Towards Dynamic Contract Extension in Supplier Development

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Abstract We consider supplier development within a supply chain consisting of a single manufacturer and a single supplier. Because investments in supplier development are usually relationship-specific, safeguard mechanisms against the hazards of partner opportunism have to be installed. Here, formal contracts are considered as the primary measure to safeguard investments. However, formal contracts entail certain risks, e.g., a lack of flexibility, particular in a dynamic and uncertain business environment. We propose a receding horizon control scheme to mitigate possible contractual drawbacks while significantly enhancing the supplier development process and, thus, to increase the overall supply chain profit. Our findings are validated by a numerical case study.

Keywords Supply chain management · supplier development · optimal control · receding horizon scheme · dynamic systems

1 Introduction

Since manufacturing firms increasingly focus on their core business activities, an efficient supply chain plays a major role in generating competitive advan-

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tages. However, suppliers too often lack the capability to perform adequately. In response, manufacturers across a wide range of industries are implementing supplier development programs to improve supply chain performance [33]. According to Krause [13, p. 206], supplier development is defined as *any effort by a buying firm to improve a supplier's performance and/or capabilities to meet the manufacturing firm's short- and/or long-term supply needs.*

In accordance with the relational view as proposed by Dyer and Singh [4], activities of supplier development, in which firms convert general-purpose resources such as money, people skills or managerial knowledge into relationship-specific resources, represent a rent-generating process. Because specializing a resource lowers its value for alternative uses, relationship-specific resources are difficult or even impossible to redeploy outside the particular business relationship [38]. Thus, firms may see resources committed to supplier development as vulnerable to opportunistic expropriation [36]. Previous research has shown that contracts, in terms of formalized, legally binding agreements that explicitly specify the obligations of each firm, are viewed as the primary formal means of safeguarding relationship-specific investments against the hazards of partner opportunism [1, 20].

Therefore, supplier development activities with high levels of asset specificity should be safeguarded with long-term contracts such that the initial investment pays off [18]. The drawback of long-term contracts is, as the degree of uncertainty increases, both specifying ex ante all possible contingencies and verifying ex post the performance of the business partner becomes increasingly difficult [38]. Therefore, firms might be reluctant to sign long-term contracts, which potentially diminishes the firms' propensity to invest in supplier development activities and thus impedes the manufacturer's initial strategy to enhance supply chain performance [25].

Given this background, the purpose of our research is to mitigate possible contractual hazards by dynamically extending the contract. In addition, we seek to answer the following questions: How does the contract period, i.e. *planning horizon*, affect firms' willingness to commit relationship-specific resources to supplier development? Does receding horizon control offer a straightforward method for dynamically extending the planning horizon, while simultaneously facilitating value generation within supplier development? Further, how should receding horizon control be arranged to optimize supply chain profit?

By answering these questions, our paper makes a threefold contribution. Firstly, we formulate a continuous time optimal control problem characterizing the supplier development investment decision. We conduct a detailed study, showing that the incentives for firms to participate in supplier development critically depends on the contract period. Secondly, given the fact that long-term contracts entail certain risks, e.g., a lack of flexibility, we utilize receding horizon control and show that the supplier development process can be enhanced by dynamically extending the contract, see Sethi and Sorger [30] for the basic idea of prediction based control. Based on this result, a one-to-one map is derived linking the contract period to the optimal level of supplier development (collaboration). The insight gained from these considerations allows to either increase the supply chain efficiency or to realize the same level of collaboration while being obliged to a shorter contract period. Finally, we present a simple strategy slightly modifying the proposed receding horizon control scheme in order to avoid pathological behaviour of the supply chain. This allows to realize the optimal level of collaboration while avoiding unnecessary transaction costs.

The remainder of this paper is structured as follows. First, the related literature is briefly reviewed in Section 2. Then, in Section 3 the basic optimal control problem is described. In the subsequent Section 4, the dependence of the control policy on the contract period is studied in detail. In Section 5, a receding horizon scheme is proposed and analysed before the effectiveness of the developed methodology is demonstrated by means of a numerical case study in Section 6 before conclusions are drawn.

2 Related Literature

The topic of supplier development has received considerable attention from researchers in the past two decades. Previous research has provided good insights into the use of certain activities [32], the antecedents [13], critical success factors [18, 34], and the prevalence of supplier development in practice [15, 29].

Supplier development have been applied in various fields of application [31]. Within the automotive industry, Toyota initially began providing on-site assistance to help suppliers implement the Toyota Production System [27]. Other manufacturers have followed this collaborative approach to develop suppliers' performance and/or capabilities, including Boeing, Chrysler, Daimler, Dell, Ford, General Motors, Honda, Nissan, Siemens and Volkswagen [23, 26]. Typically, manufacturing firms use a variety of supplier development activities, e.g., providing performance feedback, training suppliers' personnel, furnishing temporary on-site support to enhance further interaction, providing equipment and tools, or even dedicating capital resources to suppliers [32, 35].

Empirical studies support that supplier development is a key factor to attenuate inefficiencies within the supply chain and, thus, strategically contributes to strengthen the manufacturer's competitiveness [19, 28]. Benefits resulting from supplier development include, e.g., improvements in cost efficiency, product quality and/or lead time [8, 16]. However, Krause and Ellram [14] note that firms' success in supplier development varies. In particular, relationship-specific investments lead, in general, to a more satisfactory outcome. Further, Krause [13] shows that the firms' propensity to participate in supplier development activities is higher if a continuation of the relationship is expected. Here, Wagner [34] adds that supplier development is more effective in mature as opposed to initial phases of relationship life-cycles.

Although relationship-specific investments seem to be critical to the success of supplier development, the application of formal decision-making models proposed for assisting firms in contract negotiations in order to adequately safeguard such investments have received limited attention in the supplier development literature [2]. Without understanding the impact of the contract period on the firms' incentives to commit relationship-specific resources to supplier development, its return will be negligible, perhaps even leading to the premature discontinuation of such collaborative cost reduction efforts.

The trend to utilize mathematical models in general and control theory in particular in decision making within supply chains is clearly visible [9] and [7]. Here, model predictive control (MPC), also termed receding (rolling) horizon control, plays a predominant role due to its ability to deal with nonlinear constrained multi-input multi-output systems on the one hand and its inherent robustness on the other hand, see [21, 40] for details. Consequently, MPC is a well-established strategy to deal with uncertainties in supply chains, see, e.g. [22, 37] and [10]. In this paper, MPC is first used in supplier development to mitigate possible contractual hazards by dynamical extending the contract.

3 Model description

We consider a particular supply chain consisting of a single manufacturer M and a single supplier S, in which M assembles components from S and sells the final product to the market. We restrict ourselves to the linear price distribution curve $p(d) = a - bd$, which establishes a connection between the production quantity d and the sale price p, in order to streamline the upcoming analysis. Here, the coefficients $a > 0$ and $b > 0$ denote the prohibitive price and the price elasticity of the commodity, respectively. This market condition is comparable with an oligopolistic or monopolistic market structure, in which a firm can increase market demand by lowering the sale price.

3.1 Basic model

It is supposed that the decision-making process is structured such that M determines the quantity supplied to the market obeying the paradigm of profit maximization. Note that we do not distinguish market demand from the production quantity of the manufacturer because the market price is endogenous to the quantity sold. Moreover, the supplier produces the components to satisfy the demand d and thus does not decide on the production quantity. Because the manufacturer's goal is profit maximization, the production quantity d chosen by M is determined by differentiating

$$d \cdot (p(d) - c_M - c_{SC}) \tag{1}$$

with respect to d and setting the resulting expression equal to zero, i.e.,

$$p(d) - c_M - c_{SC} - bd \overset{!}{=} 0, \tag{2}$$

which yields the optimum production quantity $d^\star = \frac{a - c_M - c_{SC}}{2b}$ and the optimal sale price $p(d^\star) = \frac{a + c_M + c_{SC}}{2}$. Here, c_M and c_{SC} denote the manufacturer's unit production costs and the supply costs per unit charged by S, respectively. We further assume that the supplier wants to earn a fixed profit margin r. Thus, the supply costs c_{SC} consist of the supplier's fixed profit margin r and the supplier's unit production costs c_S, i.e., $c_{SC} = r + c_S$. Similar approaches to specify the supply costs have been proposed by Bernstein and Kök [3], Li et al. [18], and Kim and Netessine [12].

It is supposed that the manufacturer wants to decrease the supplier's unit production costs c_S by conducting supplier development projects to increase the market share if that increases the overall profit of the supply chain. To this end, the sustainable effect of supplier development on the supplier's unit production costs c_S is modelled by $c_S(x) = c_0 x^m$, where $c_0 > 0$ denotes the supplier's unit production cost at the outset, $m < 0$ characterizes the supplier's learning rate, and x defines the cumulative number of realized supplier development projects. The latter is modelled as a time-dependent function $x : [0, T] \to \mathbb{R}_{\geq 0}$ governed by the ordinary differential equation

$$\dot{x}(t) := \frac{\mathrm{d}}{\mathrm{d}t} x(t) = u(t), \qquad x(0) = x_0 = 1, \tag{3}$$

with $u \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}, [0, \omega])$. Here, $u(t)$ describes the number of supplier development projects at time t; with capacity bound $\omega > 0$ to reflect limited availability of resources in terms of time, manpower or budget. Similar models of cost reduction through learning have been proposed by Yelle [39], Fine and Porteus [5], Kim [11], Bernstein and Kök [3], and Li et al. [18].

The costs of supplier development are integrated into the proposed model by a penalization term $c_{SD} u(t)$, $c_{SD} \geq 0$. Overall, this yields the supply chain's profit function $J^{SC} : u \mapsto \mathbb{R}$

$$J_T(u; x_0) := \int_0^T \frac{(a - c_M - c_0 x(t)^m)^2 - r^2}{4b} - c_{SD} u(t) \, \mathrm{d}t \tag{4}$$

for a given time interval $[0, T]$, which must be maximized subject to the control constraints $0 \leq u(t) \leq \omega$, $t \in [0, T)$, and the system dynamics (3). The contract period T is of particular interest since investments into the cost structure of the supply chain require their amortization during the runtime of the contractual agreement. A summary of the parameters is given in Table 1.

Symbol	Description	Value
T	Contract period	60
a	Prohibitive price	200
b	Price elasticity	0.01
c_M	Variable cost per unit (M)	70
c_0	Variable cost per unit (S)	100
r	Fixed profit margin (S)	15
c_{SD}	Supplier development cost per unit	100000
ω	Resource availability	1
m	learning rate	-0.1

Table 1 List of Parameter

3.2 Solution of the Optimal Control Problem

Pontryagin's maximum principle, see, e.g. Lee and Marcus [17], is used analogously to Kim [11] to solve the optimal control problem introduced in the preceding subsection. To formulate the necessary optimality conditions, we require the so-called Hamiltonian $\mathcal{H}$, which is defined as

$$H(x,u,\lambda) := \frac{(a - c_M - c_0 x^m)^2 - r^2}{4b} - c_{SD} u + \lambda u. \tag{5}$$

From the necessary conditions, we obtain the system dynamics

$$\dot{x}^\star(t) = H_\lambda(x^\star(t), u^\star(t), \lambda(t)) = u^\star(t),$$

the so-called adjoint $\lambda : [0,T] \to \mathbb{R}$, which is characterized by

$$\dot{\lambda}(t) = -H_x(x^\star(t), u^\star(t), \lambda(t)) = \frac{mc_0 x^\star(t)^{m-1}(a - c_M - c_0 x^\star(t)^m)}{2b}, \tag{6}$$

and the transversality condition

$$\lambda(T) = 0. \tag{7}$$

The solution $u^\star : [0,T) \to [0,\omega]$ of the optimal control problem exhibits the structural property

$$u^\star(t) := \begin{cases} \omega \text{ if } t < t^\star \\ 0 \text{ if } t \geq t^\star \end{cases} \tag{8}$$

depending on the (optimal) switching time $t^\star \in [0,T]$, which is characterized by the equation

$$\frac{mc_0(x_0 + \omega t^\star)^{m-1}(a - c_M - c_0(x_0 + \omega t^\star)^m)}{2b} = \frac{c_{SD}}{(t^\star - T)}. \tag{9}$$

In the following, (9) is called switching condition. Indeed, since the cost function is (strictly) convex and the system dynamics are governed by a linear ordinary differential equation, it can be shown that this condition is necessary and sufficient for the considered problem, see [24] for a detailed derivation.

The optimal value function $V_T(x_0)$ of the problem under consideration reads

$$V_T(x_0) := \sup_{u \in \mathcal{L}^\infty([0,T),[0,\omega])} J_T^{SC}(u; x_0)$$

where the expression on the right hand side is maximized subject to $\dot{x}(t) = u(t)$, $x(0) = x_0$. $V_T : \mathbb{R}_{>0} \to \mathbb{R}$ maps the initial value x_0 to the optimal value. The index T indicates the contract period and can be considered as a parameter — an interpretation, which is crucial for the upcoming analysis.

Evidently, investments (in the cost structure) pay off in the long run: While all the effort is spent directly at the beginning of the collaboration, the resulting cost decreasing effect is exploited during the remainder of the contract period.

Remark 1 At the switching time $t^\star$ the marginal revenue of further investments in supplier development (given by the adjoint variable λ) equals the marginal costs (given by c_{SD}) as indicated in Figure 1. This reasoning is expressed by the switching condition (9).

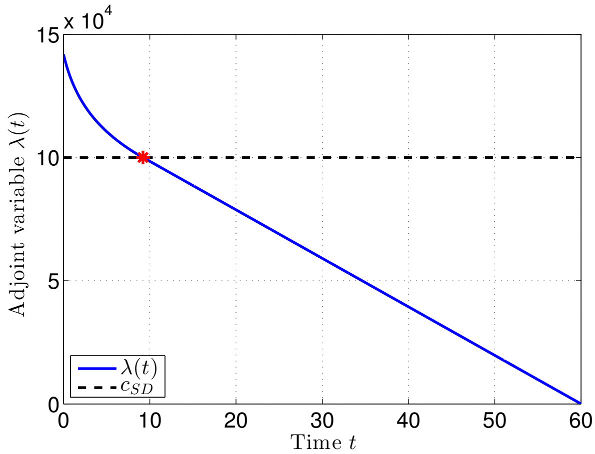

Fig. 1 The adjoint $\lambda : [0, T] \to \mathbb{R}_{\geq 0}$ computed based on the parameters given in Table 1.

4 Interplay of Switching Time and Contract Period

The contract between manufacturer M and supplier S ranges over the interval $[0, T]$. Realistically, two cases can be distinguished:

- The (optimal) switching time is given by $t^\star = 0$ meaning that investments in supplier development do not pay off during the contract period.
- A switching time $t^\star > 0$ represents the scenario where investing into supplier development amortizes during the contract period.

Within this paper, we focus on the second case. Here, from the specific structure (8) of the optimal control function we can conclude that all investments up to time $t^\star$ pay off during the contract period. Then, taking into account the already reduced supply costs given by $c_{SC}(t) = r + c_0 x(t^\star)^m$ with

$$c_0 x(t^\star)^m = c_0 \left(x_0 + \int_0^{t^\star} u^\star(s)\ \mathrm{d}t \right)^m = c_0 (1 + \omega t^\star)^m,$$

further effort in terms of $u(t) > 0$, $t \in [t^\star, T)$, does not lead to an increased profit. The latter holds true since cost reduction efforts after $t^\star$ do not amortize within the remaining time interval of at most length $T - t^\star$ and are, thus, not economically reasonable. We show that a prolongation of the contract period yields an augmentation of the investments in supplier development, which corresponds to an increased switching time $t^\star$. A proof of Lemma 1 is given in Appendix A.

Lemma 1 *Suppose that the contract period T is chosen (long enough) such that $t^\star = t^\star(T) > 0$ holds. In addition, let the condition*

$$(1-m)(a-c_M-c_0)+c_0 m \geq 0 \tag{10}$$

hold. Then prolonging the contract period $\overline{T}$, $\overline{T} > T$, implies a strictly larger switching time $t^\star = t^\star(\overline{T})$, i.e., $t^\star(\overline{T}) > t^\star(T)$.

Remark 2 The assumptions of Lemma 1 imply the inequality $a-c_M-c_0-r > 0$ as a by-product because the manufacturer cannot realize a profit per unit sold otherwise (prohibitive price is greater than the production cost per unit at time $t = 0$ from the manufacturer's point of view). Hence, the seemingly technical Condition (10) links the supplier's production costs c_0 with the difference of profit per unit $a-c_M-c_0$ by the learning rate m. Note that the assumptions of Lemma 1 can be easily verified for a given dataset of parameters.

Lemma 1 shows that investments in supplier development are extended if the contract period is prolonged. Hence, the collaboration continues after the previously determined switching time $t^\star$. As a result, the supplier's unit production costs are further decreased, the quantity offered is increased and the supply chain profit per time unit grows. The argument, that a longer contract period leads to larger switching times, can also be validated numerically as visualized in Figure 2. Here, we observe that the supply costs $c_{SC}(t) = r + c_0 x(t)^m$ are further reduced if both the manufacturer and the supplier agree on a longer contract period. The relation between the contract period T and the optimal switching time $t^\star(T)$ is almost linear.[1]

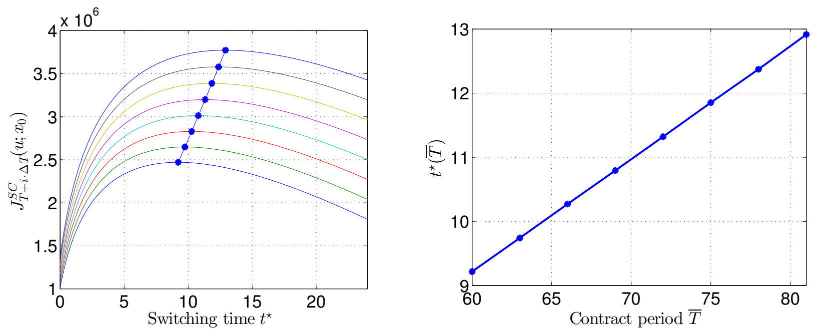

Fig. 2 Optimal switching time $t^\star = t^\star(\overline{T})$ in dependence of the length of the contract $\overline{T} = T + i \cdot \Delta T$ ($T = 60$, $\Delta T = 3$ and $i = 0, 1, \ldots, 7$).

[1] Indeed, the slope of the curve is slightly increasing.

In summary and according to the initial question *how does the contract period, i.e. planning horizon, affect firms' willingness to commit relationship-specific resources to supplier development*, the findings show that the supply chain partners' incentives to commit relationship-specific resources, i.e. to invest in cost reduction efforts, critically depends on the length of the contract period.

5 Successive Prolongation of the Contract Period

The benefits of an increased switching time come along with the inflexibility resulting from long-term contracts. In this section, we propose a methodology for assisting supply chain partners in contract negotiations to achieve the benefits of long-term contracts while rendering the collaboration time invariant. To this end, it is assumed that the manufacturer and the supplier are only content to make contracts of length T. If the collaboration is successful for a certain amount of time $[0, \Delta T)$, $\Delta T \leq t^\star$, they might agree to renew the contract on the time interval $[\Delta T, T + \Delta T]$.

Before we continue the discussion, let us briefly sketch the computation of the (optimal) control function $u^\star : [\Delta T, T + \Delta T) \to [0, \omega]$. Here, the profit function has to be maximized based on the new (initial) state $x(\Delta T)$, i.e., $J_T^{SC}(\cdot; x(\Delta T))$ is considered. Since $\Delta T \leq t^\star$ holds by assumption, the new initial state $x(\Delta T)$ is given by

$$x(\Delta T) = x(0) + \int_0^{\Delta T} u^\star(s) \; \mathrm{d}t = x_0 + \Delta T \cdot \omega \tag{11}$$

in view of Property (8). Hence, the profit on the new contract period $[\Delta T, T + \Delta T]$ is determined by maximizing

$$J_T(u; x(\Delta T)) = \int_0^T \frac{(a - c_M - c_0 \tilde{x}(t)^m)^2 - r^2}{4b} - c_{SD} u(t) \; \mathrm{d}t$$

subject to $u(t) \in [0, \omega]$, $t \in [0, T)$ and the differential equation (3) with initial condition $\tilde{x}(0) = x(\Delta T) = x_0 + \omega \Delta T$. Here, we used the notation $\tilde{x}$ to distinguish the previously computed (state) trajectory $x(\cdot; x_0)$ and its counterpart $\tilde{x}(\cdot; x(\Delta T))$ depending on the new initial condition $x(\Delta T)$. Another option is to use the time invariance of the linear differential equation $\dot{x}(t) = u(t)$, which allows to rewrite the profit functional as

$$\int_{\Delta T}^{T+\Delta T} \frac{(a - c_M - c_0 x(t)^m)^2 - r^2}{4b} - c_{SD} u(t) \; \mathrm{d}t$$

with initial value $x(\Delta T)$ given by (11) at initial time ΔT. We point out that the resulting trajectory deviates from the previously computed one already before time T. In conclusion, the implemented control strategy on $[0, T+\Delta T)$ is given by

$$u(t) := \begin{cases} u^\star(t) \text{ maximizing } J_T^{SC}(\cdot; x_0) & t \in [0, \Delta T) \\ u^\star(t) \text{ maximizing } J_T^{SC}(\cdot; x(\Delta T)) & t \geq \Delta T \end{cases}, \tag{12}$$

i.e., the first piece of the *old* policy concatenated with the newly negotiated strategy. This strategy yields an optimal policy on the time span $[0, T+\Delta T)$. Hence, the same overall supply chain profit is reached without the hazards of being committed already at the beginning (time 0) as shown in the following corollary.

Corollary 1 *Let the optimal switching time $t^\star$ determined by Condition* (9) *be strictly greater than zero. Furthermore, let ΔT, $\Delta T < t^\star$, be given. Then, the control strategy defined in* (12) *and the corresponding supply chain profit on $[0, T+\Delta T]$ equal their counterparts obtained by maximizing $J_{T+\Delta T}(u; x_0)$ with respect to $u : [0, T+\Delta T) \to [0, \omega]$*

Proof: Since the profit $J_{T+\Delta T}(u; x_0)$ on the considered time interval $[0, T+\Delta T]$ with u from (12) is the sum of

$$\int_0^{\Delta T} \frac{(a - c_M - c_0 x(t)^m)^2 - r^2}{4b} - c_{SD}\omega \, \mathrm{d}t$$

and

$$+ \int_{\Delta T}^{T+\Delta T} \frac{(a - c_M - c_0 x(t)^m)^2 - r^2}{4b} - c_{SD} u(t) \, \mathrm{d}t,$$

the dynamic programming principle yields the equality

$$J_{T+\Delta T}(u; x_0) = V_{T+\Delta T}(x_0),$$

which completes the proof. □

5.1 Receding Horizon Control

The idea of an iterative prolongation of collaboration contracts can be algorithmically formalized as receding horizon control (RHC) also known as model predictive control.

Algorithm 1 Receding Horizon Control Scheme

Given: contract period T, time step $\varDelta T$.
Set $t := 0$.

1. Measure the current state $\hat{x} := x(t)$.
2. Compute the optimal switching time $t^\star$ by solving the switching condition with $\hat{x}$ instead of x_0, i.e.
$$mc_0(\hat{x} + \omega t^\star)^{m-1}(a - c_M - c_0(\hat{x} + \omega t^\star)^m) = \frac{2bc_{SD}}{t^\star - T}.$$
3. Set
$$u^\star(s) := \begin{cases} \omega & \text{for } t \le s < \min\{t + t^\star, t + \varDelta T\} \\ 0 & \text{for } \min\{t + \varDelta T, t + t^\star\} \le s \le t + \varDelta T \end{cases}. \tag{13}$$
4. **Apply** $u^\star(s)$ for $s \in [t, t + \varDelta T)$. **Set** $t = t + \varDelta T$ and go to Step (1).

Upon start, the manufacturer M and the supplier S agree on a collaboration for a given contract period of length T. Firstly, the status quo — represented by $\hat{x}$ — is analysed. Secondly, the optimal switching time $t^\star$ is computed based on the initial state $\hat{x}$ and T, cf. Step (2). This yields the optimal control strategy defined by (13), of which the first piece $u^\star|_{[0,\varDelta T)}$ is applied. Then, the manufacturer and the supplier meet again at time $t + \varDelta T$ to negotiate a new contract. This initiates the process again, i.e. the previously described steps are repeated, which is referred to as *receding horizon principle*. Note that since the underlying system dynamics are time invariant, the newly (measured) initial state $\hat{x}$ represents all information required. In particular, no knowledge regarding the previously applied control is needed to solve the adapted switching condition of Step (2) with respect to $t^\star$. Figure 3 illustrates the outcome of Algorithm 1 with prediction horizon $T = 60$ (contract period) and control horizon $\varDelta T = 3$ (time step) based on the parameters given in Table 1.

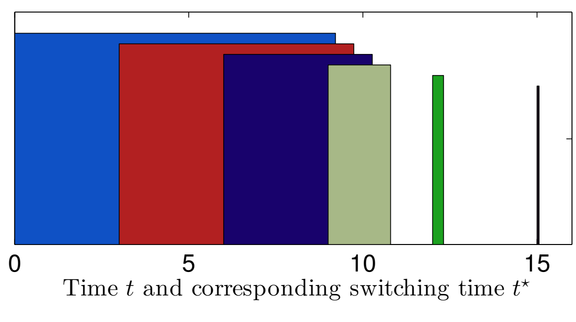

Fig. 3 Application of Algorithm 1 to compute the optimal switching times for $T = 60$ and changing initial conditions $\hat{x}$. The lengths of the collaboration intervals are decreasing.

Firstly ($t = 0$), the original optimal control problem is solved resulting in $t^\star \approx 9.21$. Then, $u^\star \equiv \omega$ is applied on the time interval $[0, \Delta T)$. Secondly ($t = \Delta T$), the collaboration is prolonged to $t^\star \approx 9.74$. Thirdly ($t = 2\Delta T$), the switching time is shifted to $t^\star \approx 10.27$. Still, $t = 3\Delta T \leq t^\star$ holds. Hence, the (measured) initial state $\hat{x}$ is given by $x_0 + t\omega = x_0 + 3\Delta T\omega$. Here, Step (2) of Algorithm (1) yields $t^\star \approx 10.79$, i.e. the collaboration stops within the time frame $[t, t + \Delta T)$. If the RHC scheme is further applied, there occur collaboration intervals of shrinking length.

As already discussed in Section 5, if the contract is not renewed, $u^\star(t)$ is set to zero for $t \geq t^\star \approx 9.21$. In contrast to that, the RHC scheme prolongs the collaboration and, thus, increases the supply chain profit. To be more precise, the profit generated by Algorithm 1 on $[0, T + i\Delta T]$, $i \in \{0, 1, 2, \ldots, T/\Delta T\}$,

$$\sum_{k=0}^{T/\Delta T+i-1} \int_{k\Delta T}^{(k+1)\Delta T} \frac{(a - c_M - c_0 x(t)^m)^2 - r^2}{4b} - c_{SD} u(t) \, \mathrm{d}t$$

is greater than its counterpart $J_T(u^\star, x_0) + V_{i\Delta T}(x^\star(T))$ consisting of the maximum of the original cost function $V_T(x_0) = J_T(u^\star, x_0)$ and a second (optimally operated) contract on $[T, T + i\Delta T]$ based on the reached cost structure represented by $x^\star(T) = x_0 + t^\star\omega \approx x_0 + 9.21\omega = 10.21$. In particular, this assertion holds in comparison to simply sticking to the cost structure based on $t^\star(T)$, i.e.

$$J_T(u^\star, x_0) + \int_T^{T+i\Delta T} \frac{(a - c_M - c_0 x(t^\star(T))^m)^2 - r^2}{4b} \, \mathrm{d}t. \qquad (14)$$

While an increased switching time $t^\star$ may already increase the profitability within a supply chain during the considered time span, the achieved cost reduction sustains. Hence, if the collaboration between the manufacturer and the supplier lasts, the obtained effect is a sustainable one.

In summary and referring to the question *how does receding horizon control offer a straightforward method for dynamically extending the planning horizon*, the findings show that dynamically extending contracts enhances the supplier development process, because value generation is facilitated while both the manufacturer and the supplier gain flexibility due to shorter contract periods.

5.2 Optimal Point of Collaboration

As observed in Figure 3, the collaboration can stop within the time interval $[t, t + \Delta T)$ meaning that the prerequisite $\Delta T \leq t^\star$ is no longer satisfied at time t. This leads to a sequence of collaboration times of shrinking length. Summing up all of these intervals on the infinite horizon yields a total collaboration time of approximately 11.18 time units. Hence, the total collaboration time is increased by 21.3%. However, since the collaboration intervals are becoming comparably short, implementing this strategy may be impracticable. Here, we propose two remedies: If the new collaboration period at time $t = k\Delta T$, i.e. $t^\star - t$, is below a certain threshold value,

1. set $t^\star = t$ in order to save negotiation costs, which would probably outweigh the achievable earning growth. For the presented example, the supplier development program stops at 10.79 (still an increase of approximately 17.2%) if the threshold is 1.
2. measure the current state $\hat{x} = x(t)$ and compute the optimal cost structure for contract periods of length T by solving

$$mc_0 T \bar{x}^{m-1}(a - c_M - c_0 \bar{x}^m) + 2bc_{SD} = 0$$

with respect to $\bar{x}$. Then, set $t^\star = t + (\bar{x} - \hat{x})/\omega$. In the considered example at time $t = 4\Delta T$, the measured state is $\hat{x} = 10.79$ while $\bar{x} \approx 11.18$. Hence, a collaboration of length 0.39 time units is fixed. At all upcoming time instants, $t^\star = t$ holds because the optimal cost structure for contract periods of length $T = 60$ is already reached.

Clearly, the threshold should be chosen such that the profit increase outweighs the negotiation costs.

Thus, Algorithm 1 allows both the manufacturer and the supplier to prolong their supplier development program without binding themselves for a time span longer than T and, thus, provides more flexibility.

Remark 3 Algorithm 1 is a simplified version. Indeed, the time step ΔT may vary in time, e.g. longer time steps in the beginning (for example $\Delta T = t^\star$ in the considered setting), and shorter ones lateron. For details on so called time varying control horizon we refer to [6].

In summary and with regard to the question *how should receding horizon control be arranged to optimize supply chain profit*, two strategies are presented in order to make the proposed receding horizon scheme, cf. Algorithm 1, applicable even if negotiation costs are taken into account.

6 Numerical Results

As seen in the previous section, applying the receding horizon Algorithm 1 dynamically extends the collaboration within the supply chain and, thus, generates additional profit within the supply chain. Next, we conduct a numerical case study to obtain further managerial insights.

To this end, we compare the outcome $J^{\star\star}$ of the proposed algorithm based on the second option presented in Subsection 5.2 and the supply chain profit resulting from the control

$$u(t) = \begin{cases} \omega & \text{for } t < t^\star(T) \\ 0 & \text{for } t \geq t^\star(T) \end{cases} \tag{15}$$

on the time interval $[0, 2T] = [0, 120]$. The control policy (15) results from the basic optimal control problem considered on $[0, 60]$ and, then, utilizing the achieved cost structure $c_{SC}(t) = x_0 + t^\star\omega$ on $[60, 120]$ without further investments in supplier development. The corresponding profit is given by (14).

To fully understand the impact of receding horizon control on the supply chain profit in depth, we first vary the following parameters of Table 1

$$\begin{aligned} a &\in \{192.5, 195, 197.5, 200, 202.5, 205, 207.5\}, \\ b &\in \{0.007, 0.008, 0.009, 0.01, 0.011, 0.012, 0.013\}, \\ c_{SD} &\in \{70000, 80000, 90000, 100000, 110000, 120000, 130000\}, \\ \omega &\in \{0.7, 0.8, 0.9, 1, 1.1, 1.2, 1.3\}, \\ m &\in \{-0.13, -0.12, -0.11, -0.1, -0.09, -0.08, -0.07\} \end{aligned}$$

resulting in a total number of $7^5 = 16807$ instances. For each parameter combination we then evaluate the respective profits.

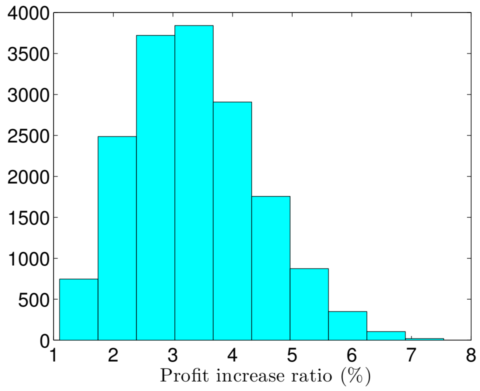

Fig. 4 Profit increase ratio in percent.

The depicted histogram in Figure 4 shows the absolute frequency with which a percentage of profit increase is observed within our parameter set. The mean value is 3.36% with a standard deviation of 1.06%. In conclusion, receding horizon control significantly improvement the profitability of the considered supply chain.

Second, we are interested in the interplay of the supplier's learning rate m and receding horizon control. Thus, based on the parameters of Table 1, we perform a sensitivity analysis with respect to the parameter m with

$$m \in \{-0.15, -0.14, -0.13, -0.12, -0.11, -0.1, -0.09, -0.08, -0.07, -0.06, -0.05\}.$$

Applying Algorithm 1 ($T = 60$, $\Delta T = 3$), Figure 5 shows both the optimal switching time $t^\star$ (without receding horizon control) compared to the optimal switching time $t^{\star\star}$ (with receding horizon control) in dependence of m (left), and the profit growth with respect to the switching time for different learning rates (right). Again, the computations are based on a simulation of 120 time units. Here, we observe that the impact of receding horizon control decreases for lower learning rates.

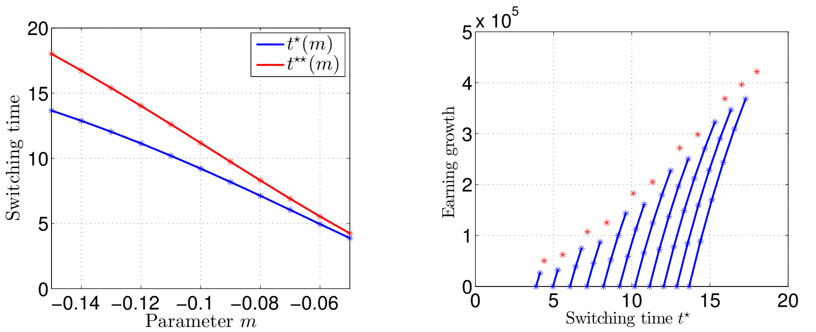

Fig. 5 Optimal switching time $t^{\star}$ and $t^{\star\star}$ with respect to the parameter m (left) and earning growth with respect to the switching time $t^{\star}$ for different values of m (right).

Hence, the results infer that especially firms in high learning industries, e.g., technology-based industries, benefit most from applying the proposed receding horizon scheme.

7 Conclusion

In this paper we investigated the impact of the contract period on supplier development. In particular, we showed that the supply chain partners' incentives to commit relationship-specific resources, i.e. to invest in cost reduction efforts, critically depends on the length of the contract period.

Given the fact that long-term contracts entail certain risks, we proposed a receding horizon control scheme to mitigate possible contractual hazards. In addition, we showed that dynamically extending contracts enhances the supplier development process, because value generation is facilitated while both the manufacturer and the supplier gain flexibility due to shorter contract periods. Furthermore, we presented two strategies in order to make the proposed receding horizon scheme, cf. Algorithm 1, applicable even if negotiation costs are taken into account.

Finally, we verified the reliability of the application by performing Algorithm 1 for an extensive parameter set and demonstrated that receding horizon control leads to a significant profit increase within the supply chain. Moreover, by means of a sensitivity analysis with respect to the learning rate, we showed that especially firms in high learning industries benefit since supplier development programs play a predominant role in order to optimize the cost structure of the supplier network.

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A Proof of Lemma 1

In this section a proof of Lemma 1 about the interplay of the contract period T and the optimal switching time $t^\star$ is given.

Proof Let the monotonic function $z : t^\star \mapsto 1+\omega t^\star$ be defined, which maps the switching time $t^\star$ to the state $x(t^\star)$ at the switching time $t^\star$. Furthermore, note that $z'(t^\star) = \omega$ holds. Then, the switching condition (9) can be rewritten as

$$(T - t^\star)z(t^\star)^{m-1}(a - c_M - c_0 z(t^\star)^m) = \frac{-2bc_{SD}}{mc_0}. \tag{16}$$

Clearly, the left and the right hand side are positive ($m < 0$). While the right hand side is independent of both T and $t^\star$, the left hand side can be interpreted as a function of the switching time $t^\star$ for a given contract period T. Let $f : [0, T] \to \mathbb{R}_{\geq 0}$ be defined by

$$f(t^\star) := (T - t^\star)z(t^\star)^{m-1}(a - c_M - c_0 z(t^\star)^m).$$

Then, the term $-f'(t^\star) \cdot z(t^\star)^{m-2}$ is a sum consisting of the positive summand $z(t^\star)(a - c_M - c_0 z(t^\star)^m)$ and

$$(T - t^\star)\omega \cdot ((1-m)(a - c_M - c_0 z(t^\star)^m) + c_0 m z(t^\star)^m)\,.$$

Here, it was used that $a - c_M - c_0 - r > 0$ holds. Hence, we investigate the term

$$(1-m)(a - c_M - c_0 z(t^\star)^m) + c_0 m z(t^\star)^m \tag{17}$$

in order to determine the sign of the second summand using that $(T - t^\star)\omega > 0$ holds. To this end, the supply chain profit $p := a - c_M - c_0 > r > 0$ per unit plays a major role: (17) equals

$$c_0 \cdot ((1-m)p/c_0 + m z(t^\star)^m) + \underbrace{(1-m)(c_0 - c_0 z(t^\star)^m)}_{\geq 0}$$

because $m < 0$ and $t^\star \geq 0$ hold. Positivity of the first summand is ensued from (10). Hence, (17) is positive and, thus, f' is (strictly) decreasing.

In conclusion, the left hand side of (16) is strictly decreasing in $t^\star$ and strictly increasing in T. As a consequence, using $\overline{T}$, $\overline{T} > T$, instead of T, i.e., considering the optimal control problem on a longer time horizon (contract period), leads a larger switching time $t^\star$ in order to ensure validity of the switching condition (9). □