)[b(6d+b(s+4-n))a_i + (4d^2+\tau_s b^2)\bar{a}_{-i}]}{(2d+2b)(2d+b)[4d^2-(2n-6+2s)db+b^2(2-2n+sn-s^2)]}
\end{aligned}
\tag{3.116}
$$

with $\tau_s = n - s - 2$.

Now fix firm $i = h$ as the firm which sold the product whose overcharges are to be allocated. The Shapley value of this "home firm" is

$$\varphi_h(N,v) = \frac{\Delta p_h}{n} + \frac{1}{n}\sum_{s=2}^{n-1}[p_h^s - p_h^{\not s}] \tag{3.117}$$

$$= \frac{\Delta p_h}{n} + \frac{1}{n}\sum_{s=2}^{n-1}\frac{b(s-1)[b(6d+b(s+4-n))a_h + (4d^2+\tau_s b^2)\bar{a}_{-h}]}{2(d+b)(2d+b)[4d^2-(2n-6+2s)db+b^2(2-2n+sn-s^2)]} \tag{3.118}$$

by Theorem 2.

To obtain the share firm h has to contribute, divide $\varphi_h(N,v)$ by Δp_h (see equ. (3.90)). This yields

$$\rho_h^* = \frac{\varphi_h}{\Delta p_h} = \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1}\frac{\frac{(s-1)b[b(6d+b(s+4-n))a_h+(4d^2+\tau_s b^2)\bar{a}_{-h}]}{2(d+b)(2d+b)[4d^2-(2n-6+2s)db+b^2(2-2n+sn-s^2)]}}{\frac{b(n-1)[b(3d+2d-bn)a_h+(2d^2+b^2n-b^2)\bar{a}_{-h}]}{2(d+b)(2d+b)(d+b-bn)(2d+b-bn)}}$$

$$= \frac{1}{n}\Bigg(1 + \sum_{s=2}^{n-1}\frac{(s-1)\Big[b(6d+b(s+4-n))a_h + (4d^2+\tau_s b^2)\bar{a}_{-h}\Big]\cdot(d+b-bn)(2d+b-bn)}{\Big(4d^2-(2n-6+2s)db+\eta_s b^2\Big)\cdot(n-1)\Big[b(3d+2b-bn)a_h + (2d^2+b^2n-b^2)\bar{a}_{-h}\Big]}\Bigg). \tag{3.119}$$

Substituting $d = \alpha(n-1)b$ and rearranging gives

$$\Big(\rho_h^* - \frac{1}{n}\Big)n$$

$$= \sum_{s=2}^{n-1}\frac{b^4(s-1)[6\alpha(n-1)+(s+4-n)+(4\alpha^2(n-1)^2+\tau_s)\frac{\bar{a}_{-h}}{a_h}](\alpha-1)(2\alpha-1)(n-1)^2}{b^4(4\alpha^2(n-1)^2-(2n-6+2s)\alpha(n-1)+\eta_s)(n-1)[(3\alpha(n-1)+2-n)+(2\alpha^2(n-1)^2+n-1)\frac{\bar{a}_{-h}}{a_h}]}$$

$$= \sum_{s=2}^{n-1}\frac{(s-1)[6\alpha(n-1)+(s+4-n)+(4\alpha^2(n-1)^2+\tau_s)\frac{\bar{a}_{-h}}{a_h}]\cdot(\alpha-1)(2\alpha-1)}{(4\alpha^2(n-1)^2-(2n-6+2s)(n-1)\alpha+\eta_s)\cdot[(3\alpha+\frac{2-n}{n-1})+(2\alpha^2(n-1)+1)\frac{\bar{a}_{-h}}{a_h}]}$$

and finally

$$\rho_h^* = \frac{1}{n} + \frac{1}{n(n-1)}\sum_{s=2}^{n-1}\frac{(s-1)\Big[6\alpha(n-1)+(s+4-n)+\Big(4\alpha^2(n-1)^2+\tau_s\Big)\frac{\bar{a}_{-h}}{a_h}\Big](\alpha-1)(2\alpha-1)}{\Big(4\alpha^2(n-1)-(2n-6+2s)\alpha+\frac{\eta_s}{n-1}\Big)\Big[(3\alpha+\frac{2-n}{n-1})+(2\alpha^2(n-1)+1)\frac{\bar{a}_{-h}}{a_h}\Big]} \tag{3.120}$$

with $\alpha = \frac{d}{(n-1)b}$, $\eta_s = s(n-s) - 2(n-1)$ and $\tau_s = n - s - 2$.

3.7.2.3 Shapley Value of Firm j

Given $h \in S$, firm j can either compete or not. The same applies for $h \notin S$. Then, it could be the case that $p^{\not{s}\not{s}} \neq p^{s\not{s}} \neq p^{\not{s}s} \neq p^{ss}$ (defined below). The first superscript indicates behavior of firm h; the second superscript behavior of firm j. $p^{s\not{s}}$ for example denotes the price when firm h is part of the coalition but firm j not; the price when both firms compete is $p^{\not{s}\not{s}}$. Prices $p^{\not{s}s}$ and p^{ss} are defined accordingly.

To derive these prices, we start with average prices given coalition size s defined in equation (3.113) with $h = i$. Note that the bracketed term after a_h and also the denominator of equation (3.113) will only depend on the fact whether firm h is part of the coalition or not. Hence, these parts stay unchanged for $p^{ss}, p^{s\not{s}}$, $p^{\not{s}s}$ and $p^{\not{s}\not{s}}$. The bracketed factor after $\bar{a}_{-h}$ can be separated into $a_j + \tilde{a}$ with $\tilde{a} = \sum_{t \neq h \neq j}^{n} a_t/(n-2)$. Prices are therefore given by

$$p^{ss} := \frac{a_h(4d^2-2bd(n+s-5)+b^2\kappa_s)+a_j(4bd-\tau_s b^2)+\tilde{a}(2bd(n+s-4)-b^2(ns-s^2+4-4n+2s))}{2(b+d)[4d^2-(2n-6+2s)bd+b^2(2-2n+sn-s^2)]}$$

$$p^{s\not{s}} := \frac{a_h(4d^2-2bd(n+s-5)+b^2\kappa_s)+a_j(2bd+2b^2)+\tilde{a}(2bd(n+s-3)-b^2(ns-s^2-3n+s+4))}{2(b+d)[4d^2-(2n-6+2s)bd+b^2(2-2n+sn-s^2)]}$$

$$p^{\not{s}s} := \frac{a_h(4d^2-2bd(n+s-4)+\mu_s b^2)+a_j(2bd+b^2)+\tilde{a}(2bd(n-2)+b^2(2n-3-sn+s^2))}{(2d+b)[4d^2-(2n-6+2s)bd+b^2(2-2n+sn-s^2)]}$$

$$p^{\not{s}\not{s}} := \frac{a_h(4d^2-2bd(n+s-4)+b^2\mu_s)+a_j(2bd+b^2(2-s))+\tilde{a}(2bd(n-2)+b^2(2n+s-4-sn+s^2))}{(2d+b)[4d^2-(2n-6+2s)bd+b^2(2-2n+sn-s^2)]}.$$

By formula (3.5), only average prices for each coalition size matter. Thus, we next derive the cardinality these prices occur, given coalition size s:

$$C_1 := \#p^{ss} = \frac{(n-2)!}{(s-2)!(n-s)!};\ C_2 := \#p^{\not{s}s} = \frac{(n-2)!}{(s-1)!(n-1-s)!};$$
$$C_3 := \#p^{s\not{s}} = \frac{(n-2)!}{(s-1)!(n-1-s)!};\ C_4 := \#p^{\not{s}\not{s}} = \frac{n!}{s!(n-s)!} - \frac{(n-2)!(s-1+2(n-s))}{(n-s)!(s-1)!}. \tag{3.121}$$

The Shapley value of firm j therefore is

$$\varphi_j(N, v) = \frac{\Delta p_h}{n} + \frac{1}{n} \sum_{s=2}^{n-1} \Big[\frac{C_1 p^{s\,s} + C_2 p^{\bcancel{s}\,s}}{C_1 + C_2} - \frac{C_3 p^{s\,\bcancel{s}} + C_4 p^{\bcancel{s}\,\bcancel{s}}}{C_3 + C_4} \Big]. \tag{3.122}$$

Substituting the terms C_1, $p^{s\,s}$, etc. and summarizing yields

$$\begin{aligned} \varphi_j(N, v) = {} & \frac{\Delta p_h}{n} \\ & + \frac{1}{2n(n-1)} \sum_{s=2}^{n-1} \Big[\frac{b(s-1)[a_h b(b(n-4-s) - 6d) + (a_j - 2\tilde{a})(4d^2 + b^2\tau_s)]}{(2d+b)(b+d)[4d^2 - bd(-6+2n+2s) + b^2(2-2n+ns-s^2)]} \Big]. \end{aligned} \tag{3.123}$$

3.7.2.4 Monotonicity of ρ_h^*

Firm h's Shapley share (3.120) in compensation to its own customers has two summands. The second one can only be non-zero for $n \geq 3$: for $n = 2$ the Shapley value entails an equal allocation by heads (which follows directly from (SYM) and (EFF)). So consider $n \geq 3$ from now on.

Let us first show that ρ_h^* is strictly decreasing in $\bar{a}_{-h}/a_h$. Some placeholders are introduced to simplify notation:

$$\begin{aligned} \rho_h^* = {} & \frac{1}{n} \\ & + \frac{1}{n(n-1)} \sum_{s=2}^{n-1} \frac{\overbrace{(s-1)}^{=:A} \Big[\overbrace{6\alpha(n-1) + (s+4-n)}^{=:B} + \Big(\overbrace{4\alpha^2(n-1)^2 + \tau_s}^{=:C} \Big) \overbrace{\frac{\bar{a}_{-h}}{a_h}}^{=:D} \Big] \overbrace{(\alpha-1)(2\alpha-1)}^{=:E}}{\underbrace{\Big(4\alpha^2(n-1) - (2n-6+2s)\alpha + \frac{\eta_s}{n-1} \Big)}_{=:F} \Big[\underbrace{\Big(3\alpha + \frac{2-n}{n-1} \Big)}_{=:G} + \underbrace{(2\alpha^2(n-1)+1)}_{=:H} \frac{\bar{a}_{-h}}{a_h} \Big]} \\ = {} & \frac{1}{n} + \frac{1}{n(n-1)} \sum_{s=2}^{n-1} \underbrace{\frac{A \cdot [B + C \cdot D] \cdot E}{F \cdot [G + H \cdot D]}}_{=:\tilde{\rho}_h^s}. \end{aligned} \tag{3.124}$$

We start by showing that all placeholders are positive. This is obvious for A, D, E, G and H given $s \geq 2$, positive saturation quantities $a_i > 0$ and $\alpha > 1$. Part B is increasing in s and α, hence

$$B = 6\alpha(n-1) + s + 4 - n \geq 6\alpha(n-1) + 6 - n \geq 5n > 0 \tag{3.125}$$

for $s \geq 2$ and $\alpha > 1$. Part C is decreasing in s and increasing in α. From $s \leq n-1$ and $\alpha > 1$ follows

$$4\alpha^2(n-1)^2 + n - s - 2 \geq 4(n-1)^2 - 1 > 0. \tag{3.126}$$

It remains to show that part F is positive. We first argue that

$$t(\alpha) := 4\alpha^2(n-1) - (2n - 6 + 2s)\alpha + \frac{s(n-s) - 2(n-1)}{n-1} \tag{3.127}$$

is increasing in α. This follows, since

$$\begin{aligned} &\frac{\partial t(\alpha)}{\partial \alpha} = 8\alpha(n-1) - 2n + 6 - 2s \geq 8\alpha(n-1) - 2n + 6 - 2(n-1) \\ \Leftrightarrow\ &\frac{\partial t(\alpha)}{4\partial \alpha} \geq 2\alpha(n-1) - n + 2 > 0. \end{aligned} \tag{3.128}$$

The last inequality is satisfied since $2\alpha > (n-2)/(n-1)$. So for all $\alpha > 1$ we can conclude

$$t(\alpha) > t(1) = 4(n-1) - (2n - 6 + 2s) + \frac{s(n-s) - 2(n-1)}{n-1}. \tag{3.129}$$

To show that $t(1) > 0$ it is sufficient to prove that

$$\tilde{t}(1) := (n-1)t(1) = 4(n-1)^2 - (2n - 6 + 2s)(n-1) + s(n-s) - 2(n-1) > 0. \tag{3.130}$$

Collecting the terms in (3.130) which depend on s yields $2s - sn - s^2$. $\tilde{t}(1)$ is therefore decreasing in s for $n \geq 3$ and $s \geq 2$. Inserting $s = n-1$ gives

$$\begin{aligned} \tilde{t}(1) &\geq 4(n-1)^2 - (2n - 6 + 2(n-1))(n-1) + (n-1)(n-(n-1)) - 2(n-1) \\ &= (n-1)[4n - 4 - 4n + 8 + 1 - 2] = (n-1)3 > 0. \end{aligned} \tag{3.131}$$

So part F and hence all parts of ρ_h^* are positive.

Next we show that each addend $\tilde{\rho}_h^s = \frac{A \cdot [B + C \cdot D] \cdot E}{F \cdot [G + H \cdot D]}$ of ρ_h^* in equation (3.124) decreases in $D = \frac{\bar{a}_{-h}}{a_h}$. In particular, taking the first derivative with respect to D yields

$$\frac{\partial \tilde{\rho}_h^s}{\partial D} = \frac{ACEF[G+HD] - A[B+CD]EFH}{F^2[G+HD]^2} = \frac{\overbrace{ACE}^{>0}[CG-BH]}{\underbrace{F[G+HD]^2}_{>0}}. \tag{3.132}$$

It remains to show that $CG - BH < 0$. Substituting the terms C, G, B and H gives

$$\begin{aligned}
CG - BH =&(4\alpha^2(n-1)^2 + n - s - 2)\Big(3\alpha + \frac{2-n}{n-1}\Big) - (6\alpha(n-1) + s + 4 - n)(2\alpha^2(n-1) + 1)\\
=&(4\alpha^2(n-1)^2+n-s-2)(3\alpha(n-1)-n+2)) - (6\alpha(n-1)+s+4-n)(2\alpha^2(n-1)^2+n-1)\\
=&(4\alpha^2n^2 - 8\alpha^2n + 4\alpha^2 + 2 - s - 2)(3\alpha n - 3\alpha - n + 2)\\
&- (6\alpha n - 6\alpha + s + 4 - n)(2\alpha^2n^2 - 4\alpha^2n + 2\alpha^2 + n - 1)\\
=&12\alpha^3n^3 - 12\alpha^3n^2 - 4\alpha^2n^3 + 8\alpha^2n^2 - 24\alpha^3n^2 + 24\alpha^3n + 8\alpha^2n^2 - 16\alpha^2n + 12\alpha^3n\\
&- 12\alpha^3 - 4\alpha^2n + 8\alpha^2 + 3\alpha n^2 - 3\alpha n + n^2 + 2n - 3\alpha ns + 3\alpha s + ns - 2s - 6\alpha n\\
&+ 6\alpha + 2n - 4 - [12\alpha^3n^3 - 12\alpha^3n^2 + 2\alpha^2n^2s + 8\alpha^2n^2 - 2\alpha^2n^3 - 24\alpha^3n^2\\
&+ 24\alpha^3n - 4\alpha^2ns - 16\alpha^2n + 4\alpha^2n^2 + 12\alpha^3n - 12\alpha^3 + 2\alpha^2s + 8\alpha^2 - 2\alpha^2n\\
&+ 6\alpha n^2 - 6\alpha n + sn + 4n - n^2 - 6\alpha n + 6\alpha - s - 4 + n]\\
=& -2\alpha^2n^3 + 4\alpha^2n^2 - 2\alpha^2n - 3\alpha n^2 + 3\alpha n - n - 3\alpha ns + 3\alpha s - s - 2\alpha^2n^2s + 4\alpha^2ns - 2\alpha^2s\\
=& -\underbrace{(n-s)}_{>0}\underbrace{(1 + 3(n-1)\alpha + 2(n-1)^2\alpha^2)}_{>0} < 0.
\end{aligned} \tag{3.133}$$

So each addend $\tilde{\rho}_h^s$ decreases in D. It follows that also ρ_h^* is decreasing in $\bar{a}_{-h}/a_h$.

3.7.2.5 Upper Bound

Since ρ_h^* is decreasing in $\bar{a}_{-h}/a_h$ we can conclude that

$$\rho_h^* \leq \lim_{\bar{a}_{-h}/a_h \to 0} \rho_h^* = \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1} \frac{(s-1)}{(n-1)} \cdot \frac{\Big[6\alpha(n-1) + (s+4-n)\Big] \cdot (\alpha-1)(2\alpha-1)}{\Big(4\alpha^2(n-1) - (2n-6+2s)\alpha + \frac{\eta_s}{n-1}\Big) \cdot \Big(3\alpha + \frac{2-n}{n-1}\Big)} \tag{3.134}$$

with $\eta_s = s(n-s) - 2(n-1)$. The right-most fraction, with numerator $[\ldots]\cdot(\alpha-1)(2\alpha-1)$, is maximal for $s = n-1$, since the numerator is positive and increasing in s whereas the denominator is also positive but decreasing in s: positivity of the denominator follows from equations (3.127)–(3.131) and $(3\alpha + (2-n)/(n-1)) > 0$. To see that the denominator is decreasing in s note that the parts in the denominator which depend on s can be collected to

$$\mathcal{D}_s := \frac{s \cdot (-2\alpha(n-1) + n - s)}{n-1}. \tag{3.135}$$

The factor in parentheses in equation (3.135) is negative and decreasing in s for $n \geq 3$, $s \geq 2$ and $\alpha > 1$. Higher s therefore lead to more negative values of $\mathcal{D}_s$. The denominator in (3.134)'s right-most fraction is therefore decreasing in s. We can conclude that a greater s leads to a larger value of the right-hand side of (3.134).

Inserting $s = n-1$ into (3.134)'s right-most fraction gives

$$\rho_h^* \leq \lim_{\bar{a}_{-h}/a_h \to 0} \rho_h^* \leq \bar{\rho}_h^* := \frac{1}{n} + \frac{1}{n} \frac{\left[6\alpha(n-1)+3\right] \cdot (\alpha-1)(2\alpha-1)}{\left(4\alpha^2(n-1) - (4n-8)\alpha - 1\right) \cdot \left(3\alpha + \frac{2-n}{n-1}\right)} \cdot \sum_{s=2}^{n-1} \frac{(s-1)}{(n-1)}. \tag{3.136}$$

Next we will show that $\bar{\rho}_h^*$ is strictly increasing in α. It is sufficient to establish that

$$z(\alpha) := \frac{\overbrace{\left[6\alpha(n-1)+3\right] \cdot (\alpha-1)(2\alpha-1)}^{=:\mathcal{N}}}{\underbrace{\left(4\alpha^2(n-1) - (4n-8)\alpha - 1\right) \cdot \left(3\alpha + \frac{2-n}{n-1}\right)}_{=:\mathcal{E}}} \tag{3.137}$$

strictly increases in α. The first derivative with respect to α gives

$$\frac{\partial z(\alpha)}{\partial \alpha} = \frac{[6(n-1)(2\alpha^2 - 3\alpha + 1) + (6\alpha(n-1)+3)(4\alpha-3)] \cdot \mathcal{E}}{\mathcal{E}^2} \tag{3.138}$$

$$- \frac{\mathcal{N} \cdot [(8\alpha(n-1) - (4n-8))(3\alpha + \frac{2-n}{n-1}) + (4\alpha^2(n-1) - (4n-8)\alpha - 1)3]}{\mathcal{E}^2}. \tag{3.139}$$

Since $\mathcal{E}^2$ is always positive, we have to show that

$$\begin{aligned}\bar{z}(\alpha) :=& [6(n-1)(2\alpha^2 - 3\alpha + 1) + (6\alpha(n-1)+3)(4\alpha-3)] \cdot \mathcal{E} \\ &- \mathcal{N} \cdot \left[(8\alpha(n-1) - (4n-8))\left(3\alpha + \frac{2-n}{n-1}\right) + (4\alpha^2(n-1) - (4n-8)\alpha - 1)3\right] > 0.\end{aligned} \tag{3.140}$$

Substituting $\mathcal{E}$ and $\mathcal{N}$ and simplifying gives

$$\begin{aligned}\bar{z}(\alpha) =& 3(n-1)\Big\{[12\alpha^2 n - 12\alpha n + 2n - 12\alpha^2 + 16\alpha - 5] \\ &\cdot [12\alpha^3 n^2 - 24\alpha^3 n + 12\alpha^3 - 16n^2\alpha^2 + 48n\alpha^2 - 32\alpha^2 - 19\alpha n + 19\alpha + 4n^2\alpha - 2 + n] \\ &- [4\alpha^3 n - 6\alpha^2 n + 2\alpha n - 4\alpha^3 + 8\alpha^2 - 5\alpha + 1] \\ &\cdot [36\alpha^2 n^2 - 72\alpha^2 n + 36\alpha^2 - 32\alpha n^2 + 96\alpha n - 64\alpha - 19n + 19 + 4n^2]\Big\}. \end{aligned} \tag{3.141}$$

Since $3(n-1)$ is always positive for $n \geq 3$ it suffices to show that

$$\begin{aligned}\hat{z}(\alpha) :=& [12\alpha^2 n - 12\alpha n + 2n - 12\alpha^2 + 16\alpha - 5] \\ &\cdot [12\alpha^3 n^2 - 24\alpha^3 n + 12\alpha^3 - 16n^2\alpha^2 + 48n\alpha^2 - 32\alpha^2 - 19\alpha n + 19\alpha + 4n^2\alpha \\ &- 2 + n] - [4\alpha^3 n - 6\alpha^2 n + 2\alpha n - 4\alpha^3 + 8\alpha^2 - 5\alpha + 1] \\ &\cdot [36\alpha^2 n^2 - 72\alpha^2 n + 36\alpha^2 - 32\alpha n^2 + 96\alpha n - 64\alpha - 19n + 19 + 4n^2] > 0. \end{aligned} \tag{3.142}$$

Expansion of the product gives

$$\begin{aligned}\hat{z}(\alpha) =& 144\alpha^5 n^3 - 288\alpha^5 n^2 + 144\alpha^5 n - 192\alpha^4 n^3 + 576\alpha^4 n^2 - 384\alpha^4 n - 228\alpha^3 n^2 + 228\alpha^3 n \\ &+ 48n^3\alpha^3 - 24\alpha^2 n + 12\alpha^2 n^2 - 144\alpha^4 n^3 + 288\alpha^4 n^2 - 144\alpha^4 n + 192\alpha^3 n^3 - 576\alpha^3 n^2 \\ &+ 384\alpha^3 n + 228\alpha^2 n^2 - 228\alpha^2 n - 48\alpha^2 n^3 + 24\alpha n - 12\alpha n^2 - 144\alpha^5 n^2 + 288\alpha^5 n - 144\alpha^5 \\ &+ 192\alpha^4 n^2 - 576\alpha^4 n + 384\alpha^4 + 228\alpha^3 n - 228\alpha^3 - 48\alpha^3 n^2 + 24\alpha^2 - 12\alpha^2 n + 24\alpha^3 n^3 \\ &- 48\alpha^3 n^2 + 24\alpha^3 n - 32\alpha^2 n^3 + 96\alpha^2 n^2 - 64\alpha^2 n - 38\alpha n^2 + 38\alpha n + 8\alpha n^3 - 4n + 2n^2 \\ &+ 192\alpha^4 n^2 - 384\alpha^4 n + 192\alpha^4 - 256\alpha^3 n^2 + 768\alpha^3 n - 512\alpha^3 - 304\alpha^2 n + 304\alpha^2 + 64\alpha^2 n^2 \\ &- 32\alpha + 16\alpha n - 60\alpha^3 n^2 + 120\alpha^3 n - 60\alpha^3 + 80\alpha^2 n^2 - 240\alpha^2 n + 160\alpha^2 \\ &+ 95\alpha n - 95\alpha - 20\alpha n^2 + 10 - 5n \\ &- [144\alpha^5 n^3 - 288\alpha^5 n^2 + 144\alpha^5 n - 128\alpha^4 n^3 + 384\alpha^4 n^2 - 256\alpha^4 n - 76\alpha^3 n^2 + 76\alpha^3 n \\ &+ 16\alpha^3 n^3 - 216\alpha^4 n^3 + 432\alpha^4 n^2 - 216\alpha^4 n + 192\alpha^3 n^3 - 576\alpha^3 n^2 + 384\alpha^3 n + 114\alpha^2 n^2 \\ &- 114\alpha^2 n - 24\alpha^2 n^3 + 72\alpha^3 n - 144\alpha^3 n^2 + 72\alpha^3 n - 64\alpha^2 n^3 + 192\alpha^2 n^2 - 128\alpha^2 n - 38\alpha n^2 \\ &+ 38\alpha n + 8\alpha n^3 - 144\alpha^5 n^2 + 288\alpha^5 n - 144\alpha^5 + 128\alpha^4 n^2 - 384\alpha^4 n + 256\alpha^4 + 76\alpha^3 n \\ &- 76\alpha^3 - 16\alpha^3 n^2 + 288\alpha^4 n^2 - 576\alpha^4 n + 288\alpha^4 - 256\alpha^3 n^2 + 768\alpha^3 n - 512\alpha^3 - 152\alpha^2 n \\ &+ 152\alpha^2 + 32\alpha^2 n^2 - 180\alpha^3 n^2 + 360\alpha^3 n - 180\alpha^3 + 160\alpha^2 n^2 - 480\alpha^2 n + 320\alpha^2 + 95\alpha n \\ &- 95\alpha - 20\alpha n^2 + 36\alpha^2 n^2 + 72\alpha^2 n + 36\alpha^2 - 32\alpha n^2 + 96\alpha n - 64\alpha - 19n + 19 + 4n^2]. \end{aligned} \tag{3.143}$$

Aggregating and collecting powers α^i with $i \in \{0, 1, 2, 3, 4\}$ yields

$$\hat{z}(\alpha) = -9 + 10n - 2n^2 + \alpha(32 - 56n + 20n^2) + \alpha^2(-20 + 74n - 54n^2 + 8n^3) + \alpha^3(-32 + 16n + 32n^2 - 16n^3) + \alpha^4(32 - 56n + 16n^2 + 8n^3). \tag{3.144}$$

We will show in several steps that $\hat{z}(\alpha)$ is strictly increasing in α and that $\hat{z}(1) > 0$ from which we will be able to conclude that $\hat{z}(\alpha)$ is strictly positive for $\alpha > 1$.

The first derivative of $\hat{z}(\alpha)$ with respect to α is

$$\hat{z}'(\alpha) = \frac{\partial \hat{z}(\alpha)}{\partial \alpha} = 32 - 56n + 20n^2 + 2\alpha(-20 + 74n - 54n^2 + 8n^3) + 3\alpha^2(-32 + 16n + 32n^2 - 16n^3) + 4\alpha^3(32 - 56n + 16n^2 + 8n^3). \tag{3.145}$$

To see that $\hat{z}'(\alpha) > 0$ consider

$$\begin{aligned}\hat{z}'(1) &= 32 - 56n + 20n^2 - 40 + 148n - 108n^2 + 16n^3 - 96 + 48n + 96n^2 - 48n^3 + 128 \\ &\quad - 224n + 64n^2 + 32n^3 \\ &= 72n^2 - 84n + 24 > 0\end{aligned} \tag{3.146}$$

and

$$\hat{z}''(\alpha) = \frac{\partial \hat{z}'(\alpha)}{\partial \alpha} = 2(-20 + 74n - 54n^2 + 8n^3) + 6\alpha(-32 + 16n + 32n^2 - 16n^3) + 12\alpha^2(32 - 56n + 16n^2 + 8n^3). \tag{3.147}$$

For this we have

$$\begin{aligned}\hat{z}''(1) &= -40 + 148n - 108n^2 + 16n^3 - 192 + 96n + 192n^2 - 96n^3 + 384 - 672n \\ &\quad + 192n^2 + 96n^3 \\ &= 16n^3 + 276n^2 - 428n + 152 > 0 \quad \text{for} \quad n \geq 3\end{aligned} \tag{3.148}$$

and

$$\hat{z}^{(3)}(\alpha) = \frac{\partial \hat{z}''(\alpha)}{\partial \alpha} = 6(-32 + 16n + 32n^2 - 16n^3) + 24\alpha(32 - 56n + 16n^2 + 8n^3) \tag{3.149}$$

where

$$\begin{aligned}\hat{z}^{(3)}(1) =& -192 + 96n + 192n^2 - 96n^3 + 768 - 1344n + 384n^2 + 192n^3 \\ =& 96n^3 + 576n^2 - 1248n + 576 > 0 \;\text{ for }\; n \geq 3\end{aligned} \tag{3.150}$$

and

$$\hat{z}^{(4)}(\alpha) = \frac{\partial \hat{z}^{(3)}(\alpha)}{\partial \alpha} = 24(32 - 56n + 16n^2 + 8n^3) > 0 \;\text{ for }\; n \geq 3. \tag{3.151}$$

So $\hat{z}^{(3)}(\alpha)$ is positive and increasing for all $\alpha > 1$; this extends to $\hat{z}''(\alpha)$ by (3.147)–(3.148), and to $\hat{z}'(\alpha)$ by (3.145)–(3.146). It therefore remains to check that $\hat{z}(1) > 0$. We have

$$\begin{aligned}\hat{z}(1) =& -9 + 10n - 2n^2 + (32 - 56n + 20n^2) + (-20 + 74n - 54n^2 + 8n^3) \\ &+ (-32 + 16n + 32n^2 - 16n^3) + (32 - 56n + 16n^2 + 8n^3).\end{aligned} \tag{3.152}$$

Simplifying yields

$$\hat{z}(1) = -29 - 12n + 12n^2 > 0 \;\text{ for }\; n \geq 3. \tag{3.153}$$

This concludes the proof that $\bar{\rho}_h^*$ is strictly increasing in α. The Shapley share of firm h is therefore bounded above by

$$\rho_h^* \leq \lim_{\alpha\to\infty} \bar{\rho}_h^* = \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1} \frac{s-1}{n-1} = \frac{1}{2}. \tag{3.154}$$

3.7.2.6 Lower Bound

Next we will show that $1/n$ is a lower bound to the Shapley share ρ_h^*. It follows from equations (3.125)–(3.133) that ρ_h^* is decreasing in $\bar{a}_{-h}/a_h$ and therefore

$$\rho_h^* \geq \lim_{\bar{a}_{-h}/a_h \to\infty} \rho_h^* = \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1} \frac{(s-1)}{(n-1)} \cdot \frac{\Big[4\alpha^2(n-1) + n - s - 2\Big]\cdot(\alpha-1)(2\alpha-1)}{\Big(4\alpha^2(n-1) - (2n-6+2s)\alpha + \frac{\eta_s}{n-1}\Big)\cdot\Big(2\alpha^2(n-1)+1\Big)}. \tag{3.155}$$

It remains to establish that the right-hand side of (3.155) is increasing in α; then a lower bound follows from considering $\alpha \to 1$. The numerator as well as the denominator of the fraction in (3.155) which depends on α are positive and decreasing in s (see equation (3.135)). Hence, inserting $s = n-1$ in the numerator and $s = 2$ in the denominator yields:

$$\rho_h^* \geq \lim_{\bar{a}_{-h}/a_h \to \infty} \rho_h^* \geq \underline{\rho}_h^* := \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1} \frac{(s-1)}{(n-1)} \cdot \frac{[4\alpha^2(n-1)-1](\alpha-1)(2\alpha-1)}{(4\alpha^2(n-1)-(2n-2)\alpha-\frac{3}{n-1})(2\alpha^2(n-1)+1)}. \tag{3.156}$$

To show that $\underline{\rho}_h^*$ is increasing in α it is sufficient to show that

$$v(\alpha) := \frac{\overbrace{[4\alpha^2(n-1)-1](\alpha-1)(2\alpha-1)}^{=:\mathcal{M}}}{\underbrace{(4\alpha^2(n-1)-(2n-2)\alpha-\frac{3}{n-1})(2\alpha^2(n-1)+1)}_{=:\mathcal{F}}} \tag{3.157}$$

strictly increases in α. The first derivative with respect to α yields

$$\begin{aligned}\frac{\partial v(\alpha)}{\partial\alpha} &= \frac{[(8\alpha n-8\alpha)(2\alpha^2-3\alpha+1)+(4\alpha^2 n-4\alpha^2-1)(4\alpha-3)]\cdot\mathcal{F}}{\mathcal{F}^2} \\ &- \frac{2\mathcal{M}[(4\alpha n-4\alpha-n+1)(2\alpha^2 n-2\alpha^2+1)+2\alpha(2\alpha(2\alpha n-2\alpha-n+1)-\frac{3}{n-1})(n-1)]}{\mathcal{F}^2}.\end{aligned} \tag{3.158}$$

We have to show that

$$\begin{aligned}\bar{v}(\alpha) :&= [(8\alpha n-8\alpha)(2\alpha^2-3\alpha+1)+(4\alpha^2 n-4\alpha^2-1)(4\alpha-3)]\cdot\mathcal{F} \\ &-\mathcal{M}\cdot[(8\alpha n-8\alpha-2n+2)(2\alpha^2 n-2\alpha^2+1) \\ &+(4\alpha^2 n-4\alpha^2-2n\alpha+2\alpha-\frac{3}{n-1})(4\alpha n-4\alpha)] > 0.\end{aligned} \tag{3.159}$$

Substituting $\mathcal{M}$ and $\mathcal{F}$ and simplifying gives

$$\begin{aligned}\bar{v}(\alpha) =&(n-1)[-32\alpha^3+36\alpha^2-12\alpha+32\alpha^3 n-36\alpha^2 n+8\alpha n+3]\cdot[-8\alpha^4+4\alpha^3+10\alpha^2\\&-2\alpha+8\alpha^4 n^3-4\alpha^3 n^3-24\alpha^4 n^2+12\alpha^3 n^2+4\alpha^2 n^2-2\alpha n^2+24\alpha^4 n-12\alpha^3 n\\&-14\alpha^2 n+4\alpha n-3]-[-8\alpha^4+12\alpha^3-6\alpha^2+3\alpha+8\alpha^4 n-12\alpha^3 n+4\alpha^2 n-1]\\&\cdot[-32\alpha^3+12\alpha^2+20\alpha+32\alpha^3 n^3-12\alpha^2 n^3-96\alpha^3 n^2+36\alpha^2 n^2+8\alpha n^2-2n^2\\&+96\alpha^3 n-36\alpha^2 n-28\alpha n+4n-2].\end{aligned}\tag{3.160}$$

Since $n-1>0$ we have to prove that

$$\begin{aligned}\hat{v}(\alpha) =&[-32\alpha^3+36\alpha^2-12\alpha+32\alpha^3 n-36\alpha^2 n+8\alpha n+3]\cdot[-8\alpha^4+4\alpha^3+10\alpha^2-2\alpha\\&+8\alpha^4 n^3-4\alpha^3 n^3-24\alpha^4 n^2+12\alpha^3 n^2+4\alpha^2 n^2-2\alpha n^2+24\alpha^4 n-12\alpha^3 n-14\alpha^2 n\\&+4\alpha n-3]-[-8\alpha^4+12\alpha^3-6\alpha^2+3\alpha+8\alpha^4 n-12\alpha^3 n+4\alpha^2 n-1]\cdot[-32\alpha^3\\&+12\alpha^2+20\alpha+32\alpha^3 n^3-12\alpha^2 n^3-96\alpha^3 n^2+36\alpha^2 n^2+8\alpha n^2-2n^2+96\alpha^3 n\\&-36\alpha^2 n-28\alpha n+4n-2]>0.\end{aligned}\tag{3.161}$$

Expanding and aggregating powers α^i with $i \in \{0,1,2,3,4,5,6\}$ yields

$$\begin{aligned}\hat{v}(\alpha) =&-11+4n-2n^2+\alpha(56-52n+8n^2)+\alpha^2(-114+82n+52n^2-20n^3)\\&+\alpha^3(-8+216n-312n^2+104n^3)+\alpha^4(264-736n+696n^2-240n^3+16n^4)\\&+\alpha^5(-256+736n-768n^2+352n^3-64n^4)\\&+\alpha^6(64-256n+384n^2-256n^3+64n^4).\end{aligned}\tag{3.162}$$

We want to show that $\hat{v}(\alpha)$ is increasing in α. Taking the first derivative with respect to α gives

$$\begin{aligned}\hat{v}'(\alpha) =&\frac{\partial\hat{v}(\alpha)}{\partial\alpha}=(56-52n+8n^2)+2\alpha(-114+82n+52n^2-20n^3)\\&+3\alpha^2(-8+216n-312n^2+104n^3)+4\alpha^3(264-736n+696n^2-240n^3+16n^4)\\&+5\alpha^4(-256+736n-768n^2+352n^3-64n^4)\\&+6\alpha^5(64-256n+384n^2-256n^3+64n^4)\end{aligned}\tag{3.163}$$

with

$$\hat{v}'(1)=4(-9-10n+106n^2-116n^3+32n^4)>0 \text{ for } n\geq 3.\tag{3.164}$$

Doing five iterations along the lines of the derivations in equations (3.145)–(3.151), where the value of each corresponding derivative can be shown to be positive for $\alpha = 1$, it follows from

$$\hat{v}^{(6)} = 720 \cdot 64(n-1)^4 > 0 \;\text{ for }\; n \geq 3 \tag{3.165}$$

that the derivative $\hat{v}'(\alpha)$ of $\hat{v}(\alpha)$ and all intermediate derivatives $\hat{v}''(\alpha), \ldots, \hat{v}^{(5)}(\alpha)$ are positive, too. Inserting $\alpha = 1$ into $\hat{v}(\alpha)$ and simplifying gives

$$\hat{v}(1) = -5 - 6n + 58n^2 - 60n^3 + 16n^4 > 0 \;\text{ for }\; n \geq 3. \tag{3.166}$$

Hence $\hat{v}(\alpha) > 0$ and $\bar{v}(\alpha) > 0$ for $\alpha > 1$. We can conclude that $\underline{\rho}_h^*$ is increasing in α. The Shapley share of firm h is therefore bounded below by

$$\rho_h^* \geq \lim_{\alpha \to 1} \underline{\rho}_h^* = \frac{1}{n}. \tag{3.167}$$

Chapter 4

Simple Games and Cartel Damage Proportioning

A *simple game* partitions the set of all possible coalitions among a given set N of players into two categories: a coalition $S \subseteq N$ is either 'winning' (denoted by $v(S) = 1$) or 'losing' ($v(S) = 0$). Simple games are a subclass of cooperative games with transferable utility, i.e., *TU games*. They received an entire chapter's attention already by von Neumann and Morgenstern (1953, ch. 10). More recently, Taylor and Zwicker (1999) devoted a full-length monograph to them, and the investigation of their properties – e.g., the dimensionality of representations of a given characteristic function v by means of integer weight vectors and weight thresholds (Kurz and Napel 2016) – still goes on.

The long list of applications of simple games is dominated by voting bodies, such as the US Electoral College, the Board of Governors or the Board of Directors of the International Monetary Fund, the EU Council of Ministers, shareholder meetings, etc. The typical concern is the distribution of voting power implied by a given function v.[1] Simple games also play an important technical role in cooperative game theory because their subclass of unanimity games forms a basis of the vector space of general TU games. Shapley (1953b) provided the first axiomatic characterization using this observation, and many later authors proceed similarly in their axiomatic work.

We here propose a new domain of application for simple games: they can serve as useful first approximations in the proportioning of cartel damages. The European Commission has passed its *Directive on Antitrust Damages Actions* (2014/104/EU) in

[1]See Napel (2019), for instance, for an overview. For an application of simple games and power indices without voting context see Kovacic and Zoli (2018).

order to reduce legal and procedural hurdles for cartel victims to reclaim antitrust damages. The Directive is based on the observation that "... total annual cost for hardcore cartels in the EU can be estimated to range from approximately €25 billion [...] to approximately €69 billion..." and only a small share of accrued damages are reclaimed later (see SWD/2013/203/Final, recitals 65, 67).

When a detected cartel is litigated, all infringers are *jointly liable* in the EU. That means a customer who has suffered a cartel-related price overcharge on its purchases can sue any cartel member for any desired share of their total compensation – independently of whether the purchases were made from this or another firm.[2] A co-defendant that was sued and convicted to pay compensation to victims however has a right to force other cartel members to contribute "... if it has paid more compensation than its harm" (Directive 2014/104/EU, recital 37). Or when litigants should settle sequentially, "... the claim of the injured party should be reduced by the settling infringer's share of the harm caused to it ..." (Directive 2014/104/EU, recital 51). A firm's share is explicitly tied to its *"... relative responsibility for the harm caused by the infringement of competition law"* by Article 11(5) of the Directive.

How to economically quantify this norm was already discussed by Schwalbe (2013), Napel and Oldehaver (2015) and in Chapter 3. The *Shapley value* is argued to be most suitable for allocating damage by relative responsibility. In particular, Subsection 3.2.2 highlights desirable properties that a responsibility-based allocation should satisfy: first, it is indisputable that the entire damage should be allocated among cartel members (*efficiency*) and that a firm's damage share should not depend on currency choice, interest or on whether all customers simultaneously or sequentially act against former cartel members (*linearity*). To reflect responsibility, it is additionally sensible to require that infringers who have identical influence on a victims damage, should contribute the same to the compensation (*symmetry*) and that a firm which has not caused any damage need not contribute at all (*null-player*). Most importantly, *marginality*, introduced by Young (1985), reflects that the causal relation between the size of a customer's damage and firm i's cartel membership crucially determines relative responsibility of firm i. The Shapley value is the *unique* value which satisfies *all* these properties.

That it makes good economic sense to use the Shapley value for allocating cartel damages is accepted by legal practitioners: "*Shapley values consistently deliver an apportionment according to the relative responsibility for the harm*" (Bornemann 2018,

[2]Litigants can also strive to find out-of-court settlements, or settle with some firms and take the remaining ones to court.

recital 123). Nevertheless, a major disadvantage of applying the Shapley value is that many market parameters are needed for its calculation. Bornemann (2018, recital 124) holds that: "*. . . for almost all real-life cases, such a data panel will be exceedingly difficult or downright impossible to obtain*". Thus, simple heuristics which are highly incongruent with relative responsibility – for instance an allocation based on cartel or competitive sales or revenues – have been suggested. As shown in Subsection 3.3.4, selecting an appropriate heuristic depends on the kind of asymmetry between firms, and even the heuristic which is closest to the Shapley value is in some cases far off. In order to enable the Shapley value to be useful in legal practice, a central question has to be answered: is there a simple approximation which on the one hand reflects responsibility reasonable well but on the other hand can do without estimating a wide range of market parameters?

We answer this affirmatively question by pointing to the use of *simple games* in cartel damage contexts. We propose to categorize and normalize the damage of a (partial) cartel S as either 1, in case this illicit conduct caused huge damage, or 0 when the caused damage was rather small. In Section 4.3 we investigate such dichotomous damage scenarios, that is, scenarios where different coalitions cause either a unit damage or none. We elaborate for $n \leq 5$, where n is the number of co-defendants, all 179 dichotomous damage scenarios which can arise. The Shapley value, respectively the Shapley-Shubik index, of the normalized damage allocation scenario yields an allocation which is based on relative responsibility in the approximation of the underlying market structure.

In Section 4.4, we compare this heuristic allocation based on dichotomous damage scenarios with several ad hoc heuristics suggested by legal practitioners for two claim scenarios. First, we assume that only customers of one former cartel member reclaim antitrust damages. We then extend the analysis to market-wide claim scenarios. Even though the discretization heuristic is easy to apply, it continuously outperforms most other heuristics with respect to the accuracy of approximating the Shapley damage shares in the original market model in both claim scenarios. In particular, from a market-wide perspective, it is the unique heuristic which is always close to an apportionment by non-approximated Shapley shares no matter whether firms differ in size, efficiency or other market parameters.

4.1 Illustration

Before formally introducing simple games and the Shapley value as a tool for allocating damages in Section 4.2, let us illustrate the damage apportionment problem.

Consider a market with three producers of differentiated goods. Their respective costs be $C_1(q_1) = 30q_1$, $C_2(q_2) = 20q_2$ and $C_3(q_3) = 10q_3$ and they compete à la Bertrand. Let demands be $D_1(p) = 100 - 4p_1 + 3p_2 + 0.4p_3$, $D_2(p) = 100 - 4p_2 + 3p_1 + 0.4p_3$ and $D_3(p) = 150 - 3p_3 + 0.4(p_1 + p_2)$. So products 1 and 2 constitute closer substitutes than product 3 and collaboration by firms 1 and 2 is likely to have the largest price effect.

The individual maximization of profits yields Bertrand equilibrium prices $p^B = (44.7; 41.0; 35.7)$ rounded to one decimal place. The corresponding equilibrium outputs are $q^B = (58.7; 84.2; 77.1)$, with revenues of $R^B = (2622.5; 3453.7; 2755.1)$ and profits of $\Pi^B = (861.5; 1770.6; 1983.7)$. If the firms form a cartel and maximize total industry profit, prices rise to $p^C = (82.2; 77.2; 47.9)$ while quantities fall to $q^C = (22; 57; 70)$. Ignoring potential side payments, individual profits in the cartelized market are $\Pi^C = (1147.6; 3258.4; 2653.7)$ from revenues of $R^C = (1807.6; 4398.4; 3353.7)$.

Profits increase but each unit of good i which was purchased involved an *overcharge damage* of $\Delta p_i = p_i^C - p_i^B$ (referred to as *damnum emergens* in the legal literature). Here, overcharges are $\Delta p = (37.5; 36.1; 12.2)$ per unit, resulting in product-specific total overcharge damages of $D = q^C \cdot \Delta p = (824.8; 2059.1; 853.7)$.[3]

Suppose now that a customer k who purchased $x_1^k = 10$ units from firm 1 at p_1^C, and nothing else, sues. The customer may take firm 2 to court because the plaintiff is free to choose; perhaps k perceives the best odds for enforcing his claim against the profit champion. If k is then granted compensation for his total overcharges $O^k = 375$, firm 2 must pay out O^k. But it is entitled to reclaim some of this from firms 1 and 3. Table 4.1 lists the respective shares of the three firms suggested by several allocation rules ρ that have been discussed by legal practitioners.[4]

The table illustrates two shortcomings of ad hoc allocation rules. First, differences in the compensation share for the firms seem arbitrary and there is wide scope for a firm's share of the compensation. It can increase by up to 20 percentage points when switching from one rule to another (compare, e.g., ρ^0, ρ^2 and ρ^7). Without further

[3]Further harm relates to deadweight losses: customers who would have made (additional) purchases, and thus would have enjoyed surplus had prices only been p^B, failed to do so (*lucrum cessans*). We are unaware of cases in which compensation for it has successfully been claimed. We will disregard those damages in what follows.

[4]The allocation rule by cartel benefits is derived by normalizing the relative profit increases of the cartel members.

Allocation rule (ρ)	%-allocation
per head (ρ^0)	(33.3%; 33.3%; 33.3%)
by cartel revenue (ρ^1)	(18.9%; 46.0%; 35.1%)
by cartel sales (ρ^2)	(14.8%; 38.3%; 47.0%)
by competitive revenue (ρ^3)	(29.7%; 39.1%; 31.2%)
by competitive sales (ρ^4)	(26.7%; 38.4%; 35.1%)
by cartel profits (ρ^5)	(16.3%; 46.2%; 37.6%)
by competitive profits (ρ^6)	(18.7%; 38.3%; 43.0%)
by cartel benefits (ρ^7)	(22.0%; 55.6%; 22.4%)

Table 4.1 Allocation rules discussed by legal practitioners

economic analysis, there is no good reason why one allocation rule, e.g., by cartel revenues, should be preferred over another allocation rule, e.g., a per head allocation.

Second, all rules assign firm 1 a share equal to or smaller than 33.3%. Thus, the firm which sold the product has to contribute only a rather small share. This is noteworthy since own-price demand parameters in this illustrative example are larger than the sum of cross-price parameters: the behavior of the firm which sold the product has the strongest effect on the price, that its customers pay. None of the allocation shares in Table 4.2 reflect this fact.

An allocation based on Shapley shares $\rho^* = (46.0\%; 44.7\%; 9.3\%)$, which is derived in Section 4.2, takes into account that – without an infringement – competition is closest between the first two firms. This is missed by the ad hoc heuristics in Table 4.1. Competitive revenue shares $\rho^3 = (29.7\%; 39.1\%; 31.2\%)$ happen to have smallest $\|\cdot\|_1$-distance to ρ^*. This distance is a measure of a heuristic's aggregated mis-allocation relative to Shapley shares. It amounts to 43.8% in the example. Thus, the allocation of damages by ad hoc heuristics does not seem to be useful at all when each firm's share is meant to reflect its relative responsibility for harm. We will show that a heuristic allocation based on the Shapley share in an appropriate binary approximation of the market can do much better.

4.2 Preliminaries

We first introduce simple games, that is, transferable utility games where the worth of any coalition is normalized to 0 or 1. Then, we turn to the Shapley value as a tool for allocating antitrust damages.

4.2.1 Notation and Setup

To discuss how to allocate cartel damages among co-defendants, we introduce *transferable utility* (TU) games. In TU games, surplus or costs are divided among a set of players $N = \{1, \ldots, n\}$. In our context of cartel damage allocation we consider a successfully reclaimed damage of an harmed antitrust victim, which has to be allocated among the set of former cartel members N. In the EU, cartel members also have to compensate for umbrella losses, i.e., they have to compensate customers who suffered a damage but bought a product of firms $i \notin N$ (see CJEU C-557/12 2014). This influences absolute compensation payments but not the set of co-defendants N.

The real number $v(S)$ assigns a (partial) cartel or *coalition* $S \subseteq N$ of co-defendants the damage or negative *worth* caused by this coalition. The *characteristic function* $v\colon 2^N \to \mathbb{R}$ of the TU game (N, v) is generally customer harm specific.

A customer's observed damage is $v(N)$. This damage naturally differs from when a coalition $S \subset N$ had formed. Additionally, as long as firm $i \in N$ is not a *null-player*, a customer's damage will depend on whether firm i is part of coalition S or not. The decrease in a customer's damage when firm $i \in S$ leaves the cartel, that is $v(S) - v(S \setminus \{i\})$, determines a firm's economic responsibility.[5] See Subsection 3.2.1 on a detailed discussion why all counterfactual market scenarios play a role in assessing a firm's responsibility and how the analysis might be extended when several partial cartels operated simultaneously.

Needless to say that damages in a cartel context can only be positive when at least two firms coordinated strategies, that is, $v(S) = 0$ for $S = \emptyset$ or $\#S = 1$.[6] Whenever a partial cartel of size $\#S > 1$ formed while remaining firms $j \in N \setminus S$ acted competitively, a full-blown merger simulation analysis is needed to specify $v(S)$. Although potential price effects caused by mergers are frequently predicted by discussing appropriate oligopoly models (see, e.g., Peters 2006 or Garmon 2017), only few papers determine a cartel's damage by using a simulation approach (see, e.g., Roos 2006). Determining the Shapley value for the TU game (N, v) is frequently not feasible for data reasons and the general complexity of merger simulations. Thus, an appropriate approximation is needed.

[5] We assume that firms' roles in the cartel were identical. Thus, we do not consider ringleaders or immunity recipients. One can adapt the analysis to these issues by considering weighted Shapley values, which were introduced by Shapley (1953a). See Section 3.4 for their discussion in a cartel context.

[6] With $n = 2$, both firms are needed to cause damage. Thus, each firm has to pay half of the overcharge damage. See Hart and Mas-Colell (1989) for a detailed discussion.

A first step to facilitate the computational complexity is to categorize and normalize overcharge damages to $\tilde{v}(S) \in \{0, 1\}$. That is, a useful way to approximate the damage caused by a partial cartel S is to assume that its damage was either large (which can be normalized to 1), or that its damage is comparatively small (normalized to 0). This procedure suggests to use *(monotonic) simple game* $(N, \tilde{v})$. They are defined by the monotonicity condition $S \subseteq T \Rightarrow \tilde{v}(T) \geq \tilde{v}(S)$ and the restrictions $\tilde{v}(\emptyset) = 0$ and $\tilde{v}(N) = 1$ in cooperative game theory. Such games have been studied extensively (see, e.g., Taylor and Zwicker 1999) and often arise in the context of voting and election rules. Hence a coalition S such that $\tilde{v}(S) = 1$ is typically referred to as *winning* and one with $\tilde{v}(S) = 0$ as *losing*. In our application, a winning coalition $S \subseteq N$ corresponds to a (partial) cartel which could profitably have imposed a big overcharge.

The assumption that if a given partial cartel S can 'win', so does a larger cartel T which contains S, could be restrictive in very specific setups[7] but generally is innocuous. It allows to fully define the mapping $\tilde{v}$ by the list $\mathcal{M}(\tilde{v}) = \{S \subseteq N : \tilde{v}(S) = 1 \text{ and } T \subset S \Rightarrow \tilde{v}(T) = 0\}$ of *minimal winning coalitions* (MWC). Any coalition $S \in \mathcal{M}(\tilde{v})$ and all its supersets cause damage classified as large; collaboration by a strict subset of S does not.

4.2.2 Shapley Value

Several solution concepts for coalition games, like the core or the nucleolus, could be used to allocate damages among co-defendants. However, as shown by Shapley (1953b) and Young (1985), only the Shapley value satisfies *all* properties listed in the introduction. The Shapley value of game (N, v), which is is a single-valued solution concept, is given by

$$\Phi_i(N, v) = \varphi_i(N, v) := \sum_{S \subseteq N} \frac{(s-1)!(n-s)!}{n!} \cdot \left[v(S) - v(S \setminus \{i\}) \right] \tag{4.1}$$

where $s = \#S$ denotes the cardinality of coalition S.

As illustration, re-consider the overcharge damage which accrued to the exemplary purchaser of 10 units of good 1 in Section 4.1. If we want to proportion this to the three firms in line with relative responsibility, we need to check their marginal contributions to overcharge Δp_1 and weight them according to eq. (4.1). Table 4.2 collects the damages for all conceivable cartel scenarios S implied by the indicated

[7]Think of some cartel members producing complements rather than substitutes, which generates non-monotonicities. Such cases seem rare but possible (e.g., the 1992–2004 bathroom fittings cartel).

market model, and lists the respective differences that participation by a given firm makes.

S	$v(S)$	$v(S) - v(S \setminus \{1\})$	$v(S) - v(S \setminus \{2\})$	$v(S) - v(S \setminus \{3\})$
$\emptyset, \{1\}, \{2\}, \{3\}$	0	0	0	0
$\{1,2\}$	28.20	28.20	28.20	0
$\{1,3\}$	1.67	1.67	0	1.67
$\{2,3\}$	0.73	0	0.73	0.73
$\{1,2,3\}$	37.49	36.76	35.82	9.29

Table 4.2 Marginal contributions to Δp_1

The numbers confirm that cooperation by firm 1 and 2 is the main driver of overcharges on product 1. Firm 3's participation has an effect, too; but mainly when 1 and 2 are already collaborating. So, as economic intuition about collaboration of firm 1 and firm 2 being the key driver of price increases had it, firm 3's responsibility for k's damage is small. Those of firms 1 and 2 are similar to another, with a slightly bigger average contribution for 1. Aggregating the figures according to eq. (4.1) yields $\varphi(N, v) = (17.2; 16.8; 3.5)$; normalization gives $\rho^* := 100\% \cdot \varphi(N, v)/v(N) = (46.0\%; 44.7\%; 9.3\%)$.

Similar computations yield allocations of compensation owed to customers of firms 2 and 3. Conveniently, linearity of the Shapley value permits to focus on the damage associated with a single unit of the respective good i. Obligations of the three firms for compensating a plaintiff with purchases of $x = (x_1, x_2, x_3)$ then follow from the matrix multiplication

$$(x_1, x_2, x_3) \cdot \begin{pmatrix} \varphi(N, v^1) \\ \varphi(N, v^2) \\ \varphi(N, v^3) \end{pmatrix}$$

where characteristic function v^i reflects the price overcharge on a *single unit* of good i.

4.3 Dichotomous Approximation

A dichotomous approximation is straightforward in cases like the above where damages can naturally be categorized as either big or small, with minor differences within each category. For instance, exact damages $v(S) = 1.67$ and 0.73 for coalitions $\{1,3\}$ and $\{2,3\}$ are small compared to the damage values of 28.20 and 37.49 for coalitions

$\{1,2\}$ and $\{1,2,3\}$ in Table 4.2. Dividing by $v(N)$ yields normalized values of 0.04, 0.02, 0.75 and 1.00, respectively. A corresponding binary approximation of v by $\tilde{v}$ with

$$\tilde{v}(S) = \begin{cases} 1 & \text{if } \{1,2\} \subseteq S, \\ 0 & \text{otherwise} \end{cases} \tag{4.2}$$

is not far off. It could in practice be derived from a qualitative assessment which finds firms 1 and 2 competing a lot more closely with each other than firm 3 – without full estimates of cost and demand functions. The corresponding normalized Shapley shares $\rho^D := 100\% \cdot \varphi(N, \tilde{v}) = (50\%; 50\%; 0)$ are pretty close to $\rho^* = (46.0\%; 44.7\%; 9.3\%)$ computed for (N, v) with $\sum_{i=1}^{3} \mid \rho_i^D - \rho_i^* \mid = 18.6\%$. ρ^D is much closer to ρ^* than Section 4.1's heuristic suggestions $\rho^0, \rho^1, \ldots$ derived from market or profit shares.

It is noteworthy that ρ^D is a product specific share whereas the heuristics introduced in Table 4.1 do not depend on the firm which sold its product to a particular victim. If for example firm 3 sold the product, heuristic suggestions would still be unchanged whereas the Shapley value in the original game (N, v^3) would change to $\rho^* = (35.5\%; 37.2\%; 27.3\%)$. In this case, the binary approximation $\tilde{v}^3$ which is closest to the Shapley value in the original game would lead to normalized Shapley shares $\rho^D = (33.\bar{3}\%; 33.\bar{3}\%; 33.\bar{3}\%)$.

As illustration of how MWC can be used in order to approximate a given cartelized market by a simple game, consider a market with $N = \{\text{A, B, C, D}\}$. Assume that collaboration by firm A with at least one other firm implies a unit damage. The corresponding set of MWC is

$$\mathcal{M}(\tilde{v}) = \{\text{AB, AC, AD}\}.$$

Here we write AB as shorthand for {A, B}, etc. Firm A's participation is essential for overcharges; non-participation by up to two other cartelists would not noticeably change things. Many people's intuition is probably that the singular importance of A – with essentially a veto position – entails greater responsibility for compensating victims. But how much greater? Operationalizing responsibility in a systematic way yields the answer. Presuming one deems the Shapley properties desirable, the allocation should be

$$\rho^D = 100\% \cdot \varphi(N, \tilde{v}) = (75\%; 8.\bar{3}\%; 8.\bar{3}\%; 8.\bar{3}\%)\,.$$

$\mathcal{M}(\tilde{v})$	$100\% \cdot \varphi(N, \tilde{v})$	$\mathcal{M}(\tilde{v})$	$100\% \cdot \varphi(N, \tilde{v})$
1. AB	(50%; 50%; 0%; 0%)	11. AB, ACD, BCD	($33.\bar{3}$%; $33.\bar{3}$%; $16.\bar{6}$%; $16.\bar{6}$%)
2. AB, AC	($66.\bar{6}$%; $16.\bar{6}$%; $16.\bar{6}$%; 0%)	12. AB, AC, AD, BC, BD	($33.\bar{3}$%; $33.\bar{3}$%; $16.\bar{6}$%; $16.\bar{6}$%)
3. AB, AC, BC	($33.\bar{3}$%; $33.\bar{3}$%; $33.\bar{3}$%; 0%)	13. AB, BC, CD	($16.\bar{6}$%; $33.\bar{3}$%; $33.\bar{3}$%; $16.\bar{6}$%)
4. ABC	($33.\bar{3}$%; $33.\bar{3}$%; $33.\bar{3}$%; 0%)	14. AB, AC, AD, BC	($41.\bar{6}$%; 25.0%; 25.0%; $8.\bar{3}$%)
5. ABC, ABD	($41.\bar{6}$%; $41.\bar{6}$%; $8.\bar{3}$%; $8.\bar{3}$%)	15. ABC, ABD, ACD, BCD	(25%; 25%; 25%; 25%)
6. ABCD	(25%; 25%; 25%; 25%)	16. AB, AC, AD, BCD	(50%; $16.\bar{6}$%; $16.\bar{6}$%; $16.\bar{6}$%)
7. AB, AC, BCD	($41.\bar{6}$%; 25.0%; 25.0%; $8.\bar{3}$%)	17. AB, AC, AD, BC, BD, CD	(25%; 25%; 25%; 25%)
8. AB, AC, AD	(75.0%; $8.\bar{3}$%; $8.\bar{3}$%; $8.\bar{3}$%)	18. AC, AD, BC, BD	(25%; 25%; 25%; 25%)
9. AB, CD	(25%; 25%; 25%; 25%)	19. ABC, ABD, ACD	(50%; $16.\bar{6}$%; $16.\bar{6}$%; $16.\bar{6}$%)
10. AB, ACD	($58.\bar{3}$%; 25%; $8.\bar{3}$%; $8.\bar{3}$%)	continued for $n = 5$ in Appendix B	

Table 4.3 Shapley allocations for all dichotomous damage scenarios with $n \leq 4$ firms

One can easily generalize the idea of dichotomous approximation to bigger scenarios: assume that one large firm A and $n-1$ small firms operate. Let us also presume that a unit damage accrues if and only if firm A and at least one more firm cooperate. Determining the normalized Shapley value with $v(S) = 1$ if $A \in S$ and $v(S) = 0$ if $A \notin S$ yields[8]

$$\varphi_{\mathrm{A}}(N, \tilde{v}) = \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1}(1-0) = \frac{n-1}{n} \quad \text{and} \quad \varphi_i(N, \tilde{v}) = \frac{1}{n(n-1)} \text{ for } i \neq \mathrm{A}. \tag{4.3}$$

The first part of (4.3), that is, $\varphi_{\mathrm{A}}(N, \tilde{v}) = (n-1)/n$, also defines an upper bound for the contribution share of firm A in all TU games (N, v) (see Appendix A). Needless to say that a firm which bears no responsibility, i.e., a null player, does not have to contribute at all. Thus, the Shapley share $\Phi_i(N, v)$ is bounded by $[0, (n-1)/n]$. This broad interval is insufficient as a first order approximation. However, for small cartels, it is possible to enumerate *all* dichotomous damage scenarios which can arise. They correspond to simple games $(N, \tilde{v})$ with n players such that $\tilde{v}(S) = 1$ implies $\#S \geq 2$. Exactly 19 such scenarios exist for $n \leq 4$ firms, up to relabeling. They are listed in Table 4.3 with the corresponding Shapley shares.[9]

[8]The easiest way to determine the Shapley value in this scenario is to use equation (3.5) in Subsection 3.2.3. A related non-dichotomous scenario would have all player *pairs* $\{A, i\} \subseteq S$ with $i \in \{B, C, \ldots\}$ cause *incremental* unit damages, independently of each other. The corresponding mapping v with $v(S) = s-1$ if $A \in S$ and $v(S) = 0$ otherwise, assumes more than two values and is no simple game. Still, it is not hard to conclude that $\varphi_{\mathrm{A}}(N, v) = \frac{1}{2}v(N)$ and $\varphi_i(N, v) = \frac{1}{2(n-1)}v(N)$ for $i \neq \mathrm{A}$.

[9]The median number of firms in price fixing cartels is 4 in the US analysis by Levenstein and

For instance, scenario 1 approximates situations in which only cooperation by firms A and B is critical for the overcharges in question; then A and B share responsibility for any damage 50 : 50. We saw that this is a reasonable approximation for the example in Section 4.1. Scenario 2 corresponds to the big firm-small firm situation in eq. (4.3) with $n = 3$. Here, firm D is – with the caveat that we deal with a binary approximation of the real market – a null player; hence it bears no responsibility for damages and need not contribute. In scenario 3, cooperation by any two firms from {A, B, C} causes damage; while that of all three is necessary and sufficient for damage in scenario 4; etc.

The number of distinct scenarios involving n firms is related to the *Dedekind numbers* in discrete mathematics. These grow at a doubly exponential rate. A list of all dichotomous damage scenarios with $n = 5$ non-null players already involves 160 entries. They are collected in Appendix B.[10] A comprehensive categorization may be useful for ballpark assessments of responsibility in contribution settlements. The key practical advantage is that binary approximations just require a big-or-small classification of damages, not a full-blown market simulation. Even if some approximation error cannot be avoided, the corresponding Shapley allocations actually reflect marginal contributions to harm and hence responsibility, in contrast to profits or market shares.

4.4 Comparisons to Other Heuristics in Linear Market Environments

Several simple heuristics have been proposed by legal and economic practitioners. Baker (2004) suggested that an allocation by "...sales of the product during the conspiracy ..." can be an appropriate benchmark. This was adopted in Portugal: co-defendants are liable according to their average cartel market shares in the affected market (see Lei n.[0] 23/2018, Art. 5(5)). We compare how a heuristic (ρ^D) based on

Suslow (2016), which encompasses 329 cases.

[10]See Straffin (1983) for $n \leq 4$ and Baldan (1992) for $n = 5$. We have fewer games because $\tilde{v}(S) = 1$ requires $\#S \geq 2$ in a cartel context. Appendix B corrects several hidden typos in Baldan's list. Note that some games in the list, such as scenario 9, would be considered as *improper* in the context of voting: they involve disjoint winning coalitions. If we think of A and B as two producers and of C and D as their retailers, damage may plausibly arise already if the producers or the retailers cooperate. Presuming little scope for further marginalization by vertical coordination, $\mathcal{M}(\tilde{v}) = \{AB, CD\}$ makes good sense.

approximation by dichotomous damage scenarios performs compared to this and other ad hoc heuristics in the linear setting introduced in Subsection 4.4.1. We first analyse symmetric firms in Subsection 4.4.2; thereafter, in Subsection 4.4.3, firms are assumed to be asymmetric.

ρ^D assigns to a set of MWC, i.e., a DDS, a vector of percentage numbers which determines firms' contribution shares.[11] It builds on a dichotomous approximation of a coalition‘s damage on a product-specific level. Thus, coalitions that caused use damage (damage then is normalized to 1) are identified for each product sold by a cartel member. This allows to derive a heuristic which is product-specific, while ad hoc heuristics, e.g., based on market shares or revenues, do not depend on the considered product.

We distinguish two claim scenarios in our analysis. First, we assume that only customers who bought a product produced by one specific firm act against former cartel members. Due to linearity, it is then irrelevant for firms' compensation shares whether plaintiffs demand compensation for one product unit or for several product units. This claim scenario is particularly useful when firms supply specific groups of consumers, e.g., when a cartel is composed of firms that produce premium and non-premium products, respectively. Then, it could be the case that only some customers (e.g., those who bought the premium product) ask for compensation. Calculating contribution shares on an aggregated perspective would then be biased. Second, we consider a market-wide perspective, i.e., all customers act against former cartel members. This claim scenario can (due to linearity) be adopted when a uniform share of all customers reclaim damages. Moreover, considering a market-wide perspective is a good a priori assessment: without knowing more details about customers, it is a reasonable assumption to say that a uniform share of all customers act against former cartel members. This could, e.g., be useful when discussing the deterrent effect of private antitrust enforcement.

4.4.1 Linear Market Model

Firms simultaneously set prices à la Bertrand. Best-responding non-cartel members $j \notin S$ maximize own profits whereas cartel members $i \in S \subseteq N$ simultaneously maximize joint collusive profits.

[11] It may be the case that the contribution shares for firms coincide although different DDS are considered, as illustrated in Table 4.3. Below, we will discuss in more detail which DDS is used to approximate the Shapley shares of the original game.

Each firm $i \in N = \{1, \ldots, n\}$ with $n \geq 3$ produces a single good. Products are differentiated substitutes. Firm-specific unit costs γ_i are constant with $\gamma_i \geq 0$. A firm's demand function is assumed to be linear and decreasing in its own price. Let $p = (p_1, \ldots, p_n)$ be the price vector. Firm i's demand then is

$$D_i(p) = a_i - d_i \cdot p_i + \sum_{j \in N \setminus \{i\}} b_{ij} \cdot p_j \text{ for } a_i > \gamma_i,\ d_i > 0, \text{ and } b_{ij} > 0 \text{ for all } j \neq i. \tag{4.4}$$

Private antitrust enforcement would be superfluous if firm i does not face positive demand even when pricing at costs. Hence, we assume $D_i(\gamma) > 0$. To ensure existence and uniqueness of a Nash equilibrium we will additionally assume that the well-known dominant diagonal conditions are satisfied,[12] i.e., we assume

$$\alpha_i := d_i / \sum_{j \neq i} b_{ij} > 1 \text{ and } d_i > \sum_{j \neq i} b_{ji} \text{ for all } i \in N. \tag{4.5}$$

Parameter α_i measures the degree of differentiation between the product produced by firm i and products of the remaining market participants $j \neq i \in N$. With $\alpha_i \approx 1$, products are close substitutes and cartel behavior causes huge damage. With increasing values of α_i, substitutability and also the price overcharge damage of a cartel decrease.

The price vector $p^S = (p_1^S, \ldots, p_n^S)$ denotes the Nash equilibrium resulting when firms in $S \subseteq N$ coordinate their strategies and remaining firms in $N \setminus S$ act competitively. See Subsection 3.3.1 for a derivation of p^S assuming that firms are symmetric and Davis and Garcés (2009, ch. 8) for the basic procedure to determine p^S in a general linear market environment. Recall that $\Delta p = p^C - p^B$ is the price overcharge vector. If, e.g., a customer who bought one unit of product 1 acts against former cartel members, overcharge Δp_1 has to be allocated among co-defendants.

4.4.2 Symmetric Firms

When firms are symmetric, that is, when $a_i = a$, $d_i = d$, $b_{ij} = b$ and $\gamma_i = \gamma$, Subsection 3.3.2 shows that Shapley shares solely depend on the differentiation parameter α and on the number of co-defendants n. The Shapley share of the firm which sold the product (in the following referred to as the *home firm*) lies in $(1/n, 1/2)$ whereas remaining firms $i \neq h$ have to contribute equally with $\rho_i^* \in (0.5/(n-1), 1/n)$.

[12] See Vives (1999, Sec. 6.2) and Federgruen and Pierson (2011, Cor. 4.6).

When precise estimates of own and cross price elasticities are not known, some heuristic has to be used to divide damages and, ideally, to approximate a firm's Shapley share. First, assume that a single customer who bought one unit of a cartelized product, say produced by firm 1, acts against its home firm. Firm 1 then asks for internal contribution. All ad hoc heuristics $\rho^0 = \rho^1 = \rho^2 = \ldots =: \bar{\rho}$ discussed in Subsection 3.3.4 call for an equal per head allocation if firms are symmetric at the market level. However, for a specific customer k, firms are *not* symmetric even when products are (symmetrically) differentiated. Thus, with increasing heterogeneity, that is, with increasing values of α, heuristic $\bar{\rho}$ does not reflect relative responsibility. This is illustrated in panel (a) of Figure 4.1 with $n = 4$, $a = 30$, $\gamma = 2$, $d = 3$ and $b = d/(3\alpha)$. It shows (i) the Shapley share of the firm which sold the product (ρ^*_1) is increasing with an increasing differentiation parameter α, (ii) the Shapley share of the remaining firms ($\rho^*_{i\neq 1}$) is decreasing in α, and (iii) the shares based on ad hoc heuristics ($\bar{\rho}$) are equal to 25%.[13]

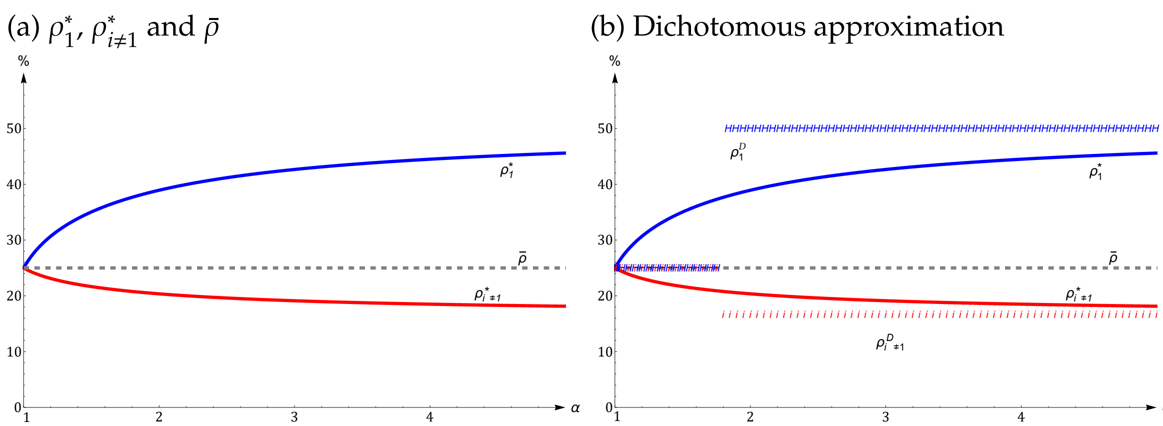

Figure 4.1 Shapley shares vs. ad hoc heuristic shares and dichotomous approximation

Panel (b) of Figure 4.1 adds the Shapley shares in the dichotomous damage scenarios which respectively minimize $\sum_{i\in N} \mid \rho^*_i - \rho^D_i \mid$ for given α. When α is small, the Shapley allocation in the dichotomous damage scenario coincides with $\bar{\rho}$ since firms' Shapley shares in the original (non-dichotomous) game (N, v), $\rho^*_i(N, v)$, are relatively close to $1/n$. However, with $\alpha \geq 1.8$, the dichotomous scenario (50%; 16.$\bar{6}$%; 16.$\bar{6}$%; 16.$\bar{6}$%) fits best. Whereas deviations between $\rho^*(N, v)$ and $\bar{\rho}$ increase with increasing values of α, the highest deviation using the discretization heuristic is reached when $\alpha \approx 1.8$.

[13]Note that a firm's Shapley share ρ^*_i and all heuristics are independent of a and γ when firms are symmetric. However, these baseline parameters will matter for asymmetric firms below.

Thus, with symmetric firms, ρ^D is always at least as good as $\bar{\rho}$ with respect to the accuracy of deviation.

Second, assume that all customers act against former cartel members. Then, aggregated overcharge damages are $O := \sum_{i\in N} q_i^C \cdot \Delta p_i$ where q_i^C is the cartel quantity sold by firm i. The damage for each single good j then can be allocated according to some heuristic or according to the Shapley allocation $\rho^*(N, v^j)$. Let Φ_i be firm i's aggregated Shapley payments, i.e.,

$$\Phi_i := \sum_{j\in N} \varphi_i(N, v^j) = \sum_{j\in N} q_j^C \cdot \Delta p_j \cdot \rho_i^*(N, v^j), \tag{4.6}$$

and let H_i^ρ be i's aggregated payments when heuristic ρ is applied. Firm i therefore contributes too much compared to its relative responsibility when $H_i^\rho - \Phi_i > 0$ and too little when $H_i^\rho - \Phi_i < 0$.

From a market-wide perspective, we are interested in the aggregated net mis-allocation of damages, that is, in $\sum_{i\in N} \mid \Phi_i - H_i^\rho \mid$. Dividing the total mis-allocation by the total overcharge damage O will make different numerical simulations comparable and gives $M^\rho := \sum_{i\in N} \mid \Phi_i - H_i^\rho \mid / O$.

When firms are symmetric, M^ρ is by definition zero: although Shapley shares ρ_h^* and ρ_i^* differ among co-defendants, these differences cancel each other out when all customers sue. An equal per head allocation applies. As the following subsection will show, results do not generalize to firms that are asymmetric from a market-wide perspective.

4.4.3 Asymmetric Firms

When firms differ in their saturation quantity a_i or in their efficiency parameter γ_i, Subsection 3.3.3 shows that bounds similar to the symmetric case exist. A home firm's Shapley share still lies in $\rho_h^* \in (1/n, 1/2)$ but the shares between remaining firms $i \neq h$ can range more widely with $\rho_i^* \in (0, 1/2)$. When firms differ in own or cross price parameters, not even the derived bound for the home firm stays valid. Thus, reliable heuristics are particularly important in the asymmetric case.

4.4.3.1 Product-Specific Perspective

We start by assuming that Δp_i has to be allocated among former cartel members. Can ad hoc heuristics now be used to approximate Shapley shares $\rho_i^*(N, v)$ when firms are asymmetric, say when they differ in size? To answer this question we adapt the example introduced in Subsection 4.4.2 with $a_1/3 = a_2/3 = a_3 = a_4 = 30$, $\gamma = 2$, $d = 3$, $n = 4$ and $b = d/(3\alpha) = 1/\alpha$. Out of the 19 dichotomous damage scenarios that exist for $n = 4$ (see Table 4.3) we choose the respective approximation to minimize $\sum_{i \in N} \mid \rho_i^* - \rho_i^D \mid$.[14]

Figure 4.2 illustrates results. Panels (a1) and (a2) depict the Shapley share ρ_h^*, the discretization heuristic ρ_h^D and ad hoc heuristics for the home firm. When a large (**small**) firm sold the product, it is assumed w.l.o.g. that it is firm 1 (3). A capital D (small d) is used in the dichotomous scenario when the home firm is large (small). Interestingly, Shapley shares ρ_h^* with $h \in \{1, 3\}$ are quite similar whereas ad hoc heuristics significantly differ among small and large (home) firms when damage is not allocated per head for which $\rho_1^0 = \rho_3^0$ holds. Ad hoc heuristics give a good approximation of the Shapley share ρ_1^* but completely fail to apply for approximating ρ_3^*. This differs for the discretization heuristic which takes a firm's ability to influence the market price into account. Approximations ρ_1^D and ρ_3^D are rather good.

Panels (b1) and (b2) consider a large firm $i \neq h$. The Shapley share ρ_i^* with $i \neq h \in \{1, 2\}$ then varies around $1/n$. This makes ρ_i^0, which coincides with ρ_i^D when the small firm sold the product the best approximation. Other ad hoc heuristics are rather far off, in particular when the small firm sold the product. Dichotomous scenarios are in a reasonable range, also for a large home firm.

Panels (c1) and (c2) illustrate a small firm i's contribution share when $i \neq h$. Heuristic ρ_i^0 now is only a good approximation when α is small. Other heuristics, like an allocation based on competitive revenues ρ_i^3, do much better for larger α. The discretization heuristic is also pretty close to the Shapley share evaluated for (N, v), in particular when a large firm sold the product.

To sum up: there is usually some ad hoc heuristic that can be used to approximate a firm's Shapley share $\rho_i^*(N, v)$ with $i \in N$ reasonably well except for the case where $\rho_h^*(N, v)$ with $h \in \{3, 4\}$ has to be approximated. However, no single heuristic is always close to $\rho_i^*(N, v)$. Thus, ad hoc heuristics are not a good approximation for damage allocation by relative responsibility when only a single customer acts against former cartel members. An allocation based on dichotomous damage scenarios is rather well suited for use, irrespectively whether the large or the small firm sold the product.

[14]We rerun the analysis using the DDS which leads to the second closest distance to Shapley shares. This robustness test leads to similar results.

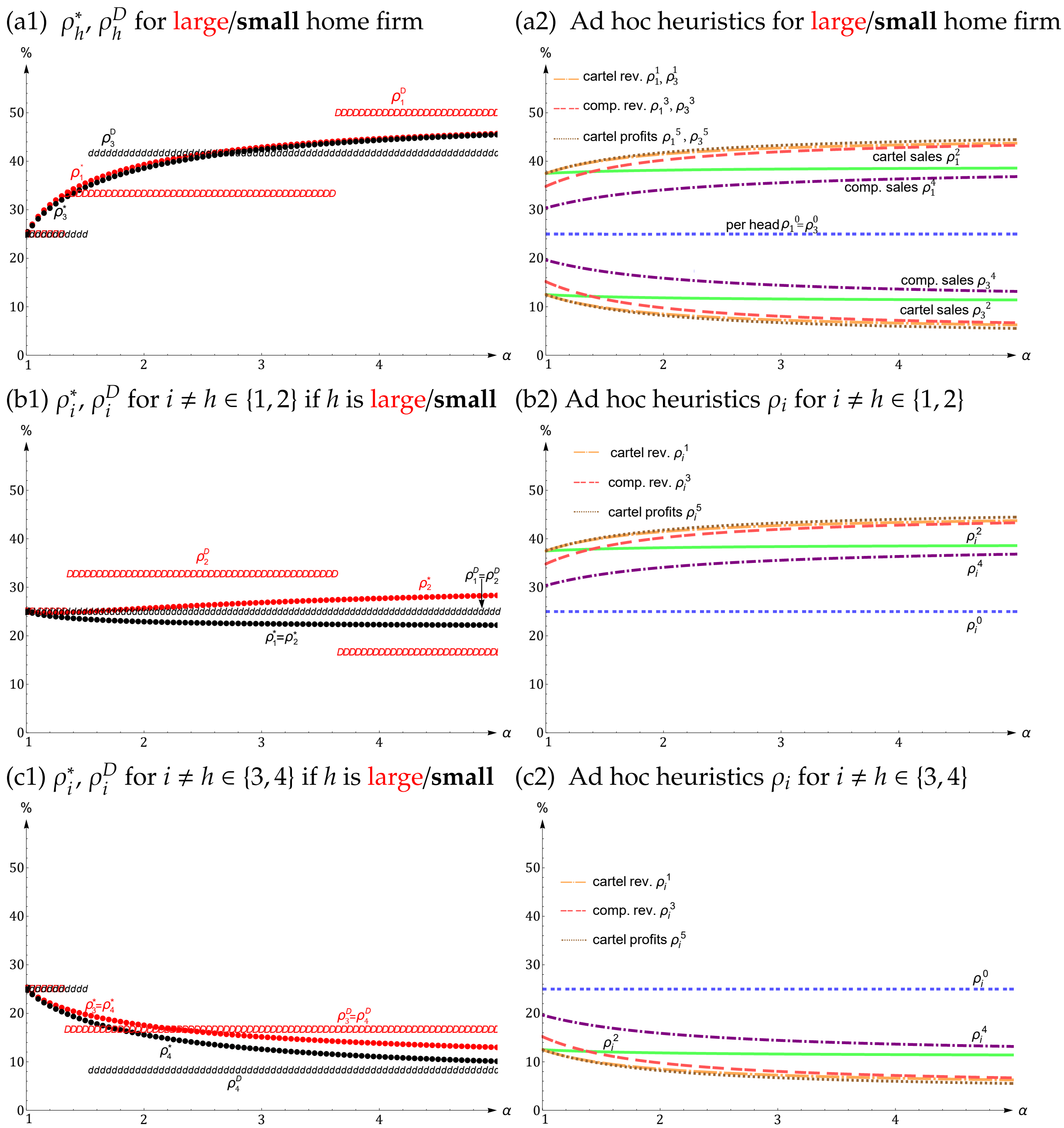

Figure 4.2 Shapley shares ρ_i^* vs. ad hoc heuristic shares and dichotomous approximation with $a_1/3 = a_2/3 = a_3 = a_4$

4.4.3.2 Market-Wide Perspective

Contrary to the symmetric case, aggregate deviations between product-specific Shapley shares $\rho^*(N, v)$ and other heuristics do generally not vanish when firms are asymmetric and all customers act against former cartel members. Note, however, that differences between an allocation based on the Shapley share $\rho^*(N, v)$ and other heuristics can partially cancel out across products.

In Figure 4.3 we illustrate how ad hoc heuristics and the discretization heuristic fit, i.e., the percentage mis-allocation M^ρ, for different kinds of asymmetries when baseline parameters are $a = 30$, $\gamma = 2$, $d = 3$, $n = 4$, $b = d/(3\alpha)$.

In the two top panels (a) and (b), we allow that firms differ in their market saturation quantity a_i. Panel (a) reconsiders the example discussed in Subsection 4.4.3.1. The discretization heuristic ρ^D outperforms ad hoc heuristics for $\alpha > 1.4$. With increasing values of α, only an allocation based on competitive sales ρ^4 comes close to the Shapley based allocation $\rho^*(N, v)$ in both panels. However, ρ^D does much better and achieves smaller mis-allocation M^{ρ^D} for almost all levels of differentiation.

The discontinuous jumps occurring in the product-wise discretization heuristic can be most easily explained by using panel (c) where two firms are twice as efficient as their competitors. When $\alpha = 1.55$, aggregated overcharge damages are roughly $O = 321$. Then, using a dichotomous approximation requires each firm to contribute share $1/n$, that is, $H_i^{\rho^D} = 80.3$. Now fix an inefficient firm, say firm 3. Firm 3's Shapley shares $\rho_3^*(N, v)$ are $(21\%; 21\%; 35.4\%; 20.6\%)$ in the four price overcharges $\Delta p1$, $\Delta p2$, $\Delta p3$ and $\Delta p4$; $\Phi_3 = 77.2$. Thus, $\rho_3^D = 25\%$ significantly underestimates firm 3's own part but slightly overestimates its contribution share when another firm sold the product. Differences between over and underestimation of a firm's contribution share partially cancel and $H_3^{\rho^D} - \Phi_3 = 3.1$. Hence, $M^{\rho^D} = 4 \cdot 3.1/321 = 4.3\%$.[15] Next, consider $\alpha = 1.6$; then, $O = 286$. Dichotomous damage shares for a high cost firm abruptly changes to $(16.\bar{6}\%; 16.\bar{6}\%; 25\%; 25\%)$ with $H_3^{\rho^D} = 58$. Its respective Shapley shares $\rho_3^*(N, v)$ are $(20.8\%; 20.8\%; 35.9\%; 20.4\%)$ with $\Phi_3 = 68.7$. The product-specific dichotomous approximations now always underestimate firm 3's shares (except for the case in which firm 4 sold the product), with a huge difference when its home customers sue. The deviation increases to $\Phi_3 - H_3^{\rho^D} = 10.7$; $M^{\rho^D} = 4 \cdot 10.7/286 = 14.9\%$. Thus, the sum over the dichotomous approximations that minimize the product-specific deviations – which change continuously in α – only minimizes market-wide deviations by coincidence, not in general. In particular, on a market-wide perspective, the "second-best" dichotomous approximation used in our robustness analysis sometimes outperforms the approximation that minimizes $\sum_{i \in N} | \rho_i^* - \rho_i^D |$.

In panels (c) and (d), an allocation based on competitive revenue shares ρ^3 is very close to Φ_i and frequently outperforms the discretization heuristic. In panel (c), ρ^D is

[15] Note that two firms are efficient and two firms inefficient each. When the efficient firms pay too much, the inefficient firms have to pay too little since all discussed allocation rules satisfy efficiency.

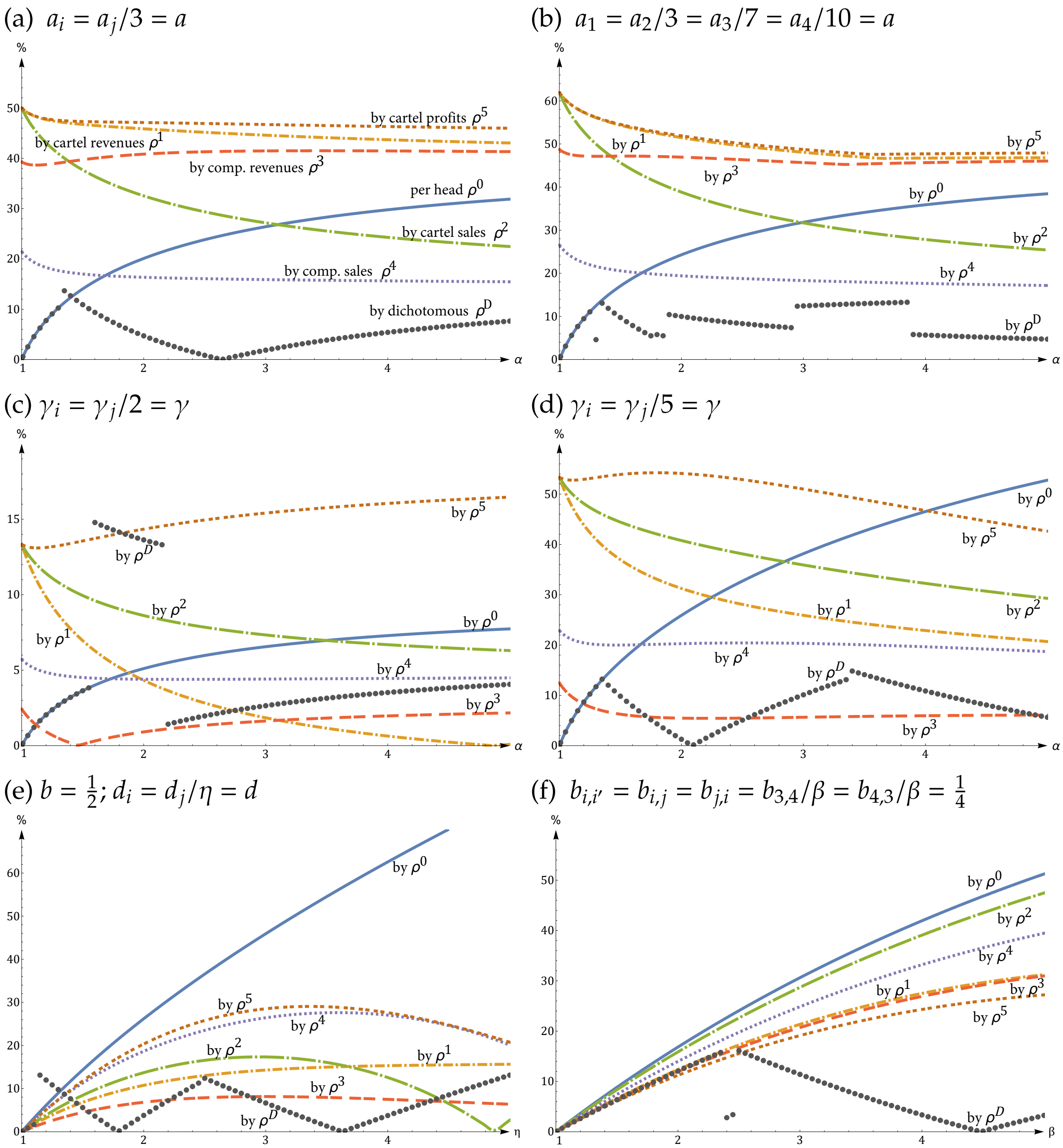

Figure 4.3 M^ρ for different heuristics considering $i \in \{1, 2\}$ and $j \in \{3, 4\}$

worst for some values of α. In this case, however, all heuristics could be used since normalized deviations in panel (c) are all rather small.

When own-price effects of two firms increase by factor η (see panel (e)), some ad hoc heuristics, in particular the one based on competitive revenue shares ρ^3, perform well. The discretization heuristic is sometimes best, but can also be worst for a very small range of η. Nevertheless, ρ^D is close to the Shapley share $\rho^*(N, v)$ for all relevant values of η; M^{ρ^D} is consistently below 15%.

Finally, panel (f) involves two firms with increasing cross-price effects. With more intense competition, that is, with increasing values of β, only the discretization heuristic is close to the Shapley value in the original game. With rather symmetric firms, an allocation based on profit shares ρ^5 performs relatively good.

To sum up: there is no ad hoc heuristic which is always close to the correct Shapley shares evaluated for (N, v). This differs for a heuristic allocation based on dichotomous damage scenarios. It does not only reflect responsibility, but M^{ρ^D} is additionally always significantly below 20% in the discussed cases. Sometimes, it is the only heuristic which is close to the Shapley value of the original game (N, v).[16]

4.5 Concluding Remarks

This chapter has argued that a responsibility-based allocation of cartel damages is feasible even without a full-blown merger simulation analysis. This should receive increasing attention in the EU since Directive 2014/104/EU has been transposed into national law throughout the EU.

The question how to economically quantify the relative responsibility of a firm is answered in Chapter 3: ideally use the Shapley value. Ad hoc heuristics could be used to approximate the Shapley share in view of the former's strong data requirements. But no heuristic always outperforms the others. Thus, more details on the firms at hand have to be known to argue which heuristic indeed fits best. Then, however, the additional expense to answer the question which coalitions actually caused (or would have caused, as counterfactual market scenarios are evaluated) huge damage, is small. This leads to the proposal of a new heuristic, ρ^D, based on approximation by dichotomous damage scenarios. Heuristic ρ^D has two main advantages compared to ad hoc heuristics. First, it reflects relative responsibility and second, it rather robustly offers a good approximation of the Shapely value in all considered market scenarios; from a product but also from a market-wide perspective.

[16] Interestingly, although baseline parameters in panels (a) – (f) differ from the ones used in the simulations in Subsection 3.3.4, the ad hoc heuristic which is closest to the Shapley value in the original game stays unchanged. Thus, baseline parameters are rather unimportant to evaluate and select ad hoc heuristics; the asymmetry at hand matters.

4.6 Appendix A Upper Bound of $\Phi_i(N, v)$

We want to show that the upper bound of the Shapley share is $(n-1)/n$ for all cartel damage allocation games. Recall that the Shapley value is given by

$$\Phi_i(N, v) = \sum_{S \subseteq N} \frac{(s-1)!(n-s)!}{n!} \cdot \Big[v(S) - v(S \setminus \{i\})\Big].$$

In view of Φ_i's linearity, it is without loss of generality to normalize damages to $v(N) = 1$, $v(S) \in [0, 1]$ and $v(\emptyset) = v(\{i\}) = 0$ for $i \in N$. The highest damage caused by a coalition is therefore 1. The absolute Shapley values in the normalized game can be interpreted as Shapley shares in the original cartel damage allocation game.

Next, consider firm A. The Shapley value of firm A (thus, its contribution payment) is maximal when firm A's marginal contribution to all coalitions is maximal, that is, when firm A is a dictator player: assuming that all coalitions including A could cause a unit damage ($v(S) = 1 \Leftrightarrow \mathrm{A} \in S$) would then lead to a Shapley-(Shubik) power index of $\Phi^{Dic}_{\mathrm{A}} = 1$. However, from the assumption that $v(\{i\}) = 0$ for $i \in N$ follows that firm A causes zero damage when all firms compete. Then, firm A's marginal contribution is zero, that is, $v(\{\mathrm{A}\}) - v(\emptyset) = 0$. Thus, the highest Shapley value of a firm in cartel damage allocation games is reached by subtracting $\frac{(s-1)!(n-s)!}{n!}\,|_{s=1} \cdot [v(\{\mathrm{A}\}) - v(\emptyset)] = \frac{(s-1)!(n-s)!}{n!}\,|_{s=1} \cdot 1$ from Φ^{Dic}_{A}. This yields

$$\Phi^{max}_{A} = \Phi^{Dic}_{\mathrm{A}} - \frac{(n-1)!}{n!} = 1 - \frac{1}{n} = \frac{n-1}{n}$$

as the highest possible Shapley share in damages in markets in which at least two players must cooperate to cause an overcharge.

4.7 Appendix B
All Dichotomous Damage Scenarios with n = 5 Firms

$\mathcal{M}(\tilde{v})$	$60 \cdot \varphi(N, \tilde{v})$	$\mathcal{M}(\tilde{v})$	$60 \cdot \varphi(N, \tilde{v})$
1.–19. see Table 4.3 on p. 122		71. AB, AC, ADE, BCDE	(30, 10, 10, 5, 5)
20. AB, AC, AD, AE	(48, 3, 3, 3, 3)	72. AB, AC, ADE, BDE, CDE	(24, 9, 9, 9, 9)
21. AB, AC, AD, AE, BC	(28, 13, 13, 3, 3)	73. AB, AC, BC, ADE	(22, 17, 17, 2, 2)
22. AB, AC, AD, AE, BC, BD	(23, 18, 8, 8, 3)	74. AB, AC, BC, ADE, BDE	(19, 19, 14, 4, 4)
23. AB, AC, AD, AE, BC, BD, BE	(21, 21, 6, 6, 6)	75. AB, AC, BC, ADE, BDE, CDE	(16, 16, 16, 6, 6)
24. AB, AC, AD, AE, BC, BD, BE, CD	(16, 16, 11, 11, 6)	76. AB, AC, BC, DE	(14, 14, 14, 9, 9)
25. AB, AC, AD, AE, BC, BD, BE, CD, CE	(14, 14, 14, 9, 9)	77. AB, AC, BCD, BCE	(22, 17, 17, 2, 2)
26. AB,AC,AD,AE,BC,BD,BE,CD,CE,DE	(12, 12, 12, 12, 12)	78. AB, AC, BCD, BCE, BDE	(19, 19, 14, 4, 4)
27. AB, AC, AD, AE, BC, BD, BE, CDE	(18, 18, 8, 8, 8)	79. AB, AC, BCD, BCE, BDE, CDE	(16, 16, 16, 6, 6)
28. AB, AC, AD, AE, BC, BD, CD	(18, 13, 13, 13, 3)	80. AB, AC, BCD, BDE	(22, 17, 12, 7, 2)
29. AB, AC, AD, AE, BC, BD, CE	(18, 13, 13, 8, 8)	81. AB, AC, BCD, BDE, CDE	(19, 14, 14, 9, 4)
30. AB, AC, AD, AE, BC, BD, CE, DE	(16, 11, 11, 11, 11)	82. AB, AC, BCDE	(28, 13, 13, 3, 3)
31. AB, AC, AD, AE, BC, BD, CDE	(20, 15, 10, 10, 5)	83. AB, AC, BD, ADE	(22, 17, 7, 12, 2)
32. AB, AC, AD, AE, BC, BDE	(25, 15, 10, 5, 5)	84. AB, AC, BD, ADE, BCE	(19, 19, 9, 9, 4)
33. AB, AC, AD, AE, BC, BDE, CDE	(22, 12, 12, 7, 7)	85. AB, AC, BD, ADE, BCE, CDE	(16, 16, 11, 11, 6)
34. AB, AC, AD, AE, BC, DE	(20, 10, 10, 10, 10)	86. AB, AC, BD, ADE, CDE	(19, 14, 9, 14, 4)
35. AB, AC, AD, AE, BCD	(33, 8, 8, 8, 3)	87. AB, AC, BD, CD, ADE	(17, 12, 12, 17, 2)
36. AB, AC, AD, AE, BCD, BCE	(30, 10, 10, 5, 5)	88. AB, AC, BD, CD, ADE, BCE	(14, 14, 14, 14, 4)
37. AB, AC, AD, AE, BCD, BCE, BDE	(27, 12, 7, 7, 7)	89. AB, AC, BD, CDE	(17, 17, 12, 12, 2)
38. AB,AC,AD,AE,BCD,BCE,BDE,CDE	(24, 9, 9, 9, 9)	90. AB, AC, BD, CE	(12, 17, 17, 7, 7)
39. AB, AC, AD, AE, BCDE	(36, 6, 6, 6, 6)	91. AB, AC, BD, CE, ADE	(14, 14, 14, 9, 9)
40. AB, AC, AD, BC, BD, CDE	(17, 17, 12, 12, 2)	92. AB, AC, BD, CE, DE	(12, 12, 12, 12, 12)
41. AB, AC, AD, BC, BD, CE	(15, 15, 15, 10, 5)	93. AB, AC, BDE	(25, 15, 10, 5, 5)
42. AB, AC, AD, BC, BD, CE, DE	(13, 13, 13, 13, 8)	94. AB, AC, BDE, CDE	(22, 12, 12, 7, 7)
43. AB, AC, AD, BC, BDE	(22, 17, 12, 7, 2)	95. AB, AC, DE	(22, 7, 7, 12, 12)
44. AB, AC, AD, BC, BDE, CDE	(19, 14, 14, 9, 4)	96. AB, AC, DE, BCD	(19, 9, 9, 14, 9)
45. AB, AC, AD, BC, BE	(20, 20, 10, 5, 5)	97. AB, AC, DE, BCD, BCE	(16, 11, 11, 11, 11)
46. AB, AC, AD, BC, BE, CDE	(17, 17, 12, 7, 7)	98. AB, ACD, ACE	(37, 12, 7, 2, 2)
47. AB, AC, AD, BC, BE, DE	(15, 15, 10, 10, 10)	99. AB, ACD, ACE, ADE	(39, 9, 4, 4, 4)
48. AB, AC, AD, BC, DE	(17, 12, 12, 12, 7)	100. AB, ACD, ACE, ADE, BCD	(24, 14, 9, 9, 4)
49. AB, AC, AD, BCD, BCE	(27, 12, 12, 7, 2)	101. AB, ACD, ACE, ADE, BCD, BCE	(21, 16, 11, 6, 6)
50. AB, AC, AD, BCD, BCE, BDE	(24, 14, 9, 9, 4)	102. AB, ACD, ACE, ADE, BCD, BCE, BDE	(18, 18, 8, 8, 8)
51. AB, AC, AD, BCD, BCE, BDE, CDE	(21, 11, 11, 11, 6)	103. AB,ACD,ACE,ADE,BCD,BCE,BDE,CDE	(15, 15, 10, 10, 10)
52. AB, AC, AD, BCDE	(33, 8, 8, 8, 3)	104. AB, ACD, ACE, ADE, BCD, BCE, CDE	(18, 13, 13, 8, 8)
53. AB, AC, AD, BCE	(30, 10, 10, 5, 5)	105. AB, ACD, ACE, ADE, BCD, CDE	(21, 11, 11, 11, 6)
54. AB, AC, AD, BCE, BDE	(27, 12, 7, 7, 7)	106. AB, ACD, ACE, ADE, BCDE	(27, 12, 7, 7, 7)
55. AB, AC, AD, BCE, BDE, CDE	(24, 9, 9, 9, 9)	107. AB, ACD, ACE, ADE, CDE	(24, 9, 9, 9, 9)
56. AB, AC, AD, BE	(25, 15, 5, 5, 10)	108. AB, ACD, ACE, BCD	(22, 17, 12, 7, 2)
57. AB, AC, AD, BE, BCD	(22, 17, 7, 7, 7)	109. AB, ACD, ACE, BCD, BCE	(19, 19, 14, 4, 4)
58. AB, AC, AD, BE, BCD, CDE	(19, 14, 9, 9, 9)	110. AB, ACD, ACE, BCD, BCE, CDE	(16, 16, 16, 6, 6)
59. AB, AC, AD, BE, CDE	(22, 12, 7, 7, 12)	111. AB, ACD, ACE, BCD, BDE	(19, 19, 9, 9, 4)
60. AB, AC, AD, BE, CE	(20, 10, 10, 5, 15)	112. AB, ACD, ACE, BCD, BDE, CDE	(16, 16, 11, 11, 6)
61. AB, AC, AD, BE, CE, BCD	(17, 12, 12, 7, 12)	113. AB, ACD, ACE, BCD, CDE	(19, 14, 14, 9, 4)
62. AB, AC, AD, BE, CE, DE	(18, 8, 8, 8, 18)	114. AB, ACD, ACE, BCDE	(25, 15, 10, 5, 5)
63. AB, AC, AD, BE, CE, DE, BCD	(15, 10, 10, 10, 15)	115. AB, ACD, ACE, BDE	(22, 17, 7, 7, 7)
64. AB, AC, ADE	(42, 7, 7, 2, 2)	116. AB, ACD, ACE, BDE, CDE	(19, 14, 9, 9, 9)
65. AB, AC, ADE, BCD	(27, 12, 12, 7, 2)	117. AB, ACD, ACE, CDE	(22, 12, 12, 7, 7)
66. AB, AC, ADE, BCD, BCE	(24, 14, 14, 4, 4)	118. AB, ACD, BCD, CDE	(17, 17, 12, 12, 2)
67. AB, AC, ADE, BCD, BCE, BDE	(21, 16, 11, 6, 6)	119. AB, ACD, BCDE	(23, 18, 8, 8, 3)
68. AB, AC, ADE, BCD, BCE, BDE, CDE	(18, 13, 13, 8, 8)	120. AB, ACD, BCE	(20, 20, 10, 5, 5)
69. AB, AC, ADE, BCD, BDE	(24, 14, 9, 9, 4)	121. AB, ACD, BCE, CDE	(17, 17, 12, 7, 7)
70. AB, AC, ADE, BCD, BDE, CDE	(21, 11, 11, 11, 6)	122. AB, ACD, CDE	(20, 15, 10, 10, 5)

	$\mathcal{M}(\tilde{v})$	$60 \cdot \varphi(N, \tilde{v})$
123.	AB, ACDE	(33, 18, 3, 3, 3)
124.	AB, AC, ADE, BDE	(27, 12, 7, 7, 7)
125.	AB, ACDE, BCDE	(21, 21, 6, 6, 6)
126.	AB, CD, ACE	(17, 12, 17, 12, 2)
127.	AB, CD, ACE, ADE	(19, 9, 14, 14, 4)
128.	AB, CD, ACE, ADE, BCE	(16, 11, 16, 11, 6)
129.	AB, CD, ACE, ADE, BCE, BDE	(13, 13, 13, 13, 8)
130.	AB, CE, ACE, BDE	(14, 14, 14, 14, 4)
131.	AB, CDE	(18, 18, 8, 8, 8)
132.	ABC, ABD, ABE	(27, 27, 2, 2, 2)
133.	ABC, ABD, ABE, ACD	(32, 12, 7, 7, 2)
134.	ABC, ABD, ABE, ACD, ACE	(34, 9, 9, 4, 4)
135.	ABC, ABD, ABE, ACD, ACE, ADE	(36, 6, 6, 6, 6)
136.	ABC, ABD, ABE, ACD, ACE, ADE, BCD	(21, 11, 11, 11, 6)
137.	ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE	(18, 13, 13, 8, 8)
138.	ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE	(15, 15, 10, 10, 10)
139.	ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE	(12, 12, 12, 12, 12)
140.	ABC, ABD, ABE, ACD, ACE, ADE, BCDE	(24, 9, 9, 9, 9)
141.	ABC, ABD, ABE, ACD, ACE, BCD	(19, 14, 14, 9, 4)
142.	ABC, ABD, ABE, ACD, ACE, BCD, BCE	(16, 16, 16, 6, 6)
143.	ABC, ABD, ABE, ACD, ACE, BCD, BDE	(16, 16, 11, 11, 6)
144.	ABC, ABD, ABE, ACD, ACE, BCD, BDE, CDE	(13, 13, 13, 13, 8)
145.	ABC, ABD, ABE, ACD, ACE, BCDE	(22, 12, 12, 7, 7)
146.	ABC, ABD, ABE, ACD, ACE, BDE	(19, 14, 9, 9, 9)
147.	ABC, ABD, ABE, ACD, ACE, BDE, CDE	(16, 11, 11, 11, 11)
148.	ABC, ABD, ABE, ACD, BCD	(17, 17, 12, 12, 2)
149.	ABC, ABD, ABE, ACD, BCD, CDE	(14, 14, 14, 14, 4)
150.	ABC, ABD, ABE, ACD, BCDE	(20, 15, 10, 10, 5)
151.	ABC, ABD, ABE, ACD, BCE	(17, 17, 12, 7, 7)
152.	ABC, ABD, ABE, ACD, BCE, CDE	(14, 14, 14, 9, 9)
153.	ABC, ABD, ABE, ACD, CDE	(17, 12, 12, 12, 7)
154.	ABC, ABD, ABE, ACDE	(30, 15, 5, 5, 5)
155.	ABC, ABD, ABE, ACDE, BCDE	(18, 18, 8, 8, 8)
156.	ABC, ABD, BCE	(30, 10, 10, 5, 5)
157.	ABC, ABD, ABE, CDE	(15, 15, 10, 10, 10)
158.	ABC, ABD, ACD, BCE	(15, 15, 15, 10, 5)
159.	ABC, ABD, ACD, BCE, BDE	(12, 17, 12, 12, 7)
160.	ABC, ABD, ACD, BCE, BDE, CDE	(9, 14, 14, 14, 9)
161.	ABC, ABD, ACD, BCDE	(18, 13, 13, 13, 3)
162.	ABC, ABD, ACE, ADE	(32, 7, 7, 7, 7)
163.	ABC, ABD, ACE, ADE, BCDE	(20, 10, 10, 10, 10)
164.	ABC, ABD, ACE, BCDE	(18, 13, 13, 8, 8)
165.	ABC, ABD, ACE, BDE	(15, 15, 10, 10, 10)
166.	ABC, ABD, ACE, BDE, CDE	(12, 12, 12, 12, 12)
167.	ABC, ABD, ACDE	(28, 13, 8, 8, 3)
168.	ABC, ABD, ACDE, BCDE	(16, 16, 11, 11, 6)
169.	ABC, ABD, CDE	(13, 13, 13, 13, 8)
170.	ABC, ABDE	(23, 23, 8, 3, 3)
171.	ABC, ABDE, ACDE	(26, 11, 11, 6, 6)
172.	ABC, ABDE, ACDE, BCDE	(14, 14, 14, 9, 9)
173.	ABC, ADE	(28, 8, 8, 8, 8)
174.	ABC, ADE, BCDE	(16, 11, 11, 11, 11)
175.	ABCD, ABCE	(18, 18, 18, 3, 3)
176.	ABCD, ABCE, ABDE	(21, 21, 6, 6, 6)
177.	ABCD, ABCE, ABDE, ACDE	(24, 9, 9, 9, 9)
178.	ABCD, ABCE, ABDE, ACDE, BCDE	(12, 12, 12, 12, 12)
179.	ABCDE	(12, 12, 12, 12, 12)

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