

LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

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Abstract: Constant dimension codes are e.g. used for error correction and detection in random linear network coding, so that constructions for these codes have achieved wide attention. Here, we improve over 150 lower bounds by describing better constructions for subspace distance 4.

Keywords: constant dimension codes, multilevel construction, Echelon–Ferrers construction, linkage, network coding

1. INTRODUCTION

Let q be a prime power and \mathbb{F}_q be the finite field with q elements. For two integers $0 \leq k \leq n$ we denote by $\mathcal{G}_q(n, k)$ the set of all k -dimensional subspaces in \mathbb{F}_q^n . The so-called subspace distance $d_S(U, W) := \dim(U) + \dim(W) - 2 \dim(U \cap W) = 2k - 2 \dim(U \cap W)$ defines a metric on $\mathcal{G}_q(n, k)$. A subset $\mathcal{C} \subseteq \mathcal{G}_q(n, k)$ is called a *constant dimension code* (CDC), its elements are also called codewords, and $d_S(\mathcal{C}) = \min\{d_S(U, W) : U, W \in \mathcal{C}, U \neq W\}$ is the corresponding *minimum (subspace) distance*. We also call \mathcal{C} an $(n, M, d, k)_q$ CDC if \mathcal{C} has cardinality M and $d_S(\mathcal{C}) \geq d$. The maximum possible cardinality of an $(n, M, d, k)_q$ CDC is denoted by $A_q(n, d; k)$. Constant dimension codes are e.g. applied in random linear network coding, see e.g. [14], and the determination of bounds for $A_q(n, d; k)$ is one of the main problems. Here we improve more than 150 of the previously best known constructions for CDCs. An online table for bounds for $A_q(n, d; k)$ can be found at `subspacecodes.uni-bayreuth.de`, see the corresponding technical manual [11].

The remaining part of this paper is structured as follows. In Section 2 we introduce the necessary preliminaries and describe constructions for CDCs from the literature. Our theoretical and algorithmical results are the topic of Section 3. The resulting numerical improvements for lower bounds for $A_q(n, d; k)$ are listed in Appendix A. Extensive computational data about the details of the underlying constructions are given in Appendix B.

2. PRELIMINARIES

Given a CDC \mathcal{C} we first consider the question how to represent its codewords, i.e., k -dimensional subspaces $U \in \mathcal{G}_q(n, k)$. Starting from a generator matrix whose k rows form a basis of U the application of the Gaussian elimination algorithm gives a unique

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generator matrix in *reduced row echelon form* denoted by $E(U)$. In the other direction we write $U = \langle E(U) \rangle$. By $v(U) \in \mathbb{F}_2^n$ we denote the characteristic vector of the pivot columns in $E(U)$, which is also called *identifying vector*. The *Ferrers tableaux* $T(U)$ of U arises from $E(U)$ by removing the zeroes from each row of $E(U)$ left to the pivots and afterwards removing all pivot columns. If we then replace all remaining entries by dots we obtain the *Ferrers diagram* $\mathcal{F}(U)$ of U which only depends on the identifying vector $v(U)$. As an example we consider

$$U = \left\langle \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \right\rangle \in \mathcal{G}_2(10, 4),$$

where we have

$$E(U) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix},$$

$v(U) = 1011001000 \in \mathbb{F}_2^{10}$, and

$$\mathcal{F}(U) = \begin{array}{cccccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ & \bullet & \bullet & \bullet & \bullet & \bullet \\ & \bullet & \bullet & \bullet & \bullet & \bullet \\ & & & \bullet & \bullet & \bullet \end{array}.$$

The *Hamming distance* $d_H(u, w) = \#\{1 \leq i \leq n : u_i \neq w_i\}$, for $u, w \in \mathbb{F}_q^n$, can be used to lower bound the subspace distance between two codewords $U, W \in \mathcal{G}_q(n, k)$:

Lemma 1. ([5, Lemma 2])

For $U, W \in \mathcal{G}_q(n, k)$ we have $d_S(U, W) \geq d_H(v(U), v(W))$.

If the identifying vectors of two codewords coincide, then we can utilize the *rank distance* $d_R(A, B) := \text{rank}(A - B)$ for matrices $A, B \in \mathbb{F}_q^{m \times l}$:

Lemma 2. ([20, Corollary 3])

For $U, W \in \mathcal{G}_q(n, k)$ with $v(U) = v(W)$ we have $d_S(U, W) = 2d_R(E(U), E(W))$.

Since d_R is a metric, we call a subset $C \subseteq \mathbb{F}_q^{m \times l}$ of matrices a *rank-metric code*. If C is a linear subspace of $\mathbb{F}_q^{m \times l}$ we call the code *linear*. Given a Ferrers diagram \mathcal{F} with m dots in the rightmost column and l dots in the top row, we call a rank-metric code $C_{\mathcal{F}}$ a *Ferrers diagram rank-metric* (FDRM) code if for any codeword $M \in \mathbb{F}_q^{m \times l}$ of $C_{\mathcal{F}}$ all entries not in \mathcal{F} are zero. By $d_R(C_{\mathcal{F}})$ we denote the minimum rank distance, i.e., the minimum of the rank distance between pairs of different codewords.

Definition 1. ([21])

Let \mathcal{F} be a Ferrers diagram and $C_{\mathcal{F}} \subseteq \mathbb{F}_q^{k \times (n-k)}$ be an FDRM code. The corresponding

lifted FDRM code $\mathcal{C}_{\mathcal{F}}$ is given by

$$\mathcal{C}_{\mathcal{F}} = \{U \in \mathcal{G}_q(n, k) : \mathcal{F}(U) = \mathcal{F}, T(U) \in C_{\mathcal{F}}\}.$$

We remark that the bijection between the codewords of the CDC $\mathcal{C}_{\mathcal{F}}$ and the FDRM code $C_{\mathcal{F}}$ generalizes the construction of lifted *maximum rank distance* (MRD) codes. An MRD code corresponds to the case of a Ferrers diagram \mathcal{F} with k dots in each column and $n - k$ dots in each row (more details below).

Directly from Lemma 2 and Definition 1 we can conclude:

Lemma 3. ([5, Lemma 4])

Let $C_{\mathcal{F}} \subseteq \mathbb{F}_q^{k \times (n-k)}$ be an FDRM code with minimum rank distance δ , then the lifted FDRM code $\mathcal{C}_{\mathcal{F}} \subseteq \mathcal{G}_q(n, k)$ is an $(n, \#C_{\mathcal{F}}, 2\delta, k)_q$ CDC.

Let $v(\mathcal{F})$ be the identifying vector of a given Ferrers diagram \mathcal{F} . In general, we denote by $A_q(n, d; k; v)$ the maximum cardinality M of an $(n, M, d, k)_q$ CDC where all codewords have $v \in \mathbb{F}_2^n$ as identifying vector. We also speak of an $(n, M, d, k, v)_q$ CDC. With this (and Lemma 3) the upper bound for the cardinality of $\mathcal{C}_{\mathcal{F}}$ from [5, Theorem 1]¹ can be rewritten to:

Theorem 1.

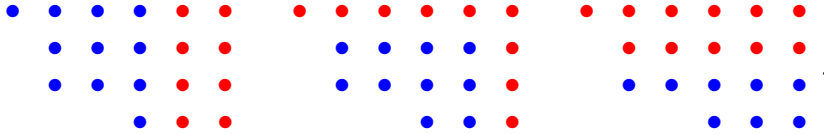
$$A_q(n, d; k; v(\mathcal{F})) \leq q^{\min\{\nu_i : 0 \leq i \leq d/2-1\}},$$

where ν_i is the number of dots in \mathcal{F} , which are neither contained in the first i rows nor contained in the last $\frac{d}{2} - 1 - i$ columns.

If we choose a minimum subspace distance of $d = 6$, then we obtain

$$A_2(10, 6; 4; 1011001000) \leq 2^8$$

due to



The Hamming weight of a vector $v \in \mathbb{F}_2^n$ is the Hamming distance $d_H(v, \mathbf{0})$ of v to the zero vector.

Theorem 2. ([5, Theorem 3])

If $\mathcal{S} \subseteq \mathbb{F}_2^n$ is a set of binary vectors with Hamming weight k that has minimum Hamming distance d and $\mathcal{C}_v \subseteq \mathcal{G}_q(n, k)$ is an $(n, \star, d, k, v)_q$ CDC for each $v \in \mathcal{S}$, then $\mathcal{C} = \cup_{v \in \mathcal{S}} \mathcal{C}_v$ is an $(n, \star, d, k)_q$ CDC with cardinality $\sum_{v \in \mathcal{S}} \#\mathcal{C}_v$.

¹In the cited paper the upper bound was stated for linear FDRM codes only. However, the statement is also true without this assumption, as observed by e.g. the same authors later.

Choosing the \mathcal{C}_v as lifted FDRM codes, the underlying construction is called *multilevel construction* in [5] and *Echelon-Ferrers construction* in some other papers. The binary code $\mathcal{S} \subseteq \mathbb{F}_2^n$ is called *skeleton code*. Using our notation, a given skeleton code \mathcal{S} with minimum Hamming distance $\frac{d}{2}$ gives

$$A_q(n, d; k) \geq \sum_{v \in \mathcal{S}} A_q(n, d; k; v).$$

The upper bound of Theorem 1 is attained in many cases including $d \leq 4$ and rectangular Ferrers diagrams. For other cases we refer e.g. to [1, 18] and the references mentioned therein. Indeed, for finite fields no strict improvement of Theorem 1 is known so that it is conjectured that the upper bound can always be attained. If $2 \leq 2k \leq n$ and \mathcal{F} is the rectangular Ferrers diagrams with k dots in each column and $n - k$ dots in each row, then a rank-metric code $C_{\mathcal{F}} \subseteq \mathbb{F}_q^{k \times (n-k)}$ attaining the maximum possible cardinality $q^{(n-k)(k-d/2+1)}$ for a given minimum subspace distance $d \leq 2k$ is called *maximum rank distance* (MRD) code. Even linear MRD codes exist for all parameters, so that lifting gives the well-known lower bound

$$A_q(n, d; k) \geq q^{(n-k)(k-d/2+1)}$$

(assuming $2k \leq n$), which is at least half the optimal value for $d \geq 4$, see e.g. [12, Proposition 8].

Instead of starting with an FDRM code $C_{\mathcal{F}}$ and lift it to a CDC $\mathcal{C}_{\mathcal{F}}$ one can also start from an $(m, N, d, k)_q$ CDC \mathcal{C} and an MRD code $\mathcal{M} \subseteq \mathbb{F}_q^{k \times (n-m)}$ with minimum rank distance $d/2$. With this we can construct a CDC

$$\mathcal{C}' = \{\langle E(U)|M \rangle : U \in \mathcal{C}, M \in \mathcal{M}\} \subseteq \mathcal{G}_q(n, k)$$

with $d_S(\mathcal{C}') = d$ and $\#\mathcal{C}' = \#\mathcal{C} \cdot \#\mathcal{M}$, where $A|B$ denotes the concatenation of two matrices A and B with the same number of rows. This lifting variant was called *Construction D* in [21, Theorem 37], cf. [6, Theorem 5.1]. By construction, the identifying vectors of the codewords of \mathcal{C}' contain their k ones in the first m positions. More generally, we denote by

$$\binom{n_1}{k_1} \cdots \binom{n_l}{k_l}$$

the set of binary vectors which contain exactly k_i ones in positions $1 + \sum_{j=1}^{i-1} n_j$ to $\sum_{j=1}^i n_j$ for all $1 \leq i \leq l$. With this, we write $A_q\left(n, d; k; \binom{n_1}{k_1} \cdots \binom{n_l}{k_l}\right)$ for the maximum cardinality of an $(n, \star, d, k)_q$ CDC whose codewords all have identifying vectors in this set and state:

Theorem 3. *For each $0 \leq \Delta < n$ we have*

$$A_q(n, d; k) \geq A_q\left(n, d; k; \binom{n-\Delta}{k}, \binom{\Delta}{0}\right) \geq q^{\Delta(k-d/2+1)} A_q(n-\Delta, d; k).$$

The special structure of the identifying vectors can be used to add further codewords. In our notation the *linkage construction* from [7, Theorem 2.3], [21, Corollary 39] can be written as

$$A_q(n, d; k) \geq A_q\left(n, d; k; \binom{n-\Delta}{k}, \binom{\Delta}{0}\right) + A_q\left(n, d; k; \binom{n-\Delta}{0}, \binom{\Delta}{k}\right),$$

which was improved to

$$\begin{aligned} A_q(n, d; k) \geq & A_q\left(n, d; k; \binom{n-\Delta}{k}, \binom{\Delta}{0}\right) \\ & + A_q\left(n, d; k; \binom{n-\Delta-k+d/2}{0}, \binom{\Delta+k+d/2}{k}\right) \end{aligned}$$

in [12, Theorem 18, Corollary 4], taking Theorem 3 and $A_q(n, d; k; \binom{n-m}{0}, \binom{m}{k}) = A_q(m, d; k)$ into account. By using the notation $\binom{n-n'}{\leq k-k'}, \binom{n'}{\geq k'}$ for the set of vectors in \mathbb{F}_2^n with at most $k - k'$ ones in the first $n - n'$ positions and at least k' ones in the last n' positions we can denote by $A_q(n, d; k; \binom{n-n'}{\leq k-k'}, \binom{n'}{\geq k'})$ the maximum cardinality of an $(n, \star, d, k)_q$ CDC whose codewords have identifying vectors in this set, so that Lemma 1 gives:

Lemma 4. *For each $0 \leq \Delta < n$ we have*

$$\begin{aligned} A_q(n, d; k) \geq & A_q\left(n, d; k; \binom{n-\Delta}{k}, \binom{\Delta}{0}\right) \\ & + A_q\left(n, d; k; \binom{n-\Delta}{\leq k-d/2}, \binom{\Delta}{\geq d/2}\right). \end{aligned}$$

We remark that Lemma 4 was implicitly contained in the proofs of many papers improving lower bounds for $A_q(n, d; k)$, see e.g. [3, 22] and the references cited therein. In [16] the quantity $A_q\left(n, d; k; \binom{n-\Delta}{\leq k-d/2}, \binom{\Delta}{\geq d/2}\right)$ was introduced as $B_q(n, \Delta, d; k)$. For the special case $\Delta = k$ a lower bound for $A_q\left(n, d; k; \binom{n-\Delta}{\leq k-d/2}, \binom{\Delta}{\geq d/2}\right)$ was constructed in [22] via

$$\{\langle M | I_k \rangle : M \in \mathcal{M}, \text{rank}(M) \leq k - d/2\},$$

where I_k denotes the $k \times k$ unit matrix and $\mathcal{M} \subseteq \mathbb{F}_q^{k \times (n-k)}$ is a rank metric code with $d_R(\mathcal{M}) \geq d/2$. By replacing I_k by $E(U)$ for all codewords of a $(\Delta, \star, d, k)_q$ CDC we obtain yet another variant of the lifting idea. One of the most general versions can be found in [3, Lemma 4.1].

Of course we can also utilize the multilevel/Echelon-Ferrers construction from Theorem 2 to obtain lower bounds for $A_q\left(n, d; k; \binom{n-\Delta}{\leq k-d/2}, \binom{\Delta}{\geq d/2}\right)$ by restricting the skeleton code \mathcal{S} to subsets of the set $\binom{n-\Delta}{\leq k-d/2}, \binom{\Delta}{\geq d/2}$ of identifying vectors. More generally, we can define the Hamming distance $d_H(\mathcal{S}, \mathcal{S}')$ between two sets $\mathcal{S}, \mathcal{S}' \subseteq \mathbb{F}_2^n$ as

$$d_H(\mathcal{S}, \mathcal{S}') = \min \{d_H(v, v') : v \in \mathcal{S}, v' \in \mathcal{S}'\}$$

and slightly generalize Theorem 2 to:

Theorem 4. *Let $\mathcal{S}_i \subseteq \mathbb{F}_2^n$ and \mathcal{C}_i be $(n, \star, d, k, \mathcal{S}_i)_q$ CDCs for all $1 \leq i \leq n$ such that $d_H(\mathcal{S}_i, \mathcal{S}_j) \geq d$ for all $1 \leq i < j \leq l$. Then $\mathcal{C} = \cup_{1 \leq i \leq l} \mathcal{C}_i$ is an $(n, \star, d, k)_q$ CDC with cardinality $\#\mathcal{C} = \sum_{i=1}^l \#\mathcal{C}_i$.*

If we choose $n = n_1 + n_2$, $\mathcal{S}_1 = \left(\binom{n_1}{k}, \binom{n_2}{0} \right) \subseteq \mathbb{F}_2^n$, $\mathcal{S}_2 = \left(\binom{n_1}{0}, \binom{n_2}{k} \right) \subseteq \mathbb{F}_2^n$, and $\mathcal{S}_3, \dots, \mathcal{S}_l \subseteq \binom{n}{k} \cap \left(\binom{n_1}{\geq d/2}, \binom{n_2}{\geq d/2} \right) \subseteq \mathbb{F}_2^n$ of cardinality $\#\mathcal{S}_j = 1$ (for $3 \leq j \leq l$) with $d_H(\mathcal{S}_3, \dots, \mathcal{S}_l) \geq d$, then the conditions of Theorem 4 are satisfied. This is [17, Theorem 3.1].² More examples where constant dimension codes with different sets of identifying vectors are combined can be found in [13].

3. RESULTS

We want to apply Theorem 4 in order to obtain improved lower bounds for $A_q(n, d; k)$. In order to avoid to explicitly deal with the existence and construction of FDRM codes we restrict ourselves to subspace distance $d = 4$, where the upper bound of Theorem 1 can be always attained. As an introductory example we state:

Proposition 1. *We have $A_2(13, 4; 5) \geq 4\,796\,825\,069$ and $A_q(13, 4; 5) \geq q^{32} + q^{28} + q^{26} + 8q^{24} + q^{23} + 2q^{22} + 3q^{21} + 5q^{20} + 3q^{19} + 3q^{18} + 3q^{17} + 8q^{16} + 8q^{15} + 9q^{14} + 5q^{13} + 8q^{12} + 9q^{11} + q^{10} + 7q^9 + 2q^8 + 2q^7 + 2q^6 + q^5 + 3q^4 + 2q^3 + 3q^2 + q^0$, i.e., the coefficient for q^{22} is only 2 instead of 3, which pays off for $q = 2$ since the coefficient for q^{21} is 3 instead of 1 and the coefficient for q^{20} is 5 instead of 4.*

Proof. We apply Theorem 2 with skeleton codes $\mathcal{S}_{13,4,5}^1$ and $\mathcal{S}_{13,4,5}^2$, see Appendix B. \square

For $q \in \{2, 3\}$ the previously best lower bounds were $A_2(13, 4; 5) \geq 4\,796\,417\,559$ and $A_3(13, 4; 5) \geq 1\,880\,918\,023\,783\,990$ [8], while our parametric lower bound yields $A_3(13, 4; 5) \geq 1\,880\,918\,252\,176\,932$. The only algorithmical challenge is to find good skeleton codes. Note that $\mathcal{S}_{13,4,5}^1$ gives $A_q(13, 4; 5) \geq q^{32} + q^{28} + q^{26} + 8q^{24} + q^{23} + 2q^{22} + 3q^{21} + 5q^{20} + 3q^{19} + 3q^{18} + 3q^{17} + 8q^{16} + 8q^{15} + 9q^{14} + 5q^{13} + 8q^{12} + 9q^{11} + q^{10} + 7q^9 + 2q^8 + 2q^7 + 2q^6 + q^5 + 3q^4 + 2q^3 + 3q^2 + q^0$, i.e., the coefficient for q^{22} is only 2 instead of 3, which pays off for $q = 2$ since the coefficient for q^{21} is 3 instead of 1 and the coefficient for q^{20} is 5 instead of 4.

If the maximum sizes of the FDRM codes are known, as it is the case for subspace distance $d = 4$, for given parameters n, d, k , and q the problem of determining the best lower bound for $A_q(n, d; k)$ based on Theorem 2 can be easily formulated as maximum weighted clique problem. To this end we denote by $\mathcal{G}_{n,d,k,q} = (\mathcal{V}_{n,d,k,q}, \mathcal{E}_{n,d,k,q})$ the graph consisting of vertices corresponding to the binary vectors in \mathbb{F}_2^n with Hamming weight k . W.l.o.g. we label the elements of $\mathcal{V}_{n,d,k,q}$ from 1 to $\binom{n}{k}$ such that $w(1) \geq w(2) \geq \dots$, where $w(v)$ denotes the weight of v that is given by the maximum possible cardinality of the corresponding FDRM code. For two different vertices $v, v' \in \mathcal{V}_{n,d,k,q}$ the edge $\{v, v'\}$ is in $\mathcal{E}_{n,d,k,q}$ iff the Hamming distance between v and v' is at least d . With

²In [9] it was claimed that [17, Theorem 3.1] is incorrect. However, the stated ‘‘counterexample’’ is flawed since the example for \mathcal{C}_2 is not of the form specified in [17, Theorem 3.1] since there are e.g. non-zero entries in the first four columns of a_1 .

this, the feasible skeleton codes are in bijection to the cliques of $\mathcal{G}_{n,d,k,q}$ and we are searching for the maximum weight cliques. Looping over all cliques or all inclusion maximal cliques of $\mathcal{G}_{n,d,k,q}$ becomes computationally intractable even for moderate sized parameters due to the quickly increasing number of vertices. Given an upper bound ub for the maximum clique size in $\mathcal{G}_{n,d,k,q}$, see e.g. [2] and <https://www.win.tue.nl/~aeb/codes/Andw.html>, we can compute an upper bound on the weight $w(\mathcal{S}') := \sum_{v \in \mathcal{S}'} w(v)$ of every clique that contains a subclique:

Lemma 5. *Let $\mathcal{S}, \mathcal{S}'$ be cliques in $\mathcal{G}_{n,d,k,q}$ with $\mathcal{S} \subseteq \mathcal{S}'$ and $\max\{w(v) : v \in \mathcal{S}\} < \min\{w(v) : v \in \mathcal{S}' \setminus \mathcal{S}\}$, where $w(i) \geq w(j) \geq 0$ for all $i \leq j$. Then, we have $w(\mathcal{S}') \leq \Omega$, where Ω is the value computed by Algorithm 1 applied to $\mathcal{G}_{n,d,k,q}$, w , and \mathcal{S} .*

Proof. By *cand* we denote the set of vertices $m+1 \leq v \leq \#\mathcal{V}$ with $\left\{ \{x, v\} : x \in \mathcal{S} \right\} \subseteq \mathcal{E}$, where m , \mathcal{V} , and \mathcal{E} are as in Algorithm 1. Since $\mathcal{S} \subseteq \mathcal{S}'$ and \mathcal{S}' is a clique, we have $\left\{ \{x, v\} : x \in \mathcal{S} \right\} \subseteq \mathcal{E}$ for all vertices $v \in \mathcal{S}' \setminus \mathcal{S}$. Due to our assumption $\max\{w(v) : v \in \mathcal{S}\} < \min\{w(v) : v \in \mathcal{S}' \setminus \mathcal{S}\}$ we also have $m+1 \leq v \leq \#\mathcal{V}$ for all $v \in \mathcal{S}' \setminus \mathcal{S}$. Thus, we have $\mathcal{S}' \subseteq \mathcal{S} \cup \text{cand}$. If $\#\mathcal{S} + \#\text{cand} \leq ub$, then $\hat{\mathcal{S}} = \mathcal{S} \cup \text{cand}$ and $w(\mathcal{S}') \leq w(\mathcal{S} \cup \text{cand}) = w(\hat{\mathcal{S}}) = \Omega$. Otherwise we have $\#\mathcal{S}' \leq ub = \#\hat{\mathcal{S}}$, set $s = \#(\mathcal{S}' \setminus \mathcal{S})$, $\hat{s} = \#(\hat{\mathcal{S}} \setminus \mathcal{S})$, and write $\mathcal{S}' = \mathcal{S} \cup \{a_1, \dots, a_s\}$, $\hat{\mathcal{S}} = \mathcal{S} \cup \{b_1, \dots, b_{\hat{s}}\}$, where we assume that the sequences a_i and b_i are increasing. Due to our assumption on the weight function w and the choice of $\hat{\mathcal{S}} \setminus \mathcal{S}$ as those vertices with the smallest elements in *cand* we have $w(a_i) \leq w(b_i)$ for all $1 \leq i \leq s$, so that

$$w(\mathcal{S}') = w(\mathcal{S}) + \sum_{i=1}^s w(a_i) \leq w(\mathcal{S}) + \sum_{i=1}^s w(b_i) \leq w(\mathcal{S}) + \sum_{i=1}^{\hat{s}} w(b_i) = w(\hat{\mathcal{S}}) = \Omega.$$

□

We can use Algorithm 1 to determine a maximum weight clique of $\mathcal{G}_{n,d,k,q}$ without explicitly traversing all cliques:

Proposition 2. *Algorithm 2 determines a maximum weight clique \mathcal{U} in $\mathcal{G}_{n,d,k,q}$.*

Proof. Let \mathcal{U}' be a maximum weight clique in $\mathcal{G}_{n,d,k,q}$ and \mathcal{U}'_i be the subset of the smallest i elements of \mathcal{U}' for $1 \leq i \leq \#\mathcal{U}'$. Now let m be the largest index such that `Dive` is called with $\mathcal{S} = \mathcal{U}'_m$. Since `Dive` is initially called with $\mathcal{S} = \emptyset = \mathcal{U}'_0$, $0 \leq m \leq \#\mathcal{U}'$ is well defined. If $m = \#\mathcal{U}'$ then either \mathcal{U} is set to \mathcal{U}' or we already have $w(\mathcal{U}) \geq w(\mathcal{U}')$. Note that every replacement of \mathcal{U} strictly increases the value of $w(\mathcal{U})$. In the remaining cases we assume $m < \#\mathcal{U}$ and set $v' = \mathcal{U}'_{m+1} \setminus \mathcal{U}'_m$. By construction we have $l \leq v'$, so that the for loop of Algorithm 3 attains $v = v'$. Since \mathcal{U}'_{m+1} is a clique we have $\left\{ \{x, v\} : x \in \mathcal{S} \right\} \subseteq \mathcal{E}$ and since `Dive` is not called with $\mathcal{U}'_{m+1} = \mathcal{S} \cup \{v\}$ we have $\text{UB}(\mathcal{G}, \mathcal{S} \cup \{v\}, ub) \leq w(\mathcal{U})$, so that Lemma 5 gives $w(\mathcal{U}) \geq w(\mathcal{U}')$. Since

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Input: graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  with weight function  $w: \mathcal{V} \rightarrow \mathbb{N}$  such that  $\mathcal{V} \subseteq \mathbb{N}$  and
 $w(i) \geq w(j)$  if  $i \leq j$ , a clique  $\mathcal{S}$  in  $\mathcal{G}$ , and an upper bound  $ub$  on the
maximum clique size in  $\mathcal{G}$ 
Output: An upper bound  $\Omega$  for the weight of every clique extension of  $\mathcal{S}$ 
 $\Omega \leftarrow w(\mathcal{S});$ 
 $\hat{\mathcal{S}} \leftarrow \mathcal{S};$ 
 $m \leftarrow 0;$ 
if  $\mathcal{S} \neq \emptyset$  then
   $m \leftarrow \max\{v : v \in \mathcal{S}\};$ 
for  $v$  from  $m + 1$  to  $\#\mathcal{V}$  do
  if  $\{\{x, v\} : x \in \mathcal{S}\} \subseteq \mathcal{E}$  and  $\#\hat{\mathcal{S}} < ub$  then
     $\Omega \leftarrow \Omega + w(v);$ 
     $\hat{\mathcal{S}} \leftarrow \hat{\mathcal{S}} \cup \{v\};$ 
return  $\Omega;$ 

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Algorithm 1: UB: upper bound for the weight of an extended clique

every replacement of \mathcal{U} strictly increases the value of $w(\mathcal{U})$ the proposed statement follows. \square

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Input: graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  with weight function  $w: \mathcal{V} \rightarrow \mathbb{N}$  such that  $\mathcal{V} \subseteq \mathbb{N}$  and
 $w(i) \geq w(j)$  if  $i \leq j$ , a clique  $\mathcal{S}$  in  $\mathcal{G}$ , and an upper bound  $ub$  on the
maximum clique size in  $\mathcal{G}$ 
Output: a weight maximum clique  $\mathcal{U}$  with respect to  $w$ 
// global data structures:
 $\mathcal{U} \leftarrow \emptyset;$ 
// local data structures:
 $\mathcal{S} \leftarrow \emptyset;$ 
Dive ( $\mathcal{G}, w, \mathcal{S}, ub$ );
return  $\mathcal{U};$ 

```

Algorithm 2: Framework for the maximum weight clique algorithm

As an application of Proposition 2 we remark that for $(n, d, k) = (13, 4, 5)$ and $q \in \{2, \dots, 9\}$ the lower bound stated in Proposition 1 is indeed the optimal multilevel/Echelon-Ferrers construction. So, based on Lemma 5, an exhaustive search is indeed possible, while in [8] only a heuristic was used. However, we also have to use heuristics for larger parameters and replace Algorithm 2 by Algorithm 4. (The enlargement of \mathcal{L}' has to be implemented in a modified version of Dive to be technically correct.)

We can iteratively apply Algorithm 4 in order to heuristically find a clique of large weight in $\mathcal{G}_{n,d,k,q}$. Starting from $\mathcal{L} = \emptyset$ we can use the determined list \mathcal{L}' in the next

Input: clique $\mathcal{S} \subseteq \mathcal{V}$ and the input data from Algorithm 2

Output: -

if $w(\mathcal{S}) > w(\mathcal{U})$ **then**

└ $\mathcal{U} \leftarrow \mathcal{S}$;

Let $1 \leq l \leq \#V$ be the smallest index such that the elements in \mathcal{S} have strictly smaller indices; return if no such index exists;

for v from l to $\#\mathcal{V}$ **do**

└ **if** $\{\{x, v\} : x \in \mathcal{S}\} \subseteq \mathcal{E}$ **then**

└ **if** $\text{UB}(\mathcal{G}, w, \mathcal{S} \cup \{v\}, ub) > w(\mathcal{U})$ **then**

└└ $\text{Dive}(\mathcal{G}, w, \mathcal{S} \cup \{v\}, ub)$;

return;

Algorithm 3: Subroutine Dive

Input: graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with weight function $w : \mathcal{V} \rightarrow \mathbb{N}$ such that $\mathcal{V} \subseteq \mathbb{N}$ and $w(i) \geq w(j)$ if $i \leq j$, a clique \mathcal{S} in \mathcal{G} , a list \mathcal{L} of cliques in \mathcal{G} , and parameters Δ_1, Δ_2, ub

Output: a weight maximum clique \mathcal{U} with respect to w and a list \mathcal{L}' of cliques

// global data structures:

$\mathcal{U} \leftarrow \emptyset$;

$\mathcal{L}' \leftarrow \emptyset$;

for each $\mathcal{S} \in \mathcal{L}$ **do**

└ $\text{Dive}(\mathcal{G}, w, \mathcal{S}, ub)$;

└ **if** Dive is called with $\#\mathcal{S} \geq ub - \Delta_2$ **then** add the smallest $ub - \Delta_2 - \Delta_1$ elements of \mathcal{S} as a clique to \mathcal{L}' ;

return \mathcal{U} ;

Algorithm 4: Framework a heuristic for the maximum weight clique algorithm

round increasing ub incrementally, say by 10 in each iteration, until no further improvement is found. The advantage of that approach is that using the parameters Δ_1 and Δ_2 we can control the extend to which we want to perform an exhaustive search. The iterative process also partially prevents from stalling in local neighborhoods of partial cliques that are very diverse to a global optimum. As an example for a result obtained by the application of Algorithm 4 we state:

Proposition 3. $A_q(17, 4; 6) \geq q^{55} + q^{51} + q^{49} + 8q^{47} + 3q^{45} + 3q^{44} + 5q^{43} + q^{42} + 5q^{41} + 9q^{40} + 22q^{39} + 7q^{38} + 11q^{37} + 13q^{36} + 19q^{35} + 3q^{34} + 17q^{33} + 15q^{32} + 69q^{31} + 20q^{30} + 49q^{29} + 22q^{28} + 33q^{27} + 15q^{26} + 23q^{25} + 20q^{24} + 38q^{23} + 17q^{22} + 29q^{21} + 24q^{20} + 40q^{19} + 19q^{18} + 20q^{17} + 15q^{16} + 28q^{15} + 15q^{14} + 13q^{13} + 8q^{12} + 7q^{11} + 5q^{10} + 3q^9 + 10q^8 + q^7 + 2q^6 + q^5 + 2q^4 + q^3 + q^2 + q^1 + q^0$.

Proof. We apply Theorem 2 with skeleton codes $\mathcal{S}_{17,4,6}$, see Appendix B. \square

In Table 1 we list the cases where Algorithm 2 or Algorithm 4 yields a skeleton code such that Theorem 2 gives a strictly better constructive lower bound for $A_q(n, 4; k)$. In all cases we obtain improvements for all $q \in \{2, \dots, 9\}$, see Table 3 in Appendix A. The utilized skeleton codes are listed in Appendix B. Some of them consist of over 1000 identifying vectors. Whenever we state two skeleton codes, for given parameters n and k , the first code gives the lower bound for $A_2(n, 4; k)$ and the second code gives the lower bound for general field sizes q , which is larger for $q \geq 3$, see Proposition 1 for an example. Whenever we only list one skeleton code, it yields the utilized lower bound for all field sizes $q \geq 2$, see Proposition 3 for an example. We remark that for $d = 4$ all explicitly stated numerical results of [8, 19] have been strictly improved. (The other results of [8] depend on the unproven assumption, nevertheless the author states otherwise, that the upper bound of Theorem 1 is attained for $d = 6$.)

k	n
5	13, 14
6	14, 15, 16, 17
7	16, 17, 18, 19
8	18, 19
9	19

TABLE 1. Parameters where improved codes for $A_q(n, 4; k)$ have been found using Algorithm 2 or Algorithm 4.

Another approach, to obtain improved lower bounds, is based on Theorem 3 and Lemma 4:

Proposition 4. $A_q(11, 4; 3) \geq q^8 \cdot A_q(7, 4; 3) + q^4 + q^3 + 2q^2 + q + q^0$.

Proof. We apply Theorem 3 with $\Delta = 4$ to deduce $A_q(11, 4; 3; \binom{7}{3}, \binom{4}{0}) \geq q^8 \cdot A_q(7, 4; 3)$ and Theorem 2 with the skeleton code $\mathcal{S}_{11,7,4,3}$, see Appendix B, to deduce

$A_q\left(11, 4; 3; \binom{7}{\leq 1}, \binom{4}{\geq 2}\right) \geq q^4 + q^3 + 2q^2 + q + q^0$. With this, the stated lower bound follows from Lemma 4. \square

For $q \geq 3$ Proposition 4 yields a strictly larger lower bound than the previous record from [10]. Using $\Delta = 3$ and skeleton code

$$\mathcal{S}_{10,7,4,3} = \{0000100110, 0000010101, 0000001011\} \hat{=} \{38, 21, 11\},$$

where the integers give the binary identifying vectors reading them in base 2 representation, gives $A_q(10, 4; 3) \geq q^6 \cdot A_q(7, 4; 3) + q^2 + q + 1$, which equals the lower bound from [10]. For the naming of the skeleton codes $\mathcal{S}_{n,m,d,k}$ for $A_q(n, d; k)$ we use $m = n - \Delta$ as an abbreviation.

k	n
3	11(7), 12(7), 13(7), 14(11,7), 15(11,7), 16(13,7), 17(13,7), 18(13,7), 19(13,7)
4	13(8), 14(8), 15(8), 17(12), 18(12), 19(12)

TABLE 2. Parameters where improved codes for $A_q(n, 4; k)$ have been found using Theorem 3 and Lemma 4.

In Table 2 we list the parameters n and k where the approach based on Theorem 3 and Lemma 4 yields strict improvements. The corresponding value of m is stated in brackets, where the last is for general field sizes q and the last but one for $q = 2$. In some cases, when n is rather small, better lower bounds for $A_2(n, 4; k)$ have been found using integer linear programming and prescribed automorphisms, see e.g. [15] for an introductory paper and [11] for the precise reference per specific instance. The numerical improvements are listed in Table 6 in Appendix A and the corresponding skeleton codes in Appendix B. We remark that for $k = 4$ and $n \in \{12, 16\}$ the obtained codes are inferior compared to those from [3]. We remark that for $d = 4$ all numerical results of [17] have been strictly improved.

In order to find the skeleton codes Algorithm 2 and Algorithm 4 are applied by modifying the input graphs $\mathcal{G}_{n,m,d,k,q}$, where $m = n - \Delta$, accordingly. Since Lemma 5 and Algorithm 1 significantly speed up the solution process by cutting parts of the search tree as early as possible, we also want to use “good” upper bounds on the maximum clique size in $\mathcal{G}_{n,m,d,k,q}$. While of course the maximum clique size in $\mathcal{G}_{n,d,k,q}$ is an upper bound this can usually be improved.

Proposition 5. *Let $1 \leq k \leq m \leq n$ and $2 \leq d/2 \leq k$ be integers. Then, the maximum cardinality of a set $\mathcal{S} \subseteq \mathbb{F}_2^n$ of binary vectors with Hamming weight k that have at most $k - d/2$ ones in their first m coordinates with minimum Hamming distance d is upper bounded by $\sum_{j \in J} c_j$, where $J = \{j \in \mathbb{N} : j \geq k - n + m, j \leq k - d/2\}$ and the c_j are integers satisfying the constraints*

$$\sum_{j \in J} \binom{j}{i} \cdot \binom{k-j}{k-d/2+1-i} c_j \leq \binom{m}{i} \binom{n-m}{k-d/2+1-i}$$

for all $0 \leq i \leq k - d/2 + 1$. Moreover, we have $c_0 \leq A_1(n - m, d; k)$, which denotes the maximum cardinality of a subset of binary vectors in \mathbb{F}_2^{n-m} with Hamming weight k and minimum Hamming distance d .

Proof. Given \mathcal{S} we let c_j be the number of elements in \mathcal{S} that have exactly j ones in their first m coordinates. It can be easily checked that $j \in J$ implies $0 \leq j \leq m$, $0 \leq k - j \leq n - m$, and $j \leq k - d/2$, i.e., the counts c_j are well defined. Since \mathcal{S} has minimum Hamming distance d every subset $I \subseteq \mathbb{F}_2^n$ of cardinality $k - d/2 + 1$ is contained in the support of at most one element in \mathcal{S} . For a given integer $0 \leq i \leq k - d/2 + 1$ we consider all those sets I containing exactly i elements in $\{1, \dots, m\}$.

There are exactly $\binom{m}{i} \binom{n-m}{k-d/2+1-i}$ of those sets and every element of \mathcal{S} with j ones in the first m coordinates has a support that contains exactly $\binom{j}{i} \cdot \binom{k-j}{k-d/2+1-i}$ of those sets. Thus, the stated inequalities are valid. The additional upper bound for c_0 follows directly from our choice of the count c_0 . \square

Given fixed parameters the integer linear program of Proposition 5 can be solved easily. As an example we will show that also parametric upper bounds can be concluded. To this end, let $k = 4$, $d = 4$, and $n - m \geq 4$, so that $J = \{0, 1, 2\}$. For $i \in \{0, 1, 2\}$ the constraints of Proposition 5 read

$$\begin{aligned} 4c_0 + c_1 &\leq \binom{m}{0} \binom{n-m}{3} = \binom{n-m}{3}, \\ 3c_1 + 2c_2 &\leq \binom{m}{1} \binom{n-m}{2} = \frac{m(n-m)(n-m-1)}{2}, \\ 2c_2 &\leq \binom{m}{2} \binom{n-m}{1} = \frac{m(m-1)(n-m)}{2}, \end{aligned}$$

noting that that the constraint for $i = 3 = k - d/2 + 1$ is trivially satisfied. Since c_1 and c_2 are non-negative integers, dividing the second constraint by two gives

$$c_1 + c_2 \leq \left\lfloor \frac{m(n-m)(n-m-1)}{4} \right\rfloor,$$

so that $c_0 \leq A_1(n-m, 4; 4)$ gives

$$c_0 + c_1 + c_2 \leq \left\lfloor \frac{m(n-m)(n-m-1)}{4} \right\rfloor + A_1(n-m, 4; 4).$$

For $(n, m) = (13, 8)$ we obtain an upper bound of $40 + 1 = 41$, which indeed is attained by $\mathcal{S}_{13,8,4,4}$, see Appendix B, while $A_1(13, 4; 4) = 65$. Similarly, for $(n, m) = (14, 8)$ we obtain an upper bound of $60 + 3 = 63$, which indeed is attained by $\mathcal{S}_{14,8,4,4}$, while $A_1(14, 4; 4) = 91$ is much larger.

We remark that Proposition 5 mimics [16, Lemma 4.1], which proves an upper bound for $A_q\left(n, d; k; \binom{n-\Delta}{\leq k-d/2}, \binom{\Delta}{\geq d/2}\right)$. Possibly the quantity $A_1(n, m, d; k)$ upper bounded in Proposition 5, generalizing $A_1(n, d; k)$, is interesting on its own and similar techniques as for the (partial) determination of $A_1(n, d; k)$ can be applied. To this end, we remark that the upper bound of Proposition 5 can surely be improved in most cases but was sufficiently good for our purpose.

REFERENCES

- [1] J. Antrobus and H. Gluesing-Luerssen. Maximal Ferrers diagram codes: constructions and genericity considerations. *IEEE Transactions on Information Theory*, 65(10):6204–6223, 2019.
- [2] A. E. Brouwer, L. B. Shearer, N. Sloane, and D. S. Warren. A new table of constant weight codes. *IEEE Transactions on Information Theory*, 36(6):1334–1380, 1990.

- [3] A. Cossidente, S. Kurz, G. Marino, and F. Pavese. Combining subspace codes. *arXiv preprint 1911.03387*, 2019.
- [4] T. Etzion, E. Gorla, A. Ravagnani, and A. Wachter-Zeh. Optimal Ferrers diagram rank-metric codes. *IEEE Transactions on Information Theory*, 62(4):1616–1630, 2016.
- [5] T. Etzion and N. Silberstein. Error-correcting codes in projective spaces via rank-metric codes and Ferrers diagrams. *IEEE Transactions on Information Theory*, 55(7):2909–2919, 2009.
- [6] H. Gluesing-Luerssen, K. Morrison, and C. Troha. Cyclic orbit codes and stabilizer subfields. *Advances in Mathematics of Communications*, 9(2):177–197, 2015.
- [7] H. Gluesing-Luerssen and C. Troha. Construction of subspace codes through linkage. *Advances in Mathematics of Communications*, 10(3):525–540, 2016.
- [8] X. He. A hierarchical-based greedy algorithm for Echelon-Ferrers construction. *arXiv preprint 1911.00508*, 2019.
- [9] X. He and Y. Chen. A note on Echelon-Ferrers construction. *arXiv preprint 2002.10035*, 2020.
- [10] D. Heinlein. Generalized linkage construction for constant-dimension codes. *arXiv preprint 1910.11195*, 2019.
- [11] D. Heinlein, M. Kiermaier, S. Kurz, and A. Wassermann. Tables of subspace codes. *arXiv preprint 1601.02864*, 2016.
- [12] D. Heinlein and S. Kurz. Asymptotic bounds for the sizes of constant dimension codes and an improved lower bound. In *5th International Castle Meeting on Coding Theory and Applications*, pages 1–30, 2017. *arXiv preprint 1705.03835*.
- [13] D. Heinlein and S. Kurz. Coset construction for subspace codes. *IEEE Transactions on Information Theory*, 63(12):7651–7660, 2017.
- [14] R. Koetter and F. Kschischang. Coding for errors and erasures in random network coding. *IEEE Transactions on Information Theory*, 54(8):3579–3591, Aug. 2008.
- [15] A. Kohnert and S. Kurz. Construction of large constant dimension codes with a prescribed minimum distance. In *Mathematical methods in computer science*, pages 31–42. Springer, 2008.
- [16] S. Kurz. A note on the linkage construction for constant dimension codes. *arXiv preprint 1906.09780*, 2019.
- [17] F. Li. Construction of constant dimension subspace codes by modifying linkage construction. *IEEE Transactions on Information Theory*, to appear.
- [18] S. Liu, Y. Chang, and T. Feng. Constructions for optimal Ferrers diagram rank-metric codes. *IEEE Transactions on Information Theory*, 65(7):4115–4130, 2019.
- [19] S. Liu, Y. Chang, and T. Feng. Parallel multilevel constructions for constant dimension codes. *arXiv preprint 1911.01878*, 2019.
- [20] N. Silberstein and T. Etzion. Large constant dimension codes and lexicodes. *Advances in Mathematics of Communications*, 5(2):177–189, 2011.
- [21] N. Silberstein and A.-L. Trautmann. Subspace codes based on graph matchings, Ferrers diagrams, and pending blocks. *IEEE Transactions on Information Theory*, 61(7):3937–3953, 2015.
- [22] L. Xu and H. Chen. New constant-dimension subspace codes from maximum rank distance codes. *IEEE Transactions on Information Theory*, 64(9):6315–6319, 2018.

APPENDIX A. IMPROVED LOWER BOUNDS

In Table 3 we compare the numerical improvements for $A_q(n, 4; k)$ based on Theorem 2. The previously best known lower bounds are mostly from [8]. However, the lower bounds for $A_q(19, 4; 7)$ from [8] are flawed, i.e., some magnitudes too small.

$A_q(n, 4; k)$	New	Old
$A_2(13, 4; 5)$	4796825069	4796417559 [8]
$A_3(13, 4; 5)$	1880918252176932	1880918023783990 [8]
$A_4(13, 4; 5)$	18525690519076963184	18525690479132333173 [8]
$A_5(13, 4; 5)$	23322304251254415373950	23322304248923865096456 [8]
$A_7(13, 4; 5)$	1104898620940893642387898640	1104898620939789578683671514 [8]
$A_8(13, 4; 5)$	79247846163928378274466378432	79247846163915655208442806985 [8]
$A_9(13, 4; 5)$	3434214279120463268840947142394	3434214279120353599762054717228 [8]
$A_2(14, 4; 5)$	76749681496	76745404672 [8]
$A_3(14, 4; 5)$	152354381482802889	152354354408240436 [8]
$A_4(14, 4; 5)$	4742576773448941977744	4742576757171205745408 [8]
$A_5(14, 4; 5)$	14576440157067948253027775	14576440154794852120820500 [8]
$A_7(14, 4; 5)$	2652861588879102858398037208053	2652861588875282767080909163052 [8]
$A_8(14, 4; 5)$	324599177887450844354761706030144	324599177887378338095324360943616 [8]
$A_9(14, 4; 5)$	22531879885309361371329789150871659	22531879885308389971852673089028988 [8]
$A_2(13, 4; 6)$	38327432465	38325131657 [8]
$A_3(13, 4; 6)$	50782269101589019	50782269101569336 [8]
$A_4(13, 4; 6)$	1185639430145591548865	1185639430145591024577 [8]
$A_5(13, 4; 6)$	2915286427121720403833501	2915286427121720397974126 [8]
$A_7(13, 4; 6)$	378980216844611802379905892367	378980216844611802379704124332 [8]
$A_8(13, 4; 6)$	40574896910456482568428114359809	40574896910456482568427309053441 [8]
$A_9(13, 4; 6)$	2503542202545027727509322118254705	2503542202545027727509319406311282 [8]
$A_2(14, 4; 6)$	1227267234053	1227203232293 [8]
$A_3(14, 4; 6)$	12340234566815274820	12340234566810426241 [8]
$A_4(14, 4; 6)$	1214095649227435851637265	1214095649227435312865809 [8]
$A_5(14, 4; 6)$	9110270832108553596766282526	9110270832108553578429954401 [8]
$A_7(14, 4; 6)$	6369520523839151727825310046433656	6369520523839151727821917674342793 [8]
$A_8(14, 4; 6)$	1329558223045414729174436187566776385	1329558223045414729174409793499570241 [8]
$A_9(14, 4; 6)$	147831663555770209444761899682581639650	147831663555770209444761739522908440329 [8]
$A_2(15, 4; 6)$	39273527139056	39267675031563 [8]
$A_3(15, 4; 6)$	2998677038028861083854	2998676636295383433055 [8]
$A_4(15, 4; 6)$	1243233944887303700963929029	1243233943362040432057180581 [8]
$A_5(15, 4; 6)$	28469596350369158409165204552256	28469596349440811610995019309681 [8]
$A_7(15, 4; 6)$	107052531444164864809019626364096185842	107052531444149851093995761837649623043 [8]
$A_8(15, 4; 6)$	43566963852752158504511827652740340200969	43566963852751451455533741586485379191945 [8]
$A_9(15, 4; 6)$	8729311901304675301507926588885778379185444	8729311901304654112388972101947942905023381 [8]
$A_2(16, 4; 6)$	1256765678235469	1256703351587805 [8]
$A_3(16, 4; 6)$	728678523485248028210242	728678523483522880513165 [8]
$A_4(16, 4; 6)$	1273071559584675915665964748625	1273071559584674249524907514705 [8]
$A_5(16, 4; 6)$	88967488594921579438014390756229526	88967488594921579094529973785226401 [8]
$A_7(16, 4; 6)$	1799231895982079405219048786740663875885338	1799231895982079405217983712681675184889205 [8]
$A_8(16, 4; 6)$	1427602271526982761008299726644512165936067137	1427602271526982761008273790284715743116587585 [8]

TABLE 3. Improvements based on Theorem 2.

LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES 15

$A_q(n, 4; k)$	New	Old
$A_9(16, 4; 6)$	515457138460139773028759563895406751585607978830	515457138460139773028759130033330465293577720745 [8]
$A_2(17, 4; 6)$	40214593296350543	40210734642430233 [8]
$A_3(17, 4; 6)$	177068881245303116902339546	177068857538981556600415147 [8]
$A_4(17, 4; 6)$	1303625277015154268047530588005973	1303625275416014562978042328889121 [8]
$A_5(17, 4; 6)$	278023401859130594858143611346347770156	278023401850065051300001033963426380051 [8]
$A_7(17, 4; 6)$	30239690475770808604751881256801280488268311358	30239690475766567624866522189453324986367024243 [8]
$A_8(17, 4; 6)$	46779671233396171116050537713696416375261813580361	46779671233395411929881098933346105696388708610177 [8]
$A_9(17, 4; 6)$	30437228568932793457735943799142491471855909627410848	30437228568932719575938776421624397010040042569507531 [8]
$A_2(15, 4; 7)$	313939996903443	313923840120169 [8]
$A_3(15, 4; 7)$	80962390735680572668348	80962387333738514962426 [8]
$A_4(15, 4; 7)$	79566863776146089092873059065	79566863724904828874349525569 [8]
$A_5(15, 4; 7)$	3558699030838267109468908231148586	3558699030750375431966367668488876 [8]
$A_7(15, 4; 7)$	36719018111675892766467253005230234472800	36719018111669485366326051726236511270134 [8]
$A_8(15, 4; 7)$	22306285465490553868002054600166649426159313	22306285465490013824377105083555006005322241 [8]
$A_9(15, 4; 7)$	6363668373716727919196827279641011245211439310	6363668373716701033966177001673180121197368344 [8]
$A_2(16, 4; 7)$	20093092605969267	20090530823175168 [11]
$A_3(16, 4; 7)$	59021599901810630842384564	59021591907098096238648717 [8]
$A_4(16, 4; 7)$	325905875863710195123556939265157	325905875503966895183219736444928 [8]
$A_5(16, 4; 7)$	55604672371466910516074031340231456506	55604672369921077974140644073486328125 [8]
$A_7(16, 4; 7)$	4319955761831483147392111643370405301140304392	4319955761830991620286839274911971343679804101 [8]
$A_8(16, 4; 7)$	5847458897067883289789161262085069896599613506569	5847458897067807309822598445696990677571597238272 [8]
$A_9(16, 4; 7)$	3381914284196654468636744719790551399185806891914222	3381914284196647958706309890495614396898957540722053 [8]
$A_2(17, 4; 7)$	1285973764408635208	1285780755925958656 [8]
$A_3(17, 4; 7)$	43026746493711590523056074275	43026740586030433477849264332 [8]
$A_4(17, 4; 7)$	1334910467554985714047715561596194368	1334910466075153545917778842397704192 [8]
$A_5(17, 4; 7)$	868823005804308136696506169077607676476375	868823005780116978008300065994262695312500 [8]
$A_7(17, 4; 7)$	508238475423712269410606273308115865319675271592683	508238475423654428658806537376355439210673571016516 [8]
$A_8(17, 4; 7)$	1532876265112963221100230608239000778944194589626442240	1532876265112943302025972486322657421692282703427141632 [8]
$A_9(17, 4; 7)$	1797287909107754250279842359793367953350214940243153381903	1797287909107750790631435583972090558986192090028734124332 [8]
$A_2(18, 4; 7)$	82302571633443282819	82291970549255555624 [11]
$A_3(18, 4; 7)$	31366498204356725852831211333673	31366481390109307710095048570592 [8]
$A_4(18, 4; 7)$	5467793275108157387543615361694729718789	5467793218060404819993105682513015078912 [8]
$A_5(18, 4; 7)$	13575359465692365684898552467453540627598430631	13575359458996180137997725978493690490722656250 [8]
$A_7(18, 4; 7)$	59793748395124324914384236296038210886457090991018946109	59793748394835040265326746482178302037689516829976915564 [8]
$A_8(18, 4; 7)$	401834315641772630677971972159869730106935844085497257345033	401834315641464759976904424139464794128095421048885275000832 [8]
$A_9(18, 4; 7)$	955152483704134026531065483605449770742908421375268890568749019	955152483703990438603592115845721099567855543645243756034671222 [8]

TABLE 4. Improvements based on Theorem 2 cont.

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$A_q(n, 4; k)$	New	Old
$A_2(19, 4; 7)$	5267367924445148864092	5058097205000347197549 [21]
$A_3(19, 4; 7)$	22866177190621892757679222318915925	22813524065165704375162588681706875 [21]
$A_4(19, 4; 7)$	22396081254840251014738769405257047429902 630	22388542052518188728938813857333763516548 177 [21]
$A_5(19, 4; 7)$	21211499165144290480179096294676880646602 6993016307	21209813680090451272236350632742053588039 4984750151 [21]
$A_7(19, 4; 7)$	70346747049379816915369395320383414712645 47397843085217471817	70346075648600428027267630601996366936175 78982540335678343419 [21]
$A_8(19, 4; 7)$	10533845483959684448674419012790949807456 1619935428942997712801930	10533801523606741064564183358274781734572 1344118067187280665018945 [21]
$A_9(19, 4; 7)$	50760719109220869118965528769591000765261 3420811027477177039803795175	50760616504319115174434295841792410847448 925685291954483933945104727 [21]
$A_2(18, 4; 8)$	1316667538397101428149	1264601087568682942805 [21]
$A_3(18, 4; 8)$	2540681546251523549771895302399518	2534836537121138399731153735006570 [21]
$A_4(18, 4; 8)$	13997550036505113139933121791886674834127 53	13992838834950406313750769467311428427492 01 [21]
$A_5(18, 4; 8)$	84845996396175179770599196313863292814169 28756276	84839254734261831517960672499611973762512 21110026 [21]
$A_7(18, 4; 8)$	14356478989082867395087999913040607219209 1727223436678152394	14356341969123295746286010290803218889077 1110979148690800890 [21]
$A_8(18, 4; 8)$	16459133568567622026207934780783177598554 08541359722889311735873	16459064880639275271231203508694685723605 96441738049974310277185 [21]
$A_9(18, 4; 8)$	62667554455729492107715380469110602507032 42571284112409913460035888	62667427783112758078838097157679059423642 52612630126994566337005626 [21]
$A_2(19, 4; 8)$	168534060204346081643054	161868939208791416732918[21]
$A_3(19, 4; 8)$	5556470543990117093920796733024932097	5543687506683929680212033218489009029[21]
$A_4(19, 4; 8)$	22933585979825739912419408751656551857477 750740	22925867147182745704449260695243044338837 225537[21]
$A_5(19, 4; 8)$	66285934684512179124945660759025164232422 7507808667905	66280667761142055873406775390321854501962 6771492187626[21]
$A_7(19, 4; 8)$	11823177776106271896222089221602557589481 1105121482204632859666129	11823064934277706348783619772918955293567 2313049101078765671318222[21]
$A_8(19, 4; 8)$	34517304881588725672548617783010034714431 24750791181363571603492943432	34517160832562417413613060900666077554631 69548983834979971097956450817[21]
$A_9(19, 4; 8)$	29973697026756603314019362623281023278094 694159378234061636310178009770385	29973636439636704539558217472416705317244 006954378905881075605724274378842[21]
$A_2(19, 4; 9)$	1348002146261417447406857	1289520797394170812563456 [4]
$A_3(19, 4; 9)$	150024595081884393007012600338236544880	149656439781495352647490043484854812672 [4]
$A_4(19, 4; 9)$	14677494853604982256349239711358766939534 33631137	14672330163008228150280295382725315655876 80198656 [4]
$A_5(19, 4; 9)$	82857418316609426935359109104035114463034 035070083878426	82850622250904212594169271527415901448337 330052257546240 [4]
$A_7(19, 4; 9)$	40553499771898134480982178003920834382400 027150097709434199076454452	40553105686976525598537903185027802877404 377818341526587313754210304 [4]
$A_8(19, 4; 9)$	17672860099364199843916651708091969078559 21985408931899768042741180322945	17672785292630676941870781019917737428555 76219388246358086384262766919680 [4]
$A_9(19, 4; 9)$	21850825132503488679494454968297144272741 734751304289430201060161991357689262	21850780456810680205404210364952225732272 903279105184451471542312700846538752 [4]

TABLE 5. Improvements based on Theorem 2 cont.

In Table 3 we compare the numerical improvements for $A_q(n, 4; k)$ based on Theorem 2, Theorem 3, and Lemma 4. We also list a few cases which result in the same

lower bound that was previously known. For consistency reasons we also list the obtained lower bounds for $A_q(12, 4; 4)$ and $A_q(16, 4; 4)$ which are inferior to the lower bounds obtained in [3].

$A_q(n, 4; k)$	New	Old
$A_3(10, 4; 3)$	5086975	5086975 [10]
$A_4(10, 4; 3)$	273727509	273727509 [10]
$A_5(10, 4; 3)$	6162421906	6162421906 [10]
$A_7(10, 4; 3)$	680487816156	680487816156 [10]
$A_8(10, 4; 3)$	4407724867657	4407724867657 [10]
$A_9(10, 4; 3)$	22911698562814	22911698562814 [10]
$A_3(11, 4; 3)$	45782788	45782686 [10]
$A_4(11, 4; 3)$	4379640165	4379639873 [10]
$A_5(11, 4; 3)$	154060547681	154060547001 [10]
$A_7(11, 4; 3)$	33343902991701	33343902989195 [10]
$A_8(11, 4; 3)$	282094391530121	282094391525889 [10]
$A_9(11, 4; 3)$	1855847583588025	1855847583581293 [10]
$A_3(12, 4; 3)$	412045132	412044676 [10]
$A_4(12, 4; 3)$	70074242725	70074241065 [10]
$A_5(12, 4; 3)$	3851513692181	3851513687561 [10]
$A_7(12, 4; 3)$	1633851246593749	1633851246571461 [10]
$A_8(12, 4; 3)$	18054041057928329	18054041057886353 [10]
$A_9(12, 4; 3)$	150323654270630845	150323654270557225 [10]
$A_3(13, 4; 3)$	3708406309	3708405100 [10]
$A_4(13, 4; 3)$	1121187883941	1121187877725 [10]
$A_5(13, 4; 3)$	96287842305306	96287842283141 [10]
$A_7(13, 4; 3)$	80058711083096502	80058711082943685 [10]
$A_8(13, 4; 3)$	1155458627707417737	1155458627707087193 [10]
$A_9(13, 4; 3)$	12176215995921105826	12176215995920451445 [10]
$A_2(14, 4; 3)$	6241671	6241671 [10]
$A_3(14, 4; 3)$	33375657145	33375648396 [10]
$A_4(14, 4; 3)$	17939006144421	17939006056956 [10]
$A_5(14, 4; 3)$	2407196057636556	2407196057148120 [10]
$A_7(14, 4; 3)$	3922876843071748206	3922876843065022206 [10]
$A_8(14, 4; 3)$	73949352173274772617	73949352173255598072 [10]
$A_9(14, 4; 3)$	986273495669609638336	986273495669561209956 [10]
$A_3(15, 4; 3)$	300380915398	300380802505 [10]
$A_4(15, 4; 3)$	287024098316197	287024096598921 [10]
$A_5(15, 4; 3)$	60179901440933431	60179901426410736 [10]
$A_7(15, 4; 3)$	192220965310515799351	192220965310140305694 [10]
$A_8(15, 4; 3)$	4732758539089585747081	4732758539088207997713 [10]
$A_9(15, 4; 3)$	79888153149238381303087	79888153149234028941436 [10]
$A_2(16, 4; 3)$	102223687	102223687 [10]
$A_3(16, 4; 3)$	2703428238582	2703427322125 [10]
$A_4(16, 4; 3)$	4592385573059152	4592385547188501 [10]
$A_5(16, 4; 3)$	1504497536023335775	1504497535674194406 [10]
$A_7(16, 4; 3)$	9418827300215274168199	9418827300197242691478 [10]
$A_8(16, 4; 3)$	302896546501733487813184	302896546501646667812937 [10]
$A_9(16, 4; 3)$	647094040508830885550047	6470940405087960642352846 [10]

TABLE 6. Improvements based on Theorem 2, Theorem 3, and Lemma 4.

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$A_q(n, 4; k)$	New	Old
$A_2(17, 4; 3)$	408894755	408894729 [10]
$A_3(17, 4; 3)$	24330854147239	24330847680853 [10]
$A_4(17, 4; 3)$	73478169168946433	73478168809292217 [10]
$A_5(17, 4; 3)$	37612438400583394376	37612438392939375961 [10]
$A_7(17, 4; 3)$	461522537710548434241752	461522537709756797516943 [10]
$A_8(17, 4; 3)$	19385378976110943220043777	19385378976105915500991697 [10]
$A_9(17, 4; 3)$	524146172812153019729553808	524146172812127278980234877 [10]
$A_2(18, 4; 3)$	1635579035	1635578957 [10]
$A_3(18, 4; 3)$	218977687325155	218977629126520 [10]
$A_4(18, 4; 3)$	1175650706703142933	1175650700948669781 [10]
$A_5(18, 4; 3)$	940310960014584859406	940310959823484378906 [10]
$A_7(18, 4; 3)$	22614604347816873277845856	2261460434778083078190614 [10]
$A_8(18, 4; 3)$	1240664254471100366082801737	1240664254470778592063165001 [10]
$A_9(18, 4; 3)$	42455839997784394598093858458	42455839997782309597398420706 [10]
$A_2(19, 4; 3)$	6542316171	6542316059 [10]
$A_3(19, 4; 3)$	1970799185926408	1970798662145206 [10]
$A_4(19, 4; 3)$	18810411307250286949	18810411215178781533 [10]
$A_5(19, 4; 3)$	23507774000364621485181	23507773995587109861266 [10]
$A_7(19, 4; 3)$	1108115613043026790614447001	1108115613041126070837091728 [10]
$A_8(19, 4; 3)$	79402512286150423429299311241	79402512286129829892059310425 [10]
$A_9(19, 4; 3)$	3438923039820535962445602535189	3438923039820367077389315073958 [10]
$A_2(12, 4; 4)$	19674269	19676797 [3]
$A_3(12, 4; 4)$	288648673507	288648887023 [3]
$A_4(12, 4; 4)$	283104148286289	283104153226065 [3]
$A_5(12, 4; 4)$	59732550564570151	59732550620930151 [3]
$A_7(12, 4; 4)$	191677878196845899475	191677878199060649103 [3]
$A_8(12, 4; 4)$	4723722950504908124737	4723722950514423444033 [3]
$A_9(12, 4; 4)$	79780441020720237308359	79780441020754680563815 [3]
$A_2(13, 4; 4)$	157396313	157332190 [3]
$A_3(13, 4; 4)$	7793514240823	7793495430036 [3]
$A_4(13, 4; 4)$	18118665490931521	18118664249474716 [3]
$A_5(13, 4; 4)$	7466568820575245751	7466568787180077320 [3]
$A_7(13, 4; 4)$	65745512221518213208951	65745512216555289614188 [3]
$A_8(13, 4; 4)$	2418546150658513179095553	2418546150622126921477496 [3]
$A_9(13, 4; 4)$	58159941504105053602711351	58159941503893673245551936 [3]
$A_2(14, 4; 4)$	1259181253	1258757174 [3]
$A_3(14, 4; 4)$	210424885316173	210424421624298 [3]
$A_4(14, 4; 4)$	1159594591440766481	1159594516050838620 [3]
$A_5(14, 4; 4)$	933321102572187566901	933321098538702991570 [3]
$A_7(14, 4; 4)$	22550710691980761977054117	22550710690309028764671498 [3]
$A_8(14, 4; 4)$	1238295629137158820145963073	1238295629118788686643907448 [3]
$A_9(14, 4; 4)$	42398597356492584370698238141	42398597356340204444957848530 [3]
$A_2(15, 4; 4)$	10073483841	10071464646 [3]
$A_3(15, 4; 4)$	5681471907358915	5681463153275925 [3]
$A_4(15, 4; 4)$	74214053852334056793	74214050169101548368 [3]
$A_5(15, 4; 4)$	116665137821525417847661	116665137415279661027650 [3]
$A_7(15, 4; 4)$	7734893767349401492485075543	7734893766857015258769289566 [3]
$A_8(15, 4; 4)$	634007362118225316643319878225	634007362109986775858834010688 [3]
$A_9(15, 4; 4)$	30908577472883094009493870125553	30908577472784286989399940957138 [3]
$A_2(16, 4; 4)$	80596312221	80596320222 [3]
$A_3(16, 4; 4)$	153399853228893616	153399853246113244 [3]
$A_4(16, 4; 4)$	4749699529175914867537	4749699529177178098513 [3]
$A_5(16, 4; 4)$	14583142241438194910273276	14583142241438230122883276 [3]
$A_7(16, 4; 4)$	2653068562231495127604331943392	2653068562231495132921600978204 [3]
$A_8(16, 4; 4)$	324611769405185201386954976457281	324611769405185201425928435085889 [3]
$A_9(16, 4; 4)$	22532352977741502934357344934823692	22532352977741502934583323007263948 [3]

TABLE 7. Improvements based on Theorem 2, Theorem 3, and Lemma 4 cont.

$A_q(n, 4; k)$	New	Old
$A_2(17, 4; 4)$	644770526929	644769492958 [3]
$A_3(17, 4; 4)$	4141796037183639766	4141796035658667783 [3]
$A_4(17, 4; 4)$	303980769867258670043393	303980769866940801875612 [3]
$A_5(17, 4; 4)$	1822892780179774365686344376	1822892780179753492675780445 [3]
$A_7(17, 4; 4)$	910002516845402828768417638272482	910002516845402816852352967620731 [3]
$A_8(17, 4; 4)$	166201225935454823110121665687982081	166201225935454822961083064550157688 [3]
$A_9(17, 4; 4)$	16426085320773555639146507673669565108	16426085320773555637759638858654591537 [3]
$A_2(18, 4; 4)$	5158164354661	5158157544758 [3]
$A_3(18, 4; 4)$	111828493003995506044	111828492966430770027 [3]
$A_4(18, 4; 4)$	19454769271504557075730961	19454769271485256959951964 [3]
$A_5(18, 4; 4)$	227861597522471795764877751276	227861597522469274830565882195 [3]
$A_7(18, 4; 4)$	312130863277973170267574410726205300	312130863277973166253742271546610147 [3]
$A_8(18, 4; 4)$	85095027678952869432382343292909789249	85095027678952869357138271983639907192 [3]
$A_9(18, 4; 4)$	11974616198843922060937804378020488874436	11974616198843922059938039662012799420059 [3]
$A_2(19, 4; 4)$	41265315376833	41265282958278 [3]
$A_3(19, 4; 4)$	3019369311108187930600	3019369310399000457648 [3]
$A_4(19, 4; 4)$	1245105233376291684834977113	1245105233375348762973895504 [3]
$A_5(19, 4; 4)$	28482699690308974471842016207036	28482699690308720567594897355775 [3]
$A_7(19, 4; 4)$	107060886104344797401778345454852597360	107060886104344796219558793302396711773 [3]
$A_8(19, 4; 4)$	43568654171623869149379762750201287709265	43568654171623869115634697686019566694976 [3]
$A_9(19, 4; 4)$	8729495208957219182423659412931786239553508	8729495208957219181775386539037823737728819 [3]

TABLE 8. Improvements based on Theorem 2, Theorem 3, and Lemma 4 cont.

APPENDIX B. SKELETON CODES FOR THE MULTILEVEL CONSTRUCTION

In this appendix we list the used skeleton codes from the results of Section 3. For the ease of a more compact representation we replace each vector $v \in \mathbb{F}_2^n$ by the integer $\sum_{i=1}^n v_i \cdot 2^{n-i}$. As an example, the integer 6168 corresponds to the vector $1100000011000 \in \mathbb{F}_2^{13}$. Starting from an integer, the value of n needs to be clear from the context.

$S_{13,4,5}^1 = \{7936, 7360, 6816, 1984, 3488, 3680, 5776, 6496, 6544, 6736, 7216, 3720, 5456, 5704, 3400, 5000, 5508, 2948, 4912, 5416, 5668, 7180, 2856, 3604, 4932, 1816, 2882, 6666, 1826, 3346, 4802, 2753, 4336, 5218, 5281, 6406, 6409, 6661, 7171, 1256, 1380, 2706, 2833, 3217, 4514, 4545, 5258, 740, 929, 1618, 2264, 2388, 2636, 3206, 3333, 4705, 696, 1236, 2274, 2442, 3114, 1417, 1669, 2228, 4300, 4426, 4636, 4867, 5189, 376, 466, 841, 1202, 1329, 1577, 2598, 5142, 5145, 428, 618, 790, 1347, 1550, 2161, 2217, 4389, 1084, 2339, 405, 4154, 597, 4243, 651, 563, 2078, 2123, 118, 199, 109, 283, 1063, 4111\}$

$S_{13,4,5}^2 = \{7936, 7360, 6816, 3008, 3488, 3680, 5776, 6496, 6544, 6736, 7216, 3720, 5456, 5512, 5704, 3400, 1840, 1924, 5668, 7180, 3604, 4904, 4932, 4994, 1858, 2840, 2852, 5410, 6666, 3346, 1264, 1698, 4516, 4801, 5281, 6406, 6409, 6661, 7171, 1380, 1473, 2706, 3217, 4328, 4706, 4748, 4881, 740, 929, 1801, 2264, 2388, 2444, 2636, 3333, 5254, 696, 1617, 2274, 3114, 3142, 4308, 1228, 2228, 2609, 2819, 4426, 5146, 5189, 376, 466, 714, 1308, 4274, 4630, 426, 1550, 2217, 2245, 4209, 409, 617, 4156, 661, 355, 1078, 1081, 309, 333, 2131, 4235, 391, 1099, 2078, 583, 110, 539, 2087\}$

$S_{14,4,5}^1 = \{15872, 14720, 13632, 3968, 7456, 11072, 11456, 12992, 13088, 13472, 14432, 11536, 6816, 7312, 5904, 6920, 10896, 3680, 6736, 7240, 9992, 14360, 5768, 9808, 11304, 9860, 13332, 5508, 5700, 6468, 10788, 992, 2864, 3608, 9602, 12812, 12818, 13322, 14342, 1488, 1712, 2760, 3394, 4994, 5232, 5666, 6338, 9508, 10530, 3236, 4528, 4552, 6418, 8872, 9089, 9368, 9761, 10508, 10762, 12561, 1448, 1730, 1857, 4712, 5313, 6284, 7173, 8560, 8644, 8980, 10324, 10401, 920, 2288, 2408, 3590, 4756, 6196, 9314, 11267, 12425, 2452, 2497, 3210, 3337, 6659, 9292, 3122, 3153, 4324, 4450, 5164, 6186, 10313, 12357, 844, 850, 2821, 5379, 4657, 8372, 8402, 8522, 8774, 12323, 628, 721, 810, 1308, 1329, 2601, 5145, 4393, 8729, 678, 1557, 4188, 4250, 421, 1114, 1129, 1174, 2150, 2201, 4366, 234, 345, 1187, 1547, 8250, 8334, 403, 611, 653, 310, 4179, 8455, 542, 2183, 8237, 1095\}$

20 LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

$S_{14,4,5}^2 = \{15872, 14720, 13632, 6016, 6976, 7360, 11552, 12992, 13088, 13472, 14432, 7440, 10912, 11024, 11408, 6800, 3680, 3848, 11336, 14360, 7208, 9808, 9864, 9988, 3716, 5680, 5704, 10820, 13332, 6692, 992, 3396, 9032, 9602, 10562, 12812, 12818, 13322, 14342, 2760, 2946, 3457, 5412, 6434, 8656, 9412, 9496, 9762, 1480, 1858, 3602, 4528, 4776, 4888, 5272, 6412, 9089, 10433, 10762, 12561, 1392, 3234, 4548, 6228, 6305, 6665, 8616, 10380, 11269, 1729, 2288, 2408, 2456, 2468, 2849, 5218, 5638, 6282, 7171, 8852, 10292, 10505, 12425, 1428, 1809, 2616, 5385, 8548, 9313, 852, 908, 4705, 5201, 9260, 12357, 2641, 3100, 3121, 4434, 4869, 5253, 8418, 1234, 8312, 8498, 8753, 12323, 690, 1322, 1577, 4739, 710, 806, 4204, 8771, 10259, 618, 2326, 4262, 8346, 451, 665, 677, 2138, 2150, 8462, 220, 233, 316, 345, 1166, 2197, 4154, 8278, 779, 1078, 2567, 589, 1287, 4149, 542, 2093, 4171, 8327, 115, 8221, 1051\}$

$S_{13,4,6}^1 = \{8064, 7776, 7504, 6624, 6864, 6960, 6984, 7344, 7368, 7464, 7704, 3908, 2016, 3752, 5828, 5924, 5954, 6820, 3778, 3874, 6552, 7686, 3492, 3732, 5032, 5794, 3521, 3857, 5524, 6794, 7429, 1944, 3474, 5057, 5537, 5777, 5897, 2956, 2977, 4948, 5010, 5452, 5514, 6264, 6534, 6915, 7299, 2898, 3402, 3657, 5330, 5426, 6697, 6725, 2516, 3188, 3356, 5292, 1656, 1925, 2738, 3621, 2668, 4722, 4764, 5226, 5233, 6246, 948, 2474, 2673, 3225, 4340, 6293, 970, 1478, 1814, 3130, 3214, 4878, 504, 1385, 1638, 3171, 4412, 4442, 4697, 6227, 1244, 1621, 2236, 2266, 2281, 2393, 2405, 6174, 867, 1253, 1675, 2358, 4282, 4493, 4661, 825, 1333, 1363, 1587, 2695, 4302, 4323, 5149, 5191, 723, 845, 1206, 1326, 2589, 3095, 4395, 435, 685, 4205, 4375, 414, 606, 2319, 63\}$

$S_{13,4,6}^2 = \{8064, 7776, 7504, 6624, 6864, 6960, 6984, 7344, 7368, 7464, 7704, 3908, 6824, 2016, 5828, 5924, 5954, 3748, 3778, 3874, 6552, 7686, 5794, 3521, 3857, 5524, 7429, 1944, 2964, 3468, 3474, 5057, 5537, 5777, 5897, 2977, 3284, 3380, 3721, 4948, 5004, 5010, 5452, 5514, 6264, 6534, 6789, 6915, 7299, 2898, 2954, 3402, 5330, 5426, 2764, 2860, 4788, 5236, 5292, 1656, 1925, 2676, 2738, 3180, 3186, 3242, 4810, 4906, 4716, 4722, 5226, 6246, 2666, 1478, 1814, 2396, 2417, 3161, 6229, 504, 1265, 1638, 4316, 4412, 4442, 4457, 4697, 5177, 934, 1381, 1621, 1678, 2236, 2266, 2281, 2362, 2617, 6174, 6189, 6195, 6219, 867, 4282, 4325, 1363, 1581, 1587, 1611, 3143, 723, 821, 845, 1206, 1229, 1326, 4679, 5159, 469, 1309, 2599, 435, 459, 683, 795, 374, 429, 669, 1118, 1179, 2327, 414, 4247, 4367, 238, 574, 2191, 123\}$

$S_{14,4,6}^1 = \{16128, 15552, 15008, 13248, 13728, 13920, 13968, 14688, 14736, 14928, 15408, 4032, 7816, 11600, 11656, 11848, 11908, 13640, 7556, 7748, 13104, 15372, 5968, 6984, 7464, 3888, 7042, 7490, 7714, 11048, 11076, 11556, 13588, 14858, 6948, 10114, 11137, 11794, 11809, 6017, 7256, 7697, 9896, 10008, 10904, 10946, 11426, 12528, 13068, 13830, 13833, 14598, 14601, 14853, 15363, 5796, 5826, 6804, 6849, 7314, 7329, 9668, 10049, 13394, 13450, 3312, 3852, 5032, 5528, 6376, 6712, 11409, 5476, 5898, 6484, 9124, 9812, 10468, 10804, 13445, 1896, 9432, 9570, 11334, 12492, 12625, 13059, 1940, 2914, 3425, 4824, 5332, 6450, 8680, 10360, 10570, 12586, 1008, 2520, 3276, 3843, 4961, 5660, 9042, 9396, 9441, 10524, 10545, 2897, 4578, 5425, 6342, 10441, 12454, 12457, 12581, 1745, 2484, 2732, 2738, 3354, 4472, 4724, 4934, 5321, 6257, 9324, 9513, 12348, 12483, 1490, 3178, 4785, 6297, 8564, 8626, 10390, 972, 1452, 1650, 1829, 2412, 2474, 2652, 2838, 3132, 3267, 3349, 4714, 4889, 8817, 8901, 9486, 1690, 2501, 2665, 5178, 5389, 8806, 8981, 9491, 10325, 10339, 963, 1372, 1699, 4502, 4750, 5198, 5219, 6190, 6221, 8601, 8762, 12339, 1593, 1677, 8851, 10285, 828, 1254, 2451, 2699, 3123, 4325, 4691, 9291, 1334, 1419, 1582, 4653, 8525, 9245, 10267, 726, 819, 1607, 2166, 4427, 6167, 8583, 9255, 12303, 1141, 2599, 252, 3087, 8286, 243, 359, 783, 1175, 207, 63\}$

$S_{14,4,6}^2 = \{16128, 15552, 15008, 13248, 13728, 13920, 13968, 14688, 14736, 14928, 15408, 4032, 7816, 13648, 11656, 11848, 11908, 7496, 7556, 7748, 13104, 15372, 11588, 3888, 7042, 7714, 11048, 14858, 5928, 6936, 6948, 10114, 11074, 11137, 11554, 11794, 11809, 5954, 6017, 6568, 6760, 6977, 7442, 7457, 7697, 9896, 10008, 10020, 10904, 11028, 12528, 13068, 13578, 13830, 13833, 14598, 14601, 14853, 15363, 5796, 5908, 6804, 10049, 10660, 10852, 11537, 3312, 3852, 5528, 5720, 9576, 10472, 10584, 13573, 5352, 5476, 6360, 6372, 6484, 9620, 9812, 9432, 9444, 10452, 12492, 13059, 5332, 1008, 2786, 3276, 3843, 4792, 6322, 12458, 4578, 4818, 4833, 8632, 8824, 8884, 9394, 10354, 10417, 2732, 2769, 3242, 4472, 4532, 4724, 5234, 5297, 6257, 8658, 8673, 8906, 12348, 12378, 12390, 12393, 12438, 12441, 12453, 12483, 1489, 1737, 2506, 8564, 9329, 972, 1452, 1644, 1692, 1734, 2412, 2460, 2652, 3132, 3162, 3174, 3177, 3222, 3225, 3237, 3267, 4553, 6286, 8901, 12373, 2501, 9358, 10318, 10381, 937, 963, 1372, 1699, 2618, 3157, 5198, 5261, 6221, 12339, 874, 922, 934, 1593, 2467, 8803, 8851, 9293, 828, 857, 869, 917, 1338, 1379, 1590, 1619, 2358, 2361, 2613, 2699, 3123, 4499, 4654, 6187, 854, 1333, 1419, 2387, 4683, 4743, 8494, 8734, 8749, 9259, 10267, 10279, 819, 2631, 4382, 4397, 4637, 5147, 5159, 6167, 8523, 8583, 12303, 4423, 8477, 9239, 252, 3087, 243, 783, 207, 63\}$

$S_{15,4,6}^1 = \{32256, 31104, 30016, 26496, 27456, 27840, 27936, 29376, 29472, 29856, 30816, 8064, 15632, 23200, 23312, 23696, 23816, 27280, 15112, 15496, 26208, 30744, 11936, 13968, 14928, 7776, 14084, 14980, 15428, 22096, 22152, 23112, 27176, 29716, 13896, 20228, 22274, 23588, 23618, 12034, 14512, 15394, 19792, 20016, 21808, 21892, 22852, 25056, 26136, 27660, 27666, 29196, 29202, 29706, 30726, 11592, 11652, 13608, 13698, 14628, 14658, 19336, 20098, 26788, 26900, 6624, 7704, 10064, 11056, 12752, 13424, 14881, 22818, 28945, 3792, 10952, 11796, 11841, 12968, 13185, 14529, 18248, 19624, 19841, 20936, 21608, 22049, 26890, 3880, 7489, 18864, 19140, 21313, 21697, 22668, 24984, 25250, 26118, 27141, 28809, 2016, 5828, 6850, 9648, 10664, 12900, 17360, 20720, 21140, 25172, 25745, 25865, 26705, 5040, 6552, 7329, 7686, 9922, 11320, 18084, 18792, 18882, 19233, 19977, 21048, 21090, 23043, 28741, 5794, 9156, 10017, 10850, 12684, 13827, 19553, 20882, 20897, 22577, 24908, 24914, 25162, 26755, 2980, 3428, 3490, 3724, 3857, 4968, 5464, 6708, 6801, 8944, 9448, 9665, 10642, 12514, 12641, 13445, 18648, 19026, 24696, 24966, 25137, 25347, 28707, 1944, 2904, 3009, 5474, 5897, 6356, 6917, 12594, 17128, 17252, 20780, 25637, 25667, 3658, 4824, 4948, 5304, 5521, 5676, 6264, 6534, 6698, 9428, 9778, 9865, 13337, 14357, 17634, 17802, \}$

18972, 2744, 3252, 5002, 5330, 10356, 10778, 17612, 17962, 18982, 20650, 20678, 1844, 3356, 4833, 6441, 9004, 9542, 10396, 10438, 10565, 12380, 12442, 14347, 17202, 17524, 17989, 19477, 21517, 21523, 24678, 1656, 1925, 2508, 2886, 3186, 3273, 3621, 9033, 9498, 10521, 17702, 20570, 1713, 2668, 3226, 4902, 5398, 5413, 5653, 6246, 8650, 9382, 10345, 10531, 18582, 1452, 1859, 2673, 6419, 8982, 9493, 11271, 17050, 17177, 18490, 18701, 20534, 20747, 1393, 4332, 4549, 5262, 8869, 12334, 12371, 12551, 17091, 17166, 17497, 18510, 24606, 504, 937, 1638, 1686, 2282, 2723, 3339, 6221, 8846, 2453, 3118, 4686, 4749, 6174, 8851, 9293, 16572, 1253, 2358, 5195, 8409, 16725, 16739, 17543, 4659, 9259, 16941, 486, 725, 2259, 4277, 8733, 8775, 1563, 2221, 4381, 16563, 1181, 2583, 619, 1118, 365, 411, 574, 207, 1079}

$S_{15,4,6}^2 = \{32256, 31104, 30016, 26496, 27456, 27840, 27936, 29376, 29472, 29856, 30816, 8064, 15632, 27296, 23312, 23696, 23816, 14992, 15112, 15496, 26208, 30744, 23176, 7776, 14084, 22096, 23620, 29716, 11856, 13872, 13896, 14916, 15396, 15426, 20228, 22148, 22274, 11908, 12034, 13136, 13520, 13954, 19792, 20016, 20040, 21808, 22056, 23076, 23106, 23586, 25056, 26136, 27156, 27660, 27666, 29196, 29202, 29706, 30726, 11592, 11816, 13608, 14882, 20098, 21320, 21704, 6624, 7704, 11056, 11440, 14657, 19152, 20944, 21168, 21889, 27146, 28945, 10704, 10952, 11649, 12720, 12744, 12968, 13185, 19240, 19624, 22721, 22817, 14497, 18864, 18888, 19329, 20904, 24984, 26118, 26769, 26889, 28809, 2016, 10664, 3524, 6552, 7686, 9584, 12644, 13409, 18241, 24916, 25681, 25861, 28741, 9921, 10017, 17264, 17348, 17648, 17768, 17828, 17858, 18788, 19553, 20708, 20834, 21089, 2980, 3010, 3490, 3745, 3857, 5464, 5524, 6484, 7249, 7429, 8944, 9064, 9448, 10468, 10594, 10849, 12514, 13445, 13571, 21253, 24696, 24756, 24780, 24786, 24876, 24882, 24906, 24966, 25137, 25161, 25221, 25347, 25641, 25731, 26661, 26691, 28707, 1944, 2904, 3288, 3384, 5026, 5777, 5897, 9108, 9612, 9618, 11013, 17128, 18658, 19589, 19715, 4824, 4920, 5004, 5304, 5514, 6264, 6324, 6348, 6354, 6444, 6450, 6474, 6534, 6705, 6729, 6789, 6915, 7209, 7299, 11333, 12572, 13337, 14357, 17298, 18057, 21123, 21541, 21571, 24746, 1876, 2744, 9098, 10883, 12837, 12867, 18716, 19013, 19481, 20636, 20762, 21017, 22541, 22547, 3188, 5446, 5701, 6314, 10396, 10522, 10777, 11299, 12442, 14347, 24678, 1656, 1716, 1740, 1746, 1836, 1842, 1866, 1925, 3651, 4724, 5228, 5234, 9030, 9414, 9510, 18586, 18979, 1489, 2668, 2674, 3178, 3350, 4806, 5286, 5667, 6246, 9308, 9749, 12374, 17190, 1706, 3597, 4714, 4886, 8870, 16988, 17468, 17498, 17558, 17678, 17939, 18518, 20534, 20558, 945, 969, 1449, 2417, 2710, 2830, 3214, 4549, 8764, 8794, 9274, 9739, 10294, 10318, 12334, 24606, 504, 1638, 4337, 4457, 8613, 8643, 16954, 17038, 18478, 1381, 2281, 2453, 6174, 8537, 16803, 17485, 867, 4499, 16601, 16697, 16781, 741, 1251, 2381, 2443, 4405, 8377, 8405, 16723, 16941, 486, 1566, 1309, 2227, 4269, 4299, 5143, 8491, 16501, 795, 2247, 2343, 8301, 8307, 669, 1179, 4189, 4623, 8471, 16491, 414, 2109, 2139, 599, 4155, 8335, 1071, 126}$

$S_{16,4,6}^1 = \{64512, 62208, 60032, 52992, 54912, 55680, 55872, 58752, 58944, 59712, 61632, 16128, 31264, 46400, 46624, 47392, 47632, 54560, 30224, 30992, 52416, 61488, 23872, 27936, 29856, 15552, 28168, 29960, 30856, 44192, 44304, 46224, 54352, 59432, 27792, 40456, 44548, 47176, 47236, 24068, 29024, 30788, 39584, 40032, 43616, 43784, 45704, 50112, 52272, 55320, 55332, 58392, 58404, 59412, 61452, 23184, 23304, 27216, 27396, 29256, 29316, 38672, 40196, 53576, 53800, 13248, 15408, 20128, 22112, 25504, 26848, 29762, 45636, 57890, 7584, 21904, 23592, 23682, 25936, 26370, 29058, 36496, 39248, 39682, 41872, 43216, 44098, 53780, 4032, 7760, 14978, 27713, 37728, 38280, 40065, 42626, 42753, 43394, 45336, 45441, 49968, 50500, 52236, 54282, 57618, 57633, 57873, 61443, 11656, 13700, 19296, 21328, 22273, 25800, 26241, 27009, 34720, 41440, 42280, 50344, 51490, 51730, 51745, 53410, 10080, 13104, 14658, 14913, 15372, 19844, 22640, 36168, 37584, 37764, 38466, 39954, 42096, 42180, 46086, 46089, 53521, 54277, 57482, 11588, 18312, 20034, 21700, 23569, 25368, 27654, 39106, 41764, 41794, 45154, 49816, 49828, 50324, 53345, 53510, 3888, 5960, 6856, 6980, 7448, 7714, 9936, 10928, 13416, 13602, 17888, 18896, 19330, 21284, 22721, 22817, 25028, 25282, 25409, 26890, 28817, 36417, 37296, 38052, 49392, 49932, 50274, 50694, 50697, 51345, 51465, 52227, 53385, 57414, 57417, 57477, 5808, 6018, 7041, 10948, 11794, 11809, 12712, 13834, 14497, 25188, 28933, 34256, 34504, 41560, 41665, 45137, 51274, 51334, 7316, 9648, 9896, 10608, 11042, 11352, 12528, 13068, 13396, 13585, 14601, 15363, 18856, 19556, 19730, 26674, 27139, 28714, 35268, 35604, 36129, 37944, 39429, 43145, 43269, 3688, 5488, 6504, 10004, 10660, 13829, 20712, 21153, 21556, 35224, 35924, 37964, 41300, 41356, 43057, 51269, 3312, 3852, 5825, 6712, 9666, 11025, 12882, 18008, 19084, 20792, 20876, 21130, 24760, 24884, 25649, 28694, 34404, 35048, 35978, 37641, 38954, 43034, 43046, 45093, 49356, 49923, 5016, 5772, 6372, 6546, 7242, 18066, 18996, 19593, 21042, 21577, 26649, 26661, 34570, 35404, 38147, 41140, 2904, 3426, 5336, 6452, 9804, 10796, 10826, 11306, 11397, 12492, 13059, 17300, 18764, 20690, 21062, 25129, 37164, 37425, 3473, 3718, 5346, 7237, 12838, 14357, 17964, 18986, 19017, 22542, 34100, 34354, 36980, 37402, 37507, 41068, 41494, 42005, 42019, 1008, 2786, 3276, 3843, 8664, 9098, 9441, 10524, 12825, 17738, 21539, 24668, 24742, 25102, 25613, 34182, 34332, 34437, 34994, 35865, 37020, 37189, 49212, 49347, 1873, 4564, 5446, 5673, 6678, 9545, 12442, 17617, 17692, 17989, 20803, 41485, 2529, 2769, 4906, 6236, 9093, 9372, 9498, 9795, 12348, 12483, 17702, 17705, 17795, 18586, 21005, 33144, 33969, 35139, 2506, 2857, 3605, 4716, 5413, 6297, 10345, 10390, 16818, 18709, 24675, 33450, 33478, 34981, 35086, 41107, 41227, 972, 1452, 3132, 3267, 4553, 5651, 6667, 8817, 9318, 17573, 18518, 18595, 20565, 33882, 33897, 49203, 1477, 1689, 4518, 4885, 6243, 8554, 16753, 17466, 17550, 33193, 34061, 963, 1699, 2661, 4442, 6407, 8762, 10323, 12339, 16793, 20615, 33126, 36939, 1622, 2362, 5166, 5259, 6189, 8549, 8611, 9479, 33173, 33365, 33379, 745, 828, 2445, 3123, 8505, 16979, 20507, 32985, 49167, 730, 5149, 8405, 33863, 36887, 819, 1206, 2222, 4217, 4277, 12303, 1141, 8365, 16494, 16935, 17431, 252, 3087, 243, 414, 663, 783, 343, 207, 63}$

$S_{16,4,6}^2 = \{64512, 62208, 60032, 52992, 54912, 55680, 55872, 58752, 58944, 59712, 61632, 16128, 31264, 54592, 46624, 47392, 47632, 29984, 30224, 30992, 52416, 61488, 46352, 15552, 28168, 44192, 47240, 59432, 23712, 27744, 27792, 29832, 30792, 30852, 40456, 44296, 44548, 23816, 24068, 26272, 27040, 27908, 39584, 40032, 40080, 43616, 44112, 46152, 46212,$

22 LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

47172, 50112, 52272, 54312, 55320, 55332, 58392, 58404, 59412, 61452, 23184, 23632, 27216, 29764, 40196, 42640, 43408, 13248, 15408, 22112, 22880, 29314, 38304, 41888, 42336, 43778, 54292, 57890, 21408, 21904, 23298, 25440, 25488, 25936, 26370, 27393, 38480, 39248, 45442, 45634, 45697, 4032, 28994, 29057, 29249, 37728, 37776, 38658, 39681, 41808, 42753, 49968, 52236, 53538, 53778, 53793, 57618, 57633, 57873, 61443, 21328, 22273, 45377, 7048, 13104, 15372, 19168, 25288, 26818, 36482, 49832, 51362, 51722, 53521, 57482, 7554, 7746, 7809, 10120, 11080, 11140, 34528, 35296, 35536, 37576, 39106, 41416, 41668, 42178, 43201, 3888, 5960, 6020, 6980, 10928, 11586, 11649, 11810, 11841, 12968, 14498, 14858, 17888, 18128, 18896, 19240, 20936, 21188, 21698, 22721, 25028, 25793, 26890, 27142, 42506, 43529, 49392, 49512, 49560, 49572, 49752, 49764, 49812, 49932, 50274, 50322, 50337, 50442, 50694, 50697, 51282, 51297, 51345, 51462, 51465, 51717, 52227, 53322, 53382, 53385, 57414, 57417, 57477, 5808, 6576, 6768, 7489, 10052, 19746, 19986, 20001, 22026, 23049, 34256, 34600, 35608, 35620, 37316, 38081, 39178, 39430, 3752, 9648, 9840, 10608, 12528, 12648, 12696, 12708, 12888, 12900, 12948, 13068, 13410, 13458, 13473, 13578, 13830, 13833, 14418, 14433, 14481, 14598, 14601, 14853, 15363, 18200, 18212, 19220, 22666, 25144, 25865, 26117, 26674, 26761, 28714, 36114, 36129, 36369, 42246, 43082, 43142, 43269, 49492, 50257, 50437, 53317, 5488, 19729, 21766, 22789, 25674, 25734, 34580, 37432, 38026, 38153, 38405, 38962, 41272, 41524, 42034, 42121, 43057, 45082, 45094, 45097, 3312, 3432, 3480, 3492, 3672, 3684, 3732, 3852, 6376, 11402, 12628, 13393, 13573, 19084, 20792, 21044, 21554, 22577, 22598, 24884, 25649, 26693, 28694, 28697, 28709, 38985, 39045, 49356, 49923, 9448, 10456, 10468, 21577, 21637, 34444, 35212, 35404, 37172, 37937, 37958, 42053, 45077, 2978, 3412, 5336, 5348, 6356, 6700, 7210, 12492, 13059, 17804, 17996, 18616, 18764, 24748, 1953, 9428, 9772, 10540, 10780, 11305, 33976, 34124, 34936, 34996, 35878, 37036, 41068, 41116, 1008, 1890, 1938, 2898, 2913, 2961, 3276, 3843, 4834, 5420, 5660, 6428, 7190, 7193, 7205, 10883, 17290, 17528, 17588, 18548, 20588, 20636, 24668, 49212, 49347, 1873, 8674, 8914, 8929, 9500, 11285, 18051, 18819, 19011, 33610, 33670, 33673, 33908, 36956, 2762, 4562, 4577, 4817, 4906, 6691, 12348, 12483, 17074, 17222, 17225, 17285, 17578, 18538, 24739, 25099, 34179, 34371, 34970, 34985, 35139, 1737, 8657, 8986, 8998, 9001, 10531, 17731, 18582, 18597, 33202, 33394, 33457, 33605, 37027, 37387, 41059, 41107, 41227, 41479, 972, 1482, 1734, 2502, 2505, 2757, 3132, 3267, 4762, 4886, 4889, 4901, 5411, 5651, 6419, 8810, 16754, 16817, 17009, 17561, 18521, 20579, 20627, 20747, 20999, 24659, 24839, 33882, 33897, 49203, 1477, 4713, 8857, 8981, 9491, 17494, 17509, 33137, 33941, 34901, 36947, 37127, 963, 2618, 4454, 8598, 8846, 12339, 33002, 1593, 4494, 4501, 4686, 4749, 8549, 16602, 16614, 16617, 828, 1338, 1590, 2358, 2361, 2613, 3123, 8378, 8526, 8589, 8781, 16942, 32982, 32985, 32997, 49167, 1333, 4218, 4278, 4281, 4429, 10267, 16597, 33070, 33310, 33325, 819, 2222, 8310, 8313, 8373, 9255, 12303, 16670, 16685, 16925, 1197, 4213, 33053, 252, 683, 1134, 1182, 2142, 2157, 2205, 3087, 423, 1117, 243, 363, 411, 603, 615, 663, 783, 343, 207, 63}

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2956, 2962, 3466, 6264, 6534, 9524, 9772, 9778, 9802, 10822, 10885, 11302, 12617, 13379, 17620, 25125, 25155, 25635, 33508, 33620, 33634, 34018, 35122, 35909, 37042, 37073, 41137, 41158, 41254, 41494, 41998, 49318, 49445, 49678, 67794, 68145, 70182, 70213, 82467, 98406, 5002, 9426, 9987, 10793, 17234, 17714, 18982, 34346, 34988, 35018, 37923, 38925, 38931, 66274, 67882, 68227, 69802, 73894, 74254, 1873, 3188, 5425, 6469, 7189, 8908, 12835, 17076, 17196, 20572, 20677, 20803, 24678, 24853, 34076, 35365, 66004, 66769, 69925, 98389, 1656, 1925, 4724, 5228, 5234, 8988, 9002, 9372, 9498, 17098, 17578, 33204, 33228, 33234, 67139, 1713, 1737, 1833, 2668, 2674, 3178, 3241, 6246, 6309, 6339, 6435, 7179, 16756, 20643, 24717, 24723, 24843, 33562, 35093, 65964, 65970, 65994, 66652, 70663, 98334, 98349, 98355, 98379, 2883, 3597, 3603, 4714, 8436, 8556, 8562, 8858, 9381, 12342, 12366, 17468, 18518, 19463, 33194, 2396, 16620, 16626, 16746, 24606, 33433, 33881, 37127, 49181, 66108, 66138, 66325, 66618, 504, 1637, 4883, 8426, 16954, 16985, 41003, 66701, 2645, 4284, 4314, 4410, 4749, 6174, 9355, 1366, 2234, 2265, 2361, 67655, 1357, 18459, 20503, 483, 723, 822, 846, 1206, 1230, 1326, 1566, 10263, 65717, 4381, 683, 65819, 8285, 407, 2319, 32911, 16463, 4143, 119}

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$S_{15,4,7}^2 = \{32512, 31936, 31392, 29632, 30112, 30304, 30352, 31072, 31120, 31312, 31792, 28296, 30032, 12224, 15944, 23944, 24196, 27976, 28036, 28228, 29488, 31756, 15240, 16002, 20384, 24104, 15656, 15908, 23368, 23876, 24130, 27432, 27522, 28194, 31242, 8032, 8080, 14212, 15172, 15682, 15745, 20304, 12080, 15512, 15636, 22216, 22402, 23332, 23425, 23842, 26392, 26404, 26434, 26497, 27412, 27457, 27922, 27937, 28177, 28912, 29452, 29962, 30214, 30217, 30982, 30985, 31237, 31747, 13768, 13992, 15138, 23192, 23640, 14145, 14756, 14996, 15041, 22292, 22978, 23314, 23698, 23713, 23825, 26856, 29957, 14098, 14680, 14904, 15121, 15457, 21416, 21864, 22305, 11504, 12044, 13160, 13668, 13908, 19396, 20161, 21953, 22756, 22840, 23137, 25816, 25828, 26836, 28876, 29443, 11682, 11937, 14562, 19184, 20234, 7400, 7945, 11858, 13016, 13028, 13217, 20020, 21346, 21688, 21716, 21730, 22066, 22097, 25272, 25314, 26802, 28842, 5872, 6640, 7942, 13432, 13492, 13522, 13873, 14452, 17904, 20229, 20952, 22737, 5976, 9200, 11601, 12035, 12728, 12756, 14513, 18316, 19660, 19756, 21112, 21172, 21202, 22642, 24952, 25012, 25042, 25057, 25204, 25297, 25714, 25777, 26737, 28732, 28762, 28774, 28777, 28822, 28825, 28837, 28867, 7708, 7749, 10122, 10956, 11082, 11436, 11466, 12978, 3800, 3812, 11141, 13509, 13593, 19116, 19146, 19249, 19626, 19843, 20852, 20914, 21317, 26766, 28757, 3048, 5932, 6604, 7366, 7450, 7461, 7715, 10922, 11356, 12658, 13603, 18249, 21273, 3512, 3540, 3553, 6764, 6825, 6979, 9836, 9884, 9926, 9929, 10025, 10604, 10652, 10694, 10697, 11366, 11414, 13454, 17866, 19036, 19561, 19609, 21605, 22606, 22669, 25678, 25741, 26701, 28723, 3762, 5073, 5553, 6570, 6822, 7317, 9132, 10857, 10905, 11322, 14382, 14419, 14475, 18026, 18074, 18086, 18089, 18794, 18842, 18854, 18857, 19046, 19094, 19542, 21134, 21155, 22581, 2018, 2996, 3026, 3697, 5066, 5532, 5737, 5785, 5797, 5827, 6505, 6553, 6745, 7225, 9578, 10005, 12878, 12899, 12941, 17260, 17756, 19002, 20885, 21550, 21643, 25134, 25227, 26667, 3442, 5542, 5734, 5782, 6502, 6550, 6742, 6803, 7222, 9052, 9532, 9637, 9667, 10806, 10837, 11317, 12691, 13357, 13387, 14365, 17766, 17814, 18069, 18195, 18777, 18789, 1908, 6332, 6362, 6485, 9062, 9110, 9558, 9875, 10469, 10553, 10595, 12597, 12621, 17242, 17317, 17347, 17722, 19027, 19507, 20819, 21037, 21067, 21533, 22555, 24862, 24877, 24907, 24967, 25117, 25159, 25627, 25639, 26647, 28687, 1989, 2979, 3723, 9018, 18659, 3469, 3662, 5454, 6451, 12827, 13335, 17206, 17717, 20779, 1516, 2958, 3283, 3403, 3629, 4581, 9043, 21015, 2893, 4999, 7183, 8444, 8885, 17101, 1497, 3607, 4725, 4894, 5235, 9387, 10767, 16634, 17607,$

24 LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

1831, 2759, 2843, 3239, 4345, 17935, 18703, 953, 4342, 9487, 16629, 8435, 1627, 2286, 2398, 875, 1598, 2269, 1246, 2235, 1213, 4303, 471, 687, 2167, 446, 1135, 8351, 381, 16479, 4159}

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14048, 14160, 23428, 23848, 26500, 28225, 29506, 43696, 43816, 44228, 44578, 45736, 46401, 46604, 47626, 50640, 50888, 52500, 52545, 52754, 54465, 54801, 55394, 58049, 61701, 75568, 77332, 79393, 80034, 86804, 87140, 87562, 107064, 108594, 110634, 115028, 115793, 115973, 118853, 15048, 27304, 27812, 29844, 40228, 51073, 52324, 59529, 76208, 76568, 77121, 83400, 87180, 92675, 8072, 11984, 12104, 22192, 22960, 23320, 23704, 26032, 26224, 28170, 28912, 29080, 29452, 29794, 30858, 31747, 38696, 39746, 42788, 43376, 45476, 45668, 46346, 53652, 53816, 54537, 57656, 57908, 58418, 59441, 61481, 72484, 76392, 78424, 79121, 80134, 88114, 94234, 94246, 102712, 102964, 103474, 104497, 106804, 107569, 110614, 110617, 110629, 114892, 115459, 7648, 13768, 14760, 14788, 15634, 15649, 15889, 20292, 20354, 23361, 27284, 29330, 39528, 39576, 39588, 40098, 42408, 42600, 42648, 43416, 43608, 45400, 51852, 57556, 59418, 69296, 71280, 71448, 71553, 75428, 76577, 76898, 76946, 78056, 84417, 84536, 90296, 90444, 90668, 90698, 91178, 14114, 15878, 21872, 22978, 23825, 26968, 26980, 27730, 27910, 27913, 36272, 38340, 43240, 43457, 44129, 44170, 45833, 46595, 50772, 51602, 57681, 71080, 72088, 72788, 72865, 74992, 75160, 75532, 84364, 86410, 87089, 87299, 88105, 92185, 100536, 100908, 101418, 106668, 108558, 7876, 14904, 22290, 24069, 25448, 26049, 26273, 29265, 29769, 38296, 38488, 38548, 39508, 40012, 42388, 45282, 50082, 53548, 54298, 54310, 69060, 69410, 71330, 72290, 75992, 76116, 76426, 78036, 78597, 83256, 83508, 83532, 84276, 86136, 86196, 86598, 90228, 91158, 7956, 11714, 11876, 15434, 15497, 19696, 20056, 20257, 21736, 23092, 23122, 23185, 25816, 25940, 28876, 28978, 36200, 36620, 38561, 39308, 39329, 42578, 43340, 43654, 44291, 50360, 50508, 50732, 51320, 51380, 51782, 53364, 53426, 55317, 57452, 57638, 58382, 58389, 72268, 73219, 74596, 74644, 75169, 82849, 83156, 84298, 85018, 85030, 88078, 99448, 99508, 99628, 99868, 100468, 100636, 101398, 101401, 101413, 102508, 102556, 103438, 104461, 106588, 107533, 114748, 114883, 11916, 13624, 13876, 18344, 21348, 23622, 25994, 37360, 38242, 38538, 41826, 41868, 50566, 51498, 51753, 68840, 68952, 69393, 70872, 72033, 72273, 72329, 75090, 75345, 76169, 76358, 78108, 86553, 6896, 7826, 14516, 14561, 14673, 15107, 15429, 19810, 22150, 22153, 25314, 26182, 26802, 26833, 29221, 35732, 36056, 36497, 42629, 43589, 45257, 47123, 50481, 53571, 53774, 55307, 57869, 70540, 71890, 74956, 82822, 83270, 83587, 84076, 84502, 85013, 86309, 90389, 98994, 99114, 100899, 106659, 107019, 3952, 6052, 7074, 11448, 11572, 11665, 12037, 13194, 13522, 19172, 21834, 22700, 26822, 36426, 37708, 39241, 40003, 42100, 45169, 45210, 45589, 51397, 51493, 53413, 69002, 70482, 71978, 72069, 74210, 74450, 74465, 75290, 78350, 79883, 7753, 9200, 10130, 11090, 13484, 13701, 19660, 21788, 22822, 26947, 28732, 28867, 34532, 35681, 37586, 37766, 38469, 39110, 39194, 41426, 43286, 49778, 49841, 49946, 50723, 53337, 57443, 57611, 68308, 70985, 74104, 74569, 75046, 75973, 78115, 82354, 82757, 84133, 84259, 86179, 90259, 98674, 98737, 98929, 99094, 99097, 99109, 99619, 99859, 100627, 102499, 102547, 102667, 102919, 106579, 106759, 114739, 5985, 7288, 7377, 7558, 9700, 11817, 14428, 18900, 19244, 19337, 19845, 21194, 21290, 21297, 21829, 22641, 25004, 25475, 29195, 33768, 35634, 36012, 38275, 42150, 42265, 43068, 43110, 43173, 50281, 71190, 76301, 78855, 82586, 83097, 10121, 12660, 12914, 12956, 12998, 17368, 18130, 19626, 20945, 22678, 25180, 25366, 25369, 25891, 26131, 34641, 35954, 38166, 45134, 49577, 49941, 68764, 70762, 70853, 71740, 74812, 74857, 77965, 82289, 82537, 86099, 98538, 98958, 100491, 3809, 6953, 9930, 11046, 12721, 14605, 18249, 21189, 25701, 34630, 35274, 35498, 35529, 35715, 37290, 37948, 39013, 50317, 66488, 68721, 69134, 70086, 71181, 74393, 74901, 5076, 10865, 12650, 12905, 13475, 17332, 18076, 18214, 19050, 19516, 19651, 25146, 25230, 25657, 26670, 26701, 37667, 38966, 41321, 41747, 42131, 45191, 49370, 49382, 66932, 67025, 67270, 67273, 67369, 68262, 68419, 70309, 70339, 75019, 75829, 82573, 98518, 98521, 98533, 98638, 98701, 98893, 99403, 99463, 100423, 114703, 4984, 5554, 5740, 6508, 6514, 10051, 10947, 11353, 12694, 20841, 21651, 24809, 33652, 34249, 35238, 39175, 43083, 49803, 66506, 68186, 68877, 69873, 70202, 74126, 74294, 82989, 2018, 3020, 3025, 3866, 5065, 5364, 6926, 7435, 9585, 19107, 19219, 20822, 21091, 25735, 34499, 35057, 35925, 36359, 37517, 41518, 50247, 67942, 67990, 68197, 70795, 82222, 3497, 6378, 7221, 8940, 9157, 10554, 10659, 13342, 19029, 20877, 21255, 28695, 34138, 34213, 34362, 37205, 41082, 41145, 42027, 66396, 68243, 69942, 74375, 1784, 1969, 2552, 5689, 6547, 7214, 9020, 11315, 14375, 18745, 20665, 21547, 24757, 33500, 34571, 35381, 35411, 37451, 66028, 67171, 74029, 82471, 2981, 3429, 3670, 3723, 5715, 6707, 6791, 8668, 9550, 9805, 10782, 16874, 66746, 67335, 67790, 67811, 69806, 71703, 73838, 73931, 82014, 82203, 3302, 3411, 6349, 6685, 12615, 17230, 17530, 17590, 18550, 18718, 20590, 33212, 33998, 2748, 4570, 4581, 5421, 12571, 16764, 34974, 49303, 66846, 1934, 4790, 5671, 10397, 11279, 24667, 33251, 66355, 69725, 1749, 4725, 5277, 8883, 9003, 18587, 18959, 73787, 1494, 3179, 4779, 6235, 8438, 9495, 17501, 36923, 41015, 1709, 1821, 2503, 3223, 4702, 10327, 17679, 33447, 65963, 8807, 16627, 16807, 16957, 33179, 66647, 2397, 5199, 34863, 877, 919, 65725, 859, 1243, 1339, 2359, 4311, 8317, 33823, 719, 66079, 8367}

$S_{18,4,7} = \{260096, 255488, 251136, 237056, 240896, 242432, 242816, 248576, 248960, 250496, 254336, 226368, 240256, 126016, 128032, 163328, 193088, 223808, 225824, 235904, 254048, 64768, 127296, 187456, 193552, 97024, 97408, 121408, 125472, 127504, 190784, 192800, 219456, 220176, 225552, 249936, 64128, 179232, 186912, 190992, 191496, 112192, 113680, 121120, 121864, 125200, 162176, 189632, 190624, 211136, 211232, 211472, 211976, 219296, 219656, 223376, 223496, 225416, 231296, 235616, 239696, 241712, 241736, 247856, 247880, 249896, 253976, 120000, 123584, 175680, 177472, 186640, 112800, 118288, 120976, 124048, 124168, 125064, 178696, 183584, 185504, 185864, 192644, 214848, 218628, 222468, 239656, 246852, 105792, 109376, 112904, 113156, 127106, 178320, 182976, 184768, 186504, 189192, 63584, 89632, 92032, 95752, 110088, 116512, 117184, 119560, 120322, 123652, 170816, 174880, 176800, 177668, 178436, 183812, 189698, 206528, 206624, 208388, 210434, 214688, 216578, 221828, 221954, 222338, 231008, 235544, 238148, 238628, 238658, 246308, 246338, 246818, 253958, 55168, 59200, 96336, 111876, 118020, 158992, 161032, 182032, 185092, 48320, 62600, 94624, 105232, 107968, 108192, 108304, 110992, 111240, 126988, 145280, 151424, 160400, 161840, 161864, 169664, 169760, 171272, 189058, 202176, 202512, 204036, 210177, 214416, 216321, 217476, 217857, 218241, 230736, 233796, 234516, 234561, 238114, 246036, 246081, 246801,$

26 LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

249861, 44928, 60930, 62977, 95490, 96296, 102080, 109700, 112706, 116360, 117380, 172992, 173472, 173712, 176520, 181152, 192522, 30400, 30496, 60464, 62210, 62532, 63512, 73600, 80992, 91744, 92512, 93697, 103360, 103840, 104080, 115600, 158344, 158468, 158852, 159968, 167360, 167584, 171140, 175234, 178241, 181640, 182660, 189473, 199616, 200096, 200336, 200456, 201632, 202376, 203396, 203522, 203906, 205712, 206216, 207236, 207617, 208001, 209282, 209537, 213896, 215426, 215681, 229856, 230096, 230192, 230216, 230576, 230600, 230696, 230936, 231620, 231716, 231746, 231956, 232001, 232460, 232466, 232481, 233636, 233666, 233762, 233996, 234002, 234017, 234506, 235526, 237716, 237761, 237836, 237842, 237857, 238097, 238601, 239621, 241667, 245900, 245906, 245921, 246026, 246281, 247811, 16192, 54880, 59976, 61648, 61992, 94788, 119169, 123044, 123170, 146016, 146512, 154192, 154882, 157024, 157264, 160804, 169360, 28096, 28320, 46856, 47696, 53000, 56450, 59012, 60161, 60545, 87392, 87632, 88456, 89156, 91472, 92802, 93208, 95252, 101280, 101776, 105000, 105090, 105508, 105601, 108930, 109602, 110788, 116098, 123402, 151136, 154664, 158786, 160068, 173608, 173825, 174280, 175124, 181889, 214128, 217188, 221268, 230056, 231586, 231946, 233745, 237706, 24336, 30096, 54608, 55624, 59688, 80456, 102146, 104193, 104648, 116289, 119058, 123909, 152416, 153136, 154242, 166800, 170625, 174360, 174465, 184849, 185350, 188689, 15776, 16016, 23488, 44384, 45920, 46640, 47408, 52064, 52448, 56340, 57824, 58160, 58904, 59588, 61716, 63494, 77392, 79492, 85576, 86752, 90952, 91336, 92545, 92692, 107304, 107632, 109074, 115312, 115816, 116836, 118882, 122962, 144968, 152784, 156848, 158242, 160268, 176228, 188468, 188492, 188933, 205424, 205928, 206948, 208994, 213608, 215138, 221228, 221234, 221258, 229784, 230918, 231569, 231689, 233609, 24200, 27536, 29520, 29576, 31268, 31298, 31778, 31809, 40708, 46722, 54568, 56353, 58660, 79056, 79152, 79176, 80196, 84816, 85200, 85296, 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28 LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

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180809, 181289, 197480, 197528, 197540, 197988, 198228, 201260, 201798, 215054,

LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES 29

229913, 265696, 266056, 268072, 268996, 269185, 273601, 274882, 281176, 283913, 287074, 290885, 295876, 296356, 297745, 298066, 299320, 299596, 300106, 304177, 313411, 315427, 329108, 332129, 336172, 344390, 344617, 360649, 360745, 394072, 394084, 394132, 394552, 394594, 394642, 394804, 394834, 395572, 395602, 397612, 397642, 397702, 397852, 397894, 398362, 398374, 399382, 401692, 401734, 402454, 409882, 409894, 410134, 426262, 458812, 458842, 458854, 458857, 458902, 458905, 458917, 458947, 20168, 23652, 23820, 25520, 26433, 27032, 27732, 30981, 38785, 43794, 46342, 47148, 52300, 52362, 52486, 57745, 68576, 76226, 77321, 78180, 78420, 84417, 84578, 86356, 88134, 90444, 90761, 99896, 100106, 100961, 102732, 103497, 107651, 115749, 137026, 144012, 144138, 144529, 149140, 149908, 156805, 166665, 168585, 169002, 169093, 172572, 172586, 172677, 173125, 199218, 272036, 272162, 275090, 275508, 275553, 276530, 276554, 283281, 296532, 299654, 301097, 303497, 304154, 311580, 311601, 311685, 330060, 330122, 336515, 337934, 360995, 7632, 14996, 15010, 22180, 27409, 29740, 29746, 29777, 36632, 43426, 43660, 47363, 50580, 50594, 53601, 69232, 69505, 74664, 78982, 80005, 85129, 87173, 88195, 90337, 90418, 90449, 90652, 100753, 103450, 106794, 115075, 115222, 137996, 142601, 143672, 149090, 149265, 153669, 156209, 156230, 157733, 164584, 172361, 198218, 199196, 199241, 200988, 213084, 265168, 266020, 267184, 268052, 269585, 273057, 275971, 276508, 279490, 281861, 283676, 284698, 288790, 303276, 303685, 312334, 328546, 328609, 329266, 330310, 332326, 336931, 458837, 6112, 14113, 14568, 23697, 27188, 28856, 28898, 29066, 29318, 29955, 37744, 37800, 39690, 44106, 44113, 44293, 50020, 50953, 53368, 53554, 55333, 57546, 58390, 69060, 71268, 72084, 72966, 76172, 82776, 83532, 84306, 85020, 86570, 99540, 99921, 100682, 101414, 102609, 102789, 104462, 104469, 106821, 134512, 134977, 140737, 142380, 144133, 144412, 155946, 156186, 164706, 165217, 166092, 166449, 166470, 168108, 180750, 181267, 198277, 204969, 205326, 213523, 229518, 271592, 272145, 272972, 276611, 280977, 281673, 282849, 283779, 287270, 290835, 295640, 296492, 297080, 298053, 300054, 303398, 328152, 329937, 332058, 7784, 7970, 11504, 12044, 14676, 14738, 14930, 21400, 21848, 25828, 26257, 26374, 36514, 39266, 39308, 39569, 41921, 42401, 45260, 49640, 51740, 57500, 57626, 67528, 71308, 72330, 75106, 76084, 77059, 78154, 78595, 82660, 82834, 83170, 84358, 84613, 86321, 87065, 90310, 91150, 98788, 99718, 102556, 134564, 136596, 136850, 137825, 138374, 141538, 143570, 149624, 151878, 152613, 155973, 165044, 166531, 166979, 168261, 198833, 213141, 213155, 213261, 265624, 267682, 271756, 272596, 279764, 279884, 279946, 280780, 280876, 281137, 296161, 296581, 299557, 301075, 329285, 344153, 458803, 11172, 11874, 11922, 13962, 14985, 23594, 25428, 27674, 38290, 39985, 41700, 45172, 45361, 45594, 46147, 57510, 57513, 57877, 83505, 90405, 91155, 99026, 99530, 99612, 102678, 114746, 134922, 136780, 136969, 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70931, 73962, 74089, 75915, 98428, 98862, 133364, 133484, 135845, 139685, 139877, 149555, 151595, 164026, 164237, 165917, 263857, 265276, 266842, 268373, 270742, 278716, 295126, 5914, 6934, 7257, 11043, 13390, 17361, 17862, 18089, 18780, 18801, 22611, 33714, 35058, 37462, 37465, 41302, 41358, 41550, 41739, 42030, 49486, 50251, 66986, 67353, 68186, 68366, 68750, 68877, 70041, 70710, 73916, 74339, 82275, 98486, 132886, 135402, 135529, 135738, 140341, 140427, 140551, 172047, 264857, 267027, 270650, 271443, 295097, 9923, 10810, 12707, 17770, 21077, 25165, 34028, 34245, 39047, 41555, 45095, 51227, 69127, 70222, 82507, 132444, 132713, 134229, 141383, 147795, 163961, 262900, 263410, 263830, 264643, 265294, 265363, 266585, 266809, 266851, 270460, 270553, 279595, 327965, 3861, 5571, 6460, 7307, 10458, 12598, 18062, 19091, 19509, 20694, 21131, 22557, 35130, 37639, 49453, 49707, 67235, 67941, 68003, 69861, 73941, 74823, 82131, 135292, 135763, 136267, 263066, 263514, 264425, 264758, 264971, 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37917, 49269, 49331, 66289, 66499, 69934, 77839, 82078, 98475, 99343, 131804, 133895, 135350, 135353, 147566, 164039, 164123, 264589, 266635, 279067, 295027, 2921, 3641, 3670, 4965, 5013, 9402, 10579, 18003, 35029, 66894, 69753, 132654, 263509, 266451, 1958, 3659, 6699, 13335, 34187, 34334, 34990, 37163, 66445, 74011, 131546, 131558, 131897, 132218, 133406, 133451, 134171, 134183, 264499, 2787, 6323, 9006, 10413, 12403, 18759, 20763, 35367, 36973, 41067, 41115, 66765, 131917, 132397, 132637, 132679, 135463, 147517, 263289, 263342, 264350, 1513, 1765, 2684, 3374, 4939, 5237, 16822, 20647, 49295, 66166, 67699, 135695, 262861, 263719, 2518, 8989, 9325, 17502, 24635, 66347, 67863, 69783, 262581, 262766, 956, 3229, 4525, 4766, 4807, 8875, 16633, 17053, 35087, 67645, 131445, 133271, 1459, 8647, 16958, 17175, 66141, 131499, 266295, 270383, 1691, 1807, 2459, 4446, 5179, 6223, 16999, 33341, 33879, 131315, 262507, 1438, 2703, 4317, 18479, 33175, 10271, 131639, 262375, 8375, 635, 65647, 131167, 254, 319}

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LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES 31

230520, 230580, 230700, 230940, 231540, 231708, 232470, 232473, 232485, 233580, 233628, 234510, 235533, 237660, 238605, 245820, 245955, 32265, 48394, 48646, 56582, 56585, 56837, 73168, 73540, 89185, 89233, 92754, 93258, 93318, 94540, 104082, 108114, 111747, 112707, 120853, 123925, 145000, 150896, 151316, 152816, 152996, 153176, 153356, 154004, 154196, 154210, 154258, 157076, 157268, 157282, 158801, 159956, 169304, 169316, 170578, 170634, 171089, 173396, 181556, 184436, 184604, 185369, 185381, 16176, 24264, 28104, 28356, 44834, 47672, 47714, 52464, 52584, 52632, 52644, 52824, 52836, 52884, 53004, 54504, 54924, 55512, 55524, 55692, 55884, 58274, 58584, 58596, 58764, 58956, 59604, 59724, 59978, 61644, 62211, 85666, 87384, 87396, 87650, 87713, 88404, 90856, 91793, 92386, 101608, 102028, 104033, 107236, 107362, 107921, 108748, 108937, 116908, 122986, 123046, 138696, 138948, 142788, 158049, 168680, 172520, 172946, 173702, 174634, 177219, 182915, 184617, 188585, 199586, 200020, 201940, 202060, 203845, 205538, 205706, 207491, 214132, 214300, 216085, 217180, 218125, 230066, 230186, 231971, 237731, 238091, 31882, 32005, 47506, 47881, 48265, 55945, 59782, 79416, 80184, 80289, 80436, 88710, 91041, 91489, 92497, 103128, 104010, 104785, 106968, 107850, 109125, 116355, 119062, 145762, 150760, 151180, 152916, 152930, 156376, 157900, 158278, 167128, 167140, 167308, 167500, 170188, 172756, 172897, 173260, 181420, 182380, 182428, 184853, 16040, 24004, 40722, 40737, 44817, 46392, 46497, 46644, 47412, 52564, 54114, 54162, 54177, 54484, 54604, 58194, 58257, 80465, 85208, 85220, 85346, 85388, 85580, 86504, 88268, 88394, 91340, 92486, 94378, 101588, 101708, 101770, 102884, 103250, 103628, 103814, 104998, 110746, 115820, 115868, 115882, 116828, 117059, 143114, 145034, 145592, 145964, 145970, 151137, 152292, 152970, 153825, 154755, 156132, 157257, 157994, 167561, 169545, 170310, 174281, 188517, 197616, 198498, 198546, 198561, 199506, 199521, 199569, 199884, 200451, 201186, 201426, 201441, 201546, 201606, 201609, 202371, 203139, 203331, 205266, 205281, 205521, 205638, 205641, 205701, 206211, 206403, 207171, 208956, 209091, 213426, 213618, 213681, 213786, 213798, 213801, 214563, 215331, 215571, 217251, 217611, 221283, 221331, 221451, 221703, 229746, 229809, 230001, 230166, 230169, 230181, 230691, 230931, 231699, 233571, 233619, 233739, 233991, 237651, 237831, 245811, 31814, 46306, 46673, 46853, 47329, 48197, 79156, 86740, 86881, 86929, 87250, 87433, 90580, 90954, 93219, 96267, 101958, 104730, 107718, 108057, 110694, 116886, 144289, 150740, 150860, 150929, 152024, 152780, 153158, 156497, 158083, 168404, 168785, 169258, 173338, 174358, 176233, 181340, 181571, 15600, 15720, 15768, 15780, 15960, 15972, 16020, 16140, 24120, 27960, 28212, 28422, 29880, 30252, 30840, 30900, 31020, 31260, 44216, 45962, 47276, 47306, 50160, 51938, 52106, 52428, 52995, 54661, 55621, 59939, 61500, 61635, 80425, 87621, 88613, 90802, 92329, 92693, 94358, 99304, 100066, 100834, 101074, 101194, 106930, 107290, 107813, 108604, 108649, 109579, 115354, 139014, 139017, 142562, 143109, 144564, 144684, 144690, 144774, 144924, 145524, 145692, 153894, 154137, 156332, 156997, 159897, 159909, 160779, 161799, 165770, 166790, 167241, 169494, 172652, 172721, 176277, 180650, 180842, 180902, 181353, 198481, 199370, 201169, 201541, 202051, 213361, 213781, 214291, 217171, 217351, 229610, 230030, 231563, 30921, 40337, 44617, 45778, 46618, 47653, 53962, 57802, 58054, 58057, 72888, 73260, 76920, 76980, 77100, 77340, 79020, 79980, 80028, 83794, 84705, 84873, 86854, 88345, 90540, 95239, 100233, 101253, 103593, 103957, 104601, 106860, 108693, 115113, 115305, 115365, 115798, 138578, 142729, 145606, 148456, 152393, 156826, 156838, 157756, 164824, 164836, 165601, 166369, 166609, 168364, 169157, 170044, 171019, 172444, 173116, 173158, 175111, 180889, 181397, 182357, 15700, 16131, 23860, 29812, 29980, 40056, 40116, 44148, 44421, 45894, 46188, 46236, 46374, 47196, 50658, 50898, 51018, 51078, 51666, 52038, 52041, 58659, 58899, 59667, 73289, 73349, 79145, 79397, 82904, 82916, 84778, 86636, 87566, 88124, 90482, 90716, 90902, 91196, 91301, 92218, 99284, 99794, 103484, 104549, 105479, 107203, 107578, 115034, 115046, 142034, 142508, 142889, 144585, 152220, 153690, 156081, 156453, 165702, 166697, 168540, 172821, 174137, 174347, 180630, 180822, 197580, 198090, 198342, 198345, 199110, 199113, 199365, 199740, 199875, 208947, 213210, 213222, 213225, 213390, 213582, 213645, 214155, 215115, 215175, 229590, 229593, 229605, 229710, 229773, 229965, 230475, 230535, 231495, 245775, 29893, 30243, 31011, 31251, 40579, 45521, 53702, 53705, 53957, 55459, 57797, 72820, 72988, 75489, 78940, 80149, 83658, 83785, 83845, 84805, 86428, 87317, 91225, 100134, 100165, 102748, 103203, 103513, 108039, 115093, 115285, 138418, 138629, 141748, 143818, 148436, 148961, 149201, 150186, 150217, 151916, 152177, 152636, 152726, 153869, 155996, 156355, 165329, 166723, 168613, 168643, 169229, 172483, 180569, 13296, 15564, 23724, 24131, 27756, 27804, 29801, 30821, 39602, 39722, 40134, 41912, 42674, 42698, 45734, 50124, 52284, 52419, 57914, 61491, 75562, 76234, 76486, 76582, 78442, 79001, 86723, 90563, 102851, 107062, 107149, 136632, 138348, 138396, 140664, 142428, 144069, 144474, 145497, 152355, 156729, 156813, 168506, 198085, 199226, 204986, 205358, 206891, 213205, 213325, 214087, 22993, 23217, 23337, 26409, 27058, 27249, 27417, 27459, 29097, 29338, 29971, 40259, 43434, 43441, 45721, 46166, 55379, 70584, 71137, 71377, 74616, 74676, 83398, 84378, 84582, 85515, 90681, 102969, 106809, 136820, 138773, 142486, 144485, 145549, 149273, 149957, 150181, 152003, 153653, 165289, 11240, 14153, 15555, 23644, 26282, 37752, 37812, 38322, 38346, 38514, 39282, 41844, 42354, 42441, 42693, 42774, 42819, 43670, 44174, 45402, 47367, 53562, 53814, 54411, 57654, 57909, 71338, 71450, 72346, 72470, 75418, 75430, 76901, 78422, 78499, 83606, 90763, 103051, 134072, 137894, 137897, 142115, 142394, 150795, 151047, 166293, 168246, 197436, 197571, 197946, 198198, 198201, 198966, 198969, 199221, 199731, 200826, 200886, 200889, 201006, 201246, 201261, 201771, 202779, 202791, 204918, 204921, 204981, 205086, 205101, 205341, 205851, 205863, 206871, 208911, 14001, 15130, 21937, 22309, 23317, 26389, 27045, 27795, 29017, 29030, 29077, 29269, 38681, 45413, 70516, 76182, 76563, 80007, 83514, 85075, 85255, 87115, 90421, 102709, 136114, 137585, 140146, 140953, 140965, 141925, 143929, 143971, 144019, 148826, 149846, 156211, 165461, 172339, 172615, 4080, 6120, 7128, 7140, 10200, 10212, 11220, 13260, 15420, 18360, 19320, 19380, 21240, 21420, 22182, 25080, 25332, 25452, 25500, 25962, 26202, 26970, 35576, 35756, 36458, 36506, 36518, 39333, 39513, 43238, 47147, 49980, 50115, 50533, 50787, 51386, 51758, 52275, 54343, 61455, 71914, 72038, 72041, 72971, 74986, 75110, 76387, 78179, 78227, 83363, 86579, 99004, 100474, 100654, 100894, 102574, 133044, 134004, 134570, 138003, 138297, 138339, 140198, 141075, 141710, 142389, 152199, 164771, 165050, 165422, 166070, 168327, 180519, 197941, 198830, 200821, 200989,

32 LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

201751, 11890, 13681, 13738, 19881, 20121, 22165, 27213, 29259, 39982, 41893, 43363, 43603, 43659, 44077, 50579, 57502, 67320, 67500, 68088, 68340, 68460, 68508, 74649, 75349, 75993, 83254, 86134, 99219, 99885, 136553, 137530, 141653, 143669, 143699, 149045, 149689, 166429, 6100, 7793, 11939, 13251, 14990, 15411, 18292, 20980, 21340, 22283, 25942, 27271, 34296, 34548, 34668, 34716, 34737, 35316, 35676, 35697, 36186, 36198, 36246, 37617, 39225, 41817, 50298, 50358, 50361, 50478, 50718, 51318, 51321, 51381, 51486, 51501, 53358, 53421, 53787, 70565, 70886, 70998, 71894, 74597, 86315, 99614, 100531, 133868, 133994, 134742, 136038, 136611, 140009, 140622, 140643, 141854, 141899, 143534, 147900, 148092, 148371, 149715, 164220, 166061, 168107, 172187, 196860, 197427, 197742, 197790, 197805, 198750, 198765, 198813, 199695, 13965, 14731, 19801, 20053, 21721, 23595, 25827, 25995, 35737, 36259, 36409, 38229, 39125, 39965, 45341, 57437, 67060, 67420, 69013, 69267, 70489, 71431, 75085, 75091, 75315, 78363, 82300, 83245, 83485, 84085, 99445, 132850, 133618, 134501, 136085, 136787, 140501, 143659, 148871, 151835, 155927, 166003, 4044, 11497, 13116, 13206, 13902, 14669, 14919, 18156, 18924, 19164, 19363, 22919, 26183, 26798, 37773, 41430, 41774, 42105, 42165, 42195, 42539, 45591, 49404, 49971, 50461, 51735, 52239, 70106, 71861, 71987, 72221, 82606, 86171, 98926, 132588, 132828, 133596, 133977, 135897, 137547, 139737, 139982, 141422, 141491, 148659, 151659, 155739, 168027, 197291, 197725, 11482, 20013, 21331, 23575, 28766, 34618, 35557, 35638, 36149, 39070, 53399, 70350, 90199, 99483, 102487, 106575, 134029, 135653, 136478, 140573, 148295, 165143, 4035, 7011, 7397, 7566, 7734, 10940, 11574, 11595, 11822, 13107, 13639, 15375, 17884, 22685, 26731, 34521, 35534, 37662, 37707, 38439, 41210, 49835, 68069, 68281, 70835, 76303, 133859, 140459, 149607, 172079, 196851, 196971, 197019, 197031, 197211, 197223, 197271, 197391, 3945, 4005, 6035, 10126, 11061, 11086, 11803, 21619, 23055, 26895, 41566, 67033, 69159, 70093, 70765, 90159, 132566, 132941, 140815, 3900, 5820, 5850, 5859, 6588, 6780, 6867, 7463, 9660, 9852, 9942, 10620, 10707, 12540, 13479, 18041, 35277, 37110, 37485, 37995, 42255, 49395, 49515, 49935, 67253, 68779, 74155, 74903, 136347, 137367, 196951, 6861, 18891, 19623, 24813, 26679, 73950, 132907, 164407, 3891, 5500, 5934, 9651, 10023, 12531, 13071, 18803, 21053, 21095, 37275, 38967, 38991, 49495, 70799, 82109, 83999, 132345, 133365, 133787, 134235, 135390, 148527, 196815, 6393, 10653, 11351, 17653, 19035, 24891, 34030, 34247, 43039, 67179, 67933, 73959, 131998, 136271, 3324, 3855, 5549, 20670, 25631, 34455, 35051, 49359, 68183, 82043, 147999, 5035, 5725, 6487, 6715, 9067, 18063, 20791, 20815, 66030, 66285, 67983, 3315, 3483, 5467, 8693, 12495, 33639, 34383, 37167, 41079, 132327, 133679, 196671, 2967, 5335, 8923, 9047, 33214, 131773, 1020, 3279, 6319, 16871, 17111, 49215, 133407, 4791, 33403, 1011, 1515, 8879, 9327, 12351, 1723, 975, 1655, 2415, 3135, 16799, 831, 255}

$S_{19,4,8} = \{522240, 517632, 513280, 499200, 503040, 504576, 504960, 510720, 511104, 512640, 516480, 488512, 502400, 425472, 453696, 455232, 455712, 485952, 486432, 487968, 498048, 516192, 130560, 228608, 258368, 384064, 390160, 453152, 481600, 482320, 487696, 512080, 244800, 252480, 252960, 257040, 258576, 259080, 326912, 359168, 359552, 387392, 389312, 389408, 195328, 195712, 227968, 256192, 256288, 258208, 375360, 375840, 383520, 387600, 388104, 389640, 424320, 440640, 441360, 448704, 448800, 449040, 449544, 452880, 454800, 454920, 473280, 473376, 473616, 474120, 481440, 481800, 485520, 485640, 487560, 493440, 497760, 501840, 503856, 503880, 510000, 510024, 512040, 516120, 244256, 256520, 326272, 387232, 129408, 249152, 251072, 252176, 374080, 381760, 440480, 440840, 452744, 476992, 480772, 484612, 501800, 508996, 242976, 250656, 375056, 380192, 382112, 383120, 383240, 240448, 242368, 243856, 243976, 252040, 371904, 379584, 419712, 424032, 436032, 439812, 444096, 444192, 445956, 448002, 451332, 451716, 451842, 468672, 468768, 470532, 472578, 476832, 478722, 483972, 484098, 484482, 493152, 497688, 500292, 500772, 500802, 508452, 508482, 508962, 516102, 240800, 248480, 371488, 373408, 374920, 124800, 129120, 193808, 219008, 226448, 227408, 235328, 247568, 251393, 255233, 355600, 357640, 378304, 435872, 437762, 451202, 464320, 464656, 466180, 472321, 476560, 478465, 479620, 480001, 480385, 492880, 495940, 496660, 496705, 500258, 508180, 508225, 508945, 512005, 224016, 226056, 239040, 246720, 247200, 317312, 324368, 325712, 341888, 348032, 357008, 358448, 358472, 364352, 366272, 366368, 370448, 374273, 378512, 378632, 380417, 385793, 386177, 128260, 178048, 184192, 191752, 194608, 194632, 208768, 224392, 227368, 233152, 233248, 235168, 239248, 239368, 241153, 247432, 254593, 322704, 324744, 369600, 370080, 377760, 401280, 419424, 423960, 429504, 429840, 431040, 431520, 431760, 431880, 432900, 433284, 433410, 435600, 438660, 443280, 443784, 444804, 446850, 461760, 462240, 462480, 462600, 463776, 464520, 465540, 465666, 466050, 467856, 468360, 469380, 469761, 470145, 471426, 471681, 476040, 477570, 477825, 492000, 492240, 492336, 492360, 492720, 492744, 492840, 493080, 493764, 493860, 493890, 494100, 494145, 494604, 494610, 494625, 495780, 495810, 495906, 496140, 496146, 496161, 496650, 497670, 499860, 499905, 499980, 499986, 500001, 500241, 500745, 501765, 503811, 508044, 508050, 508065, 508170, 508425, 509955, 191120, 193160, 238496, 307072, 322312, 325672, 354952, 364192, 370312, 63296, 106368, 124512, 125700, 126084, 126210, 127620, 127746, 128130, 129048, 187472, 187652, 193604, 218720, 219908, 220232, 220292, 222560, 222800, 224324, 225860, 226340, 234896, 236801, 250241, 250898, 343120, 350800, 351490, 379924, 381460, 381964, 415072, 415312, 416836, 419152, 422212, 422932, 428960, 429704, 432770, 435080, 436610, 476272, 479332, 483412, 492200, 493730, 494090, 495889, 499850, 94016, 95936, 96032, 212016, 219696, 317024, 318512, 320864, 321104, 324674, 341600, 347744, 351272, 353120, 353504, 353840, 353864, 355394, 356930, 357410, 363920, 365456, 365960, 367361, 367745, 373121, 379265, 61120, 61216, 63136, 120160, 120400, 124240, 125570, 128065, 155456, 161216, 177760, 178768, 183904, 187012, 189280, 189664, 190000, 190024, 191044, 191524, 193060, 208480, 210692, 211528, 218448, 221920, 222760, 223780, 232336, 232840, 234376, 236161, 240001, 241940, 242705, 248338, 248842, 250004, 250124, 250378, 310344, 318024, 318210, 318594, 350850, 379404, 381202, 400992, 404832, 405072, 406368, 406752, 407088, 407112, 408132, 408612, 408642, 412512, 412896, 413232, 413256, 414432, 415272, 416292, 416322, 416802, 418272, 418512, 418608, 418632, 418992, 419016, 419112, 419352, 420036, 420132, 420162, 420372, 420876, 420882, 422052, 422082, 422178, 422412, 422418, 422922,$

LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES 33

423942, 435312, 438372, 442992, 443496, 444516, 446562, 450612, 450636, 450642, 467568, 468072, 469092, 471138, 475752, 477282, 483372, 483378, 483402, 491928, 493062, 493713, 493833, 495753, 32640, 93856, 179240, 186920, 242694, 278336, 286400, 286496, 290240, 291776, 292256, 306784, 309808, 316752, 320224, 321064, 322114, 322594, 324130, 342568, 349392, 349488, 354850, 356673, 363400, 371729, 56768, 57104, 106080, 109920, 110160, 111456, 111840, 112176, 112200, 117600, 117984, 118320, 118344, 119520, 120360, 121156, 123360, 123600, 123696, 123720, 124080, 124104, 124200, 124440, 125505, 125985, 127521, 129030, 147136, 147232, 155296, 158656, 159136, 160672, 175876, 185168, 185672, 204128, 204368, 208208, 209744, 210248, 211268, 216272, 216368, 217568, 217808, 218408, 218648, 221648, 222488, 223553, 225473, 225569, 227333, 234608, 241804, 242185, 246128, 246872, 247892, 248837, 249953, 254033, 292612, 308994, 339714, 347464, 349508, 350404, 350500, 371718, 372820, 372882, 373002, 373254, 381066, 381955, 400720, 404192, 405032, 408098, 414160, 415000, 416020, 418472, 420002, 420362, 434792, 436322, 450602, 463216, 463960, 464980, 471121, 475480, 477265, 479260, 479281, 479305, 492805, 81344, 81680, 89024, 89504, 89744, 89864, 113684, 161412, 161538, 177360, 185520, 186690, 215880, 219330, 219426, 240134, 241746, 248070, 278176, 289696, 302432, 302672, 306512, 308528, 314192, 315872, 316112, 316712, 316952, 319952, 320792, 325637, 334688, 335072, 335408, 335432, 338768, 340448, 340784, 340808, 341168, 341192, 341528, 342338, 346592, 346928, 347312, 347672, 348872, 348968, 349761, 352688, 352712, 353048, 353432, 354497, 354593, 356513, 357381, 358403, 363632, 365168, 365672, 369008, 369752, 371209, 372833, 377072, 377192, 377432, 377912, 378962, 378977, 385073, 385097, 48064, 48544, 48784, 48904, 56224, 56968, 105808, 109280, 110120, 112836, 112932, 119248, 120088, 120996, 121873, 123560, 127249, 155024, 162881, 170848, 171232, 171568, 171592, 175408, 176608, 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491640, 491910, 492165, 492291, 492675, 32352, 64020, 64524, 64530, 80800, 81544, 112961, 113164, 113170, 113674, 121025, 121121, 127028, 127052, 159362, 159489, 177474, 178370, 178466, 185634, 186530, 215748, 218274, 241706, 246348, 246918, 277904, 285584, 286088, 293441, 293921, 301792, 302632, 305632, 305872, 306472, 306712, 307912, 308008, 308036, 314032, 314536, 316072, 316196, 319912, 320152, 323846, 338608, 339112, 339521, 340676, 342178, 348836, 350481, 354566, 354821, 356486, 363112, 368872, 369208, 370738, 370762, 377930, 378922, 380966, 56900, 62576, 62849, 95554, 95764, 96532, 104928, 105168, 105264, 105288, 105648, 105672, 105768, 106008, 109008, 109848, 111024, 111048, 111384, 111768, 116548, 117168, 117192, 117528, 117912, 119208, 119448, 121353, 123288, 124422, 125073, 125193, 127113, 146320, 146824, 154504, 162337, 174792, 174888, 175300, 177316, 181712, 184772, 185026, 192646, 192773, 203216, 204056, 207528, 208066, 209698, 210194, 211217, 214472, 214820, 216332, 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198608, 201668, 202148, 202388, 202529, 204293, 206609, 208133, 209948, 214244, 214673, 214793, 215378,

34 LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

217208, 217292, 217733, 219158, 221573, 230753, 230993, 234533, 245957, 274224, 277284, 277794, 278028, 278034, 286218, 289380, 289554, 290058, 291220, 291594, 304833, 304929, 306314, 308305, 312993, 313994, 315724, 316489, 319794, 319818, 331540, 333580, 340741, 342085, 345738, 352554, 361300, 361684, 361780, 361804, 363589, 365125, 368965, 377125, 396962, 398740, 402338, 403082, 404102, 426722, 427562, 428582, 434342, 434702, 459220, 459601, 459985, 460060, 460081, 460105, 461125, 461845, 463045, 463126, 463141, 463171, 463381, 463885, 463891, 467221, 475285, 475405, 475411, 491605, 31400, 58818, 59028, 59154, 59810, 60044, 61794, 80240, 81164, 81185, 88304, 88850, 89172, 92392, 94424, 94520, 108193, 109705, 117830, 143024, 146594, 146954, 153960, 154200, 154305, 157377, 158040, 158865, 160140, 160146, 160401, 169746, 173708, 175178, 177225, 182604, 182921, 185385, 185475, 200466, 202506, 205668, 206412, 207410, 209196, 211011, 213844, 216195, 230226, 230610, 230706, 230730, 232515, 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78568, 79442, 80056, 83696, 83816, 85766, 87224, 87686, 87689, 88604, 101281, 104195, 107729, 107825, 110758, 115826, 116905, 142568, 150868, 152788, 156882, 156978, 157002, 157065, 157868, 158001, 167468, 167558, 169350, 171030, 172934, 173637, 174250, 175139, 182819, 201930, 202371, 203811, 205596, 205938, 207126, 207891, 209046, 213618, 214122, 215317, 231699, 233526, 233619, 233739, 269252, 273256, 275796, 275852, 277066, 281320, 281537, 282377, 289097, 289411, 290986, 299858, 301445, 304678, 305329, 305446, 312707, 319589, 320011, 330641, 331052, 331058, 332972, 334211, 338957, 352355, 360881, 360905, 361126, 361241, 361625, 28244, 30049, 31366, 31491, 40609, 42736, 42856, 43864, 45796, 45985, 46726, 46853, 50920, 54113, 58193, 58540, 59941, 76737, 77016, 77028, 78732, 78738, 79242, 87601, 88645, 90828, 103249, 107124, 107186, 107628, 115398, 138148, 144785, 144940, 149336, 150418, 152396, 153158, 156340, 156788, 158233, 160025, 169265, 170371, 180892, 181361, 184595, 188515, 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30275, 36708, 38754, 42468, 44148, 50898, 52038, 52803, 53682, 54147, 54595, 57948,

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58022, 58458, 58659, 69104, 69352, 69464, 70640, 71384, 71396, 71480, 73034, 75448, 77129, 90902, 90947, 91301, 91406, 93191, 103484, 103619, 116813, 135105, 138450, 138546, 141153, 145514, 145521, 145701, 150405, 152010, 152730, 153123, 156273, 167081, 168306, 169050, 172485, 173331, 174347, 180570, 180630, 180645, 181515, 184395, 199770, 200946, 201066, 201306, 201786, 205146, 208947, 213210, 213306, 281894, 283461, 284867, 287429, 291335, 296312, 296837, 297586, 297673, 300197, 300323, 301157, 305237, 312461, 315437, 319517, 328568, 328952, 329541, 329603, 330353, 332393, 332438, 332441, 333923, 336233, 336473, 338003, 340043, 352283, 368663, 15945, 23857, 24195, 29137, 35824, 36328, 36568, 36664, 36754, 37864, 38584, 39604, 40108, 42889, 44566, 46358, 47164, 47194, 50132, 50657, 52390, 52750, 52757, 56327, 57586, 61523, 69474, 72396, 72492, 83740, 86386, 86809, 87130, 87142, 100596, 100716, 100722, 100956, 101667, 101907, 102636, 103513, 106716, 106812, 107574, 114876, 115758, 136664, 142723, 142885, 145939, 149969, 150725, 152357, 154123, 159797, 159821, 159883, 164728, 165112, 165763, 166597, 167182, 169109, 170083, 180883, 188443, 197368, 201485, 205369, 206893, 213557, 215111, 229685, 229709, 237583, 267748, 268084, 271800, 272820, 273222, 273564, 275737, 280018, 281713, 283057, 283081, 284821, 287337, 287893, 287907, 288013, 298044, 299580, 301203, 303885, 304227, 304395, 311529, 311638, 311694, 312403, 329964, 330153, 330300, 332122, 332854, 336060, 338055, 344293, 344405, 393720, 394854, 395484, 395580, 395610, 397500, 397530, 397626, 399390, 8088, 15569, 20196, 20321, 22241, 26232, 26417, 26441, 26501, 27337, 27461, 27929, 27971, 28195, 29340, 29478, 29849, 30821, 30883, 39626, 39722, 40050, 42780, 42822, 43466, 45482, 47253, 54330, 55353, 55437, 58425, 59655, 72370, 74722, 75562, 79961, 82913, 83316, 84394, 85142, 86438, 86723, 90682, 91283, 92299, 101059, 101434, 102970, 103075, 106841, 107086, 107107, 107783, 114905, 114998, 115001, 134120, 136674, 137041, 140748, 141746, 144579, 148908, 149100, 152677, 155868, 156323, 156435, 166250, 166490, 166677, 167013, 168156, 168170, 168549, 172250, 172346, 180410, 180782, 184359, 199014, 204986, 265688, 265912, 267224, 267980, 269482, 271841, 275267, 276538, 280940, 281180, 281254, 290867, 295672, 296726, 298133, 298507, 304181, 304205, 305197, 307271, 333063, 336523, 337963, 360659, 361511, 22988, 22994, 23834, 29097, 29349, 29846, 36682, 38609, 39532, 41932, 46181, 51862, 53849, 59467, 61483, 71497, 72873, 72982, 76209, 76454, 78438, 82890, 83377, 85093, 87307, 88327, 90709, 98808, 99942, 100586, 104478, 108615, 134584, 136120, 136988, 137905, 137926, 141766, 143782, 144470, 151910, 156301, 156935, 165212, 168505, 197852, 198245, 199001, 199267, 199757, 200921, 201017, 202781, 265953, 266065, 271314, 273510, 273513, 273603, 274801, 274885, 275979, 280170, 281241, 283299, 283726, 283747, 287790, 299238, 303321, 303417, 312363, 394582, 395450, 14211, 14964, 15132, 21880, 21940, 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36 LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

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LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES 37

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38 LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

475726, 475789, 476299, 477259, 477319, 491734, 491737, 491749, 491854, 491917, 492109, 492619, 492679, 493639, 507919, 48048, 80808, 109368, 111288, 113190, 120134, 274352, 276392, 277232, 277352, 277400, 277922, 278114, 278162, 278282, 289592, 290354, 291512, 292394, 293042, 293162, 293402, 301880, 302642, 304952, 306482, 307640, 307832, 308402, 308522, 308762, 309362, 309530, 309797, 314570, 316741, 323642, 334520, 335402, 337592, 338360, 338552, 339122, 339242, 339482, 339497, 341162, 342122, 342170, 342185, 354362, 361442, 363395, 366650, 369347, 369722, 370745, 377507, 81313, 81505, 81553, 81673, 93745, 95785, 96433, 96553, 96793, 106033, 109873, 111793, 111913, 112153, 112753, 112921, 117961, 127033, 146772, 154932, 160884, 161052, 175276, 178723, 185667, 203892, 204060, 210012, 213972, 214497, 214737, 214857, 214917, 215505, 215877, 217545, 217797, 218172, 221637, 230865, 231237, 233925, 246105, 246117, 246165, 246357, 400050, 400170, 402360, 403122, 403242, 403890, 404082, 404250, 404262, 405930, 406122, 406170, 406182, 418362, 430650, 434490, 434742, 460229, 461370, 467130, 467502, 469035, 475349, 475469, 476231, 30640, 31600, 46960, 73648, 77680, 79600, 79720, 79768, 80368, 80728, 81670, 88888, 91960, 94648, 94840, 95782, 96550, 96790, 104248, 110968, 111910, 112150, 112918, 117958, 127030, 270192, 275952, 276312, 277842, 278277, 284472, 286002, 291192, 291954, 292122, 292389, 293157, 293397, 306346, 308517, 308757, 309525, 314565, 323637, 333240, 333432, 334200, 334962, 335025, 335130, 335145, 335385, 337272, 339057, 339225, 341082, 341097, 341145, 342105, 345042, 345057, 345987, 346947, 348867, 349242, 350265, 352707, 353337, 361425, 362307, 364995, 365625, 377187, 377235, 377427, 48899, 63011, 63779, 64019, 81233, 89393, 95345, 95513, 109737, 144368, 146636, 154796, 158828, 158876, 171116, 171164, 175196, 177443, 177683, 178451, 181196, 183356, 183491, 192563, 201656, 203441, 203561, 205688, 205748, 206513, 206633, 207281, 207473, 207641, 207653, 209321, 209513, 209561, 209573, 221753, 234041, 237881, 238133, 398200, 398260, 398770, 398962, 399130, 399142, 399730, 400150, 402292, 402802, 403222, 405850, 405862, 405910, 406102, 414010, 414262, 418102, 430390, 459580, 459715, 460090, 460342, 460345, 461110, 461113, 461365, 461875, 462970, 463030, 463033, 463150, 463390, 463405, 463915, 464923, 464935, 467062, 467065, 467125, 467230, 467245, 467485, 467995, 468007, 469015, 471055, 28584, 31464, 40872, 44784, 44904, 44952, 46824, 47592, 47832, 48773, 55992, 59064, 59832, 60024, 60965, 63653, 64013, 77544, 95510, 102072, 108792, 109734, 110094, 112782, 124974, 147075, 159267, 161955, 162315, 270056, 273896, 274136, 277706, 282296, 285866, 289016, 289898, 289946, 292117, 298424, 298616, 301304, 302186, 302234, 304376, 306266, 306341, 306701, 309389, 312266, 314426, 321581, 336818, 338723, 340643, 369203, 32390, 48518, 48710, 56870, 60710, 60950, 62630, 62739, 62990, 63590, 63638, 63758, 81097, 89257, 93289, 93337, 105577, 105625, 109657, 115657, 117817, 142312, 154716, 166840, 172792, 172972, 175267, 175627, 178315, 190507, 201588, 202161, 202353, 202521, 202533, 203121, 203541, 206193, 206613, 209241, 209253, 209301, 209493, 217401, 217653, 221493, 233781, 396024, 396204, 396714, 396906, 396954, 396966, 399594, 400014, 402156, 402666, 403086, 403674, 403686, 403854, 404046, 411834, 412206, 417966, 426684, 427194, 427566, 428154, 428214, 428334, 428574, 430254, 434286, 434334, 460085, 460974, 462965, 463133, 463895, 24304, 24424, 24472, 28144, 28504, 30184, 30424, 31192, 32131, 32323, 40432, 40792, 46552, 48453, 54712, 54904, 55672, 56613, 56853, 58744, 60693, 62565, 62613, 62733, 63573, 73192, 73432, 77272, 81094, 85432, 85624, 88312, 89254, 89614, 91384, 93286, 93334, 93454, 95374, 96334, 101752, 103672, 105574, 105622, 105742, 109654, 111694, 115654, 117814, 119854, 120862, 123934, 146755, 154915, 155155, 158995, 160867, 160915, 161035, 161287, 161875, 162055, 269784, 277701, 281976, 283896, 285786, 285861, 286221, 289893, 289941, 290061, 291981, 292941, 302181, 302229, 302349, 304042, 306261, 308301, 312261, 314421, 316461, 317469, 320541, 332658, 332721, 333603, 334611, 336753, 337683, 340323, 340371, 340563, 348723, 352563, 364851, 32070, 48323, 56483, 56598, 56843, 60515, 60563, 60683, 62550, 62603, 63563, 89177, 93703, 96391, 105991, 107433, 109831, 111751, 112711, 120871, 123943, 124951, 126991, 135152, 137192, 138200, 138212, 141272, 141284, 142292, 144332, 146492, 149432, 150392, 150452, 152312, 152492, 156152, 156404, 156524, 156572, 165752, 165812, 166772, 168440, 168692, 168812, 168860, 171107, 171155, 171275, 172532, 172892, 175187, 177227, 181052, 181187, 183347, 185387, 186395, 189467, 198392, 198572, 199160, 199412, 199532, 199580, 200105, 200297, 200345, 200357, 201452, 202985, 203405, 205292, 205532, 206057, 206477, 207065, 207077, 207245, 207437, 213692, 215225, 215597, 221357, 229820, 230012, 230585, 230957, 231545, 231605, 231725, 231965, 233645, 237677, 237725, 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135116, 144188, 149228, 149996, 150236, 165356, 165596, 166364, 168803,

168851, 172883, 180476, 181043, 284423, 291143, 315671, 458995, 459115, 459163, 459175, 459355, 459367, 459415, 459535, 24261, 28101, 31125, 43973, 48173, 52133, 58085, 58253, 86870, 101094, 101262, 107214, 115374, 142275, 146475, 150435, 156387, 156555, 167559, 174279, 182439, 182799, 188559, 266186, 275258, 280298, 281066, 281306, 283477, 296426, 296666, 297434, 297701, 297869, 303821, 311546, 311981, 27590, 30293, 31783, 39878, 42950, 48158, 51110, 52070, 52118, 53990, 54099, 54158, 54663, 54855, 55623, 57830, 58070, 58190, 58695, 69577, 78649, 83689, 84457, 84697, 99817, 100057, 100825, 114937, 142012, 145991, 148956, 154151, 158231, 166627, 166795, 172747, 180907, 282247, 288967, 298375, 298567, 301255, 304327, 312487, 312847, 313447, 313495, 313615, 315535, 319567, 459095, 23491, 26563, 31278, 31771, 38853, 51045, 51093, 52053, 53733, 53973, 54093, 57813, 69574, 78646, 83686, 83854, 84454, 84694, 84814, 86734, 90574, 99814, 100054, 100174, 100822, 102862, 114934, 115054, 115102, 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21931, 22123, 22171, 22771, 22891, 22939, 23127, 25518, 25843, 25963, 26007, 26203, 26971, 28891, 36213, 38133, 38253, 38301, 38493, 39261, 42333, 50493, 53373, 68854, 68974, 69022, 69214, 70894, 71902, 74974, 83134, 84094, 99454, 134515, 134935, 136435, 136555, 136603, 136795, 136807, 136855, 137563, 137575, 137623, 140635, 140887, 141655, 143575, 148795, 149047, 149815, 150559, 151675, 151735, 155767, 265461, 265581, 265629, 265821, 267501, 268509, 269391, 271581, 279741, 280701, 296061, 7638, 19830, 20125, 21750, 21863, 21918, 22110, 22765, 22878, 25837, 25950, 26845, 36083, 36203, 36251, 36443, 38123, 38238, 39131, 42203, 50363, 51323, 69031, 69223, 69271, 69391, 71311, 71911, 72079, 72271, 74983, 75151, 75343, 75991, 76111, 78031, 83503, 84151, 84271, 84511, 86191, 90223, 90271, 99511, 99631, 99871, 100471, 100639, 102511, 102559, 106591, 114751, 132092, 164987, 14903, 21358, 21851, 36077, 134379, 164535, 164655, 263162, 265127, 271079, 271247, 13871, 14639, 19687, 19805, 21725, 36062, 66553, 68951, 70871, 70991, 83063, 83231, 86111, 137423, 140495, 148655, 149615, 279223, 279343, 295351, 295543, 295711, 11835, 13623, 14523, 19182, 19675, 66550, 134359, 148255, 164215, 263157, 264039, 264087, 265047, 266727, 266967, 267087, 270807, 7742, 11581, 13501, 14462, 33779, 68815, 77887, 133007, 133967, 278903, 7455, 11447, 13435, 17902, 33773, 132075, 164079, 264911, 7343, 11375, 17383, 18877, 25663, 33758, 133751, 137279, 278767, 295135, 7287, 17371, 18045, 132055, 132535, 147679, 263631, 9181, 66511, 68671, 139583, 9143, 264767, 270527, 5023, 9071, 3003, 4983, 8955, 263487, 266367, 1981, 2941, 4847, 8702, 66367, 1887, 4603, 1783, 2527, 65791}

$$S_{10,7,4,3} = \{38, 21, 11\}$$

$$S_{11,7,4,3} = \{44, 74, 25, 134, 69, 35\}$$

$$S_{12,7,4,3} = \{56, 84, 140, 146, 74, 273, 38, 521, 1029, 2051\}$$

$$S_{13,7,4,3} = \{112, 168, 280, 292, 148, 546, 76, 1042, 1057, 529, 2058, 4102, 4105, 2053, 67\}$$

$$S_{14,11,4,3} = \{38, 21, 11\}$$

40 LIFTED CODES AND THE MULTILEVEL CONSTRUCTION FOR CONSTANT DIMENSION CODES

$S_{14,7,4,3} = \{224, 336, 560, 584, 296, 1092, 152, 2084, 2114, 1058, 4116, 4161, 8204, 8210, 8225, 1041, 4106, 134, 2057, 261, 515\}$

$S_{15,11,4,3} = \{44, 74, 25, 134, 69, 35\}$

$S_{15,7,4,3} = \{448, 672, 1120, 1168, 592, 2184, 304, 4168, 4228, 2116, 8232, 8322, 16408, 16420, 16450, 16513, 2082, 8212, 8257, 268, 4114, 4129, 522, 2065, 1030, 1033, 517, 259\}$

$S_{16,13,4,3} = \{38, 21, 11\}$

$S_{16,7,4,3} = \{896, 1344, 2240, 2336, 1184, 4368, 608, 8336, 8456, 4232, 16464, 16644, 32816, 32840, 32900, 33026, 4164, 16424, 16514, 536, 8228, 8258, 1044, 4130, 2060, 2066, 1034, 518\}$

$S_{17,13,4,3} = \{44, 74, 25, 134, 69, 35\}$

$S_{17,7,4,3} = \{1792, 2688, 4480, 4672, 2368, 8736, 1216, 16672, 16912, 8464, 32928, 33288, 65632, 65680, 65800, 66052, 8328, 32848, 33028, 1072, 16456, 16516, 2088, 8260, 4120, 4132, 2068, 1036, 7\}$

$S_{18,13,4,3} = \{56, 84, 140, 146, 74, 273, 38, 521, 1029, 2051\}$

$S_{18,7,4,3} = \{3584, 5376, 8960, 9344, 4736, 17472, 2432, 33344, 33824, 16928, 65856, 66576, 131264, 131360, 131600, 132104, 16656, 65696, 66056, 2144, 32912, 33032, 4176, 16520, 8240, 8264, 4136, 2072, 38, 21, 11\}$

$S_{19,13,4,3} = \{112, 168, 280, 292, 148, 546, 76, 1042, 1057, 529, 2058, 4102, 4105, 2053, 67\}$

$S_{19,7,4,3} = \{7168, 10752, 17920, 18688, 9472, 34944, 4864, 66688, 67648, 33856, 131712, 133152, 262528, 262720, 263200, 264208, 33312, 131392, 132112, 4288, 65824, 66064, 8352, 33040, 16480, 16528, 8272, 4144, 44, 74, 25, 134, 69, 35\}$

$S_{12,8,4,4} = \{3084, 780, 1546, 2314, 2566, 2569, 204, 1286, 1289, 1541, 3075, 2309, 170, 771, 105, 60, 90, 102, 150, 153, 165, 195, 85, 51, 15\}$

$S_{13,8,4,4} = \{6168, 1560, 3092, 4628, 5132, 5138, 408, 2572, 2578, 3082, 6150, 4618, 212, 785, 1542, 120, 308, 332, 338, 1169, 1289, 172, 178, 202, 390, 649, 2129, 2309, 298, 4145, 4169, 4229, 4355, 581, 2089, 2179, 102, 1061, 1091, 547, 15\}$

$S_{14,8,4,4} = \{12336, 3120, 6184, 9256, 10264, 10276, 816, 5144, 5156, 6164, 12300, 9236, 240, 424, 1570, 3084, 616, 664, 676, 2338, 2578, 2593, 344, 356, 404, 780, 1298, 1313, 1553, 4258, 4618, 12291, 596, 2321, 8290, 8338, 8353, 8458, 8710, 8713, 204, 1162, 3075, 4178, 4193, 4241, 4358, 4361, 4613, 2122, 2182, 2185, 8273, 8453, 771, 1094, 1097, 1157, 2117, 60, 195, 51, 15\}$

$S_{15,8,4,4} = \{24672, 6240, 12368, 18512, 20528, 20552, 1632, 10288, 10312, 12328, 24600, 18472, 480, 848, 3140, 6168, 1232, 1328, 1352, 4676, 5156, 5186, 688, 712, 808, 1560, 2596, 2626, 3106, 8516, 9236, 9281, 24582, 1192, 4642, 16580, 16676, 16706, 16916, 16961, 17420, 17426, 17441, 408, 2324, 2369, 3089, 6150, 8356, 8386, 8482, 8716, 8722, 8737, 9226, 12293, 4244, 4289, 4364, 4370, 4385, 4625, 5129, 16546, 16906, 18437, 20483, 1542, 2188, 2194, 2209, 2314, 2569, 8465, 10243, 120, 4234, 16529, 16649, 390, 773, 8329, 1157, 1283, 643, 101, 86, 51, 46, 75, 29\}$

$S_{16,12,4,4} = \{49164, 12300, 24586, 36874, 40966, 40969, 3084, 20486, 20489, 24581, 49155, 36869, 780, 1546, 12291, 2314, 2566, 2569, 204, 1286, 1289, 1541, 3075, 2309, 170, 771, 105, 60, 90, 102, 150, 153, 165, 195, 85, 51, 15\}$

$S_{17,12,4,4} = \{98328, 24600, 49172, 73748, 81932, 81938, 6168, 40972, 40978, 49162, 98310, 73738, 1560, 3092, 12305, 24582, 4628, 5132, 5138, 18449, 20489, 408, 2572, 2578, 3082, 6150, 10249, 33809, 36869, 4618, 66065, 66569, 67589, 69635, 212, 1542, 9221, 33289, 34819, 120, 308, 332, 338, 16901, 17411, 172, 178, 202, 390, 8707, 298, 101, 15\}$

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