A Real-Time Pricing Scheme for Residential Energy Systems Using a Market Maker*

Philipp Braun[1], Lars Grüne[1], Christopher M. Kellett[2], Steven R. Weller[2], and Karl Worthmann[3]

***Abstract*— Voltage rise is an undesirable side-effect of solar photovoltaic (PV) generation, arising from the flow of surplus electrical power back into the grid when PV generation exceeds local demand. Customers deploying residential-scale battery storage are likely to further exacerbate voltage rise problems for electrical utilities unless the charge/discharge schedules of batteries is appropriately coordinated. In this paper, we present a real-time pricing mechanism for use in a network of distributed residential energy systems (RESs), each employing solar PV generation and battery storage. The pricing mechanism proposed in this paper is based on a Market Maker algorithm in which predicted power profiles and real-time pricing information is iteratively exchanged between a central entity and each of the RESs. The Market Maker formulation presented in this paper is shown via simulation studies to converge to a fixed price vector, thereby reducing the price volatility observed in an earlier formulation, while achieving the same reduction in power usage variability as a centralised model predictive control (MPC) scheme presented previously.**

I. Introduction

Widespread deployment of solar photovoltaics (PV) at the residential level can cause network difficulties in the form of voltage rise caused by households generating more power than they consume; e.g., in the middle of a sunny day while the residents are not at home. If in addition solar PV is augmented with residential battery storage, voltage rise problems may well be exacerbated by multiple batteries discharging to the grid at the same time as local generation already exceeds the local load. Hence, an important question is: how to schedule battery storage?

A natural performance metric in this context is the reduction in variation of the grid usage profile or, in other words, the achieved reduction in power demand variability relative to the average demand over some time window. In [1], [2], we presented centralised, decentralised, and distributed model predictive control (MPC) schemes aimed at reducing this deviation. Here, we differentiate between decentralised control, where any individual residence chooses how to schedule its battery without any external information, and distributed control where some level of communication between residences is permitted. Not surprisingly, in [2], it is observed that the centralised MPC scheme performs better than the distributed MPC scheme which, in turn, performs better than the decentralised MPC scheme.

As a result of the curse of dimensionality, the centralised MPC scheme presented in [2] does not scale up to a large number of residences. Hence, in [3], we developed a hierarchical distributed optimisation algorithm that recovers the performance of the centralised MPC scheme. However, the algorithm presented in [3], similar to the centralised MPC scheme of [2], is essentially a cooperative scheme whereby all residences cooperate to achieve the goal of reducing network deviation from the average.

By contrast, the distributed MPC scheme of [2] is based on a real-time pricing mechanism referred to as a *Market Maker* [4], [5]. In this scheme, an iterative process is employed whereby residences communicate predicted power profiles to a central entity that computes prices that are then broadcast to all residences. This process is repeated a number of times. A drawback of the Market Maker proposed in [2] is that this iterative process may not converge to a fixed price vector, with the possibility of marked price volatility from one iteration to the next.

In this paper, we present an alternate Market Maker formulation that, at least in simulation, appears to converge to a fixed price vector. Furthermore, using this new Market Maker, we provide simulation results which recover the performance of the centralised MPC scheme.

The paper is organised as follows. In Section II we introduce the mathematical model of the Residential Energy System (RES) and define the desired performance metrics. The centralised MPC approach is presented in Section III and our new Market Maker algorithm is described in Section IV. A simulation study using data from an Australian electricity distribution company, Ausgrid, is undertaken in Section V. Concluding remarks are provided in Section VI.

II. The Residential Energy System

We consider Residential Energy Systems (RESs) comprised of residential load, generation, battery storage, and a connection to the electricity network. Let $\mathcal{I} \in \mathbb{N}$ be the number of RESs in the local area under consideration. A simple model of RES i, $i \in \{1, \ldots, \mathcal{I}\}$, is given by

$$x_i(k+1) = x_i(k) + Tu_i(k), \tag{1}$$

$$z_i(k) = w_i(k) + u_i(k) \tag{2}$$

*C. M. Kellett is supported by ARC Future Fellowship FT1101000746. L. Grüne is supported by the Deutsche Forschungsgemeinschaft, Grant GR 1569/13-1.

[1]P. Braun and L. Grüne are with the Mathematical Institute, Universität Bayreuth, 95440 Bayreuth, Germany, e-mail: {philipp.braun, lars.gruene}@uni-bayreuth.de.

[2]C. M. Kellett and S. R. Weller are with the School of Electrical Engineering and Computer Science at the University of Newcastle, Callaghan, New South Wales 2308, Australia, e-mail: {chris.kellett, steven.weller}@newcastle.edu.au.

[3]K. Worthmann is with the Institute for Mathematics, Technische Universität Ilmenau, 99693 Ilmenau, Germany, e-mail: karl.worthmann@tu-ilmenau.de.

where x_i is the state of charge of the battery in kilowatt-hours (kWh), u_i is the battery charge/discharge rate in kilowatts (kW), w_i is the static load minus the local generation in kilowatts, and z_i is the power supplied by/to the grid in kilowatts. Here, T represents the length of the sampling interval in hours. While the system dynamics (1) is autonomous, the performance output (2) depends on the time varying quantity $w_i(\cdot)$.

The RES network is then defined by the following discrete-time system

$$\begin{aligned} x(k+1) &= f(x(k), u(k)), \\ z(k) &= h(u(k), w(k)) \end{aligned}$$

where $x, u, w, z \in \mathbb{R}^{\mathcal{I}}$, and the definitions of f and h are given componentwise by (1) and (2), respectively. For each RES $i \in \{1, \ldots, \mathcal{I}\}$, the constraints on the battery capacity and charge/discharge rates are described by the constants $C_i, \overline{u}_i \in \mathbb{R}_{>0}$ and $\underline{u}_i \in \mathbb{R}_{<0}$, i.e.,

$$0 \leq x_i(k) \leq C_i \text{ and } \underline{u}_i \leq u_i(k) \leq \overline{u}_i \qquad \forall k \in \mathbb{N}_0. \quad (3)$$

Our aim is to design a pricing mechanism so as to avoid peaks in supply and demand or, put another way, to flatten the performance output z. To this end, let

$$\Pi(k) := \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} z_i(k)$$

denote the average power demand at time k and let $\mathcal{N}$ denote the number of samples in a simulation. The performance metric of peak-to-peak (PTP) variation of the average demand of all RESs is given by

$$\left(\max_{k \in \{0,\ldots,\mathcal{N}-1\}} \Pi(k) \right) - \left(\min_{k \in \{0,\ldots,\mathcal{N}-1\}} \Pi(k) \right). \quad \text{(PTP)}$$

The second performance metric of the root-mean-square (RMS) deviation about the average is defined as

$$\sqrt{\frac{1}{\mathcal{N}} \sum_{k=0}^{\mathcal{N}-1} \left(\Pi(k) - \varUpsilon \right)^2} \quad \text{(RMS)}$$

with the average demand $\varUpsilon := \frac{1}{\mathcal{N}\mathcal{I}} \sum_{k=0}^{\mathcal{N}-1} \sum_{i=1}^{\mathcal{I}} w_i(k)$.

III. Centralised Model Predictive Control

We recall the model predictive control (MPC) algorithm for the control of a network of RESs introduced in [2] and [1], respectively. This approach is a centralised MPC (CMPC) scheme, in which full communication of all relevant variables for the entire network as well as a known model of the network are required.

MPC iteratively minimises an optimisation criterion with respect to predicted trajectories and implements the first part of the resulting optimal control sequence until the next optimisation is performed (see, e.g., [6] or [7]). To this end, we assume that we have predictions of the residential load and generation some time into the future that is coincident with the horizon of the predictive controller. In other words, given a prediction horizon $N \in \mathbb{N}$, we assume knowledge of $w_i(j)$ for $j \in \{k, \ldots, k+N-1\}$, where $k \in \mathbb{N}_0$ is the current time.

To implement the CMPC algorithm, we compute the network-wide average demand at every time step k over the prediction horizon by

$$\bar{\zeta}(k) := \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \frac{1}{N} \left(\sum_{j=k}^{k+N-1} w_i(j) - x_i(k) \right). \quad (4)$$

After the average demand is computed, the joint cost function

$$V(x(k); k) := \min_{\hat{u}(\cdot)} \sum_{j=k}^{k+N-1} \left(\bar{\zeta}(k) - \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \underbrace{(w_i(j) + \hat{u}_i(j))}_{\hat{z}_i(j)} \right)^2$$

is minimised with respect to the predicted control inputs $\hat{u}(\cdot) = (\hat{u}_1(\cdot), \hat{u}_2(\cdot), \ldots, \hat{u}_{\mathcal{I}}(\cdot))^T$ with $\hat{u}_i(\cdot) = (\hat{u}_i(j))_{j=k}^{k+N-1}$, $i \in \{1, 2, \ldots, \mathcal{I}\}$, subject to the system dynamics (1), the current state $x(k) = (x_1(k), \ldots, x_{\mathcal{I}}(k))^T$, terminal constraints $\hat{x}(k+N) = x_{\text{end}}$, and the constraints (3) for $i \in \{1, \ldots, \mathcal{I}\}$. The vector of the predicted performance output $\hat{z}(\cdot)$ is defined in the same way as the predicted control $\hat{u}(\cdot)$. Explicitly, we use the hat notation to denote predicted values, with the absence of a hat denoting the actual value. Additionally we use the notation $u(j) = (u_1(j), \ldots, u_{\mathcal{I}}(j))^T$ for a fixed time $j \in \mathbb{N}$. The same holds for the other variables x, w, and z.

IV. An Improved Market Maker Scheme

The CMPC approach contains an implicit assumption that individual RESs willingly contribute to the goal of the grid operator, i.e., reducing variance in electricity supply. However, the benefit to an individual RES of such a scheme is separate to any price mechanism. Here, we will design a real-time pricing mechanism that recovers the performance of the CMPC scheme and which corresponds to the natural behaviour of an RES, namely minimisation of cost.

We assume that the price for energy at a fixed time is given by a quadratic function $l : I \subset \mathbb{R} \to \mathbb{R}$ of the form

$$l(z; b) = \begin{cases} T \cdot p \left(z + a_1 (b - z)^2 + c_b \right), & z \leq b \\ T \cdot p \left(z + a_2 (z - b)^2 + c_b \right), & z \geq b \end{cases}$$

where $c_b \in \mathbb{R}$ is given by

$$c_b = \begin{cases} -b^2 \cdot a_1, & \text{if } b > 0 \\ -b^2 \cdot a_2, & \text{if } b \leq 0 \end{cases}$$

and depends on the parameter b while p, a_1, $a_2 \in \mathbb{R}^+$ are positive constants. Note that c_b is defined so that $l(0; b) = 0$ holds. The function l needs to be monotonically increasing to capture the fact that increasing energy usage incurs increasing costs. Hence, we restrict the domain to $I = [b - a_1^{-1}/2, \infty)$. In this case, $l(z; b) > 0$ for all $z > 0$ and $l(z; b) < 0$ for all $z < 0$; i.e., energy demand produces costs and energy production leads to a profit.

The parameter b defines a variable threshold. An energy demand smaller than the threshold is penalised with additional costs $a_1 \cdot (z - b)^2$ while an energy demand bigger than the threshold is penalised by $a_2 \cdot (z - b)^2$. In other words, both excessive usage and excessive generation are penalised, dependent on the threshold b. The constant p is not necessary for the analysis of the cost function but is used to scale the costs to realistic energy prices.

We assume that the parameter b is time dependent and can be chosen by the Market Maker. Minimising the energy costs for an individual system in an MPC context can be achieved by minimising the cost function

$$v(z_i;b) := \sum_{j=k}^{k+N-1} l(z_i(j);b(j)). \tag{5}$$

The question which has to be addressed by the Market Maker is how to choose the parameters $b(j)$ to achieve the same value of PTP and RMS as obtained by CMPC.

Assuming that $z_i(j) \geq 0$ implies costs and $z_i(j) < 0$ yields a profit, it is clear that each RES wants to have a fully discharged battery at the end of the prediction horizon, i.e., $\hat{x}_i(k+N) = 0$ in order to minimise cost or maximise profit. Hence, the average energy consumption $\frac{1}{\mathcal{I}N}\sum_{i=1}^{\mathcal{I}}\sum_{j=k}^{k+N-1}\hat{z}_i(j)$ during the prediction window corresponds to $\bar{\zeta}(k)$ given by (4), and thus, does not depend on the individual strategies of the RESs as long as each RES follows the paradigm of profit maximisation. In order to make a fair comparison between the Market Maker algorithm and CMPC, we set the terminal constraint in the CMPC scheme to be $\hat{x}(k+N) = x_{\mathrm{end}} = 0$.

The idea behind the choice of $b(j)$, $j \in \{0,\dots,N-1\}$, is to penalise the deviation from the average $\bar{\zeta}(k)$. Here, we assume that the Market Maker knows the desired power profile $\hat{z}_i = (\hat{z}_i(j))_{j=k}^{k+N-1}$. If

$$\bar{\zeta}(k) - \frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}}\hat{z}_i(j) > 0 \quad \left(\bar{\zeta}(k) - \frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}}\hat{z}_i(j) < 0\right)$$

holds, the RESs should be motivated to use more (less) energy, i.e., $b(j)$ should be increased (decreased).

Note that no information on the battery capacity, the charging rate, the load data $w(k), w(k+1), \dots, w(k+N-1)$, or the initial state of the batteries is transmitted to the Market Maker, and the energy demand of an RES is the outcome of its own optimisation. The scaling factor κ, computed in (7) below, guarantees boundedness of the threshold values.

Algorithm 4.1: Set the iteration index $\ell = 0$, the maximal iteration number $\ell_{\max} \in \mathbb{N}$, and a bound $b_{\max} \in \mathbb{R}^+$.

Initialisation: The Market Maker collects the desired power profile $\hat{z}_i^0 = (\hat{z}_i^0(j))_{j=k}^{k+N-1}$, $i \in \{1,2,\dots,\mathcal{I}\}$, from each RES i, computes $\bar{\zeta}(k)$, and sets $b^0 = (\bar{\zeta}(k),\dots,\bar{\zeta}(k))^T$.

(1) Increment the iteration index ℓ by one.

(2) The Market Maker computes the average predicted energy demand at time j, $j \in \{k, 1, \dots, k+N-1\}$:

$$\Pi^\ell(j) = \frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}}\hat{z}_i^{\ell-1}(j).$$

(3) The Market Maker sets

$$b^\ell(j) = b^{\ell-1}(j) + (\bar{\zeta}(k) - \Pi^\ell(j)), \tag{6}$$

computes the scaling factor

$$\kappa = b_{\max}\frac{\max_{j=0,\dots,N-1}|(\bar{\zeta}(k) - \Pi^\ell(j))|}{\max_{j=0,\dots,N-1}|b^\ell(j)|} \tag{7}$$

for $j = k, k+1, \dots, k+N-1$, and broadcasts $\kappa \cdot b^\ell$ to the RESs.

(4) Each RES determines its energy usage profile by solving an optimal control problem, i.e.

$$\begin{aligned}\hat{z}_i^\ell := \quad & \operatorname{argmin}\ v(\hat{z}_i; \kappa \cdot b^\ell) \\ & \text{s.t. } \hat{x}_i(k) = x_{i,0} \\ & \quad \hat{x}_i(j+1) = \hat{x}_i(j) + T\hat{u}_i(j) \\ & \quad \hat{z}_i(j) = w_i(j) + \hat{u}_i(j) \\ & \quad 0 \leq \hat{x}_i(j) \leq C_i, \quad \underline{u}_i \leq \hat{u}_i(j) \leq \overline{u}_i \\ & \quad\quad \forall\, j = k,\dots,k+N-1\end{aligned}$$

and sends $\hat{z}_i^\ell$ to the Market Maker.

(5) If $\ell < \ell_{\max}$ go to step (1). Otherwise, return $\hat{z}^\ell$.

V. Numerical Results

In this section we compare the performance of CMPC with the performance of Algorithm 4.1. Additionally we show the benefits of Algorithm 4.1 in terms of a price interpretation. All simulations use a setting of 30 RESs, initial conditions $x_i(0) = 0$[kWh], constraints $\overline{u}_i = -\underline{u}_i = 0.3$[kW] and $C_i = 2$[kWh], a discretisation of $T = 0.5$[h], and a simulation length of $\mathcal{N} = 387$. ($\mathcal{N}$ is chosen such that $x_i(387) = 0$ for all $i \in \{1,\dots,\mathcal{I}\}$ holds.)

A. Performance of the improved Market Maker scheme

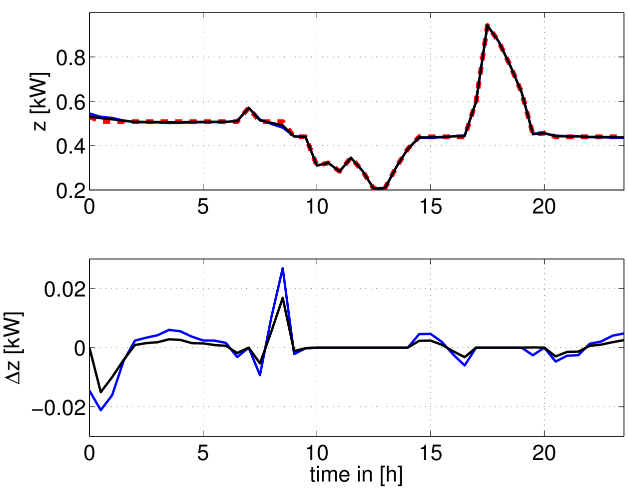

Fig. 1. *Open loop solution of the aggregated power demand $\frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}}\hat{z}_i(j)$ ($j = 0,\dots,47$) of CMPC (red) and Algorithm 4.1 (top) and the difference between the open loop CMPC solution and Algorithm 4.1 (bottom). The blue line is with $b_{\max} = 50$ and the black line with $b_{\max} = 100$.*

Figure 1 compares the (open loop) solution at a fixed time step of the aggregated power demand $\frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}}\hat{z}_i(j)$ ($j = 0,\dots,47$) solved with CMPC and Algorithm 4.1 with two values of $b_{\max}$; $b_{\max} = 50$ and $b_{\max} = 100$. We observe that the solution of Algorithm 4.1 is close to the CMPC solution and the difference appears to decrease with increasing $b_{\max}$.

Additionally, the parameters $p = 0.3$, $a_1 = 5 \cdot 10^{-3}$ and $a_2 = 2 \cdot 10^{-2}$ for $b_{\max} = 50$, and $p = 0.3$, $a_1 = 5 \cdot 10^{-4}$ and $a_2 = 2 \cdot 10^{-3}$ for $b_{\max} = 100$ are used in Algorithm 4.1 to ensure that the cost function $l(z; b)$ is monotonically increasing on the given data set. Note that according to our numerical experience the parameters p, a_1, and a_2 are not important for the performance of the algorithm as long as the monotonicity assumption holds. The specific values chosen here provide reasonable prices when $\ell(z; b)$ is assumed to be given in cents per kWh. The differences with respect to our performance metrics for the three settings are negligible (cf. Table I).

Method	**PTP**	**RMS**
Uncontrolled	1.3362	0.2281
CMPC	0.7362	0.0711
Alg. 4.1, $b_{max} = 50$	0.7362	0.0712
Alg. 4.1, $b_{max} = 100$	0.7362	0.0711

TABLE I

PERFORMANCE OF THE DIFFERENT MPC SCHEMES.

B. *Benefits for individual RES in terms of energy prices*

Algorithm 4.1 recovers the performance of CMPC by solving the problem in a distributed fashion where each RES solves its own optimal control problem. Additionally the cost function can be interpreted as actual energy prices. Figure 2 shows the time varying prices for 1[kWh] of continuous usage in the corresponding time interval of 30 minutes and the parameters connected to $b_{\max} = 50$.

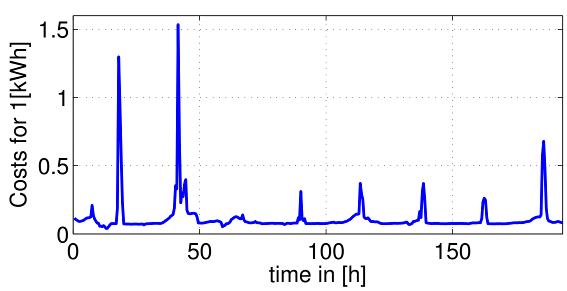

Fig. 2. *Half hourly time varying prices (in cents) for continuous usage of* 1*[kWh].*

Not only does our proposed scheme meet the goal of the grid operator, i.e., flattening the aggregated power demand, but also the individual RESs can benefit from increasing their battery capacity to lower their electricity costs. Figure 3 shows the reduction of the electricity costs of RES 1 and the impact on the other RESs, if the battery size of RES 1 is given by $C_1 = 2 \cdot c$ and charging/discharging rate $\overline{u}_1 = -\underline{u}_1 = 0.3 \cdot c$ for $c = 0, \ldots, 5$. The energy consumption of RES 1 during the simulation is 126[kWh]. The average energy consumption of the other systems is 92[kWh]. In Table II the costs for energy with changing battery sizes are given.

We observe that the installation of additional capacity provides a benefit to RES 1, albeit with a diminishing return.

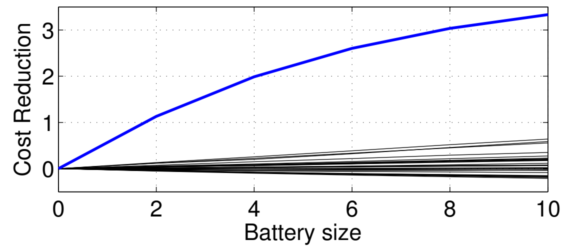

Fig. 3. *Cost reduction of the RESs if RES* 1 *(blue line) increases the battery capacity from* 0 *to* 10 *in steps of* 2*.*

The impact on the other systems is mixed. On the one hand, the installation of additional capacity results in lower prices since the overall network deviations are reduced. On the other hand, RESs who previously benefited from selling power at high prices see a reduction in their profit.

VI. CONCLUSION

In this paper, we have presented a novel real-time pricing mechanism for networks of residential energy systems (RESs) with the aim of reducing variation in network usage. This pricing mechanism is based on a central Market Maker entity that sets prices based on individual predicted usage. Simulations indicate that, when each RES acts so as to minimise its cost, this pricing mechanism recovers the performance of a centralised optimal control algorithm.

REFERENCES

[1] K. Worthmann, C. M. Kellett, L. Grüne, and S. R. Weller. Distributed control of residential energy systems using a market maker. In *19th IFAC World Congress, South Africa*, pages 11641–11646, 2014.
[2] K. Worthmann, C. M. Kellett, P. Braun, L. Grüne, and S. R. Weller. Distributed and decentralized control of residential energy systems incorporating battery storage. *IEEE Transactions on Smart Grid*, 2015. Doi: 10.1109/TSG.2015.2392081.
[3] P. Braun, L. Grüne, C. M. Kellett, S. R. Weller, and K. Worthmann. Predictive control of a smart grid: A distributed optimization algorithm with centralized performance properties. Preprint, University of Bayreuth, 2015.
[4] A. Beja and M. B. Goldman. On the dynamic behavior of prices in disequilibrium. *The Journal of Finance*, 35(2):235–248, 1980.
[5] M. B. Garman. Market microstructure. *Journal of Financial Economics*, 3:257–275, 1976.
[6] J. B. Rawlings and D. Q. Mayne. *Model Predictive Control: Theory and Design*. Nob Hill Publishing, 2009.
[7] L. Grüne and J. Pannek. *Nonlinear Model Predictive Control. Theory and Algorithms*. Springer London, 2011.

Battery Capacity C_1 [kWh]	**RES**$_1$ ($)	**av. RES**$_{2-30}$ ($)
0	42.5720	30.8757
2	41.4419	30.8617
4	40.5828	30.8474
6	39.9673	30.8283
8	39.5356	30.8048
10	39.2373	30.7848

TABLE II

ENERGY COSTS FOR RES$_1$ AND THE AVERAGE OF THE COSTS FOR THE OTHER RESS DEPENDING ON THE BATTERY CAPACITY OF RES$_1$.