On Contractual Periods in Supplier Development

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Abstract: We consider supplier development within a supply chain consisting of a single manufacturer and a single supplier. Because supplier development usually requires relationship-specific investments, firms need to protect themselves against partner opportunism. Even though contracts are viewed as the primary formal means of safeguarding transactions, they also entail certain risks, e.g., a lack of flexibility, particular in a dynamic and uncertain business environment. Thus, we propose a receding horizon control scheme to mitigate possible contractual hazards while significantly increasing the overall supply chain profit. Our findings are illustrated by a numerical example.

Keywords: Supply chain management, supplier development, optimal control, predictive control, dynamic systems.

1. INTRODUCTION

Since manufacturing firms increasingly focus on their core competencies, an efficient supply chain plays a paramount role in generating competitive advantage. In response, manufacturers across a wide range of industries are implementing supplier development programs to improve supply chain performance, see Wagner (2010). Supplier development is broadly defined as *any effort by a buying firm to improve a supplier's performance and/or capabilities to meet the manufacturing firm's short- and/or long-term supply needs*, cf. Krause (1999, p. 206), and has been applied in various fields of application with a particular focus on automotive supply chains, see, e.g., Talluri et al. (2010); Krause and Scannell (2002).

Because resources committed to supplier development activities are difficult or even impossible to redeploy and thus have little or no value in an alternative use, firms need to safeguard the respective investments against the hazards of partner opportunism, see Wang et al. (2013). Previous research has shown that contracts are viewed as the primary formal means of protecting transactions, see, inter alia, Lui et al. (2009); Artz (1999). The drawback of formal contracts is, as the degree of uncertainty increases, both specifying ex ante all possible contingencies and verifying ex post the performance of the supply chain partner become increasingly difficult, cf. Williamson (1979). Therefore, supply chain partners may be reluctant to sign long-term contracts, which potentially diminishes the firms' propensity to invest in supplier development activities and thus impedes the manufacturer's initial strategy to enhance supply chain performance, see Rokkan et al. (2003).

Even though empirical studies support the notion that relationship-specific investments are critical to the success of supplier development, see, among others, Wagner (2011); Krause et al. (2007), the application of formal decision-making models proposed for assisting firms in contract negotiations in order to adequately safeguard such investments have received limited attention in the supplier development literature. Without understanding the impact of the contract period on the firms' willingness to commit relationship-specific resources to supplier development, its return will be negligible, perhaps even leading to the premature discontinuation of such collaborative cost reduction efforts.

Given this background, the purpose of our research is to mitigate possible contractual hazards while significantly enhancing the supplier development process, and thus increasing the overall supply chain profit. Thus, the contribution of this paper is twofold: First, we investigate the impact of the contract period, i.e., the *planning horizon*, on the firms' propensity to commit relationship-specific resources to supplier development and show that the firms' willingness to participate in supplier development critically depends on the length of the planning horizon. Secondly, given the fact that long-term contracts entail certain risks, e.g., a lack of flexibility, we propose a receding horizon control scheme and show that the supplier development process can be enhanced by dynamically extending the contract, i.e., the firms are not contractually tied for unnecessarily long periods of time, see Sethi and Sorger (1991) for the basic idea of prediction based control. Here, we present a strategy that optimally balances costs and benefits of supplier development.

The paper is structured as follows. In Section 2 the mathematical model is described. This allows to study the dependence of the control policy on the contract period in the subsequent section. In Section 4, a receding horizon control scheme is proposed and analysed before the effectiveness of the developed methodology is demonstrated by means of numerical investigations in Section 5. Finally, conclusions are drawn in Section 6.

2. MODEL DESCRIPTION

We consider a supply chain consisting of a single manufacturer and a single supplier. In doing so, the decision-making process is structured such that the manufacturer M determines the quantity supplied to the (oligopolistic or monopolistic) market solely based on the leitmotif of profit maximization — without taking the outcome for the supplier S into account. Herein, we restrict ourselves to the linear price-distribution curve $p(d) = a - bd$ in order to streamline the upcoming analysis.

2.1 Basic model

It is supposed that the supplier wants to earn a constant revenue r per unit. Thus, the manufacturer's supply costs are $c_{SC} = r + x(t)^m c_0$, $\dot{x}(t) = u(t)$ with $x_0 = 1$, where the supplier's production costs per unit are modelled by $x(t)^m c_0$ depending on the learning rate $m < 0$. This means that the overall production costs may be reduced by using the control function $u \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}, [0, \omega])$ and, possibly, to increase the supply chain profit. Here, the measurable and bounded function u describes the effort invested in supplier development, e.g., by realizing inter-organizational projects. This component mimics a learning curve, cf. Yelle (1979).

The fact that increases in productivity do not typically come for free is reflected by a penalization term $c_{SD}u(t)$ that allows for integrating the costs of supplier development efforts into the proposed model. Overall, this yields the supply chain's profit function $J_T^{SC} : u \mapsto \mathbb{R}$

$$J_T^{SC}(u; x_0) := \int_0^T \frac{(a - c_M - c_0 x(t)^m)^2 - r^2}{4b} - c_{SD}u(t) \, \mathrm{d}t \tag{1}$$

during the contract period $[0, T)$ neglecting fixed costs, see Table 1 for an explanation of the individual parameters. We emphasize that investments into the cost structure of the supply chain are economically reasonable as long as these amortize during the runtime of the contract. For a detailed derivation of the model in consideration the interested reader is refered to Kim (2000).

2.2 Solution of the Optimal Control Problem

Analogously to Kim (2000), using Pontryagin's maximum principle, see, e.g. Lee and Marcus (1967), yields that

Symbol	Description	Value
T	Contract period	60
a	Prohibitive price	200
b	Price elasticity	0.01
c_M	Variable cost per unit (M)	70
c_0	Variable cost per unit (S)	100
r	Revenue per unit (S)	15
c_{SD}	Supplier development cost per unit	100000
ω	Maximal investment rate	1
m	learning rate	-0.1

Table 1. List of Parameter

the control function $u^\star : [0, T) \to [0, \omega]$ maximizing (1) exhibits the structural property

$$u^\star(t) := \begin{cases} \omega & \text{if } t < t^\star \\ 0 & \text{if } t \geq t^\star \end{cases} \tag{2}$$

depending on the (optimal) switching time $t^\star \in [0, T]$. The switching time $t^\star$ is characterized by the equation

$$\frac{mc_0(x_0 + \omega t^\star)^{m-1}(a - c_M - c_0(x_0 + \omega t^\star)^m)}{2b} = \frac{c_{SD}}{(t^\star - T)}. \tag{3}$$

In the following, (3) is called switching condition. Indeed, it can be easily shown that this condition is necessary and sufficient for the considered problem since the cost function is (strictly) convex and the system dynamics are governed by a linear ordinary differential equation.

Summarizing, the optimal value $V_T(x_0)$ of the problem in consideration is attained by

$$V_T(x_0) := \sup_{u \in \mathcal{L}^\infty([0,T),[0,\omega])} J_T^{SC}(u; x_0)$$

where the expression on the right hand side is maximized subject to $\dot{x}(t) = u(t)$, $x(0) = x_0$. Since $V_T : \mathbb{R}_{\geq 1} \to \mathbb{R}$ maps the initial state x_0 to the optimal value, V_T is called optimal value function. The index T indicates the length of the contract period and can be considered as a parameter — an interpretation, which is crucial for the upcoming analysis.

Evidently, investments (in the cost structure) pay off in the long run, i.e., all the effort is spent directly at the beginning of the collaboration. Then, the resulting cost decreasing effect is exploited during the remainder of the contract period $[0, T]$.

Remark 1. At the switching time $t^\star$ the marginal revenue (given by the adjoint variable λ) equals the marginal costs (given by c_{SD}) as indicated in Figure 1. This reasoning explains the meaning of the switching condition (3).

3. DEPENDENCE OF THE OPTIMAL SWITCHING TIME ON THE CONTRACT PERIOD

The contract between the manufacturer M and the supplier S ranges over the interval $[0, T]$. Realistically, two cases can be distinguished: on the one hand, a relationship-specific investment reducing the supply costs does not pay off during the contract period — resulting in the (optimal) switching time $t^\star = 0$, i.e., supplier development is economically not recommendable. On the other hand, $t^\star > 0$ stands for the scenario where investing into supplier development amortizes until T.

From the specific structure (2) of the optimal control function it can be concluded that this claim holds for all

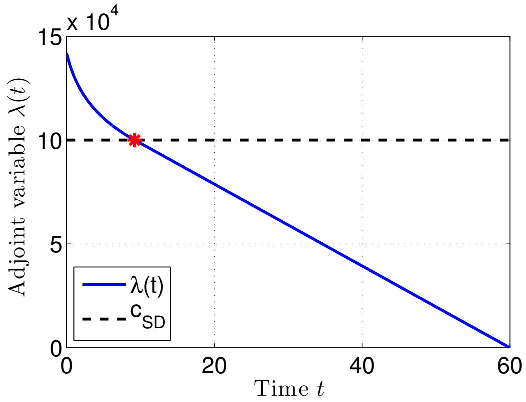

Fig. 1. The adjoint $\lambda : [0,T] \to \mathbb{R}_{\geq 0}$ computed based on the parameters given in Table 1.

investments up to time $t^\star$. Then, taking into account the already reduced supply costs given by $r + x(t^\star)^m c_0$ with optimal control $u^\star$ and

$$x(t^\star)^m \cdot c_0 = \left(x_0 + \int_0^{t^\star} u^\star(s)\ \mathrm{d}t \right)^m c_0 = (1+\omega t^\star)^m c_0$$

further effort in terms of $u(t) > 0$, $t \in [t^\star, T)$, does not lead to an increased profit since the remaining time interval of length $T - t^\star$ is too short. However, this line of argumentation already indicates what can be deduced from Condition (3): If the contract period is prolonged to a time $\overline{T}$, $\overline{T} > T$, also the switching time $t^\star$ becomes larger, see Appendix A for a proof.

Lemma 2. Suppose that the contract period T is chosen (long enough) such that $t^\star = t^\star(T) > 0$ holds. In addition, let the condition

$$(1-m)(a - c_M - c_0) + c_0 m \geq 0, \tag{4}$$

be satisfied. Then prolonging the contract period $\overline{T}$, $\overline{T} > T$, implies a strictly larger switching time $t^\star = t^\star(\overline{T})$, i.e., $t^\star(\overline{T}) > t^\star(T)$.

Remark 3. Note that the assumptions of Lemma 2 imply the inequality $a - c_M - c_0 - r > 0$ as a by-product because the manufacturer cannot realize a profit per unit sold otherwise (prohibitive price is greater than the production cost per unit at time $t = 0$ from the manufacturer's point of view). The seemingly technical condition (4) links the surplus $a - c_M - c_0$ per unit with the production costs via the learning rate and, thus, indicates whether investments are economically reasonable. Furthermore, note that the assumptions of Lemma 2 can be easily verified for a given dataset.

Lemma 2 shows that the supplier development program is extended if the contract period is prolonged. Hence, the collaboration continues after the previously determined switching time $t^\star$. As a result, the supply costs are further decreased, the quantity offered is increased and the supply chain profit increases. The argument, that longer contract periods lead to larger switching times, can also be validated numerically as visualized in Figure 2. Here, we observe that the cost structure and thus the return on investment can be further enhanced if both the manufacturer and the supplier agree on a longer contract period.

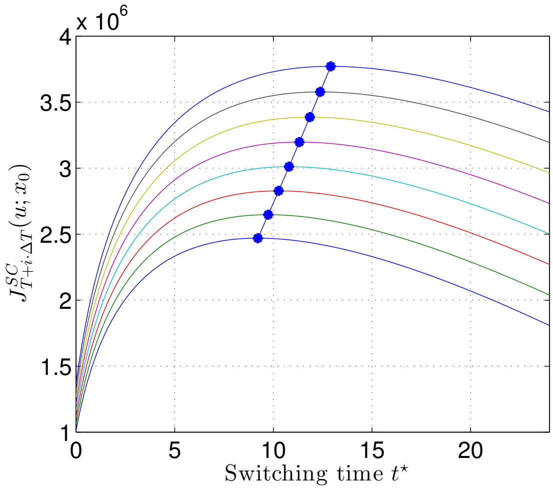

Fig. 2. Optimal switching time $t^\star = t^\star(\overline{T})$ in dependence of the length of the contract $\overline{T} = T + i \cdot \Delta T$ ($T = 60$, $\Delta T = 3$ and $i = 0, 1, \ldots, 7$).

Note that the relation between the contract period T and the optimal switching time $t^\star(T)$ is almost linear. [1]

4. SUCCESSIVE PROLONGATION OF THE CONTRACT PERIOD

The benefits of an increased switching time come along with the uncertainties of a long-term contract. In this section we investigate how the benefits of long-term contracts can be obtained while sticking to collaborations on smaller time periods. To this end, it is assumed that the manufacturer and the supplier are only content to make contracts of length T. If the collaboration is successful for a certain amount of time $[0, \Delta T)$, $\Delta T \leq t^\star$, they might agree to renew the contract on the time interval $[\Delta T, T + \Delta T]$.

Before we continue the discussion, let us briefly sketch the computation of the (optimal) control function $u^\star : [\Delta T, T + \Delta T) \to [0, \omega]$. Here, the profit function has to be maximized based on the new (initial) state $x(\Delta T)$, i.e., $J_T^{SC}(u; x(\Delta T))$ is considered. Since $\Delta T \leq t^\star$ holds by assumption, the new initial state $x(\Delta T)$ is given by

$$x(\Delta T) = x(0) + \int_0^{\Delta T} u^\star(s)\ \mathrm{d}t = x(0) + \Delta T \cdot \omega \tag{5}$$

in view of the structural property (2). Hence, the profit on the new contract period $[\Delta T, T + \Delta T]$ is determined by maximizing $J_T^{SC}(u; x(\Delta T))$, i.e.,

$$\int_0^T \frac{(a - c_M - c_0 \tilde{x}(t)^m)^2 - r^2}{4b} - c_{SD} u(t)\ \mathrm{d}t,$$

subject to $u(t) \in [0, \omega]$, $t \in [0, T)$, and the differential equation $\dot{\tilde{x}}(t) = u(t)$ with initial condition $\tilde{x}(0) = x(\Delta T)$. Here, we used the notation $\tilde{x}$ to distinguish the previously computed (state) trajectory $x(\cdot; x_0)$ and its counterpart $\tilde{x}(\cdot; x(\Delta T))$ depending on the new initial condition $x(\Delta T)$. Another option is to use the time invariance of the linear differential equation $\dot{x}(t) = u(t)$, which allows to rewrite the profit functional as

$$\int_{\Delta T}^{T+\Delta T} \frac{(a - c_M - c_0 x(t)^m)^2 - r^2}{4b} - c_{SD} u(t)\ \mathrm{d}t$$

[1] Indeed, the slope of the curve is slightly increasing.

with initial value $x(\Delta T)$ given by (5) at initial time ΔT. We point out that the resulting trajectory deviates from the previously computed one already before time T. In conclusion, the implemented control strategy on $[0, T + \Delta T)$ is given by

$$u(t) := \begin{cases} u^\star(t) \text{ maximizing } J_T^{SC}(\cdot; x_0) & t \in [0, \Delta T) \\ u^\star(t) \text{ maximizing } J_T^{SC}(\cdot; x(\Delta T)) & t \geq \Delta T, \end{cases} \tag{6}$$

i.e., the first piece of the *old* policy concatenated with the newly negotiated strategy. Using this strategy yields an optimal control policy for the complete time span $[0, T + \Delta T)$. Hence, the same overall supply chain profit is reached without the hazards of being committed already at the beginning (time 0) as shown in the following corollary.

Corollary 4. Let the optimal switching time $t^\star$ determined by Condition (3) be strictly greater than zero. Furthermore, let ΔT, $\Delta T < t^\star$, be given. Then, the control strategy defined in (6) and the corresponding supply chain profit on $[0, T + \Delta T]$ equal their counterparts obtained by maximizing $J_{T+\Delta T}^{SC}(u; x_0)$ with respect to $u : [0, T + \Delta T) \to [0, \omega]$.

Proof. Since the profit $J_{T+\Delta T}^{SC}(u; x_0)$ on the considered time interval $[0, T + \Delta T]$ with u from (6) is the sum of

$$\int_0^{\Delta T} \frac{(a - c_M - c_0 x(t)^m)^2 - r^2}{4b} - c_{SD}\omega \; \mathrm{d}t$$

and

$$+ \int_{\Delta T}^{T+\Delta T} \frac{(a - c_M - c_0 x(t)^m)^2 - r^2}{4b} - c_{SD} u(t) \; \mathrm{d}t,$$

the dynamic programming principle yields the equality $J_{T+\Delta T}^{SC}(u; x_0) = V_{T+\Delta T}(x_0)$, which completes the proof. □

4.1 Receding Horizon Control

The idea of an iterative prolongation of collaboration contracts can be algorithmically formalized as receding horizon control (RHC) aka model predictive control.

Algorithm 1 Receding Horizon Control Scheme

Given: contract period T, time step ΔT.
Set $t := 0$.

(1) Measure the current state $\hat{x} := x(t)$.
(2) Compute the optimal switching time $t^\star$ by solving the switching condition with $\hat{x}$ instead of x_0, i.e.

$$m c_0 (\hat{x} + \omega t^\star)^{m-1} (a - c_M - c_0 (\hat{x} + \omega t^\star)^m) = \frac{2 b c_{SD}}{t^\star - T}.$$

(3) Set

$$u^\star(s) := \begin{cases} \omega & \text{for } t \leq s < \min\{t + t^\star, t + \Delta T\} \\ 0 & \text{for } \min\{t + \Delta T, t + t^\star\} \leq s < t + \Delta T \end{cases} \tag{7}$$

(4) **Apply** $u^\star(s)$ for $s \in [t, t + \Delta T)$. **Set** $t = t + \Delta T$ and go to Step (1).

Beforehand, the manufacturer M and the supplier S agree on a collaboration for a given time window of length T (contract period). Firstly, the status quo — represented by $\hat{x}$ — is analysed. Secondly, the optimal switching time $t^\star$ is computed based on the initial state $\hat{x}$ and T, cf. Step (2). This yields the optimal control strategy defined by (7), of which the first piece $u^\star|_{[0,\Delta T)}$ is applied. Then, the manufacturer and the supplier meet again at time ΔT to negotiate a new contract. This initiates the process again, i.e. the previously described steps are repeated. Here, the so called *receding horizon principle* works. Note that the newly (measured) initial state $\hat{x}$ captures all information needed since the underlying system dynamics are time invariant. In particular, no knowledge about the previously applied control is needed to solve the adapted switching condition of Step (2) with respect to $t^\star$.

Figure 3 illustrates the outcome of Algorithm 1 with prediction horizon length $T = 60$ (contract period) and control horizon $\Delta T = 3$ (time step) based on the parameters given in Table 1.

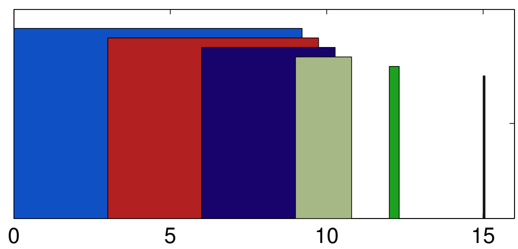

Fig. 3. Application of Algorithm 1 to compute the optimal switching times for $T = 60$ and changing initial conditions $\hat{x}$. The length of the collaboration intervals are getting smaller and smaller.

Firstly ($t = 0$), the original optimal control problem is solved resulting in $t^\star \approx 9.21$. Then, $u^\star \equiv \omega$ is applied on the time interval $[0, \Delta T)$. Secondly ($t = \Delta T$), the collaboration is prolonged to $t^\star \approx 9.74$. Thirdly ($t = 2\Delta T$), the switching time is shifted to $t^\star \approx 10.27$. Still, $t = 3\Delta T \leq t^\star$ holds. Hence, the (measured) initial state $\hat{x}$ is given by $x_0 + t\omega = x_0 + 3\Delta T\omega$. Here, Step (2) of Algorithm (1) yields $t^\star \approx 10.79$, i.e. the collaboration stops within the time frame $[t, t + \Delta T)$. If the RHC scheme is further applied, there occur collaboration intervals of shrinking length.

As already argued in Section 4, if the contract is not renewed, $u^\star(t)$ is set to zero for $t \geq t^\star \approx 9.21$. In contrast to that, the RHC scheme prolonges the collaboration and, thus, generates increased profits on arbitrary time spans, i.e. the profit generated by Algorithm 1 on $[0, T + i\Delta T]$, $i \in \{0, 1, 2, \ldots, T/\Delta T\}$,

$$\sum_{k=0}^{i-1} \int_{k\Delta T}^{(k+1)\Delta T} \frac{(a - c_M - c_0 x(t)^m)^2 - r^2}{4b} - c_{SD} u(t) \; \mathrm{d}t + V_T(x(i\Delta T))$$

is greater than its counterpart $J_T(u^\star, x_0) + V_{i\Delta T}(x^\star(T))$ and, thus, in particular than

$$J_T(u^\star, x_0) + \int_T^{T+i\Delta T} \frac{(a - c_M - c_0 x(t^\star(T))^m)^2 - r^2}{4b} \; \mathrm{d}t$$

consisting of the maximum of the original cost function $V_T(x_0) = J_T(u^\star, x_0)$ and a second (optimally operated) contract on $[T, T + i\Delta T]$ based on the reached cost structure represented by $x^\star(T) = x_0 + t^\star\omega \approx x_0 + 9.21\omega = 10.21$.

While an increased switching time $t^\star$ may already increase the profitability within a supply chain during the considered time span, the achieved cost decrease in the unit production price sustains. Hence, if the collaboration between the manufacturer and the supplier lasts, the proposed strategy generates further (additional) profits in the future.

4.2 Optimal Point of Collaboration

As observed in Figure 3, the collaboration stops within the time interval $[t, t + \Delta T)$ meaning that the prerequisite $\Delta T \leq t^\star$ was not satisfied at time t anymore. This leads to a sequence of collaboration times of shrinking length. Summing up all of these intervals on the infinite horizon yields a total collaboration time of approximately 11.18 time units. Hence, the total collaboration time is increased by 21.3%. However, implementing this strategy is highly impracticable since the collaboration intervals are becoming too short. Here, we propose two remedies: If the new collaboration period at time $t = k\Delta T$, i.e. $t^\star - t$, is below a certain threshold value,

(1) set $t^\star = t$ in order to save negotiation costs, which would probably outweigh the achievable profit growth. For the presented example, the supplier development program stops at 10.79 (still an increase of approximately 17.2%) if the threshold is 1.
(2) measure the current state $\hat{x} = x(t)$ and compute the optimal cost structure for contract periods of length T by solving
$$mc_0 T\bar{x}^{m-1}(a - c_M - c_0\bar{x}^m) + 2bc_{SD} = 0$$
with respect to $\bar{x}$. Then, set $t^\star = (\bar{x} - \hat{x})/\omega$. In the considered example at time $t = 4\Delta T$, the measured state is $\hat{x} = 10.79$ while $\bar{x} \approx 11.18$. Hence, a collaboration of length 0.39 time units is fixed. At all upcoming time instants, $t^\star = t$ holds because the optimal cost structure for contract periods of length $T = 60$ is already reached.

Clearly, the threshold should be chosen such that the profit increase outweighs the negotiation costs. In summary, Algorithm 1 allows both the manufacturer and the supplier to prolong their supplier development program without binding themselves for a time span longer than T and, thus, to provides more flexibility.

Remark 5. Algorithm 1 is a simplified version. Indeed, the time step ΔT may vary in time, e.g. longer time steps in the beginning (for example $\Delta T = t^\star$ in the considered setting), and shorter ones later on. For details on so called time varying control horizon we refer to Grüne et al. (2010).

5. NUMERICAL CASE STUDY

So far, we considered the set of parameters given in Table 1. In this section we vary some parameters in order to demonstrate their influence.

Firstly, we consider a different learning rate $m = -0.13$ and supplier development costs of $c_{SD} = 70000$, i.e. the supplier development is both less costly and more effective. Algorithm 1 prolongs the collaboration from initially 16.14 to 22.10 (an increase of 36.9 %) for $\Delta T = 3$. Here, the optimal cost structure $\bar{x}$ requires a collaboration of 23.37 time units. Hence, the outcome of the proposed RHC scheme is close to optimal. As a consequence, the profit growth is approximately increased from 896.000 ($t^\star = 16.14$) to 939.000 ($t^\star = 22.37$) without committing to contracts of length more than $T = 60$. Figure 4 illustrates the profit increase gained from the first 8 iterations of the RHC scheme. [2]

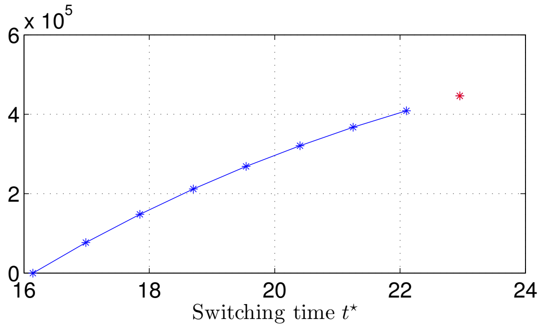

Fig. 4. Profit growth by applying Algorithm 1 ($T = 60$, $\Delta T = 3$) in comparison to the switching time $t^\star \approx 16.14$ with the parameters $m = -0.13$ and $c_{SD} = 70000$ on the interval $[0, 120]$.

Secondly, the parameter ω is varied, i.e. the available maximal resources committed to supplier development is lower or higher than in the reference scenario ($\omega = 1$, $m = -0.1$, and $c_{SD} = 100.000$, cf. Table 1). Here, the resulting prolongation of the collaboration based on RHC with strategy (2) of Subsection 4.2 is shown in the $t^{\star\star}$-column, cf. Table 2. Furthermore, the corresponding percentual profit increase is given in the last column. In conclusion, the impact of Algorithm 1 seems to grow with the starting time span of collaboration.

ω	$t^\star$	$t^{\star\star}$		Profit growth
0.3	21.79	37.25	(70.96%)	7.6268%
0.4	18.23	27.94	(53.25%)	6.4865%
0.5	15.67	22.35	(42.62%)	5.5883%
0.6	13.75	18.62	(35.52%)	4.8912%
0.7	12.23	15.96	(40.46%)	4.3417%
0.8	11.03	13.97	(26.66%)	3.9000%
0.9	10.03	12.41	(23.70%)	3.5381%
1.0	09.21	11.18	(21.32%)	3.2368%
1.1	08.51	10.16	(19.39%)	2.9821%
1.2	07.91	09.31	(17.78%)	2.7643%
1.3	07.38	08.60	(16.41%)	2.5759%

Table 2. Optimal switching times $t^\star$ for x_0 and such that the optimal cost structure $\bar{x}$ is achieved ($t^{\star\star}$) in dependence of ω ($T = 60$) and the resulting percentual profit increase.

6. CONCLUSIONS AND OUTLOOK

This paper addressed the impact of the contract period on the performance of supplier development. In particular, we showed that the supply chain partners' propensity to commit relationship-specific resources to supplier development critically depends on the length of the contract period. Given the fact that long-term contracts entail certain risks, we proposed a receding horizon control scheme to

[2] In the 8th iteration, the collaboration time is below $\Delta T = 3$.

mitigate possible contractual hazards. Herein, we showed that the supplier development process can be enhanced by dynamically extending the contract. Thus, both the manufacturer and the supplier are not contractually tied for unnecessarily long periods of time, while simultaneously facilitating value generation within supplier development. Furthermore, we introduced two strategies in order to make the proposed receding horizon control scheme, cf. Algorithm 1, compatible with industrial needs. Finally, we verified the reliability of the application by performing Algorithm 1 for varying parameters.

Future research will contain an extensive robustness analysis of the presented approach. We conjecture that the inherent robustness of receding horizon control may already mitigate consequences of badly assessed parameters. Here, the idea of dynamically renewing the contract seems to be the essential tool to generate the needed flexibility to counteract undesired effects.

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Appendix A. PROOF OF LEMMA 2

In this section a proof of Lemma 2 about the interplay of the contract period T and the optimal switching time $t^\star$ is given.

Proof. Let the monotonic function $z : t^\star \mapsto 1 + \omega t^\star$ be defined, which maps the switching time $t^\star$ to the state $x(t^\star)$ at the switching time $t^\star$. Furthermore, note that $z'(t^\star) = \omega$ holds. Then, the switching condition (3) can be rewritten as

$$(T - t^\star) z(t^\star)^{m-1} (a - c_M - c_0 z(t^\star)^m) = \frac{-2bc_{SD}}{mc_0}. \quad \text{(A.1)}$$

Clearly, the left and the right hand side are positive ($m < 0$). While the right hand side is independent of both T and $t^\star$, the left hand side can be interpreted as a function of the switching time $t^\star$ for a given contract period T. Let $f : [0, T] \to \mathbb{R}_{\geq 0}$ be defined by

$$f(t^\star) := (T - t^\star) z(t^\star)^{m-1} (a - c_M - c_0 z(t^\star)^m).$$

Then, the term $-f'(t^\star) \cdot z(t^\star)^{m-2}$ is a sum consisting of the positive summand $z(t^\star)(a - c_M - c_0 z(t^\star)^m)$ and

$$(T - t^\star)\omega \cdot ((1 - m)(a - c_M - c_0 z(t^\star)^m) + c_0 m z(t^\star)^m).$$

Here, it was used that $a - c_M - c_0 - r > 0$ holds. Hence, we investigate the term

$$(1 - m)(a - c_M - c_0 z(t^\star)^m) + c_0 m z(t^\star)^m \qquad \text{(A.2)}$$

in order to determine the sign of the second summand using that $(T - t^\star)\omega > 0$ holds. To this end, the supply chain profit $p := a - c_M - c_0 > r > 0$ per unit plays a major role: (A.2) equals

$$c_0 \cdot ((1 - m)p/c_0 + m z(t^\star)^m) + \underbrace{(1 - m)(c_0 - c_0 z(t^\star)^m)}_{\geq 0}$$

because $m < 0$ and $t^\star \geq 0$ hold. Positivity of the first summand is ensued from (4). Hence, (A.2) is positive and, thus, f' is (strictly) decreasing.

In conclusion, the left hand side of (A.1) is strictly decreasing in $t^\star$ and strictly increasing in T. As a consequence, using $\overline{T}$, $\overline{T} > T$, instead of T, i.e., considering the optimal control problem on a longer time horizon (contract period), leads a larger switching time $t^\star$ in order to ensure validity of the switching condition (3). □